Paper-1

JEE Advanced, 2015

PART I: PHYSICS

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.

2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.

3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.

4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.

5. Blank spaces are provide within this booklet for rough work.

6. Write your name and roll number in the space provided on the back cover of this booklet.

7. After breaking the seal of the booklet. Verify that the booklet contains 32 pages and that all the 60 questions along with the options are legible.

Question paper format and marking scheme :

8. The question paper has three parts: Physics, Chemistry and Mathematics. Each part has three sections.

9. Carefully read the instructions given at the beginning of each section.

10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).

    Marking scheme: +4 correct answer and 0 in all other cases.

11. Section 2 contains 10 multiple choice questions with one or more than one correct option.

    Marking scheme:+4 for correct answer, 0 if not attempted and -2 in all other cases.

12. Section 3 contains 2 " match the following" type questions and you will have to match entries in Column I with the entries in Column II.

Marking scheme:for each entry in Column I,+2 for correct answer, 0 if not attempted and -1 in all other cases.

OPTICAL RESPONSE SHEET :

13. The ORS consists of an original (top sheet) and its carbon-less copy. (bottom sheet).

14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.

15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.

16. You will be allowed to take away the carbon-less copy at the end of the examination.

17. Do not tamper with or mutilate the ORS.

18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

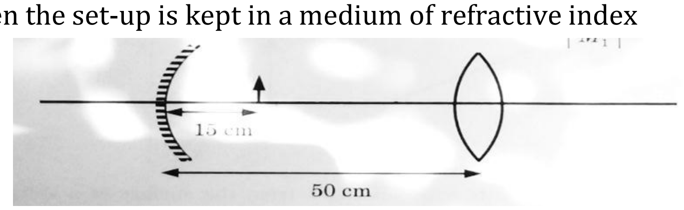
Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each questions, darkness the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  - +4 If the bubble corresponding to the answer is darkened
  - 0 In all other cases

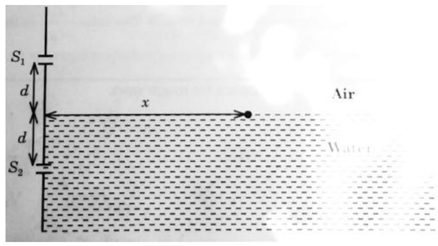
Q.1 Consider a concave mirror and a convex lens (refractive index $= 1.5$) of focal length 10 cm each, separated by a distance of 50 cm in air (refractive index $= 1$) as shown in the figure. An object is placed at a distance of 15 cm from the mirror. Its erect image formed by this combination has magnification M_1. When the set-up is kept in a medium of refractive index 7/6, the magnification becomes M_2.

The magnitude $\left|\frac{M_2}{M_1}\right|$ is



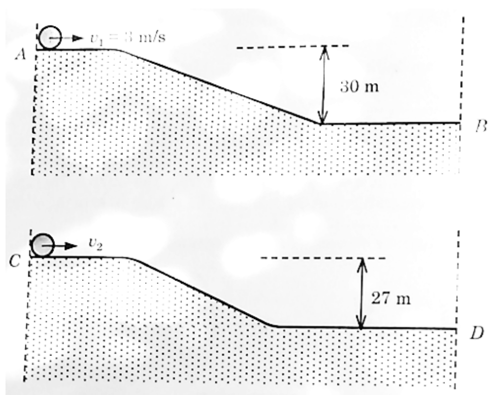
Ans.1 (7)

Q.2 A Young's double slit interference arrangement with slits S_1 and S_2 is immersed in water (refractive index = 4/3) as shown in the figure. The positions of maxima on the surface of water are given by $x^2 = p^2m^2\lambda^2 - d^2$, where λ is the wavelength of light in air (refractive index = 1), 2d is the separation between the slits and m is an integer. The value of p is



Ans.2 (3)

Q.3 Two identical uniform discs roll without slipping on two different surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$ m/s, then v_3 in m/s is ($g = 10$ m/s^2)



Ans.3 (7)

Q.4 A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere)

Ans.4 (2)

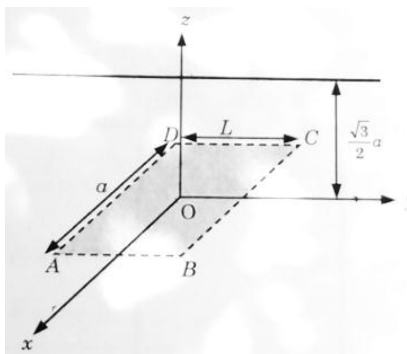
Q.5 Two spherical stars A and B emit blackbody radiation. The radius of A is 400 times that of B and A emits 10^4 times the power emitted from B. The ratio $\left(\frac{\lambda_A}{\lambda_B}\right)$ of their wavelengths λ_A and λ_B at which the peaks occur in their respective radiation curves is

Ans.5 (2)

Q.6 A nuclear power plant supplying electrical power to a village uses a radioactive material of half life T years as the fuel. The amount of fuel at the beginning is such that the total power requirement of the village is 12.5% of the electrical power available from the plant at that time. If the plant is able to meet the total power needs of the village for a maximum period of nT years, the value of n is

Ans.6 (3)

Q.7 An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = permittivity of free space), then the value of n is



Ans.7 (6)

Q.8 Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10, 4 eV, then the value of n is (hc = 1242 eV nm)

Ans.8 (2)

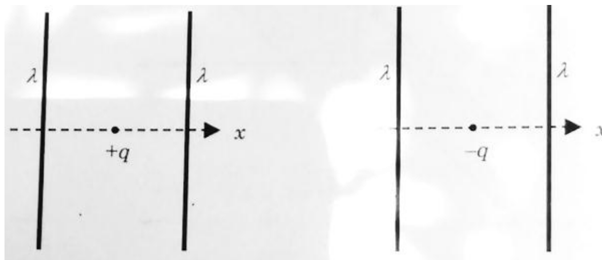
SECTION 2 (Maximum Marks:40)

- The section contains TEN questions
- Each question has FOUR options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is (are) correct
- For each question, darken the bubbles(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:

  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

  0 If none of the bubbles is darkened

Q.9 The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, Point charges q and –q are kept in equilibrium between them. The point charges are confined to move in the x direction only. If they are given a small displacement about their equilibrium positions, then the correct statements(s) is (are)



(A) Both charges execute simple harmonic motion.

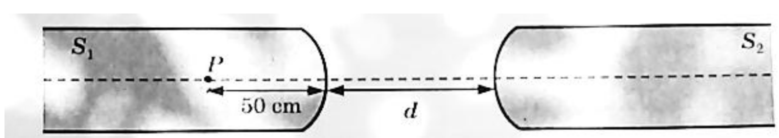
(B) Both charges will continue moving in the direction of their displacement.

(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.

(D) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

Ans.9 (C)

Q.10 Two identical glass rode S_1 and S_2 (refractive index = 1.5) have one convex end of radius of curvature 10 cm. They are placed with the curved surfaces at a distance d as shown in the figure, with their axes (shown by the dashed line) aligned. When a point source of light P is placed inside rod S_1 on its axis at a distance of 50 cm from the curved face, the light rays emanating from it are found to be parallel to the axis inside S_2. The distance d is

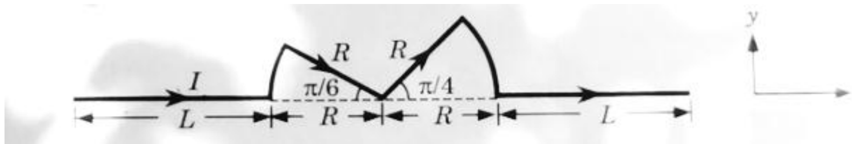


(A) 60 cm (B) 70 cm

(C) 80 cm (D) 90 cm

Ans.10 (B)

Q.11 A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a uniform magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then correct statement(s) is (are)



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$ (B)If $\vec{B}$ is along $\hat{x}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$ (D)If $\vec{B}$ is along $\hat{z}$, $F = 0$

Ans.11 (A,B,C)

Q.12 A container of fixed volume has a mixture of one mole of hydrogen and one mole of helium in equilibrium at temperature T. Assuming the gasses are ideal, the correct statement(s) is (are)

(A) The average energy per mole of the gas mixture is 2RT.

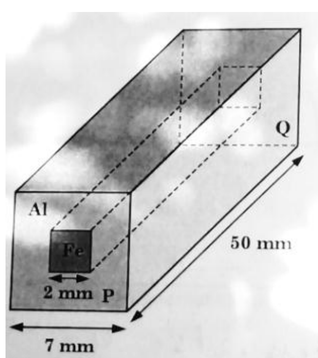
(B) The ratio of speed of sound in the gas mixture to that in helium gas is $\sqrt{6/5}$.

(C) The ratio of the rms speed of helium atoms to that of hydrogen molecules is 1/2.

(D) The ratio of the rms speed of helium atoms to that of hydrogen molecules is $1/\sqrt{2}$.

Ans.12 (A,B,D)

Q.13 In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8} \Omega$ m and 1.0×10^{-7} Ω m, respectively. The electrical resistance between the two faces P and Q of the composite bar is

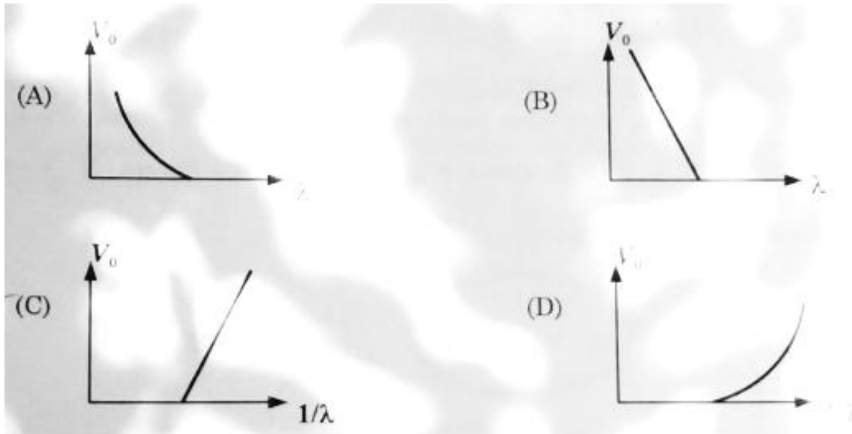


(A)$\frac{2475}{64}$ $\mu\Omega$ (B) $\frac{1875}{64}$ $\mu\Omega$

(C) $\frac{1875}{49}$ $\mu\Omega$ (D) $\frac{2475}{132}$ $\mu\Omega$

Ans.13 (B)

Q.14 For photo-electric effect with incident photon wavelength λ, the stopping potential is V_0. Identify the correct variation(s) of V_0 with λ and 1/λ.



Ans.14 (A,C)

Q.15 Consider a Verniercallipers in which each 1 cm on the main scale is divided into 8 equal divisions and a screw gauge with 100 divisions on its circular scale. In the Verniercallipers, 5 divisions of the Vernier scale coincide with 4 divisions on the main scale and in the screw gauge, one complete rotation of the circular scale moves it by two divisions on the linear scale. Then:

(A) If the pitch of the screw gauge is twice the least count of the Verniercallipers, the least count of the screw gauge is0.01 mm.

(B) If the pitch of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm.

(C) If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.01 mm.

(D)If the least count of the linear scale of the screw gauge is twice the least count of the Vernier calipers, the least count of the screw gauge is 0.005 mm.

Ans.15 (B,C)

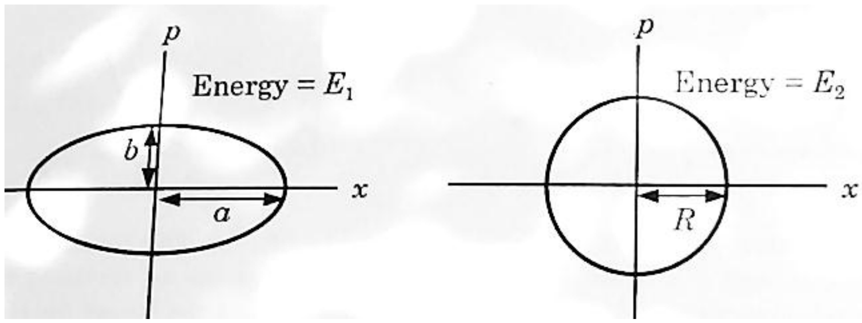
Q.16 Planck's constant h, speed of light c and gravitational constant G are used to form a unit of length L and a unit of mass M. Then the correct option(s) is (are)

(A) $M \propto \sqrt{c}$ (B) $M \propto \sqrt{G}$

(C) $L \propto \sqrt{h}$ (D)$L \propto \sqrt{G}$

Ans.16 (C,D)

Q.17 Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{\boldsymbol{R}} = n$, then the correct equation(s) is (are)



(A) $E_1\omega_1 = E_2\omega_2$

(B) $\frac{\omega_2}{\omega_1} = n^2$

(C) $\omega_1\omega_2 = n^2$

(D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

Ans.17 (B,D)

Q.18 A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular of speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$ R from O. At this instant the distance of the other mass from O is

(A) $\frac{2}{3}R$

(B) $\frac{1}{3}R$

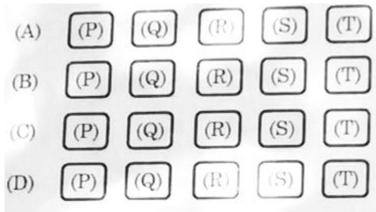
(C) $\frac{3}{5}R$

(D) $\frac{4}{5}R$

Ans.18 (D)

Section 3 (maximum marks: 16)

- This section contains TWO question
- Each question contains two columns, Column I and Column II
- Column I has four entries (A), (B), (C) and (D)
- Column II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in Column I with the entries in Column II
- One or more entries in Column I may match with one or more entries in Column II
- The ORS contains a 4 × 5 matrix whose layout will be similar to the one shown below:



- For each entry in Column I, darken the bubbles of all the matching entrires. Fore example, if entry (A) in Column I matches with entries (Q), (R) and (T), then darken these three
- Bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:

  For each entry in Column I

  +2 If only the bubbles(s) corresponding to all the correct match(es) is (are) darkened
  0 If none of the bubbles in darkened
  –1 In all other cases

Q.19 Match the nuclear processes given in column I with the appropriate option(s) in column II.

Colum I	**Column II**
(A) Nuclear fusion	(P) Absorption of thermal neutrons by ${}^{235}_{92}U$
(B) Fission in a nuclear reactor	(Q) ${}^{60}_{27}Co$ nucleus
(C) β- decay	(R) Energy production in stars via hydrogen conversion to helium
(D) γ –ray emision	(S) Heavy water
	(T) Neutrino emission

Ans.19 (A → R,T),(B → P, S, T), (C → P, Q, R), (D → P, Q, R)

Q.20 A particle of unit mass is moving along the x-axis under the influence of a force and its total energy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column II.

Column I	Column II
(A) $U_1(x) = \frac{U_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]$	(P) The force acting on the particle is zero at x = a.

(B) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$

(R) The force acting on the particle is zero at $x = -a$.

(C) $U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$

(S) The particle experiences an attractive force towards $x = 0$ in the region $|x| < a$.

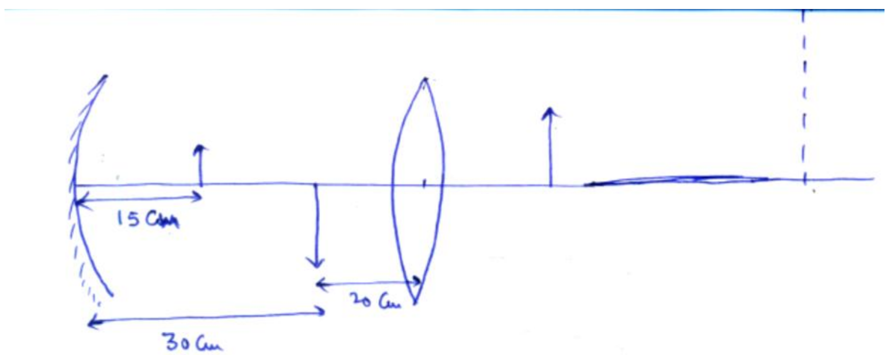
(D) $U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$

(T) The particle with total energy $\frac{U_0}{4}$ can oscillate about the point $x = -a$.

Ans.20 (A →P,R,S,T), (B → Q, S), (C →P,R,S,T), (D →P,R)

Answer Keys and Explanations

Sol.1 (7)



Dotted line shows the image formed when two system is immersed in medium of R.I = 7/6.

Image formed by mirror,

v = – 15 cm, f = –10 cm.

So, v = – 30 cm.

m=-2

As, it is at the radius of curvature of the lens images forms at 20 cm on the other side of lens and magnification now is 1. So, [Net magnification abs(M_1)= 2]

When kept in medium of R · I = 7/6.

$$\frac{1}{f_{lens}} = \left(\frac{\mu_2}{\mu_1} - 1\right)\left(\frac{1}{R_1} - \frac{1}{R_2}\right) = \frac{2}{35}$$

$f_{lens} = \frac{35}{2}\ cm.$

Focal length of mirror remains unchanged. So, new position of image,

μ = – 20 cm, $f = \frac{35}{2} cm$

so, v = 140 cm.

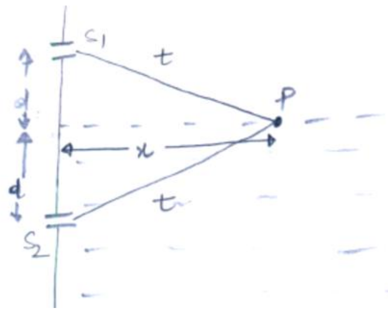
m_2= -7

Net magnification abs(M_2)= (7) × (2)

$= 14 = M_2.$

$\left|\frac{M_2}{M_1}\right| = 7$

Sol.2 (3)



At point p path difference b/ω two waves from S_1 and S_2 will be

$\Delta p = (\mu - 1)\, t$ where $\mu = \frac{4}{3}$

Now to get maxima at p (on the surface of water)

Path difference $\Delta p = m\,\lambda$

$(\boldsymbol{\mu} - 1)\, t = m\,\lambda$ $\Rightarrow$ – (1)

Now $x^2 + d^2 = t^2$ – (2)

From (1) & (2)

$(\mu - 1)^2 (x^2 + d^2) = m^2\lambda^2$

$x^2 + d^2 = \frac{1}{(\mu-1)^2} m^2\lambda^2$

$x^2 = \frac{1}{(\mu-1)^2} m^2\lambda^2 - d^2$

Now we have

$x^2 = p^2 m^2\lambda^2 - d^2$.

$\Rightarrow p^2 = \frac{1}{(\mu-1)^2}$ $\Rightarrow$ $p = \frac{1}{\mu-1}$

$p = 3$ where $\mu = \frac{4}{3}$

Sol.3 (7)

$\frac{1}{2}mv_1^2 + \frac{1}{2}Iw_1^2 + g(30) = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

$\frac{1}{2}mv_2^2 + \frac{1}{2}Iw_2^2 + g(27) = \frac{1}{2}mv^2 + \frac{1}{2}Iw^2$

$\Longrightarrow \frac{V_1^2}{2} + \frac{1}{2} \times \frac{R^2}{2} \times \frac{V_1^2}{R^2} + g \times 30 = \frac{V_2^2}{2} + \frac{1}{2} \times \frac{R^2}{2} \times \frac{V_2^2}{R^2} + g \times 27$

$\Longrightarrow \frac{3v_1^2}{4} + 30g = \frac{3v_2^2}{4} + 27g$

$\Longrightarrow \frac{3}{4}(v_2^2 - v_1^2) = 3g$

$v_2^2 - v_1^2 = \frac{4}{3} \times 30$

$v_2^2 = 40 + 9$

$v_2 = 7$

Sol.4 (2)

It is given that

$g = \frac{1}{4}g_0.$

At height h, we have variation of g as

$g = g_0\frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\frac{1}{4}g_0 = g_0 \frac{1}{\left(1+\frac{h}{R}\right)^2}$

$\left(1 + \frac{h}{R}\right)^2 = 4$

$1 + \frac{h}{R} = 2$

$\left[\frac{h}{R} = 1\right]$ $\Rightarrow [h = R]$ – (1)

we have the formula for escape velocity an

$v_{esc} = \sqrt{\frac{2GM}{R}}$ – (2)

Now applying law of conversation of energy

$\frac{1}{2}\ mv^2 - \frac{GMm}{R} = \frac{1}{2}mv_f^2 - \frac{GMm}{R+h}$

$v_f = 0$ (final velocity at max . height) v from (1)   h = R.

$\frac{1}{2}mv^2 = \frac{GMm}{R} - \frac{GMm}{2R}$.

$\frac{1}{2}mv^2 = \frac{GMm}{R}$

$v_i^2 = \frac{Gm}{R}$ $\Rightarrow$ $v_i = \sqrt{\frac{GM}{R}}$

Here initial velocity is $v_i = $ v

$V = \sqrt{\frac{GM}{R}}$ –   (3)

⇒ from (2) & (3)

$V_{ese} = \sqrt{2}\, v$

N = 2

Sol.5 (2)

Let R_A and R_B are the radii of A and B respectively and P_A and P_B are the power emitted by A and B respectively.

According to question

$R_A = 400R_B$

& $P_A = 10^4\, P_B$

Wien's displacement law

$\frac{\lambda_A}{\lambda_B} = \frac{T_B}{T_A}$ ____ (1)

Stefans – Boltzmann law

$\frac{P_A}{P_B} = \frac{A_A T_A^4}{A_B T_B^4}$

$\frac{P_A}{P_B} = \frac{R_A^2\, \lambda_B^4}{R_B^2\, \lambda_A^4}$

$10^4 = 16 \times 10^4 \frac{\lambda_B^4}{\lambda_B^4}$

$\Longrightarrow \frac{\lambda_A^4}{\lambda_B^4} = 16$ $\Longrightarrow \frac{\lambda_A}{\lambda_B} = 2$

Sol.6 (3)

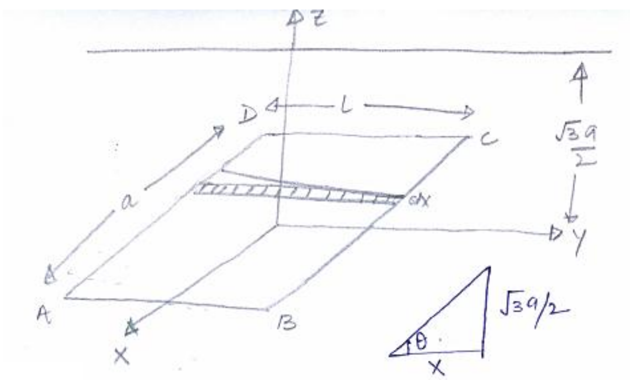
$$N = N_0\left(\frac{1}{2}\right)^{\frac{t}{t_{\frac{1}{2}}}}$$

$$\frac{125}{1000} = \left(\frac{1}{2}\right)^n$$

$$\left(\frac{1}{8}\right) = \left(\frac{1}{2}\right)^n$$

$n = 3$

Sol.7 (6)



Electric field at an element ($\perp^r$ comp.)

$$\frac{\lambda}{2\pi\infty\sqrt{\frac{3a^2}{4}+x^2}}\sin\theta$$ (x is the distance of element from origin)

$$flux = \phi = \int_{-a/2}^{a/2} E.\,ds$$

$$= \int_{-a/2}^{a/2} \frac{\lambda L}{2\pi\varepsilon_0\sqrt{\frac{3a^2}{4}+x^2}} \frac{\frac{\sqrt{3}a}{2}}{\sqrt{\frac{3a^2}{4}+x^2}}\,dx$$

$$= \frac{\lambda L}{2\pi\varepsilon_0}\left(\frac{\pi}{6}+\frac{\pi}{6}\right)$$

$= \frac{\lambda}{2\pi\varepsilon_0}\frac{2z}{6} \qquad \Rightarrow \qquad N = 6$

Sol.8 (2)

$\frac{13.6}{n^2} + 10.4 = \frac{hC}{\lambda}$

$\frac{13.6}{n^2} + 10.4 = \frac{1242}{90}$

n = 2

Sol.9 (C)

Net electric field at the mid-point of both the linear charge system, E = 0

But, on small displacement given to (+q), it will start oscillating, while (-q) would be attracted toward positive plate

Hence, option, 'C' is correct.

Sol.10 (B)

Now, for S_1

$\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$

$\frac{1}{V} - \frac{-1.5}{(-50)} = \frac{1-1.5}{(-10)}$

Solving, we get, v = 50 cm

For the S_2 tube, image should be at '∞'

Hence, $\frac{\mu_2}{v} - \frac{\mu_1}{u} = \frac{\mu_2 - \mu_1}{R}$ [as u = - d + 50]

$\frac{\mu_2}{v} - \frac{\mu_1}{(-d+50)} = \frac{\mu_2 - \mu_1}{R}$ [as calculated below]

$\mu_2 = 1.5$, $\mu_1 = 1$

as, R = +10cm, v = ∞

$\frac{\mu_2}{\infty} - \frac{\mu_1}{(-d+50)} = \frac{1.5-1}{10}$ Solving, we get d = 70 cm

Sol.11 (A,B,C)

Force experience by conductingwire,

$F = BIL\ Sin\theta$

[θ: Angle between magnetic field and length]

If $\vec{B}$ is along z axis,

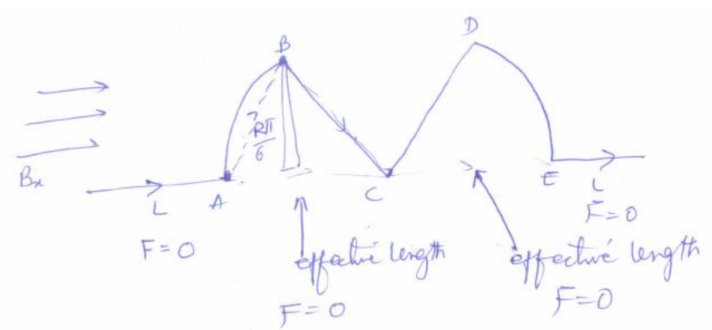
Then force $F = BILSin\ 90° + BIRSin\ 90° + BIRSin90° + BILSin\ 90°$

$|\vec{F}| \propto (L + R)$

(Component of force may vary)

Similarly, when($\vec{B}$) is along y –axis

If ($\vec{B}$) is along x – axis, net effect length is zero.



Hence A, B, and C

Sol.12 (A,B,D)

Average kinetic energy per molecule is $\frac{3}{2}nRT$& Average Energy per mole of the gas mixture would be, 2RT.

Ratio of Speed of the sound in the gas mixture is that in Helium gas would be

$$\frac{V_{mix}}{V_{He}} = \frac{\frac{\sqrt{Y_{mix}RT}}{M_{mix}}}{\frac{\sqrt{Y_{He}RT}}{M_{He}}}$$

Calculating, $\Upsilon_{\text{mix}} = \frac{(C_p)_{mix}}{(C_v)_{mix}} = \frac{C_v+R}{C_v}$

Also, $(C_v)_{mix} = \frac{n_1Cv_1+n_2Cv_2}{n_1+n_2} = \frac{\left(1\times\frac{3}{2}+1\times\frac{5}{2}\right)R}{1+1}$

$= (Cp)_{mix} = (Cv)_{mix} + R$

$(Cp)_{mix} = 3R,$

$Y = \frac{Cp}{Cv} = \frac{3R}{2R} = \frac{3}{2}$

Also, $M_{mix} = \frac{n_1M_1+n_2M_2}{n_1+n_2} = \frac{1\times4+1\times2}{1+1} = 3$

$V_{mix}\sqrt{\frac{VRT}{M}} = \sqrt{\frac{\frac{3}{2}RT}{3}} = \sqrt{\frac{1}{2}RT}$ ...(i)

$V_{He} = \sqrt{\frac{YRT}{M_{He}}} = \sqrt{\frac{\frac{5}{3}RT}{4}} = \sqrt{\frac{5}{12}RT}$ ...(ii)

$\frac{V_{mix}}{V_{He}} = \sqrt{\frac{6}{5}}$

Ratio of RMS speed of Helium atom to that of Hydrogen molecule,

$\frac{(V)_{He}}{(V)_{H2}} = \frac{\sqrt{\frac{3RT}{M_{He}}}}{\sqrt{\frac{3RT}{MH_2}}} = \frac{\sqrt{\frac{1}{4}}}{\sqrt{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$

Sol.13 (B)

Using $R = \rho\frac{L}{A}$

Calculating Resistances of Aluminum and Iron, and as they are connected in parallel,

$R_{parallel} = \frac{R_{AL}\times R_{Fe}}{R_{AL}+R_{Fe}}$ ...(i)

$R_{AL} = \frac{2.7\times10^{-8}\times50}{[7^2-2^2]} = 30\ \mu\Omega$

$R_{Fe} = \frac{1\times10^{-7}\times50}{2^2} = 1250\ \mu\Omega$

Substituting these values in eqn. (i) and getting

$\frac{1875}{64}\ \mu\Omega$

Hence, B

Sol.14 (A,C)

Using , $hv - hv^* = eV$

$V = \frac{hc}{e\lambda} = \frac{hc}{e\lambda^*}$

Finding the graph, option A and C are correct.

Sol.15 (B,C)

1 cm on MSR is divided into 8 equal divisions and a screw gauge with 100 divisions.

In Vernier calipers

Least Count = 1 MSD – 1 VSD

$= 1\text{ VSD} = \frac{4}{5}\text{MSD}$ $[\because 5\text{ VSD} = 4\text{MSD}]$

$= \frac{1}{5}\ MSD$

$= \frac{1}{5} \times \frac{1}{8} cm$

$L.C_{vs} = \frac{1}{40} cm = 0.025\ cm.$

For screw gauge,

$L.C_{sg} = \frac{Pitch}{No.of\ division\ on\ circular\ scale}$

If pitch $= 2 \times L.C_{vs} = \frac{1}{20} cm.$

$L.C_{SG} = \frac{1/20}{100} = 0.005\ cm.$

For screw gauge,

If Linear scale L.C $= 2 \times \frac{1}{40} = \frac{1}{20} cm.$

Pitch $= 2 \times \frac{1}{20} = \frac{1}{10} cm.$

$L.C._{SG} = \frac{140}{100} = 0.01\ mm.$

Sol.16 (C,D)

$L = K\,[h]^a\,[C]^b\,[G]^c$

Writing dimensional formula of each terms

$L = [ML^2T^{-1}]^a[LT^{-1}]^b[M^{-1}L^3T^{-2}]^c$

a – c = 0

2a + b + 3c = 1

-a – b – 2c = 0

Solving the equation we get

$a = \frac{1}{2}, c = \frac{1}{2}, b = \frac{-3}{2}$

Hence, L ∝√h

∝√G

Similarly, for M

$$M \propto (h)^{\frac{1}{2}}, (c)^{\frac{1}{2}}, (G)^{-\frac{1}{2}}$$

Hence, A, C, D are correct.

Sol.17 (B,D)

$E_1 = \frac{1}{2}mw_1^2a^2 = \frac{b^2}{2m} = \frac{a^2}{2m\,n^4}$ $\left[\because \frac{a}{b} = n^2\right]$

$\Rightarrow \quad m^2w_1^2 = \frac{1}{n^4}$

and ,

$E_2 = \frac{1}{2}m\,w_2^2R^2 = \frac{R^2}{2m}$

$\Rightarrow \quad E_2 = \frac{1}{2}mw_2^2 \quad \left(\frac{a}{n}\right)^2 = \frac{\left(\frac{a}{n}\right)^{\wedge}2}{2m} \left[\because \frac{a}{R} = n\right]$

$m^2w_2^2 = 1$

so, $\frac{w^2}{w_1^2} = n^2$ – (a)

Now,

$\frac{E_1}{w_1} = \frac{b^2}{2mw_1} = \frac{a^2}{2mn^4.w^1} = \frac{a^2.n^2}{2mn^4.w^2} -$

– From - (a)

$= \frac{E_2}{w_2}$

Sol.18 (D)

Initial Angular Momentum = MR^2w

As net external torque on the system is zero,

Angular momentum remains conserved.

Final $\vec{L} = \left[MR^2 + \frac{4}{8}\left(\frac{3R}{5}\right)^2 + \frac{M}{8}x^2\right]\frac{8w}{9}$

$\Rightarrow \quad x^2 = \frac{16}{25}R^2$

$\Rightarrow x = \frac{4}{5}R.$

Sol.19 (A →R,T)

In nuclear fusion Positron , neutrino and Gamma rays are released apart from the fusion products. It is responsible for the energy production in the core of the stars.

(B →P,S,T)

Fission of ^{235}U happens by absorption of thermal neutrons. Heavy water is used as modulators in controlled fission. Neutrons, neutrinos, beta rays and Gamma rays are released.

(C →P,Q,R)

Beta decays happen in fission reaction, decays of $^{60}_{27}Co$ nucleus to $^{60}_{28}N_i$

(^{60}Co is a Synthetic radio isotope of cobalt which decays to ^{60}Ni)

(D →P,Q,R)

$ß^-$ –decay is followed by emission of antineutrino and not neutrino.

γ – ray emission happens in fission,fusion, & decay of ^{60}Co.

Sol.20 (A →P,R,S,T)

$U_1(x) = \frac{v_0}{2}\left[1 - \left(\frac{x}{a}\right)^2\right]^2$

$F_1(x) = -\frac{d\,U_1(x)}{dx}$

$= -\frac{v_0}{2} \times 2\left[1 - \frac{x^2}{a^2}\right]\left[-\frac{2x}{a^2}\right]$

$= 2\frac{Uo}{G^2}.\text{x}.(1-\frac{x^2}{a^2})$

$F_1(x) = 0$ $\Rightarrow x = \pm a.$

Force towards x = 0 in the region |x| < a; f (x) = – k (x – a)

Potential Energy at x = –a,

$U_1(-a) = 0.$

So, It may oscillate about x = –a.

(B →Q,S)

$U_2\,(x) = \frac{U_0}{2}\left(\frac{x}{2}\right)^2$

$F_2\,(x) = -\frac{dU_2}{dx}$

$= -\frac{U_0}{2a^2}.(2x) = -\frac{Uo}{a^2}\;x.$

$F_2\,(x) = 0 \Rightarrow x = 0$

Hence $F_2\,(x) = \frac{-U_0}{a^2} \times x = -kx$

So, it experience attractive force towards x = 0in |x| < a.

(C →P,R,S,T)

$U_3\,(x) = \frac{U_0}{2}\left[\left(\frac{x}{z}\right)^2\right] exp.\left[-\left(\frac{x}{z}\right)^2\right]$

$$F_3(x) = \frac{-dU_3}{dx}$$

$$= \frac{-U_0}{2}\left[\frac{2x}{a^2}.\exp\left[-\left(\frac{x}{a}\right)\right] + \left(\frac{x}{a}\right)^2.\left[\frac{-2x}{a^2}\right]\left[\exp-\left(\frac{x}{a}\right)^2\right]\right]$$

$$= -U_0.exp.\left[-\left(\frac{x}{a}\right)\right]\left[\frac{x}{a^2}-\frac{x^3}{a^4}\right] = -U_0c^{-\left(\frac{x}{a}\right)}\left(\frac{x}{a}\right)\left[1-\frac{x^2}{a^2}\right]$$

For $F_3\,(x) = 0 \Longrightarrow \frac{x}{a^2} = \frac{x^3}{a^4} \Longrightarrow x^2 = a^2$

$\Longrightarrow x = \pm a$

Particle can oscillate about x = '-a' with energy $\frac{U_0}{4}$ if its energy at 'x = -a' is use than or equal to $\frac{U_0}{4}$.

$$U_3(-a) = \frac{U_0}{2e} < \frac{U_0}{4.}$$

It experiences attractive force towards x = 0.

(D →P,R)

$U_4(k) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$

$$F_4(x) = -\frac{\alpha u_4}{\alpha_x}$$

$$= -\frac{U_0}{2}\left[\frac{1}{a} - \frac{1}{3a^3} \times 3x^2\right]$$

$$= -\frac{U_0}{2a}\left[1 - \frac{x^2}{a^2}\right]$$

$F_4(x) = 0 \Longrightarrow x = \pm a$

Potential energy at x = -a,

$$V(-a) = -\frac{V_0}{3}$$

$$|v(-a)|.\frac{V_0}{4}$$

So it can't oscillate about x = -a with $\frac{V_0}{4}$.

Now,

$$F_4(x) = \frac{-V_0}{2a}\left[1 - \frac{x^2}{a^2}\right]$$

For $|x| < a\ 4 \Longrightarrow F_4(x) = tve$

So, force is acting along the same direction throughout $|x| < a$.

Paper-1

JEE Advanced, 2015

Part III –Mathematics

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provided within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet. Verify that the booklet contains 32 pages and that all the 60 questions along with the options are legible.

Question paper format and marking scheme:

8. The question paper has three parts: Physics, Chemistry and Mathematics. Each part has three sections.
9. Carefully read the instructions given at the beginning of each section.
10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).

    Marking scheme: +4 correct answer and 0 in all other cases.

11. Section 2 contains 10 multiple choice questions with one or more than one correct option.

    Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

12. Section 3 contains 2 " match the following" type questions and you will have to match entries in Column I with the entries in Column II.

Marking scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and -1 in all other cases.

OPTICAL RESPONSE SHEET :

13. The ORS consists of an original (top sheet) and its carbon-less copy. (bottom sheet).
14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.
15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.
16. You will be allowed to take away the carbon-less copy at the end of the examination.
17. Do not tamper with or mutilate the ORS.
18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Q.41 Let $F(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$ for all $x \in R$ and $f: \left[0, \frac{1}{2}\right] \to [0,\infty)$ be a continuous functions. For $\alpha \in \left[0, \frac{1}{2}\right]$, if $F'(\alpha)+2$ is the area of the region bounded by $x = 0$, $y=0$, $f(x)$ and $x = \alpha$, then $f(0)$ is

Ans.41 (3)

Q.42 A cylindrical container is to be made from certain solid material with the following constrains: It has a fixed inner volume of V mm^3, has a 2 mm thick solid wall and is open at the top. The bottom of the container is a solid circular disc of thickness 2 mm and is of radius equal to the outer radius of the container.

If the volume of the material used to make the container is minimum when the inner radius of the container is 10mm, then the value of $\frac{V}{250\pi}$ is

Ans.42 (4)

Q.43 Let n be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that all the girls stand consecutively in the queue. Let m be the number of ways in which 5 boys and 5 girls can stand in a queue in such a way that exactly four girls stand consecutively in the queue. Then the value of $\frac{m}{n}$ is

Ans.43 (5)

Q.44 The minimum number of times a fair coin need to be tossed, so that the probability of getting at least two heads is at least 0.96 is

Ans.44 (8)

Q45. If the normal the parabola $y^2 = 4x$ drawn at the end points of its latus return are tangents to the circle $(x - 3)^2 + (y + 2)^2 = r^2$, then the value of r^2 is

Ans.45 (2)

Q.46 Let f : R → R be a function defined by $f(x)=\begin{cases}[x], x \le 2\\ 0, x > 2\end{cases}$

Where [x] is the greatest integer less then or equal to x. If $l = \int_1^2 \frac{xf(x^2)}{2+f(x+1)}dx$, then the value of $(4l - 1)$ is

Ans.46 (0)

Q.47 The number of distinct solution of the equation

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$ in the interval $[0, 2\pi]$ is

Ans.47 (8)

Q.48 Let the curve C be the mirror image of the parabola $y^2 = 4x$ with respect to the $x + y + 4 = 0$. If A and B are the points of intersection of C with the line $y = -5$. Then the distance between A and B is

Ans.48 (4)

SECTION 2 (Maximum Marks: 40)

- This section contains **TEN** questions
- Each question has **FOUR** options (A), (B), (C) and (D) **ONE OR MORE THAN ONE** of these four option(s) is(are) correct.
- For each question, darken the bubbles(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:

  +4 If only the bubble(s) corresponding to all the correct option(s) is(are) darkened

  0 If none of the bubbles is darkened

  -2 In all other cases

Q.49 Let P and Q be distinct points on the parabola $y^2 = 2x$ such that a circle with PQ as diameter passes through the vertex O of the parabola. If P lies in the first quadrant and the area of the triangle ΔOPQ is $3\sqrt{2}$, then which of the following is (are) the coordinates of P?

(A) $(4,2\sqrt{2})$ (B) $(9,3\sqrt{2})$ (C) $\left(\frac{1}{4},\frac{1}{\sqrt{2}}\right)$ (D) $(1,\sqrt{2})$

Ans.49 (A,D)

Q.50 Let y(x) be a solution of the differential equation $(1 + e^x)y' + ye^x = 1$. If y (0) = 2, then which of the following statements is (are) true?

(A) y (-4) = 0

(B) y (-2) = 0

(C) y (x) hs a critical point in the interval (-1,0)

(D) y(x) has no critical point in the interval (-1,0)

Ans.50 (A,C)

Q.51 Consider the family of all circle whose centres lie on the straight line y = x. if this family of circle is represented by the differential equation Py" + Qy' + 1 = 0, where P,Q are functions of x, y and y' (here $y' = \frac{dy}{dx}, y'' = \frac{d^2y}{dx^2}$) , then which of the following statements is (are) true?

(A) P = y+x (B) P = y – x

(C) $P + Q = 1 - x + y + y' + (y')^2$ (D) $P - Q = x + y - y' - (y')^2$

Ans.51 (B,C)

Q.52 Let g: R →R be a differentiable function with g (0)= 0, g' (0)= 0 and g'(1)≠0, Let

$$f(x)=\begin{cases}\frac{x}{|x|}g(x), & x\neq 0\\ 0, & x=0\end{cases}$$

and h(x) = $e^{|x|}$ for all x ∈ R. Let (f ·h)(x) denote f(h(x)) and (h·f)(x) denote h(f(x)).

Then which of the following is (are)true?

(A) f is differentiable at x = 0

(B) h is differentiable at x = 0

(C) f · h is differentiable at x = 0

(D) h · f is differentiable at x = 0

Ans.52 (A,D)

Q.53 Let f (x) = $\sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$ for all x ∈ R and g(x)= $\frac{\pi}{2}$ sin x for all x∈ R. Let (f·g)(x) denote f(g(x)) and (g·f)(x)denote g(f(x)). Then which of the following is (are)true?

(A) Range of f is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(B) Range of f·g is $\left[-\frac{1}{2},\frac{1}{2}\right]$

(C) $\lim_{x\to 0}\frac{f(x)}{g(x)}=\frac{\pi}{6}$

(D) There is an x ∈ R such that (g·f)(x) = 1

Ans.53 (A,B,C)

Q.54 Let ΔPQR be a triangle. Let $\vec{a}=\overrightarrow{QR}$, $\vec{b}=\overrightarrow{RP}$ and $\vec{c}=\overrightarrow{PQ}$. If $|\vec{a}|=12$, $|\vec{b}|=4\sqrt{3}$ and $\vec{b}.\vec{c}=24$, then which of the following is (are)true?

(A) $\frac{|\vec{c}|^2}{2}-|\vec{a}|=12$

(B) $\frac{|\vec{c}|^2}{2}+|\vec{a}|=30$

(C) $|\vec{a}\times\vec{b}+\vec{c}\times\vec{a}|=48\sqrt{3}$

(D) $\vec{a}.\vec{b}=-72$

Ans.54 (A,C,D)

Q.55 Let X and Y be two arbitrary, 3×3, non-zero, skew-symmetric matrices and Z be an arbitrary 3×3, non-zero, symmetric matrix. Then which of the following matrices is (are) skew symmetric?

(A) $Y^3Z^4 - Z^4Y^3$

(B) $X^{44} + Y^{44}$

(C) $X^4Z^3 - Z^3X^4$

(D) $X^{23} + Y^{23}$

Ans.55 (C,D)

Q.56 Which of the following values of α satisfy the equation

$$\begin{vmatrix} (1+\alpha)^2 & (1+2\alpha)^2 & (1+3\alpha)^2 \\ (2+\alpha)^2 & (2+2\alpha)^2 & (2+3\alpha)^2 \\ (3+\alpha)^2 & (3+2\alpha)^2 & (3+3\alpha)^2 \end{vmatrix} = -648a\ ?$$

(A) -4 (B)9 (C) -9 (D)4

Ans.56 (B,C)

Q.57 In R^3, consider the planes $P_1: y = 0$ and $P_2: x+z = 1$. Let P_3 be a plane, different from P_1 and P_2, which passes through the intersection of P_1 and P_2. If the distance of the point (0,1,0) from P_3 is 1 and the distance of a point (α,β,γ) from P_3 is 2, then which of the following relations is (are) true?

(A) $2\alpha + \beta + 2\gamma + 2 = 0$

(B) $2\alpha - \beta + 2\gamma + 4 = 0$

(C) $2\alpha + \beta - 2\gamma - 10 = 0$

(D) $2\alpha - \beta + 2\gamma - 8 = 0$

Ans.57 (B,C)

Q.58 In R^3, let L be a straight line passing through the origin. Suppose that all the points on L are at a constant distance from the two planes $P_1: x+2y - z + 1= 0$ and $P_2: 2x - y + z - 1 = 0$. Let M be the locus of the feet of the perpendiculars drawn from the points on L to the plane P_1. Which of the following points lie(s) on M?

(A) $\left(0, -\frac{5}{6}, -\frac{2}{3}\right)$

(B) $\left(-\frac{1}{6}, -\frac{1}{3}, \frac{1}{6}\right)$

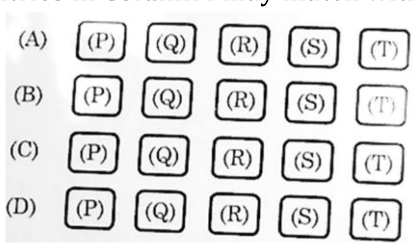
(C) $\left(-\frac{5}{6}, 0, \frac{1}{6}\right)$

(D) $\left(-\frac{1}{3}, 0, \frac{2}{3}\right)$

Ans.58 (A), (B), (C), (D)

SECTION 3 (Maximum Marks: 16)

- This section contains TWO questions
- Each question contains two columns, **Column I** and **Column II**
- **Column I** has four entries (A), (B), (C) and (D)
- **Column II** has five entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **column II**
- One or more entries in Column I may match with one or more entries in **Column II**



- For each entry in **Column I,** darken the bubbles of all the matching entries For example, if entry (A) in
  Column I matches with entries (Q), (R) and (T) then darken these three bubbles in the ORS. Similarly, for entries, (B), (C) and (D).
- Marking scheme:
  <u>For each entry in **Column I.**</u>
  +2 If only the bubble(s) corresponding to all the correct match(es) is(are) darkened
  0 If none of the bubbles is darkened
  -1 In all other cases

Q.59 **Column I** **Column II**

(A) In R^2, if the magnitude of the projection (P) 1

Vector of the vector $\alpha\hat{\imath} + \beta\hat{\jmath}$ on $\sqrt{3}\hat{\imath} + \hat{\jmath}$ is $\sqrt{3}$ and

If $\alpha = 2 + \sqrt{3}\beta$, then possible value(s) of $|\alpha|$ is (are)

(B)Let a and b be real numbers such that the function (Q) 2

$$f(x) = \begin{cases} -3ax^2 - 2, & x < 1 \\ bx + a^2, & x \geq 1 \end{cases}$$

is differentiable for all x ∈ R. then possible value(s) of a is (are)

(C) Let ω ≠1 be a complex cube root of unity. If $(3\text{-}3\omega+2\omega^2)^{4n+3}+$ (R) 3

$(2+3\omega\text{-}3\omega^2)^{4n+3} + (\text{-}3+2\omega+3\omega^2)^{4n+3} = 0$, then possible

Value(s)of n is (are)

(D) Let the harmonic mean of two positive real numbers (S) 4

a and b be 4. If q is a positive real number such that a,5,q,b

is an arithmetic progression, then value(s) of |q −a | is (are)

(T) 5

Ans.59 (A→ P.Q), (B →P, Q), (C→ P,Q,S,T), (D → Q,T)

Q.60 Column I **Column II**

(A) in a triangle ΔXYZ, let a, b and c be the (P) 1

length of the sides opposite to the angles X, Y

and Z, respectively, If $2(a^2\text{-}b^2) = c^2$ and $\lambda = \frac{\sin(X-Y)}{sinZ}$,

then possible values of n for which $\cos(n\pi\lambda) = 0$

is (are)

(B) In a triangle ΔXYZ, let a,b and c be the length (Q) 2

of the sides opposite to the angles X,Y and Z, respectively.

If 1 + cos2X – 2cos2Y = 2sinX sin Y, then possible value(s)

of $\frac{a}{b}$ is (are)

(C) In R^2,let $\sqrt{3}\hat{\imath} + \hat{\jmath}$, $\hat{\imath} + \sqrt{3}\,\hat{\jmath}$ and $\beta\hat{\imath} + (1 - \beta)\hat{\jmath}$ (R) 3

be the position vectors of X,Y and Z with respect to the

origin O, respectively, If the distance of Z from the bisector

of the acute angle of $\overrightarrow{OX}$ with $\overrightarrow{OY}$ is $\frac{3}{\sqrt{2}}$, then possible value(s) of $|\beta|$ is (are)

(D) Suppose that $F(\alpha)$ denotes that area of the region bounded (S) 5

By $x = 0$, $x = 2$, $y^2 = 4x$ and $y = |\alpha x - 1| + |\alpha x - 2| + \alpha x$, where

$\alpha \in \{0,1\}$ .then the value(s) of $F(\alpha) + \frac{8}{3}\sqrt{2}$,

when $\alpha = 0$ and $\alpha = 1$, is (are)

(T) 6

Ans.60 (A → P, R, S), (B → P), (C → P,Q), (D →S,T)

Answer Keys and Explanations

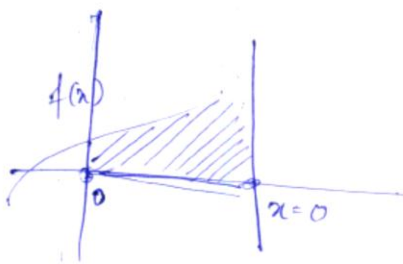
Sol.41 (3)

$$F(x) = \int_{x}^{x^2+\frac{\pi}{6}} 2\cos^2 t\,dt$$

$$F`(x) = 2\cos^2\left(x^2+\frac{\pi}{6}\right)(2x) - 2\cos^2 x$$

$$\left[\int_{h(x)}^{g(x)} f(t)\,dt = \left\{f\left(g(x)\right)g'(x) - f\left(h(x)\right)h'(x)\right\}\right]$$

$$F`(\alpha) = 4\alpha\cos^2\left(a^2+\frac{\pi}{6}\right) - 2\cos^2 a$$



Area bounded $= \int_0^{\alpha} f(x)\,dx$

Acc to question

$$\int_0^{\alpha} f(x)\,dx = F'(\alpha) + 2$$

$$\int_0^{\alpha} f(x)\,dx = 2\cos^2\left[\alpha^2+\pi/6\right]\times 2\alpha - 2\cos^2\alpha\times 1 + 2$$

Differentiating both the sides

$$f(x) = 4x\cos^2\left[x^2+\pi/6\right] + 4x\times 2\cos\left[x^2+\pi/6\right]\times -\sin$$

$$\left[x^2+\pi/6\right] + 2\times 2\cos x\times\sin x$$

$$f(0) = 4\times\frac{3}{4} + 0 = 3$$

Sol.42 (4)

Fixed volume = V   Let height be h

Let inner radius be r $V = \pi r^2 h$

Outer radius = r + 2 $h = \frac{V}{\pi r^2}$

(Volume of material = Outer Volume – Inner Volume around curved surface area)

$= \pi(r+2)^2 h - V$

Total volume of material (say V_1) $= \pi(r+2)^2 h - V + \pi(r+2)^2 \times 2$

$= \pi \frac{(r+2)^2 V}{\pi r^2} - V + 2\pi\,(r+2)^2$

$V_1 = (1+\frac{2}{r})^2 V - V + 2\pi(r+2)^2$

$V_1{}' = 2V\left(1+\frac{2}{r}\right)\left(\frac{-2}{r^2}\right) + 4\pi(r+2)$

Now V_1 is minimum at r = 10

$V_1{}' = 0\ at\ r = 10$

On solving $\frac{V}{250\pi} = 4$

Sol.43 (5)

When five girls are standing consecutively let us consider them as 1 unit. So now 1 unit of girls and 5 boys can be arrange in 6! Ways Girls themselves can be arranged in 5! Ways

Total ways n = 6! × 5!

When 4 girls are standing consecutively

First select 4 girls out of 5 in $^5C_4$ ways and then consider them as 1 unit

4 girls and other girl will be arranged in gaps between 5 Boys.

$m = {}^5C_4 5!\, {}^6C_2\, 2!\, 4!$

So, $\frac{m}{n} = 5$

Sol.44 (8)

Let n be the number of times coin is tossed

Using binomial distribution Probability At least two heads = Total – 1 head – No head

$$P(x \geq 2) = 1 - {}^nC_1 \frac{1}{2}\left(\frac{1}{2}\right)^{n-1} - {}^nC_o\left(\frac{1}{2}\right)^n$$

$$= 1 - n\left(\frac{1}{2}\right)^n - \left(\frac{1}{2}\right)^n$$

Now $P(x \geq 2) \geq 0.96$

$1 - \frac{(n+1)}{2^n} \geq 0.96$

$0.04 \geq \frac{(n+1)}{2^n}$

$(0.04)2^n \geq (n+1)$

By hit and trial.

For n = 8 (min.)

Satisfies above equation

$n = 8$

Sol.45 (2)

$y^2 = 4x$

So, a = 1, so coordinates of LR: (1, 2) and (1, -2).

So, $m_T = \frac{dy}{dx} = \frac{4}{2y} = \frac{2}{y} = \frac{2}{2} = 1$

$m_N = -1$

So, eqn. of normal is,

$y - 1 = -1(x - 2)$

$y + x = 3$

Now, solve this line with circle and put D = 0 as this time is tangent to circle so it will touch at one point only.

$(-y)^2 + (y + 2)^2 = r^2$

$2y^2 + 4y + (4 - r^2) = 0$

$D = 16 - 8(4 - r^2) = 0$

$2 = 4 - r^2$

Hence, $r^2 = 2$

Sol.46 (0)

$$f(x) = \begin{cases} [x] & x \le 2 \\ 0 & x > 2 \end{cases}$$

$$I = \int_1^2 \frac{xf(x^2)}{2+f(x+1)}\, dx$$

$$f(x^2) = \begin{cases} [x^2] & x^2 \le 2 \\ 0 & x^2 > 2 \end{cases}$$

$$= \begin{cases} [x^2] & -\sqrt{2} < x \le \sqrt{2} \\ 0 & x > \sqrt{2} \quad x < -\sqrt{2} \end{cases}$$

$-1 < x < 2$

$f(x)^2 = [x^2]\ -1 \le x \le \sqrt{2}$

$0 \qquad x > \sqrt{2}$

$$f(x+1) = \begin{cases} [x+1] & x+1 \le 2 \\ 0 & x+1 > 2 \end{cases}$$

$$f(x+1) = \begin{cases} [x+1] & x \le 1 \\ 0 & x > 1 \end{cases}$$

In (-1, 2)

$$f(x+1) = \begin{cases} [x+1] & -1 \le x \le 1 \\ 0 & x > 1 \end{cases}$$

	(-1, 0)	(0, 1)	(1, √2)	(√2, √3)	(√3, 2)
$f(x^2)$	0	0	1	0	0
$f(x+1)$	0	1	0	0	0

$$I = \int_1^2 \frac{xf(x^2)y1}{2+f(x+1)} = \int_1^{\sqrt{2}} \frac{x(1)dx}{2+0}$$

$= \frac{x^2}{4}\Big|_1^{\sqrt{2}} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

$4I - 1 = 4 \times \frac{1}{4} - 1 = 0$

Sol.47 (8)

$\frac{5}{4}\cos^2 2x + \cos^4 x + \sin^4 x + \cos^6 x + \sin^6 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x)^2 + (\sin^2 x)^2 + (\cos^2 x)^3 + (\sin^2 x)^3 = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + (\cos^2 x + \sin^2 x)^2 - 2\cos^2 x \sin^2 x + (\cos^2 x + \sin^2 x)^3 - 3\sin^2 x\cos^2 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x + 1 - 2\cos^2 x \sin^2 x + 1 - 3\cos^2 x \sin^2 x = 2$

$\Rightarrow \frac{5}{4}\cos^2 2x = 5\cos^2 x \sin^2 x$

$\Rightarrow \cos^2 2x - \sin^2 2x = 0$

$\text{Cos}4x = 0$

x lies in $[0, 2\pi]$

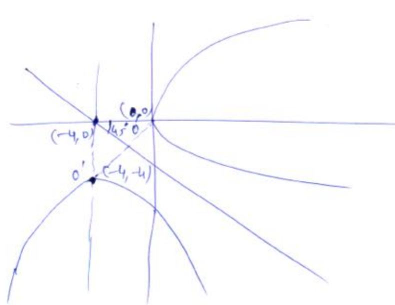
4x lies in $[0, 8\pi]$

In 0, 2π cos t = 0

At $t = \frac{\pi}{2}, \frac{3\pi}{2}$

In 0, 8π there will be 8 solutions.

Sol.48 (4)



So equation of image parabola is $(x+4)^2 = -4(y+4)$

Now solve with $y = -5$

$(x+4)^2 = -4(-5+4)$

$(x+4)^2 = 4$

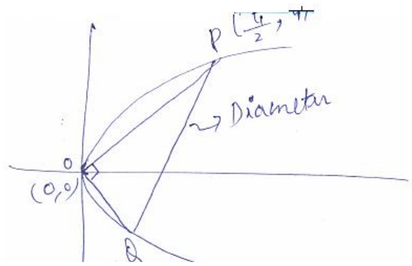
$x + 4 = \pm 2$

$x = -4 \pm 2$

$x = -6, -2$

So distance b/w then 4.

Sol.49 (A,D)



$Y^2 = 2x$

$\therefore a = \frac{1}{2}$

Now, $m_{op}\, m_{od} = -1$

$\frac{t_1}{\left(\frac{t_1^2}{2}\right)} \times \frac{t_2}{\left(\frac{t_2^2}{2}\right)} = -1$

$t_1 t_2 = -4$ ..... (1)

Area of triangle$(\Delta) = \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ \frac{t_1^2}{2} & t_1 & 1 \\ \frac{t_2^2}{2} & t_2 & 1 \end{vmatrix} = 3\sqrt{2}$ ..... (2)

Solving (1) and (2), we get,

$t_1 = 2\sqrt{2}$ and $t_2 = \sqrt{2}$

Sol.50 (A,C)

$(1 + e^x)\, y' + e^x y = 1$

$\frac{d}{dx}[(1 + e^x)y] = 1$

Integrating both sides

$(1+e^x)Y = x + c$

$Y = \frac{x+c}{1+e^x}$

$Y(0) = 2$

$\frac{c}{1+1} = 2 \Rightarrow C = 4$

$Y = \frac{x+4}{1+e^x}$

$Y(-4) = 0$

$Y' = \frac{(1+e^x)-(x+4)e^x}{(1+e^x)^2}$

$Y' = \frac{1^{-e^x}[x+3]}{(1+e^x)^2}$

$Y'(0) = \frac{1-3}{2^2} = \frac{-2}{2^2} = \frac{-1}{2}$

$Y'(-1) = \frac{1-2e^{-1}}{(1+e^{-1})^2} > 0$

So it will have critical point in (-1,0)

Sol.51 (B,C)

$(x-a)^2 + (y-a)^2 = r^2$

$2(x-a) + 2(y-a)\frac{dy}{dx} = 0$

$x - a + (y - a)\frac{dy}{dx} = 0 \Rightarrow x - a + yy' - ay' = 0$

$a = \frac{x+yy'}{1+y'}$

again differentiating

$(y - a)y'' + y'y' + 1 = 0$

$1 + (y - \frac{x+yy'}{1+y'})y'' + y'y' = 0$

$1 + y' + (y - x)y'' + y'y' + y'y'y' = 0$

$Y' + y'[1 + y' - y'\cdot y'] + 1 = 0$

$P = Y - X$

P + Q = 1 + - X + Y + Y’ + (Y’)2

Sol.52 (A,D)

$$f(x) = \begin{cases} g(x) & x > 0 \\ -g(x) & x < 0 \\ 0 & x = 0 \end{cases}$$

$$h(x) = \begin{cases} e^{x} & x > 0 \\ e^{-x} & x < 0 \end{cases}$$

$$f(h(x)) = \begin{cases} g(e^{x}) & e^{x} > 0, \quad x \geq, 0 \\ -g(e^{x}) & e^{x} < 0, \quad x \geq, 0 \quad \text{its wrong} \\ 0 & e^{x} > 0, \quad x \geq 0x \quad \text{its wrong} \end{cases}$$

$$= \begin{cases} g(e^{-x}) & e^{-x} > 0, \ x < 0 \ \text{its correct} \\ -g(e^{-x}) & e^{-x} < 0, \ x < 0 \ \text{its wrong} \\ 0 & e^{-x} > 0, \ x < 0 \ \text{its wrong} \end{cases}$$

$$f(h(x)) = \begin{cases} g(e^{x}) & x \geq 0 \\ g(e^{-x}) & x < 0 \end{cases}$$

$$(foh)' = \begin{cases} g'\left(e^{x}\right) \times e^{x} & x \geq 0 \\ -g'\left(e^{-x}\right) \times e^{-x} & x < 0 \end{cases}$$

$$h(f(x)) = \begin{cases} e^{g(x)} & g(x) > 0, \ x > 0 \\ e^{-g(x)} & -g(x) > 0, \ x < 0 \ g(x) < 0 \\ e^{0} & g(x) > 0, \ x < 0 \end{cases}$$

$$h(f(x)) = \begin{cases} e^{-g(x)} & g(x) < 0, & x > 0 & g(x) < 0 \\ e^{g(x)} & -g(x) < 0, & x < 0 & g(x) > 0, x < 0 \\ e^{-0} & 0 > 0, & x = 0 & \end{cases}$$

$$h(f(x)) = \begin{cases} e^{g(x)} & g(x) > 0 \\ e^{-g(x)} & g(x) < 0 \\ 1 & \end{cases}$$

$h(f(x)) = e^{g(x)} \times g'x$

$h(f(x)) = e^{-g(x)} \times g'(x) \times -1$

Sol.53 (A,B,C)

$f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$g\ (x) = \frac{\pi}{2}\sin x$

Now, $f(x) = \sin\left(\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin x\right)\right)$

$\theta = \frac{\pi}{2}\sin x\ \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$\sin\left(\frac{\pi}{6}\sin\theta\right) \in \left[\frac{-1}{2}, \frac{1}{2}\right]$ $\therefore$ (A)

$f\left(g(x)\right) = \sin\left[\frac{\pi}{6}\sin\frac{\pi}{2}\left(\sin\left(\frac{\pi}{2}\sin x\right)\right)\right]$

$\frac{\pi}{2}\sin x = \theta$ for $x \in R$

$\theta \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$f\ (g(x) = \sin\frac{\pi}{6}\sin\left(\frac{\pi}{2}\sin\theta\right)$

$\frac{\pi}{2}\sin\theta = \propto$

$\propto \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$

$f\left(g(x)\right) = \sin\left(\frac{\pi}{6}\sin\propto\right)$

For $\propto \in \left[\frac{-\pi}{2}, \frac{\pi}{2}\right]$ $\frac{\pi}{6}\sin\propto \in \left[\frac{-\pi}{6}, \frac{\pi}{6}\right]$

$\Rightarrow \sin\left(\frac{\pi}{6}\sin\propto\right) \in \frac{-1}{2}, \frac{1}{2}$ $\therefore$ (B)

Now, $\lim\limits_{x\to 0}\frac{\sin\left(\frac{\pi}{6}sin\left(\frac{\pi}{2}sin\, x\right)\right)}{\frac{\pi}{2}\sin x}$

$$= \lim_{x\to 0} \frac{2}{\pi} \frac{\sin\left(\frac{\pi}{6} sin\left(\frac{\pi}{2} sinx\right)\right)}{\frac{\pi}{6}\sin(\frac{\pi}{2} sinx)} \times \frac{\frac{\pi}{6}\sin(\frac{\pi}{2}\sin x)}{\frac{sinx}{x} xx}$$

$$= \lim_{x\to 0} \frac{2}{\pi} \times \frac{\pi}{6} \times \frac{\sin\left(\frac{\pi}{2} sinx\right)}{\frac{\pi}{2} sinx} \times \frac{\frac{\pi}{2} sinx}{x}$$

$\frac{1}{3} \times \frac{\pi}{2} = \frac{\pi}{6}$ $\therefore$ (C)

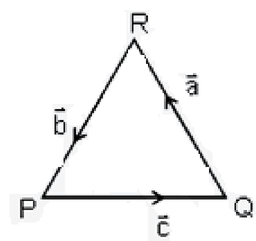
$\Rightarrow \frac{\pi}{2} \sin\left(\frac{\pi}{2}\left(\frac{\pi}{2} sinx\right)\right) = 1$

$\Rightarrow \sin\left(\frac{\pi}{6} \sin\left(\frac{\pi}{2} sinx\right)\right) = \frac{2}{\lambda} \cong \frac{2}{3.14} > \frac{1}{2}$ $\therefore$ (D)

Sol.54 (A,C,D)

$\vec{a} + \vec{b} + \vec{c} = 0$

$\Rightarrow \vec{b} + \vec{c} = -\vec{a}$



$\Rightarrow 48 + \vec{c}^2 + 48 = 144$

$\Rightarrow \vec{c}^2 = 48$

$\Rightarrow \frac{|\vec{c}|^2}{2} - |\overrightarrow{a}| = 24 - 12 = 12$ Ans (A)

Further

$\vec{a} + \vec{b} = - \vec{c}$

$\Rightarrow 144 + 48 + 2\vec{a}\vec{b} = 48$

$\Rightarrow \vec{a}\vec{b} = -72$ Ans (D)

$\because \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = 0$

$\therefore |\vec{a} \times \vec{b} + \vec{c} \times \vec{a}| = 2\,|\vec{a} \times \vec{b}| = 2.\sqrt{144.48 - (72)^2} = 48\sqrt{3}$ Ans. (C)

Sol.55 (C,D)

(A) $(Y^3Z^4 - Z^4 Y^3)^T = -Y^3Z^4 + Z^4Y^3$

$\Rightarrow Y^3Z^4 - Z^4Y^3$ is Skew-symmetric

(C) $(X^4Z^3 - Z^3X^4)^T = (X^4Z^3)^T(Z^3X^4)^T$

$= Z^3X^4 - X^4Z^3$

$= -(X^4Z^3-Z^3X^4)$

(D) $(X^{23} + Y^{23})^T = - X^{23} - Y^{23}$ $\Rightarrow X^{23} + Y^{23}$ is Skew – symmetric

Sol.56 (B,C)

$R_3 \to R_3 - R_2$

$R_2 \to R_2 - R_1$

$$\begin{vmatrix} (1+a)^2 & (1+2a)^2 & (1+3a)^2 \\ 3+2a & 3+4a & 3+6a \\ 5+2a & 5+4a & 5+6a \end{vmatrix} = -648a$$

$R_3 \to R_3 \to R_2$

$$\begin{vmatrix} (1+a)^2 & (1+2a)^2 & (1+3a)^2 \\ 3+2a & 3+4a & 3+6a \\ 2 & 2 & 2 \end{vmatrix} = -648a$$

$C_3 \to C_3 - C_2 \to C_2 - C_1$

$$\begin{vmatrix} (1+a)^2 & a(2+3a) & a(2+5a) \\ 3+2a & 2a & 2a \\ 2 & 0 & 0 \end{vmatrix} = 648a$$

$\Rightarrow 2a^2(2+3a)-2a^2(2+5a) = -324a$

$\Rightarrow -4a^3 = -324a$ $\Rightarrow a = 0, \pm 9$

Sol.57 (B,C)

$y + \lambda\,[x + z - 1] = 0$

$\left|\frac{1-\lambda}{\sqrt{\lambda^2+\lambda^2+1}}\right| = 1$

$1 + \lambda^2 - 2\lambda = 2\lambda^2 + 1$

$\lambda^2 + 2\lambda = 0$

$\lambda = 0 \,/\, \lambda = -2$

$2x + 2z - y - 2 = 0$

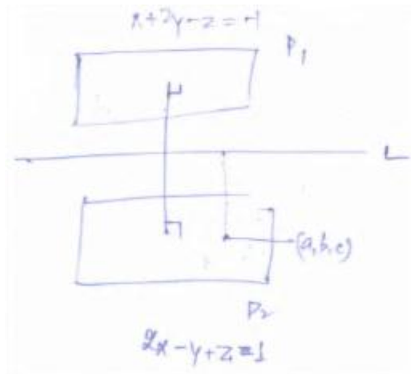
$\left|\frac{2\alpha-\beta+2\gamma-2}{\sqrt{2^2+2^2+1}}\right| = 2$

$2\alpha + 2\gamma - \beta - 2 = \pm 6$

$2\,\alpha - ß + 2\,\gamma = 8$

$2\,\alpha - ß + 2\,\gamma = -4$

Sol.58 (A), (B), (C), (D)



$$\frac{x}{a} = \frac{y}{b} = \frac{z}{c} = k$$

a + 2b – c = 0

2a – b + c = 0

3a + b = 0 $\qquad a = \frac{-b}{3}$

b = -3a

2a + 4b – 2c = 0

-2a + b – c = 0

5b – 3c = 0 $\qquad c = \frac{5b}{3}$

$\therefore \frac{x}{\frac{-b}{3}} = \frac{y}{b} = \frac{z}{\frac{5b}{3}} = k$

$\frac{x}{\frac{-1}{3}} - \frac{y}{1} = \frac{z}{\frac{5}{3}} = k'$

$\frac{x}{-1} = \frac{y}{3} = \frac{z}{5} = k$

2a – b + c = -1

Sol.59 (A→ P.Q)

(A) $\left|(\alpha\hat{\imath} + \beta\hat{\jmath}) \cdot \left(\frac{\sqrt{3}\hat{\imath} + \hat{\jmath}}{2}\right)\right| = \sqrt{3}$ $\qquad \Rightarrow \sqrt{3}\alpha + \beta = \pm 2\sqrt{3}$

$\sqrt{3}\,\alpha + \left(\frac{\alpha - 2}{\sqrt{3}}\right) = \pm 2\sqrt{3}$

$\Rightarrow 3\alpha + \alpha - 2 = \pm 6 \qquad \Rightarrow 4\alpha = 8, -4 \qquad \Rightarrow \alpha = 2, -1$

(B →P, Q)

Continuous ⇒ $-3a - 2 = b + a^2$

Differentiable ⇒ $-6a = b$ ⇒ $\qquad 6a = a^2 + 3a + 2$

$(C \to P, Q, S, T)$

Let $a = 3 - 3\omega + 2\omega^2$

$a\omega = 3\omega - 3\omega^2 + 2$

Now, $a^{4n+3}(1 + \omega^{4n+3} + (\omega^2)^{4n+3}) = 0$

$\Rightarrow$ n should not be a multiple of 3 Hence P,Q,S,T

$(D \to Q, T)$

$\frac{2ab}{a+b} = 4 \Rightarrow ab = 2a + 2b \ldots\ldots$ (i)

$q = 10 - a$ and $2q = 5 + b$

$\Rightarrow 20 - 2a = 5 + b \Rightarrow 15 = 2a + b \quad \ldots\ldots$ (II)

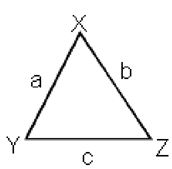
From (I) and (II) $a(15-2a) = 2a + 2(15-2a)$

$\Rightarrow 15a - 2a^2 = -2a + 30 \quad \Rightarrow \quad 2a^2 - 17a + 30 = 0 \quad \Rightarrow a = 6, \frac{5}{2}$

$\Rightarrow q = 4, \frac{15}{2} \quad \Rightarrow \quad |q - a| = 2, 5$

Sol.60 $(A \to P, R, S)$

(A)



Given $2(a^2 - b^2) = c^2$

$\Rightarrow 2(\sin^2 x - \sin^2 y) = \sin^2 z$

$\Rightarrow 2\sin(x + y)\sin(x - y) = \sin^2 z$

$\Rightarrow 2\sin(\pi - z)\sin(x - y) = \sin^2 z$

Z $\quad \Rightarrow \quad \sin(x - y) = \frac{\sin z}{2} \ldots\ldots$ (i)

Also given,

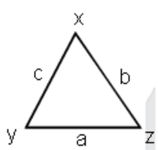
$\lambda = \frac{\sin(x-y)}{sin} = \frac{1}{2}$

Now, $\cos(n\pi\lambda) = 0$

$\Rightarrow \cos\left(\frac{n\pi}{2}\right) = 0$

$\because n = 1,3,5$ $\therefore (A \rightarrow P,R,S)$

$(B \rightarrow P)$



$1 + \cos 2x - 2\cos 2y = 2\sin x \sin y$

$2\cos^2 x - 2\cos 2y = 2\sin X \sin y$

$1 - \sin^2 x - 1 + 2\sin^2 y = \sin x \sin y$

$\text{Sin}^2 x + \sin x \sin y = 2\sin^2 y$

$\text{Sin} x(\sin x + \sin y) = 2\sin^2 y \quad \sin x = ak, \sin y = bk$

$a^2 + ab - 2b^2 = 0$

$\left(\frac{a}{b}\right)^2 + \frac{a}{b} - 2 = 0$

$\frac{a}{b} = -2, 1$

$\frac{a}{b} = 1$ $(B \rightarrow P)$

$(C \rightarrow P,Q)$

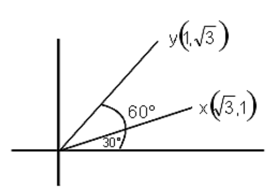
Hence equation of acute angle bisector of OX and OY is $y = x$

Hence $x - y = 0$

Now, distance of $\beta\hat{\imath} + (1 - \beta)\hat{\jmath} \equiv z\ (\beta, 1 - \beta)$ from $x - y$ is $\left|\frac{\beta-(1-\beta)}{\sqrt{2}}\right| = \frac{3}{\sqrt{2}}$

$|2\beta - 1| = 3$

$2\beta - 1 = \pm 3$



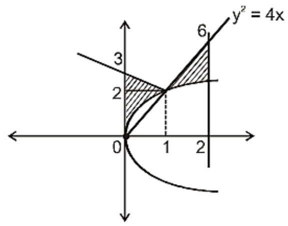
$2\beta = 4, -2$

$|\beta| = 2, 1$ Ans. (P,Q)

For α 1

$$Y = |x-1| + |x-2| + x = \begin{cases} 3-x \; ; \; x<1 \\ 1+x \; ; \; 1 \le x < 2 \\ 3x-3 \; ; \; x \ge 2 \end{cases}$$

$A = \frac{1}{2}$

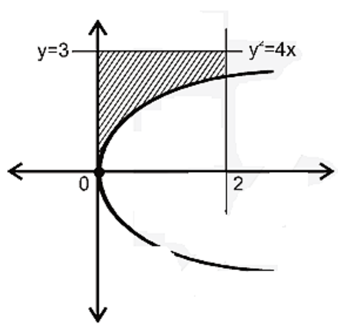


$(2+3) \times 1 + \frac{1}{2}(2+3) \times 1 - \int_0^2 2\sqrt{x\,dx}$

$A = 5 - \frac{8}{3}\sqrt{2}$

$\therefore F(1) + \frac{8}{3}\sqrt{2} = 5$

For $\alpha = 0$, $y = |-1| + |-2| = 3$



$A = 6 - \int_0^2 2\sqrt{x\,dx}$ $\Rightarrow A = 6 - \frac{8}{3}\sqrt{2}$

$\therefore F(0) + \frac{8}{3}\sqrt{2} = 6$

$\therefore (D \rightarrow S, T)$

Paper-1

JEE Advanced, 2015

Part II: Chemistry

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

Read the instructions carefully:

General:

1. This sealed booklet is your Question Paper. Do not break the seal till you are instructed to do so.
2. The question paper CODE is printed on the left hand top corner of this sheet and the right hand top corner of the back cover of this booklet.
3. Use the Optical Response Sheet (ORS) provided separately for answering the questions.
4. The ORS CODE is printed on its left part as well as the right part. Ensure that both these codes are identical and same as that on the question paper booklet. If not, contact the invigilator.
5. Blank spaces are provide within this booklet for rough work.
6. Write your name and roll number in the space provided on the back cover of this booklet.
7. After breaking the seal of the booklet. Verify that the booklet contains 32 pages and that all the 60 questions along with the options are legible.

Question paper format and marking scheme :

8. The question paper has three parts: Physics, Chemistry and Mathematics. Each part has three sections.
9. Carefully read the instructions given at the beginning of each section.
10. Section 1 contains 8 questions. The answer to each question is a single digit integer ranging from 0 to 9 (both inclusive).

    Marking scheme: +4 correct answer and 0 in all other cases.

11. Section 2 contains 10 multiple choice questions with one or more than one correct option.

    Marking scheme: +4 for correct answer, 0 if not attempted and -2 in all other cases.

12. Section 3 contains 2 " match the following" type questions and you will have to match entries in Column I with the entries in Column II.

Marking scheme: for each entry in Column I, +2 for correct answer, 0 if not attempted and -1 in all other cases.

OPTICAL RESPONSE SHEET :

13. The ORS consists of an original (top sheet) and its carbon-less copy. (bottom sheet).

14. Darken the appropriate bubbles on the original by applying sufficient pressure. This will leave an impression at the corresponding place on the carbon-less copy.

15. The original is machine-gradable and will be collected by the invigilator at the end of the examination.

16. You will be allowed to take away the carbon-less copy at the end of the examination.

17. Do not tamper with or mutilate the ORS.

18. Write your name, roll number and the name of the examination center and sign with pen in the space provided for this purpose on the original. Do not write any of these details anywhere else. Darken the appropriate bubble under each digit of your roll number.

Note: Answers have been highlighted in "Yellow" color and Explanations to answers are given at the end

SECTION 1 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- The answer to each question is a **SINGLE DIGIT INTEGER** ranging from 0 to 9, both inclusive
- For each question, darken the bubble corresponding to the correct integer in the ORS
- Marking scheme:
  +4 If the bubble corresponding to the answer is darkened
  0 In all other cases

Q.21 If the freezing point of a 0.01 molal aqueous solution of a cobalt (III) chloride – ammonia complex (which behaves as a strong electrolyte) is – 0.0558 °C, the number of chloride(s) in the coordination sphere of the complex is

[K_f of water = 1.86 K kg mol^{-1}]

Ans..21 (1)

Q.22 All the energy released from the reaction $X \rightarrow Y$, $\Delta_r G° = -193$ KJ mol^{-1} is used for oxidizing M^+ as $M^+ \rightarrow M^{3+} + 2e^-$, $E° = -0.25V$.

Under standard conditions, the number of moles of M^+ oxidized when one mole of X is converted to Y is

[$F = 96500$ C mol^{-1}]

Ans..22 (4)

Q.23 For the octahedral complexes of Fe^{3+} in SCN^- (thiocyanato) and in CN^- ligand environments, the difference between the spin-only magnetic moments in Bohr magnetons (when approximated to the nearest integer) is

[Atomic number of Fe = 26]

Ans..23 (4)

Q.24 The total number of lone pairs of electrons in N_2O_3 is

Ans..24 (8)

Q.25 Among the triatomic molecules/ions, BeCl2, N_3^-, N_2O, NO_2^+ O_3, SCl_2, ICl_2^-, I_3^- and XeF_2, the total number of linear molecule(s)/ion(s) where the hybridization of the central atom does not have contribution from the d-orbital(s) is

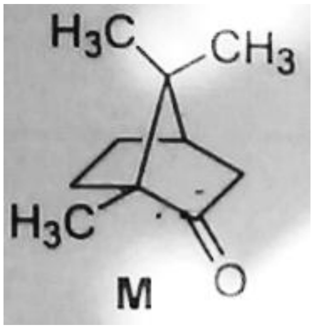
[Atomic number: S = 16, Cl = 17, I = 53 and Xe = 54]

Ans..25 (3)

Q.26 Not considering the electronic spin, the degeneracy of the second excited state (n = 3) of H atom is 9, while the degeneracy of the second excited state of H^- is

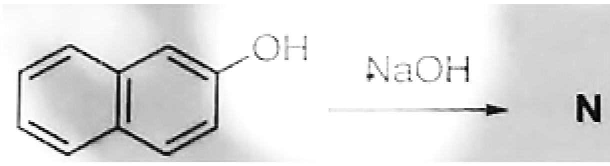
Ans..26 (3)

Q.27 The total number of stereoisomers that can exist for M is



Ans..27 (2)

Q.28 The number of resonance structures for N is



Ans.28 (9)

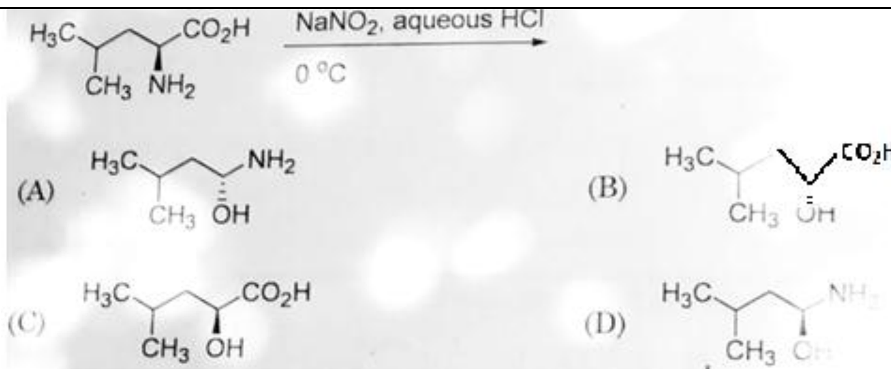
SECTION 2 (Maximum Marks: 40)

- This section contains TEN questions
- Each question has **FOUR** options (A), (B), (C) and (D). ONE OR MORE THAN ONE of these four option(s) is(are) correct.
- For each question, darken the bubble(s) corresponding to all the correct option(s) in the ORS
- Marking scheme:

  +4 if only the bubble(s) corresponding to all the correct option(s) is(are) darkened

  0 If none of the bubbles is darkened

  -2 In all other cases



Ans.29 (B)

Q.30 The correct statement(s) about Cr^{2+} and Mn^{3+} is(are)

[Atomic number of Cr = 24 and Mn = 25]

(A) Cr^{2+} is a reducing agent

(B) Mn^{3+} is an oxidizing agent

(C) Both Cr^{2+} and Mn^{3+} exhibit d^4 electronic configuration

(D) When Cr^{2+} is used as a reducing agent, the chromium ion attains d^5 electronic configuration

Ans.30 (A,B,C)

Q.31 Copper is purified by electrolytic refining of blister copper, The correct statement(s) about this process is (are)

(A) Impure Cu strip is used as cathode

(B) Acidified aqueous $CuSO_4$ is used as electrolyte

(C) Pure copper deposits at cathode

(D) Impurities settle as anode-mud

Ans.31 (B,C,D)

Q.32 Fe^{3+} is reduced to Fe^{2+} by using

(A) H_2O_2 in presence of NaOH (B) Na_2O_2 in water

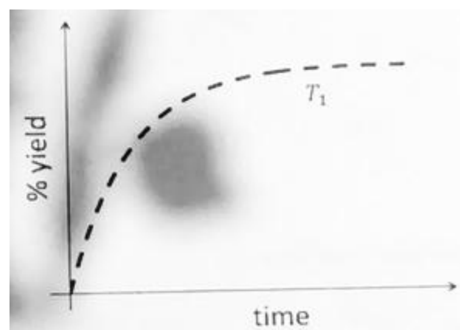
(C) H_2O_2 in presence of H_2SO_4 (D) Na_2O_2 presence of H_2SO_4

Ans.32 (A,B)

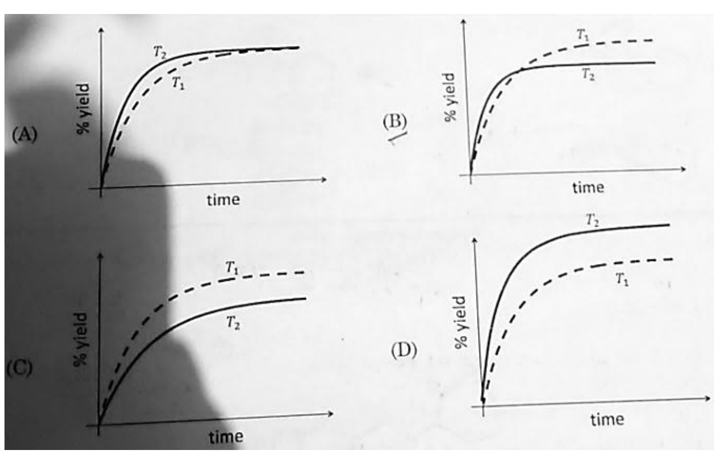
Q.33 The %yield of ammonia as a function of time in the reaction

$N_2(g) + 3H_2(g) \leftrightharpoons 2NH_3(g)$, $\Delta H < 0$

At (P, T_1) is given below.



If this reaction is conducted at (P,T_2), with $T_2 > T_1$, the %yield of ammonia as a function of time is represented by



Ans.33 (C)

Q.34 If the unit cell of a mineral has cubic close packed (ccp) array of oxygen atoms with m fraction of octahedral holes occupied by aluminium ions and n fraction of tetrahedral holes occupied by magnesium ions, m and n, respectively, are

(A) $\frac{1}{2}, \frac{1}{8}$ (B) $1, \frac{1}{4}$

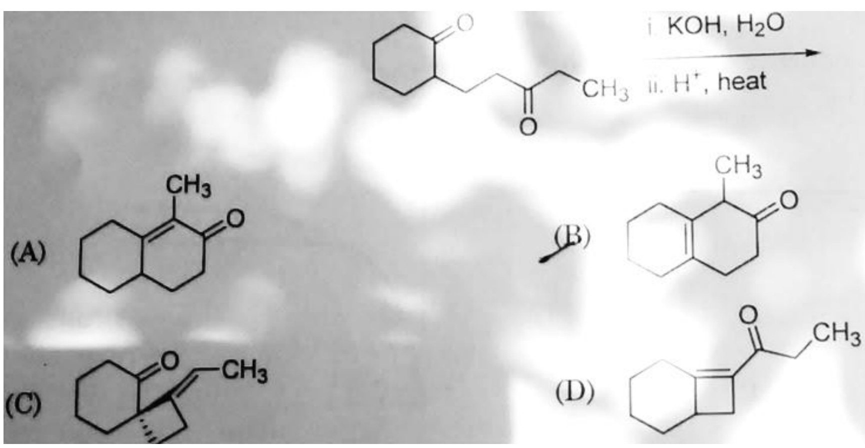
(C) $\frac{1}{2}, \frac{1}{2}$ (D) $\frac{1}{4}, \frac{1}{8}$

Ans.34 (A)

Q.35 Compound(s) that on hydrogenation produce(s) optically inactive compound(s) is(are)

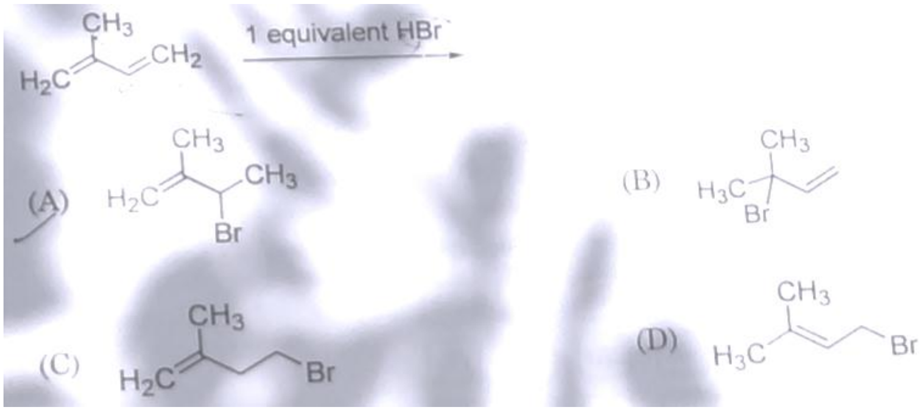
Ans.35 (B,D)

Q.36 The major product of the following reaction is



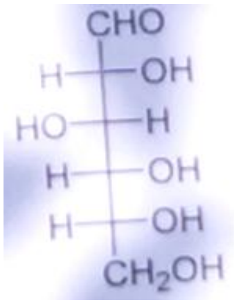
Ans.36 (B)

Q.37 In the following reaction, the major product is

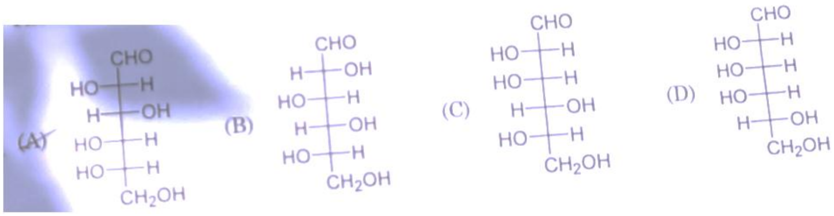


Ans.37 (B)

Q.38 The structure of D.(+)-glucose is



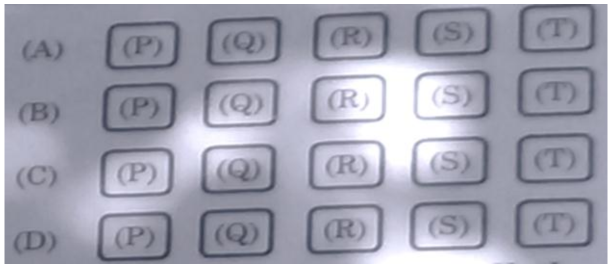
The structure of L-(-)-glucose is



Ans.38 (A)

SECTION 3 (Maximum Marks: 16)

- This section contains **TWO** questions
- Each question contains two columns, **Column I and Column II**
- Column I has four entries (A), (B), (C) and (D)
- Column II has five entries (P), (Q), (R), (S) and (T)
- Match the entries in **Column I** with the entries in **Column II**
- One or more entries in **Column I** may match will one or more entries in **Column II**
- The ORS Contains a 4 x 5 matrix whose layout will be similar to the one shown below:



- For each entry in **Column I**, darken the bubbles of all the matching entries. For example, if entry (A) in Column **I matches** with entries (Q), (R) and (T), then darken these three bubbles in the ORS. Similarly, for entries (B), (C) and (D).
- Marking scheme:

  For each entry in **Column I**

  +2 if only the bubble(s) corresponding to all the correct match(es) is(are) darkened

  0 If none of the bubbles is darkened

  -1 In all other cases

Q.39 Match the anionic species given in Column I that are present in the ore(s) given in Column II.

Column I	**Column II**
(A) Carbonate	(P) Siderite
(B) Sulphide	(Q) Malachite
(C) Hydroxide	(R) Bauxite
(D) Oxide	(S) Calamine

Ans.39 (A-P, Q), (B-T),(C-Q, R) and (D-R,S)

Q.40 Match the thermodynamic processes given Column I with the expressions given under Column II.

Column I	**Column II**
(A) Expansion of water at 273 K and 1 atm	(P) $q = 0$
(B) Expansion of 1 mol of an ideal gas into a Vacuum under isolated conditions	(Q) $w = 0$

(C) Mixing of equal volumes of two ideal gases at Constant temperature and pressure in an isolated container — (R) $\Delta S_{sys} < 0$

(D) Reversible heating of H_2 (g) at 1 atm from 300 K to 600 K, followed by reversible cooling to 300 K at 1 atm — (S) $\Delta U = 0$

(T) $\Delta U = 0$

Ans.40 (A) → Q, R, T

(B) → P, Q, S

(C) → P, Q, S

(D) → P, Q, S, T

Answer Keys and Explanations

Sol.21 (1)

Given that the complex is a strong electrolyte, therefore, it dissociates completely.

According to vant – hoff's equation,

$\Delta T_f = k_f \times m \times i$

$\Delta T_f = 0.0558°C = 0.0558\ K$

$\therefore 0.0558\ K = 1.86\ k\ kg\ mol^{-1} \times .01m \times i$

$i = 3$

$\therefore$ the no. of ions after dissociation = 3

Let the no. of chloride ions outside the coordination sphere is x

$[co\ (NH_3)\ cl_y]\ cl_x \rightarrow xcl^- + [co\ (NH_3)\ cl_y]$

$i = \frac{\text{no.of ions ofter dissociation}}{\text{no.of ions before dissociation}}$

$3 = \frac{x+1}{1}$

$x = 2$

$\therefore$ no. of chloride ions outside the sphere = 2

$\therefore$ To balance the charge on the complex, 1 chloride ion has to go inside the sphere

$\therefore$ Ans = 1

Sol.22 (4)

For the reaction

$M^+ \rightarrow M^{3+} + 2e^-$, E° = - 0 .25V

ΔG° for the cell

ΔG° = - nf E° cell

$= + 2 \times 96500\ cmol^{-1} \times 0.25V$

$= 48250\ J\ mol^{-1}$

The ΔG° we are providing by the reaction X → Y

$\Delta G^\circ_{rxn} = -193000 \text{ J mol}^{-1}$

$\therefore$ no. of moles of M^+ oxidised to M^{3+} is

$\frac{-193000\ J\ mol^{-1}}{-48250\ J\ mol^{-1}}$

$= 4$

Sol.23 (4)

Fe^{3+} electronic conf. is $3d^5$

SCN^- is a weak field ligand → no pairing

CN^- is strong field ligand → causes pairing

for CN^- | 1⇂ | 1⇂ | 1 | | | ______ case I

for SCN^- | 1 | 1 | 1 | 1 | 1 | ______ case II

magnetic moment for case I

$\sqrt{n\,(n+2)} = \sqrt{1\,(1+2)} = \sqrt{3}$

For case II

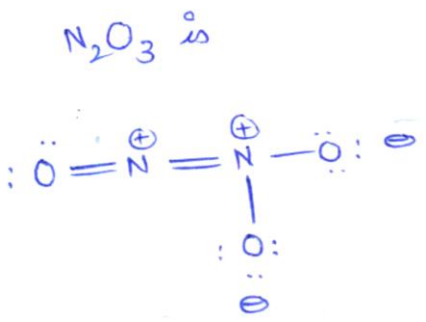
$\sqrt{5(5+2)} = \sqrt{35}$

Diff. in magnetic moment = 4.184

≈ 4

Sol.24 (8)

N_2O_3 is



No. of lone pairs is 8

Sol.25 (3)

Be $Cl_2 \rightarrow \frac{1}{2}$ $(2+2) = \frac{4}{2} = 2 \rightarrow$ sp $\rightarrow$ linear.

$N_3^- \rightarrow$ azide $\rightarrow$ Linear

$N_2O \rightarrow$ Linear

$NO_2^+ \rightarrow$ non – Linear

$O_3 \rightarrow$ non – linear

S $Cl_2 \rightarrow$ Bent – v

$ICl_2^-, I_3^-, Xef_2 \rightarrow$ all have d orbits in central atom

Sol.26 (3)

$$\text{Degeneracy} = \sum_{l=0}^{n+1}(2L+1)$$

For H atom

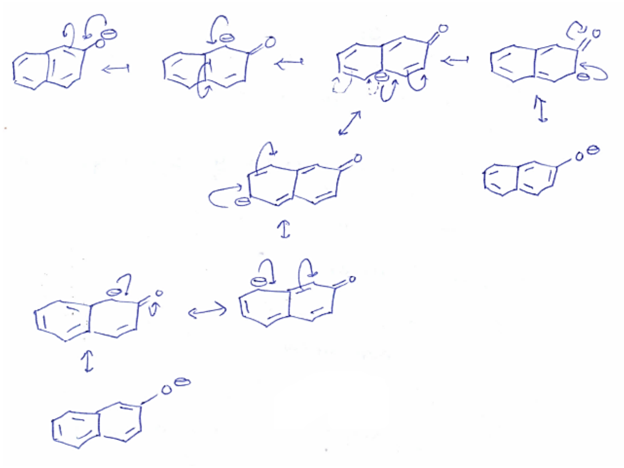
For H^- ion , degeneracy = 3 (for second excited state).

Sol.27 (2)

no. of chiral centres = 1

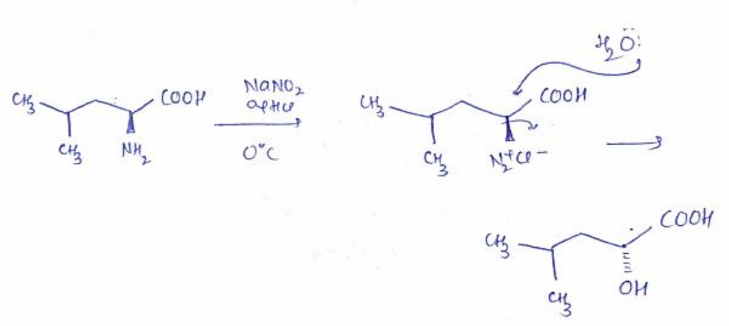
$\therefore$ Ans = $2^n = 2^1 = 2$

Sol.28 (9)



Ans = 9

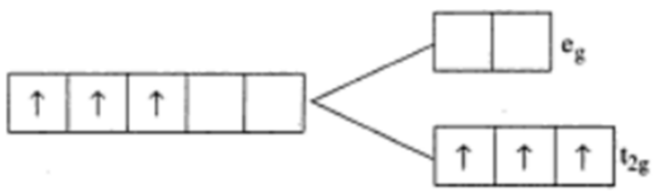
Sol.29 (B)



Ans (b)

Sol.30 (A,B,C)

Cr^{2+} is strongly reducing is nature. It has a d^4 configuration. While acting as a reducing agent, it gets oxidized to Cr^{3+} (electronic configuration, d^3). This d^3 configuration can be written as t^3_{2g} configuration, which is a more stable configuration,



In the case of Mn^{3+} (d^4) it acts as an oxidizing agent and gets reduced to Mn^{2+} (d^5). This has an exactly half – filled d-orbital and has an extra – stability.

Sol.31 (B,C,D)

Process of electrolytic refining.

Sol.32 (A,B)

Fe^{3+} can be reduced by using either Na_2O_2 or H_2O_2. While using H_2O_2, the presence of basic medium is must.

Hence, Ans. (A), (B)

Sol.33 (C)

Since the rxn is exothermic,

∴ it will be favored at low temp.

On increasing the temp, the rate of the rxn decreases.

∴ at every point of time,

The % yield at temp $T_2 < $ % yield at T_1

The graph (c) explains the answer.

Sol.34 (A)

$O^{2-} \rightarrow$ ccp (4)

$Al^{3+} \rightarrow$ octahedral void (4)

$Mg^{2+} \rightarrow$ tetrahedral void (8)

Considering the mineral of Al, O and Mg as

$MgAl_2O_4$

$O^{2-} \rightarrow 4$

$Al^{3+} \rightarrow m = \frac{2}{4} = \frac{1}{2}$

$Mg^{2+} \rightarrow n = \frac{1}{8}$

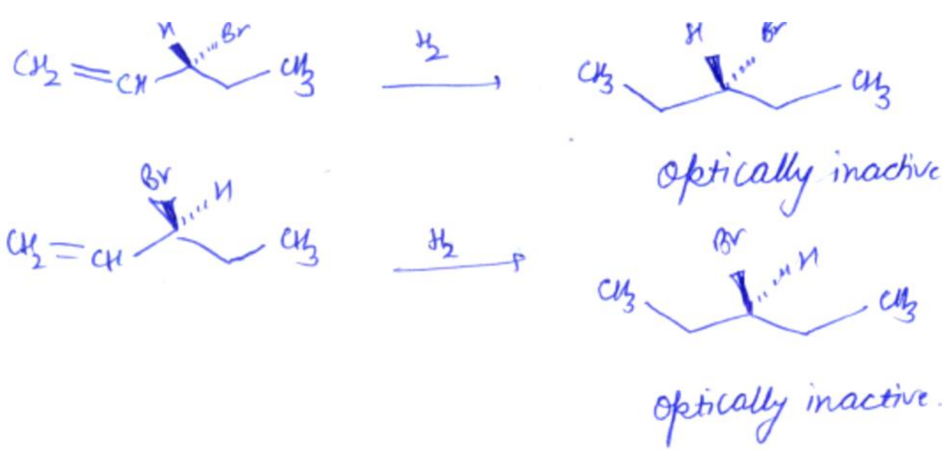
∴ the m and n values are

$\frac{1}{2}, \frac{1}{8}$

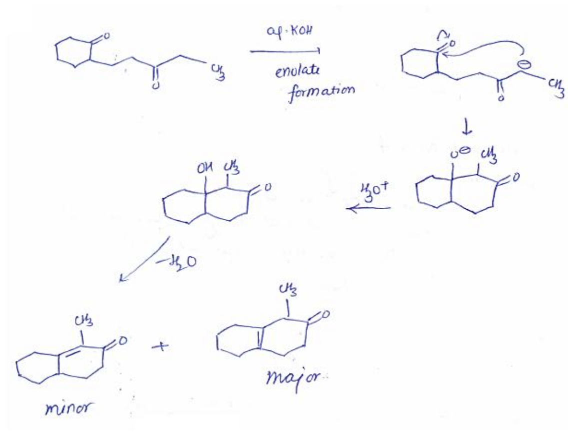
Ans (A)

Sol.35 (B,D)

The reactions give optically inactive products



Sol.36 (B)

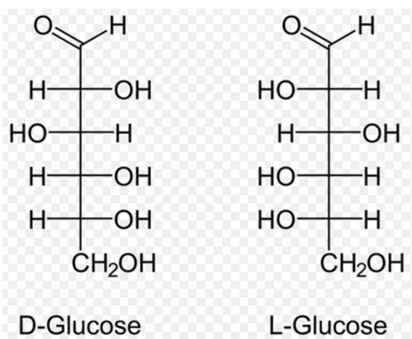


Ans (B)

Sol.37 (B)

Tertiary carbocation will be more stable.

Sol.38 (A)



Sol.39 (A-P,Q), (B-T),(C-Q,R) and (D-R,S)

The composition of the ores is:

(Feco$_3$)
(P) Siderite → carbonate.

(Q) Malachite → carbonate, hydroxide

(R) Bauxite (Al $(OH)_3$), γ – Al O (OH), α – Al O(OH)
↓
hydroxide, oxide.

(S) calamine (ZnO + ferric oxide) → oxide.

(T) Argentite → Ag_2S → sulphide

$\therefore$ Ans is $A \rightarrow P, Q$

$B \rightarrow T$

$C \rightarrow Q, R$

$D \rightarrow R, S.$

Sol.40 (A) $\rightarrow$ R, T, Q

during the phase change,

work done $= w = 0$

change in internal energy is less than O

& since the rxn. occurs at equilibrium,

$$\Delta G = O$$

(B) $\rightarrow$ P, Q, S.

(C) $\rightarrow$ P, Q, S

During the mixing of equal volumes of two ideal gases at constant T and P in an isolated container, heat change taking place is O,

Work done is zero

& $\Delta U = 0$

(D) $\rightarrow$ P, Q, S, T.