JEE ADVANCED (Paper - 1)
MATHEMATICS

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 in all other cases.

  Negative Marks : –1 in all other cases.

37. A computer producing factory has only two plants T_1 and T_2. Plant T_1 produces 20% and plant T_2 produces 80% of the total computers produced. 7% of computers produced in the factory turn out to be defective. It is known that

P (computer turns out to be defective given that it is produced in plant T_1)

= 10 P (computer turns out to be defective given that it is produced in plant T_2) ,

where P(E) denotes the probability of an event E. A computer produced in the factory is randomly selected and it does not turn out to be defective. Then the probability that it is produced in plant T_2 is

(A) $\frac{36}{73}$ (B) $\frac{47}{79}$ (C) $\frac{78}{93}$ (D) $\frac{75}{83}$

Key. (C)

Sol: Let $E_1 \Rightarrow$ computers produced in plant T_1

$E_2 \Rightarrow$ Computer produced in plant T_2

$A \Rightarrow$ Computer is non defective

We have to find out $P(E_2 \mid A) = \dfrac{P(E_2)P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$

Now it is given that 7% computers are defective.

Let P be the probability that computer produced in plant T_2 is defective then 10 p will be the probability that computer produced in plant T_1 is defective

So $10p \times \frac{20}{100} + p \times \frac{80}{100} = \frac{7}{100} \Rightarrow P = \frac{1}{40}$

So, $P(A \mid E_1) = 1 - \frac{10}{40} = \frac{3}{4}, P(A \mid E_2) = 1 - \frac{1}{40} = \frac{39}{40}$

$$P(E_2 \mid A) = \frac{\frac{4}{5} \times \frac{39}{40}}{\frac{4}{5} \times \frac{39}{40} + \frac{1}{5} \times \frac{3}{4}} = \frac{156}{186} = \frac{26}{31} = \frac{78}{93}$$

38. A debate club consists of 6 girls and 4 boys. A team of 4 members is to be selected from this club including the selection of a captain (from among these 4 members) for the team. If the team has to include at most one boy, then the number of ways of selecting the team is

(A) 380 (B) 320 (C) 260 (D) 95

Key. (A)

Sol: Total number of ways

$$= \left(6_{C_4} \times 4_{C_0} + 6_{C_3} \times 4_{C_1}\right) \times 4_{C_1}$$

$$= (15 \times 1 + 20 \times 4) \times 4$$

$$= 380$$

39. Let $-\frac{\pi}{6} < \theta < -\frac{\pi}{12}$. Suppose α_1 and β_1 are the roots of the equation $x^2 - 2x \sec\theta + 1 = 0$ and α_2 and β_2 are the roots of the equation $x^2 + 2x \tan\theta - 1 = 0$. If $\alpha_1 > \beta_1$ and $\alpha_2 > \beta_2$, then $\alpha_1 + \beta_2$ equals

(A) $2(\sec\theta - \tan\theta)$ (B) $2\sec\theta$ (C) $-2\tan\theta$ (D) 0

Key. (C)

Sol. $x^2 - 2x\sec\theta + 1 = 0$

$$x = \frac{2\sec\theta \pm \sqrt{4\sec^2\theta - 4}}{2}$$

$$= \sec\theta \pm \tan\theta$$

$$\alpha_1 = \sec\theta - \tan\theta \left(\text{as } \alpha_1 > \beta_1\right)$$

$$x^2 + 2x\tan\theta - 1 = 0$$

$$x = \frac{-2\tan\theta \pm \sqrt{4\tan^2\theta + 4}}{2}$$

$$= -\tan\theta \pm \sec\theta$$

$$\beta_2 = -\tan\theta - \sec\theta \left(\text{as } \alpha_2 > \beta_2\right)$$

So, $\alpha_1 + \beta_2 = -2\tan\theta$

40. Let $S = \left\{x \in (-\pi, \pi) : x \neq 0, \pm\frac{\pi}{2}\right\}$. The sum of all distinct solutions of the equation $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$ in the set S is equal to

(A) $-\frac{7\pi}{9}$ (B) $-\frac{2\pi}{9}$ (C) 0 (D) $\frac{5\pi}{9}$

Key. (C)

Sol. $\sqrt{3}\sec x + \operatorname{cosec} x + 2(\tan x - \cot x) = 0$

$$\frac{\sqrt{3}}{2}\sin x + \frac{\cos x}{2} = \cos^2 x - \sin^2 x$$

$$\cos\left(x - \frac{\pi}{3}\right) = \cos 2x$$

$$\cos 2x - \cos\left(x - \frac{\pi}{3}\right) = 0$$

$$-2\sin\left(\frac{3x - \frac{\pi}{3}}{2}\right) \times \sin\left(\frac{2x - x + \frac{\pi}{3}}{2}\right) = 0$$

(A) $\dfrac{3x - \frac{\pi}{3}}{2} = n\pi,$ $\qquad 3x = 2n\pi + \dfrac{\pi}{3}$

$x = \dfrac{\pi}{9}, \dfrac{7\pi}{9}, \dfrac{-5\pi}{9}$

(B) $\dfrac{x + \frac{\pi}{3}}{2} = n\pi$

$x = 2n\pi - \dfrac{\pi}{3}$

$x = \dfrac{-\pi}{3}$

Sum of distinct roots from A and B = 0

41. The least value of $\alpha \in$ ¡ for which $4\alpha x^2 + \dfrac{1}{x} \ge 1,$ for all $x > 0$ is

(A) $\dfrac{1}{64}$ (B) $\dfrac{1}{32}$ (C) $\dfrac{1}{27}$ (D) $\dfrac{1}{25}$

Key. (C)

Sol: $f(x) = 4ax^2 + \dfrac{1}{x}$

$f'(x) = 8ax - \dfrac{1}{x^2} = 0$

$x = \dfrac{1}{2}(a)^{-\frac{1}{3}}$

Now f” (x) = 8a + $\dfrac{2}{x^3}$

Which is positive for x > 0 and for positive a (as a cannot be negative otherwise f(x) ≥ 1, is not for all x > 0)

Minimum value of f(x) should be greater than or equal to 1

$f\left(\dfrac{1}{2a^{\frac{1}{3}}}\right) \ge 1$

$4a\dfrac{1}{4} \times \dfrac{1}{a^{\frac{2}{3}}} + 2a^{\frac{1}{3}} \ge 1$

$3a^{\frac{1}{3}} \ge 1$

$a \ge \dfrac{1}{27}$

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +4 if only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

  Zero Marks : 0 in all other cases.

  Negative Marks : –2 in all other cases.

- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks, and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

42. A solution curve of the differential equation $(x^2 + xy + 4x + 2y + 4)\frac{dy}{dx} - y^2 = 0$, $x > 0$, passes through the point (1, 3). Then the solution curve

(A) intersects $y = x + 2$ exactly at one point

(B) intersects $y = x + 2$ exactly at two points

(C) intersects $y = (x+2)^2$

(D) does **NOT** intersect $y = (x + 3)^2$

Sol. (A, D)

$$[(x+2)^2 + y(x+2)]\frac{dy}{dx} = y^2$$

$$y^2dx = (x + 2)^2dy + y(x + 2)dy$$

$$y[ydx - (x + 2)dy] = (x + 2)^2dy$$

$$-d\left(\frac{y}{x+2}\right) = \frac{dy}{y}$$

$$\log y = \frac{-y}{x+2} + \log k$$

This curve passes through (1, 3)

Hence, $k = 3e$

So, solution curve of given D.E. is

$$\log y = -\frac{y}{x+2} + \log 3e \qquad \text{... (i)}$$

For $y = x + 2$ (i), cut at only one point and (i) does not cut

$y = (x+2)^2$ and $y = (x+3)^2$

43. Consider a pyramid OPQRS located in the first octant ($x \ge 0$, $y \ge 0$, $z \ge 0$) with O as origin, and OP and OR along the x-axis and the y-axis, respectively. The base OPQR of the pyramid is a square with OP = 3. The point S is directly above the mid-point T of diagonal OQ such that TS = 3. Then

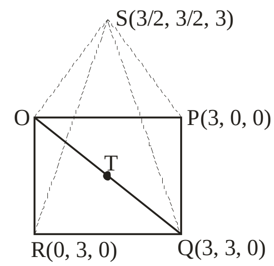
(A) the acute angle between OQ and OS is $\frac{\pi}{3}$

(B) the equation of the plane containing the triangle OQS is $x - y = 0$

(C) the length of the perpendicular from P to the plane containing the triangle OQS is $\frac{3}{\sqrt{2}}$

(D) the perpendicular distance from O to the straight line containing RS is $\sqrt{\frac{15}{2}}$

Sol. (B, C, D)



$T = \left(\frac{3}{2}, \frac{3}{2}, 0\right)$, $S = \left(\frac{3}{2}, \frac{3}{2}, 3\right)$

Let θ be the angle between OQ and OS, then

$$\tan\theta = \frac{ST}{OT} = \sqrt{2}$$

$$\theta \ne \frac{\pi}{3}$$

Equation of plane passing through O, Q and S is given by $\begin{vmatrix} x & y & z \\ 1 & 1 & 2 \\ 1 & 1 & 0 \end{vmatrix} = 0$ and is

$x - y = 0$ ... (i)

Perpendicular distance of (i), from P is $\frac{3}{\sqrt{2}}$.

Foot of perpendicular from (0, 0, 0) to line RS is given by $\left(\frac{1}{2}, \frac{5}{2}, 1\right)$, hence perpendicular distance is $\sqrt{\frac{15}{2}}$.

44. The circle $C_1 : x^2 + y^2 = 3$, with centre at O, intersects the parabola $x^2 = 2y$ at the point P in the first quadrant. Let the tangent to the circle C_1 at P touches other two circles C_2 and C_3 at R_2 and R_3, respectively. Suppose C_2 and C_3 have equal radii $2\sqrt{3}$ and centres Q_2 and Q_3, respectively. If Q_2 and Q_3 lie on the y-axis, then

(A) $Q_2Q_3 = 12$ (B) $R_2R_3 = 4\sqrt{6}$

(C) area of the triangle OR_2R_3 is $6\sqrt{2}$ (D) area of the triangle PQ_2Q_3 is $4\sqrt{2}$

Sol. (A, B, C)

For point P,

Solve parabola $x^2 = 2y$ and $x^2 + y^2 = 3$, we get value of $P(\sqrt{2}, 1)$

So equation of tangent at P is

$$\sqrt{2}x + y = 3 \qquad \text{... (i)}$$

(i) also touches $x^2 + (y - k)^2 = 12$

Hence, $k = -3, 9$

Put value in $x^2 + (y - k)^2 = 12$

$\Rightarrow 3y^2 - 4ky - 6y + 2k^2 - 15 = 0$

For tangency D = 0

$k^2 - 6k - 27 = 0$

$k = -3, 9$

So, circle C_2 and C_3 are

$x^2 + (y + 3)^2 = 12$ ... (ii)

$x^2 + (y - 9)^2 = 12$ ... (iii)

$\Rightarrow Q_2 = (0, -3)$

$Q_3 = (0, 9)$

For R_2 and R_3, solving (i), (ii) and (iii), we get $R_2(2\sqrt{2}, -1)$, $R_3(-2\sqrt{2}, 7)$

45. Let $f : R \to R$, $g : R \to R$ and $h : R \to R$ be differentiable functions such that $f(x) = x^3 + 3x + 2$, $g(f(x)) = x$ and $h(g(g(x))) = x$ for all $x \in R$. Then

(A) $g'(2) = \frac{1}{15}$ (B) $h'(1) = 666$ (C) $h(0) = 16$ (D) $h(g(3)) = 36$

Sol. (B, C)

$f : R \to R$

$f(x) = x^3 + 3x + 2$

f is invertible.

Since $g(f(x)) = x$

$\Rightarrow f(x) = g^{-1}(x)$ or $f^{-1}(x) = g(x)$

Since $g(f(x)) = x$

$\Rightarrow g'(f(x)).f^1(x) = 1$

Put x = 0

$g'(f(0)).f'(0) = 1$

$g'(2).3 = 1$

$g'(2) = \frac{1}{3}$

Also, $h(g(g(x))) = x$

$h'(g(g(x))g'(g(x))g'(x) = 1$

When x = 236

$h'(1)g'(6).g'(236) = 1$

$h'(1).\frac{1}{6}\times\frac{1}{111} = 1$

$h'(1) = 666$

and $h(g(g(x)) = x$

For x = 16 $\Rightarrow$ h(0) = 16; For x = 38 $\Rightarrow$ h(g(3)) = 38

46. Let $P=\begin{bmatrix}3 & -1 & -2\\ 2 & 0 & \alpha\\ 3 & -5 & 0\end{bmatrix}$, where $\alpha\in R$, Suppose $Q=\left[q_{ij}\right]$ is a matrix such that $PQ=kI$, where $k\in R, k\neq 0$ and I is the identity matrix of order 3. If $q_{23}=-\frac{k}{8}$ and det $\left(Q\right)=\frac{k^2}{2}$, then

(A) $\alpha=0, k=8$

(B) $4\alpha-k+8=0$

(C) $\det(P\,adj(Q))=2^9$

(D) $\det\left(Q\ adj\left(P\right)\right)=2^{13}$

Key. (B,C)

Sol. Q= $\begin{bmatrix}q_{11} & q_{12} & q_{13}\\ q_{21} & q_{22} & q_{23}\\ q_{31} & q_{32} & q_{33}\end{bmatrix}$; $PQ=kI$

Last column comparison

$3q_{13}-q_{23}-2q_{33}=0 \qquad (i)$

$2q_{13}+2q_{33}=0 \qquad (ii)$

$3q_{13}-5q_{23}=k \qquad (iii)$

Given $q_{23}=\frac{-k}{8}$

$\alpha=-1 \qquad (iv)$

Now given $PQ=kI$

$\Rightarrow |P||Q|=|kI|$

$$\left(12\alpha+20\right)\frac{k^2}{2}=k^3$$

$\Rightarrow k=4$ using EQ. (iv)

$-4-4+8=0$

(C) det $\left(Padj\left(Q\right)\right)=|P||adj\,Q|$

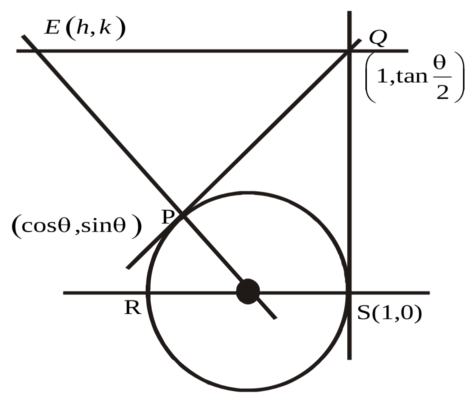
$$=\left(12\alpha+20\right)|Q|^2=\left(8\right)\left(\frac{k^2}{2}\right)^2=2^3.2^6=2^9$$

47. Let RS be the diameter of the circle $x^2+y^2=1$, where S is the point (1, 0). Let P be a variable point (other than R and S) on the circle and tangents to the circle at S and P meet at the point Q. The normal to the circle at P intersects a line drawn through Q parallel to RS at point E. Then the locus of E passes through the point(s)

(A) $\left(\frac{1}{3},\frac{1}{\sqrt{3}}\right)$ (B) $\left(\frac{1}{4},\frac{1}{2}\right)$ (C) $\left(\frac{1}{3},-\frac{1}{\sqrt{3}}\right)$ (D) $\left(\frac{1}{4},-\frac{1}{2}\right)$

Key. (A,C)

Sol.



tangent at P $x\cos\theta+y\sin\theta=1$

Tangent at 'S' is $x=1$

$\therefore$ then Q (intersection of tangent at P and S)

$\equiv\left(1,\tan\frac{\theta}{2}\right)$

Let E(h,k)

So, k = $\frac{1-\cos\theta}{\sin\theta}=\tan\frac{\theta}{2}$

K = $h\tan\theta \quad =h\left(\frac{2\tan\frac{\theta}{2}}{1-\tan^2\frac{\theta}{2}}\right)=h\left(\frac{2k}{1-k^2}\right)$

$k\left(1-k^2\right)=2hk$

Then locus of E is $y-y^3=2xy$

Now, Option A and C are correct

48. Let $f:\left(0,\infty\right)\to R$ be a differentiable function such that $f'\left(x\right)=2-\frac{f\left(x\right)}{x}$ for all $x\in\left(0,\infty\right)$ and $f\left(1\right)\neq1$. then

(A) $\lim_{x\to0+}f'\left(\frac{1}{x}\right)=1$ (B) $\lim_{x\to0+}xf\left(\frac{1}{x}\right)=2$

(C) $\lim_{x\to0+}x^2f'\left(x\right)=0$ (D) $|f\left(x\right)|\leq2$ for all $x\in\left(0,2\right)$

Key. (A)

Sol. $f'(x)=2-\frac{f(x)}{x}$.

$\Rightarrow xf(x)=x^2+c \quad \Rightarrow f(x)=x+\frac{c}{x}$

As $f(1)\neq 1$ Hence $c\neq 0$.

(A) $f\left(\frac{1}{x}\right)=\frac{1}{x}+cx$

$\left(-\frac{1}{x^2}\right)f'\left(\frac{1}{x}\right)=-\frac{1}{x^2}+c \;\Rightarrow f'\left(\frac{1}{x}\right)=1-cx^2$

$\lim\limits_{x\to 0^+} f'\left(\frac{1}{x}\right)=1$ Hence A is correct

(B) $x\ f\left(\frac{1}{x}\right)=x\left(\frac{1}{x}\right)+cx^2$

$x\ f\left(\frac{1}{x}\right)=1+cx^2$

$\lim\limits_{x\to 0^+} x\ f\left(\frac{1}{x}\right)=1$ Hence B is incorrect

(C) $f'(x)=1-\frac{c}{x^2}$

$x^2 f'(x)=x^2-c$

$\lim\limits_{x\to 0^+} x^2 f'(x)=0-c$ But $c\neq 0$

Hence C is incorrect.

(D) Option D is not possible

49. In a triangle XYZ, let x, y, z be the lengths of sides opposite to the angles X, Y, Z, respectively, and $2s=x+y+z$. if $\frac{s-x}{4}=\frac{s-y}{3}=\frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2}=\frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right)=\frac{3}{5}$

Key. (A, C, D)

Sol. As $\frac{S-x}{4}=\frac{S-y}{3}=\frac{S-z}{2}$

$\pi r^2=\frac{8\pi}{3} \Rightarrow r=\frac{2\sqrt{2}}{\sqrt{3}}$ $\quad (i)$

On Simplification S = 9, x = 5, y = 6, z = 7

(A) Area of (XYZ) = $\sqrt{9(9-5)(9-6)(9-7)}=6\sqrt{6}$

(B) $R=\frac{abc}{4\Delta}=\frac{5.6.7}{4\times 6\sqrt{6}}=\frac{35}{4\sqrt{6}}=\frac{35\sqrt{6}}{24}$

(C) $\sin\frac{X}{2}.\sin\frac{Y}{2}.\sin\frac{Z}{2}=\frac{r}{4R}=\frac{\frac{2\sqrt{2}}{\sqrt{3}}}{4\times\frac{35}{4\sqrt{6}}}=\frac{2\sqrt{2}\times\sqrt{6}}{\sqrt{3}\times 35}=\frac{4}{35}$

(D) $\sin^2\left(\frac{X+Y}{2}\right)=\cos^2\left(\frac{Z}{2}\right)=\frac{s(s-z)}{xy}=\frac{9(9-7)}{5\times 6}=\frac{18}{30}=\frac{3}{5}$

SECTION 3 (Maximum Marks: 15)

- This section contains FIVE questions
- The answer to each question is SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

Zero Marks : 0 in all other cases

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50. Let m be the smallest positive integer such that the coefficient of x^2 in the expansion of $(1+x)^2+(1+x)^3+......+(1+x)^{49}+(1+mx)^{50}$ is $(3n+1)\ {}^{51}C_3$ for same positive integer n. Then the value of n is

Key (5)

Sol. ${}^2C_2+{}^3C_2+.........+{}^{49}C_2+{}^{50}C_2m^2=(3n+1).{}^{51}C_3$

Or ${}^{50}C_3+{}^{50}C_2m^2=(3n+1)\ {}^{51}C_3$

Or $\frac{50\times 49\times 48}{3\times 2}+\frac{50\times 49}{2}m^2=(3n+1)\ {}^{51}C_3$

Or $\frac{51\times 50\times 49}{3\times 2}\left(\frac{16+m^2}{17}\right)=(3n+1)\ {}^{51}C_3$

Or ${}^{51}C_3\left(\frac{16+m^2}{17}\right)=(3n+1)\ {}^{51}C_3$

Or $\dfrac{16+m^2}{17}=3n+1$

Or $16+m^2=51n+17$

Or $\left(m^2-51n\right)=1$

For Smallest value of

$m=16$, $n=5$

51. Let $\alpha,\beta\in$ ¡ be such that $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{\alpha x-\sin x}=1$. Then 6 $(\alpha+\beta)$ equals

Key. (7)

Sol. $\lim\limits_{x\to 0}\dfrac{x^2\sin(\beta x)}{x-\sin x}=1 = \lim\limits_{x\to 0}\dfrac{x^2\left[\beta x-\dfrac{(\beta x)^3}{3!}+.....\right]}{\alpha x-\left(x-\dfrac{x^3}{3!}+.......\right)}=1$

$$\Rightarrow \lim_{x\to 0}\frac{\beta x^3-\dfrac{\beta^3x^5}{6}+...}{(\alpha-1)x+\dfrac{x^3}{6}-.....}=1$$

$$\Rightarrow \alpha-1=0 \ \ and \ \ \frac{\beta}{\frac{1}{6}}=1 \quad \Rightarrow \alpha=1, \beta=\frac{1}{6}$$

$$\alpha+\beta=\frac{7}{6} \qquad \therefore\ 6(\alpha+\beta)=7$$

52. Let $z=\dfrac{-1+\sqrt{3}i}{2}$ where $i=\sqrt{-1}$, and $r,s\in\{1,2,3\}$. Let $P=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$ and I be the identity matrix of order 2. Then the total number of ordered pairs (r,s) for which $P^2=-I$ is

Key (1)

Sol. $z=w \qquad P=\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}$

$$P^2=-I\Rightarrow\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}\begin{bmatrix}(-z)^r & z^{2s}\\ z^{2s} & z^r\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}(-z)^{2r}+z^{4s} & (-z)^r z^{2s}+z^{2s}z^r\\ (-z)^r z^{2s}+z^r z^{2s} & z^{4s}+z^{2r}\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}(-w)^{2r}+w^{4s} & (-w)^r w^{2s}+w^{2s}w^r\\ (-w)^r w^{2s}+w^r w^{2s} & w^{4s}+w^{2r}\end{bmatrix}=\begin{bmatrix}-1 & 0\\ 0 & -1\end{bmatrix}$$

$$\Rightarrow (-w)^{2r} + w^{4s} = -1 \quad and \quad w^{4s} + w^{2r} = -1$$

$$w^{2r} + w^{s} = -1 \quad and \quad w^{s} + w^{2r} = -1$$

$$r = 1, s = 1$$

53. The total number of distinct $x \in \mathbb{R}$ for which $\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$ is

Key (2)

Sol. $$\begin{vmatrix} x & x^2 & 1+x^3 \\ 2x & 4x^2 & 1+8x^3 \\ 3x & 9x^2 & 1+27x^3 \end{vmatrix} = 10$$

$$\begin{vmatrix} x & x^2 & x^3 \\ 2x & 4x^2 & 8x^3 \\ 3x & 9x^2 & 27x^3 \end{vmatrix} + \begin{vmatrix} x & x^2 & 1 \\ 2x & 4x^2 & 1 \\ 3x & 9x^2 & 1 \end{vmatrix} = 10$$

$$\Rightarrow x^3 \times x \times x^2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 8 \\ 3 & 9 & 27 \end{vmatrix} + x \times x^2 \begin{vmatrix} 1 & 1 & 1 \\ 2 & 4 & 1 \\ 3 & 9 & 1 \end{vmatrix} = 10$$

$$= x^6 2 \times 3 \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{vmatrix} + x^3 \begin{vmatrix} 1 & 0 & 0 \\ 2 & 2 & -1 \\ 3 & 6 & -2 \end{vmatrix} = 10$$

$$= 6x^6 \begin{vmatrix} 1 & 0 & 0 \\ 1 & 1 & 3 \\ 1 & 2 & 8 \end{vmatrix} + x^3(-4+6) = 10$$

$$\Rightarrow 6x^6 \times 2 \;+\; 2x^3 = 10$$

$$\Rightarrow 6x^6 + x^3 = 5$$

$x^3 = t$

Then $t = -1$, $t = 5/6$

$\Rightarrow$ $x = -1$, $x = (5/6)^{1/3}$

54. The total number of distinct $x \in [0,1]$ for which $\int_0^x \frac{t^2}{1+t^4}\,dt = 2x - 1$ is

Ans. (1)

Sol. Let $f(x) = \int_0^x \frac{t^2}{1+t^4}\,dt - (2x-1)$

$$\Rightarrow f'(x) = \frac{x^2}{1+x^4} - 2$$

$$\Rightarrow \quad f'(x) < 0$$

$f(x)$ is strictly decreasing function

$$I = \frac{1}{2}\int_0^x \frac{2t^2}{t^4+1}dt = \frac{1}{2}\int_0^x \frac{(t^2+1)+(t^2-1)}{t^4+1}dt$$

$$= \frac{1}{2}\int_0^x \frac{t^2+1}{t^4+1}dt + \frac{1}{2}\int_0^x \frac{t^2-1}{t^4+1}dt$$

$$= \frac{1}{2}\int_0^x \frac{1+\frac{1}{t^2}}{t^2+\frac{1}{t^2}}dt + \frac{1}{2}\int_0^x \frac{1-\frac{1}{t^2}}{\left(t+\frac{1}{t}\right)^2 - 2}dt$$

$$\frac{1}{2}\times\frac{1}{\sqrt{2}}\left[\tan^1\left(\frac{t-\frac{1}{t}}{\sqrt{2}}\right)\right]_0^x + \frac{1}{2}\times\ \frac{1}{2\sqrt{2}}\left[\ln\left|\frac{t+\frac{1}{t}-\sqrt{2}}{t+\frac{1}{t}+\sqrt{2}}\right|\right]_0^x$$

$$\frac{1}{2\sqrt{2}}\left[\tan^{-1}\left(\frac{t^2-1}{t\sqrt{2}}\right)\right]_0^x + \frac{1}{4\sqrt{2}}\left[\ln\left(\frac{t^2-\sqrt{2}t+1}{t^2+\sqrt{2}t+1}\right)\right]_0^x$$

$$= \frac{1}{2\sqrt{2}}\left[\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) - \tan^{-1}(-\infty)\right] + \frac{1}{4\sqrt{2}}\left[\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2}t+1}\right)\right]$$

$$= \frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{x^2-1}{x\sqrt{2}}\right) + \frac{1}{4\sqrt{2}}\ln\left(\frac{x^2-\sqrt{2}x+1}{x^2+\sqrt{2x}+1}\right) = 2x-1$$

$$f(0)=1,\ f(1)<0$$

As function is strictly decreasing, no. of root is 1.

JEE ADVANCED (Paper - 1)
CHEMISTRY

SECTION 1 (Maximum Marks: 15)

- This section contains **FIVE** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONLY ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 in all other cases.

  Negative Marks : –1 in all other cases.

19. The increasing order of atomic radii of the following Group 13 elements is

(A) Al < Ga < In < Tl (B) Ga < Al < In < Tl

(C) Al < In < Ga < Tl (D) Al < Ga < Tl < In

Ans. (B)

Sol: As atomic radii are respectively

$Al = 1.43\ \mathring{A}$

$Ga = 1.35\ \mathring{A}$

$In = 1.67\ \mathring{A}$

$Tl = 1.70\ \mathring{A}$

So Ga < Al < In < Tl

20. Among $[Ni(CO)_4]$, $[NiCl_4]^{2-}$, $[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$, Na_2O_2 and CsO_2, the total number of paramagnetic compounds is

(A) 2 (B) 3 (C) 4 (D) 5

Ans. (C)

Sol: $[NiCl_4]^{2-}$,$[Co(NH_3)_4Cl_2]Cl$, $Na_3[CoF_6]$ and CsO_2 are paramagnetic.

21. On complete hydrogenation, a natural rubber produces

(A) ethylene-propylene copolymer (B) vulcanized rubber

(C) polypropylene (D) poylbutylene

Ans. (A)

Sol: Natural rubber is a polymer of isoprene

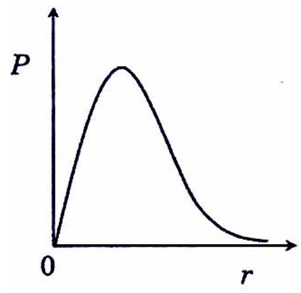
$$-\!\!\left(CH_2-\underset{\displaystyle CH_3}{\underset{|}{C}}=CH-CH_2\right)_{\!2}\!\!-$$

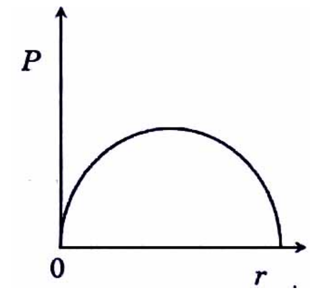
On hydrogenation it gives

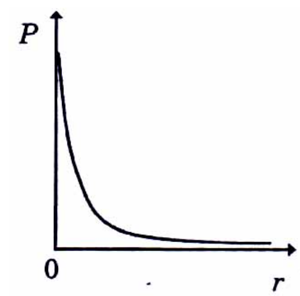
$$-\!\!\left(CH_2-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-CH_2\right)_{\!n}\!\!-$$

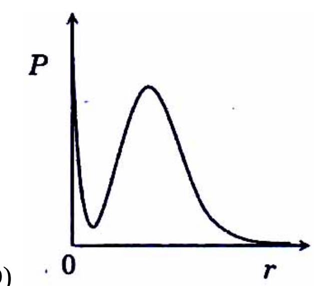
which is a copolymer of $CH_3CH = CH_2$ and $CH_2 = CH_2$

22. P is the probability of finding the 1s electron of hydrogen atom in a spherical shell of infinitesimal thickness, dr, at a distance r form the nucleus. The volume of this shell is 4 $\pi r^2 dr$. The qualitative sketch of the dependence of P on r is

(A) 

(B) 

(C) 

(D) 

Ans. (A)

Sol: Radial probability distribution of 1s orbital has only one peak and has no nodes.

23. One mole of an ideal gas at 300 K in thermal contact with surroundings expands isothermally form 1.0 L to 2.0 L against a constant pressure of 3.0 atm. In this process, the change in entropy of surrounding (ΔS_{surr}) in JK^{-1} is

(1 L atm = 101.3 J)

(A) 5.763 (B) 1.013 (C) –1.013 (D) –5.763

Ans. (C)

Sol: Work done by the gas = –3 atm l

= –303.9 J

As the temperate of the system remains constant so, heat supplied by the surrounding to the system = 303.9 J

$$\therefore \Delta S_{surr} = \frac{-303.9\text{J}}{300\text{K}} = -1.013\ \text{JK}^{-1}$$

SECTION 2 (Maximum Marks: 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D). **ONE OR MORE THAN ONE** of these four options is correct.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +4 if only the bubble(s) corresponding to all the correct option(s) is(are) darkened.

  Partial Marks : +1 For darkening a bubble corresponding **to each correct option**, provided NO incorrect option is darkened.

  Zero Marks : 0 in all other cases.

  Negative Marks : –2 in all other cases.
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks, and darkening (A) and (B) will result in –2 marks, as a wrong option is also darkened.

24. The correct statement(s) about the following reaction sequence is(are)

$$\text{Cumene}(C_9H_{12}) \xrightarrow[\text{(ii) } H_3O^+]{\text{(i) } O_2} P \xrightarrow{CHCl_3/NaOH} Q(\text{major}) + R(\text{minor})$$

$$Q \xrightarrow[PhCH_2Br]{NaOH} S$$

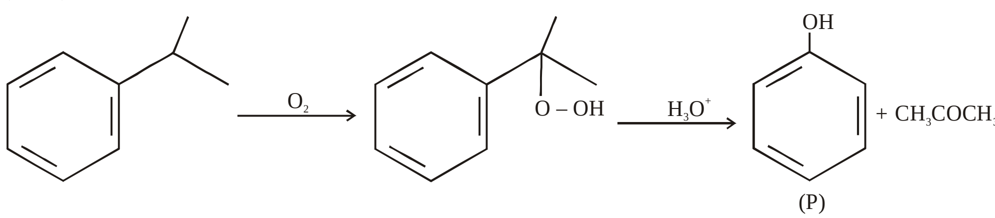
(A) R is steam volatile

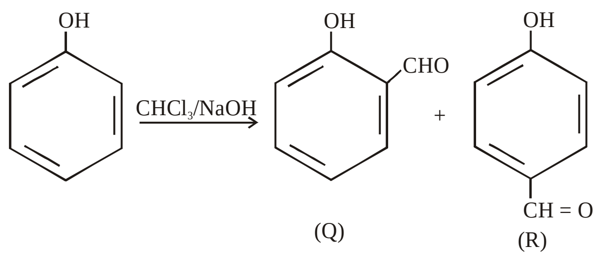
(B) Q gives dark violet coloration with 1% aqueous $FeCl_3$ solution

(C) S gives yellow precipitate with 2, 4-dinitrophenylhydrazine

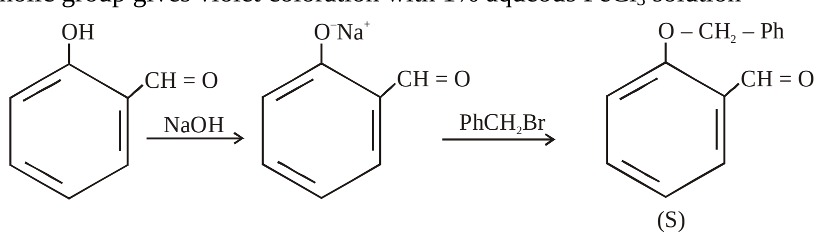
(D) S gives dark violet coloration with 1% aqueous $FeCl_3$ solution

Sol. (B, C)





Phenolic group gives violet coloration with 1% aqueous $FeCl_3$ solution



25. The compound(s) with TWO lone pairs of electrons on the central atom is(are)

(A) BrF_5 (B) ClF_3 (C) XeF_4 (D) SF_4

Sol. (B, C)

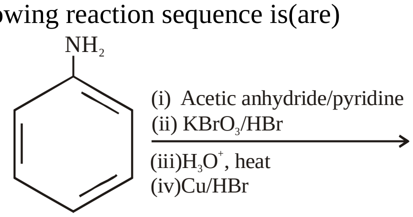
$$BrF_5 \rightarrow 7+35 = \frac{42}{8} = 5+\left(\frac{2}{2}\right) = 5 + 1 = 6;\ sp^3d^2 + 1 \text{ lone pair}$$

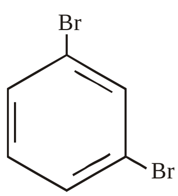
$$Cl\,F_3 \rightarrow 7+21 = \frac{28}{8} = 3 + \left(\frac{4}{2}\right) = 3 + 2 = 5;\ sp^3d + 2 \text{ lone pair}$$

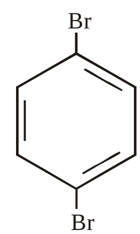
$$Xl\,F_4 \rightarrow 8+28 = \frac{36}{8} = 4+\left(\frac{4}{2}\right) = 6;\ sp^3d^2 + 2 \text{ lone pair}$$

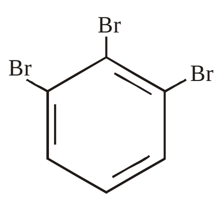
$$SF_4 \rightarrow 6 + 28 = \frac{34}{8} = 4+\left(\frac{2}{2}\right) = 5;\ sp^3d + 1 \text{ lone pair}$$

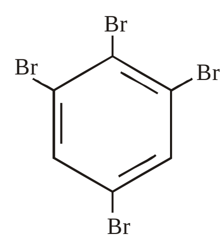
26. The product(s) of the following reaction sequence is(are)



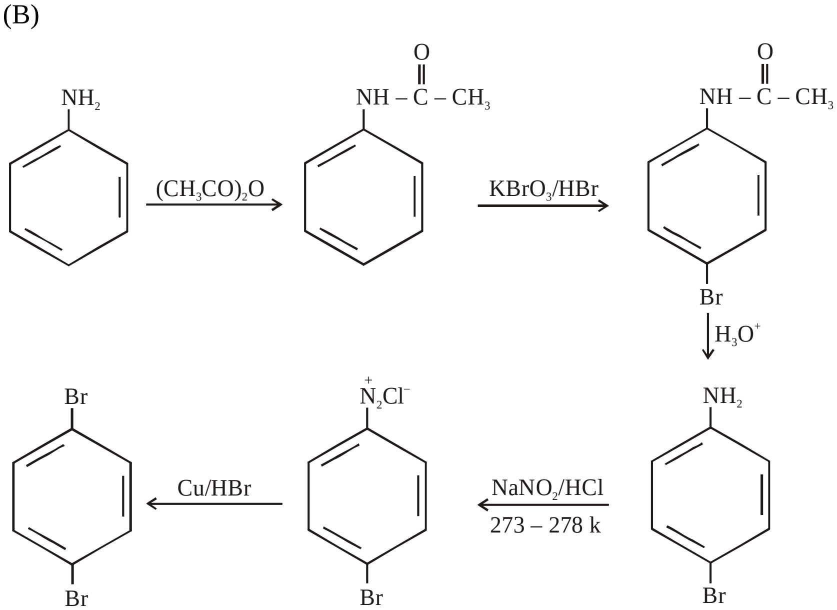
(A)


(B)


(C)


(D)


Sol. (B)



27. According to the Arrhenius equation

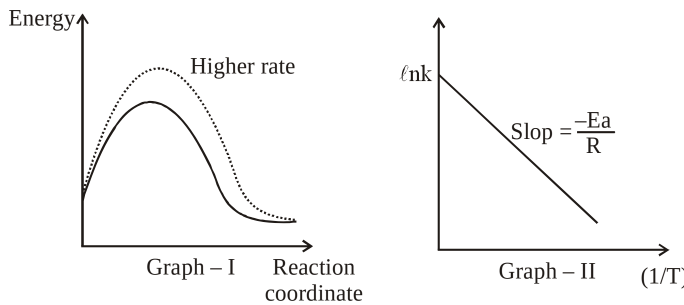
(A) a high activation energy usually implies a fast reaction

(B) rate constant increases with increase in temperature. This ia due to a greater number of collisions whose energy exceeds the activation energy

(C) higher the magnitude of activation energy, stronger is the temperature dependence of the rate constant

(D) the pre-exponential factor is a measure of the rate at which collisions occur, irrespective of their energy

Sol. (B, C, D)



According to Arrhenius equation

$$K = Ae^{-Ea/RT} \quad \text{... (i)}$$

$$\ln k = \frac{-Ea}{RT} + \ln A$$

By increasing the temperature rate constant of reaction will increases.

By differentiation of equation (i)

$$\frac{dK}{dT} = +A\left(\frac{Ea}{RT^2}\right)e^{-Ea/RT}$$

So slope $\left(\frac{dK}{dT}\right) \rightarrow$ rate of change of rate constant depends strongly on higher magnitude of activation energy (graph – I).

28. The crystalline form of borax has

(A) tetranuclear $[B_4O_5(OH)_4]^{2-}$

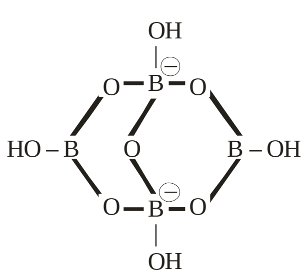
(B) all boron atoms in the same plane

(C) equal number of sp^2 and sp^3 hybridized boron atoms

(D) one terminal hydroxide per boron atom

Key. (A, C, D)

Sol. A, C, D



Negatively charged ‘B’ are sp^3 hybridised whereas other two ‘B’ are sp^2 hybridised.

29. The reagent(s) that can selectively precipitate S^{2-} from a mixture of S^{2-} and SO_4^{2-} in aqueous solution is (are)

(A) $CuCl_2$ (B) $BaCl_2$

(C) $Pb(OOCCH_3)_2$ (D) $Na_2[Fe(CN)_5NO]$

Key. (A)

Sol. $$CuCl_2 + S^{2-} \rightarrow \underset{\text{Black}}{CuS}\downarrow + 2Cl^-$$

But $$CuCl_2 + SO_4^{2-} \rightarrow \underset{\text{Soluble}}{CuSO_4} + 2Cl^-$$

$$BaCl_2 + SO_4^{2-} \rightarrow \underset{\text{white}}{BaSO_4}\downarrow + 2Cl^-$$

BaS is soluble in water

$$Pb(OOCCH_3)_2 + S^{2-} \rightarrow \underset{\text{Black}}{PbS}\downarrow + 2CH_3COO^-$$

$$Pb(OOCCH_3)_2 + SO_4^{2-} \rightarrow \underset{\text{White}}{PbSO_4}\downarrow + 2CH_3COO^-$$

$$Na_2[Fe(CN)_5NO] + S^{2-} \rightarrow \underset{\text{Violet solution}}{Na_4[Fe(CN)_5NOS]},\ \text{and}$$

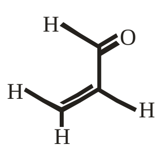
$$Na_2[Fe(CN)_5NO] \xrightarrow{h\nu} Na_2[Fe(CN)_5NO]^*$$

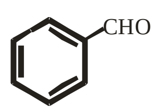
$$Na_2[Fe(CN)_5NO]^* + H_2O \xrightarrow{h\nu} Na_3[Fe(CN)_5(H_2O)] + NO^+$$

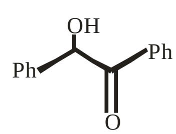
$$Na_3[Fe(CN)_5(H_2O)] + SO_4^{2-} \longrightarrow Na_5[Fe(CN)_5(SO_4)] + H_2O$$

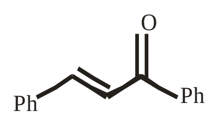
$$Na_5[Fe(CN)_5(SO_4)] + SO_4^{2-} \longrightarrow \underset{\text{Peacock blue}}{Na_6[Fe(CN)_4(SO_4)_2]}\downarrow + CN^-$$

30. Positive Tollen's test is observed for

(A) 

(B) 

(C) 

(D) 

Key. (A, B, C)

Sol. Aliphatic aldehyde **(A)**, Aromatic aldehyde **(B)** and α hydroxyl ketone gives tollen's test.

31. A plot of the number of neutrons(N) against the number of protons (P) of stable nuclei exhibit unpward deviation from linearity for atomic number, Z>20. For an unstable nucleus having N/P ratio less than 1, the possible mode(s) of decay is(are)

(A) β^--decay (β emission) (B) orbital or K-electron capture

(C) neutron emission (D) β^+-decay (positron emission)

Key. (B, D)

Sol. If $\frac{N}{P}$ ratio is less than one in Z > 20

Then possible modes of decay are K electron capture and positron decay.

$${}_{Z}A^{M} + {}_{-1}e^{0} \rightarrow {}_{(z-1)}B^{M}\ \text{(K capture)}$$

$${}_{Z}A^{M} \longrightarrow {}_{(z-1)}B^{M} + {}_{+1}\beta^{0}\ \text{(Positron emission)}$$

SECTION 3 (Maximum Marks: 15)

- This section contains FIVE questions
- The answer to each question is SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories:

  Full Marks : +3 if only the bubble corresponding to the correct answer is darkened.

  Zero Marks : 0 in all other cases

32. The diffusion coefficient of an ideal gas is proportional to its mean free path and mean speed. The absolute temperature of an ideal gas is increased 4 times and pressure is increased 2 times. As a result, the diffusion coefficient of this gas increases x times. The value of x is

Ans. (4)

Sol. $K \propto \frac{T^{3/2}}{P} \Rightarrow \frac{K_2}{K_1} = \frac{(4)^{3/2}}{2}$

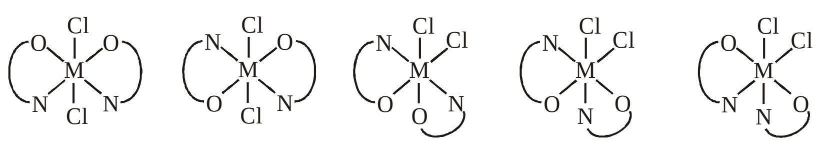
$= \frac{8}{2} = 4$

33. The number of geometric isomers possible for the complex $[CoL_2Cl_2]^-$ ($L = H_2NCH_2CH_2O^-$) is

Ans. (5)

Sol. Compound $[ML_2B_2]$ has total geometrical isomers = 3, (L = Bidentate ligand)

cis = 2 and trans = 1



34. The mole fraction of a solute in a solution is 0.1. At 298 K, molarity of this solution is the same as its molality. Density of this solution at 298 K is 2.0 g cm^{-3}. The ratio of the molecular weights of the solute and solvent, $\left(\frac{MW_{solute}}{MW_{solvent}}\right)$, is

Ans. (9)

Sol. $\chi_{solute} = \frac{1}{10}$

If $n_{solute} = 1$ then $n_{solvent} = 9$

Let MW of solute = x and solvent = y

Mass of solution = x + 9y

Let total volume of solution = v

Molality = molarity

$$\frac{1}{9y} = \frac{1}{V} \Rightarrow \frac{V}{y} = 9 \quad ....(i)$$

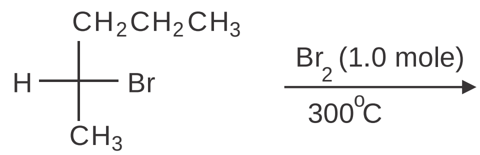
as density = 2

$$\Rightarrow \frac{x + 9y}{V} = 2 \quad .....(ii)$$

from equation (i) and (ii)

$$\frac{x}{y} = 9$$

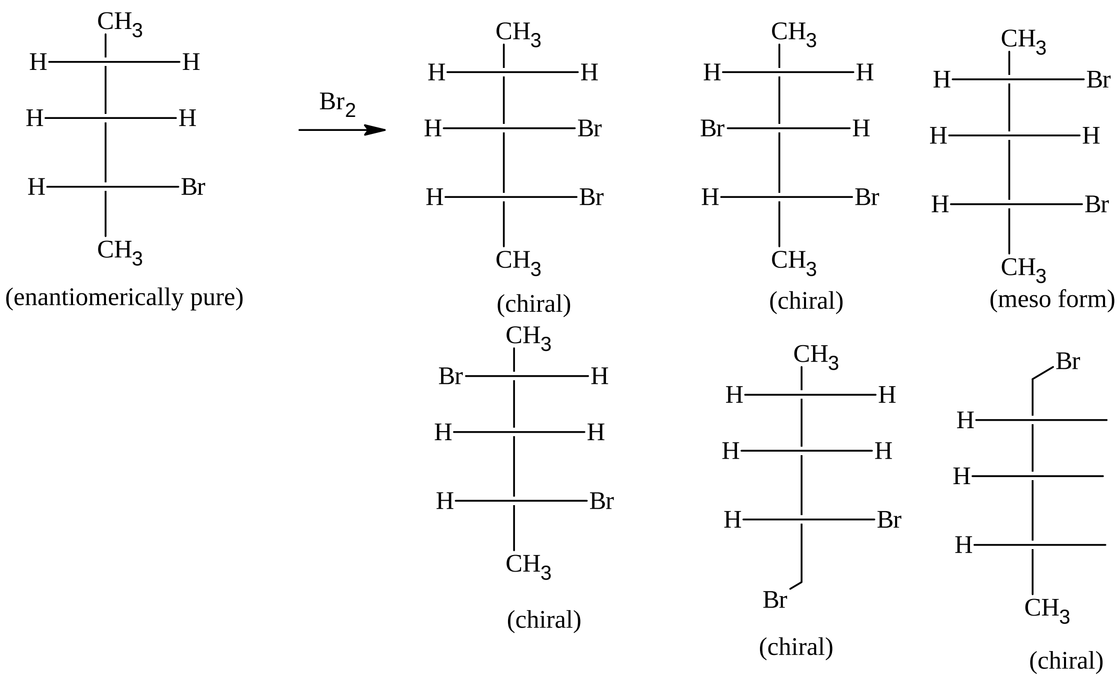
35. In the following monobromination reaction, the number of possible chiral products is



(1.0 mole)
(enantiomerically pure)

Ans. (5)

Sol.



36. In neutral or faintly alkaline solution, 8 moles of permanganate anion, quantitatively oxidize thiosulphate anions to produce X moles of a sulphur containing product. The magnitude of X is

Ans. (6)

Sol.

$$3S_2O_3^{2-} + 8MnO_4^- + H_2O \rightarrow 6SO_4^{2-} + 8MnO_2 + 2OH^-$$

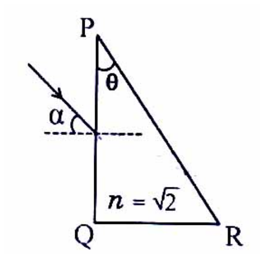
JEE ADVANCED (Paper - 1)

PHYSICS

Section 1 (Maximum Marks : 15)

- This section contains **FIVE** questions
- Each question has Four options (A), (B), (C) and (D) only one of these four options is correct.
- For each question, darken the bubble corresponding to the correct option in ORS.
- For each question, marks will be awarded in one of the following categories. :
  Full Marks : +3 if only the bubble corresponding to the correct option is darkened
  Zero Marks : 0 if none of the bubbles is darkened.
  Negative Marks : -1 in all other cases

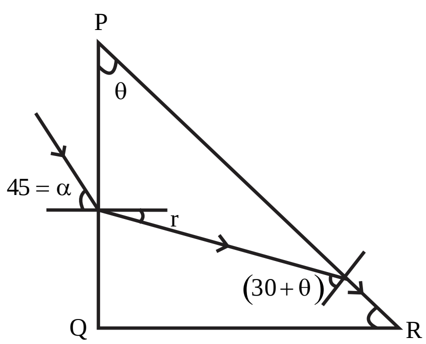
1. A parallel beam of light is incident from air at an angle α on the side PQ of a right angled triangular prism of refractive index $n = \sqrt{2}$. Light undergoes total internal reflection in the prism at the face PR when α has a minimum value of 45^o. the angle θ of the prism is



(A) 15^o (B) 22.5^o (C) 30^o (D) 45^o

Ans. (A)

Sol: at PQ



$$\frac{\sin 45^o}{\sin r} = \sqrt{2}$$

$r = 30^o$

at PR

$$\frac{\sin(30+\theta)}{\sin 90} = \frac{1}{\sqrt{2}}$$

$\theta = 15^o$

Hence (A) is correct.

2. In a historical experiment to determine Planck's constant, a metal surface was irradiated with light of different wavelengths. The emitted photoelectron energies were measured by applying a stopping potential. The relevant data for the wavelength (λ) of incident light and the corresponding stopping potential (V_o) are given below:

$\lambda(\mu m)$	V_o(Volt)
0.3	2.0
0.4	1.0
0.5	0.4

Given that $c = 3 \times 10^8$ m s^{-1} and $e = 1.6 \times 10^{-19}$C, Planck's constant (in units of J s) found from such an experiment is

(A) 6.0×10^{-34} (B) 6.4×10^{-34} (C) 6.6×10^{-34} (D) 6.8×10^{-34}

Ans. (B)

Sol: $\frac{hc}{\lambda} = W + eV_o$

Using reading 1 & 2

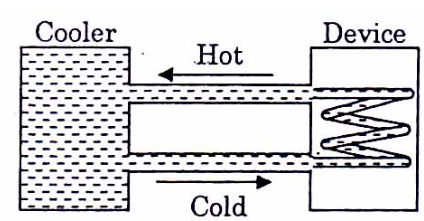
$$\frac{hc}{10^{-7}}\left[\frac{1}{3}-\frac{1}{4}\right] = e(2-1)$$

We get h = 6.4×10^{-34}

Similarly using reading 2 & 3 or 3 & 1 we get the same value

Hence (B) is correct.

3. A water cooler of storage capacity 120 litres can cool water at a constant rate of P watts. In a closed circulation system (as shown schematically in the figure), the water from the cooler is used to cool an external device that generates constantly 3 kW of heat (thermal load). The temperature of water fed into the device cannot exceed 30^{o}C and the entire stored 120 litres of water is initially cooled to 10^{o}C. The entire system is thermally insulated. The minimum value of P (in watts) for which the device can be operated for 3 hours is



(Specific heat of water is 4.2 kJ kg^{-1}K^{-1} and the density of water is 1000 kg m^{-3})

(A) 1600 (B) 2067 (C) 2533 (D) 3933

Ans. (B)

Sol: Total heat generated in 3 hr

$H = 3 \times 10^3 \times 3 \times 60 \times 60$

Heat used to raise the temperature of cooler

Q= m C. ΔT = $120 \times 4.2 \times 10^3 \times 20$

Now heat required to eject

$Q^1 = H - Q$

and $P = \frac{Q^1}{t} = \frac{3\times10^3 \times 3\times60\times60 - 120\times4.2\times10^3 \times 20}{3\times60\times60}$

= 3000 – 933.33

= 2067 watt

4. A uniform wooden stick of mass 1.6 kg and length l rests in an inclined manner on a smooth, vertical wall of height $h(<l)$ such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30^o with wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/l and the frictional force f at the bottom of the stick are

($g = 10\ ms^{-2}$)

(A) $\frac{h}{l} = \frac{\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

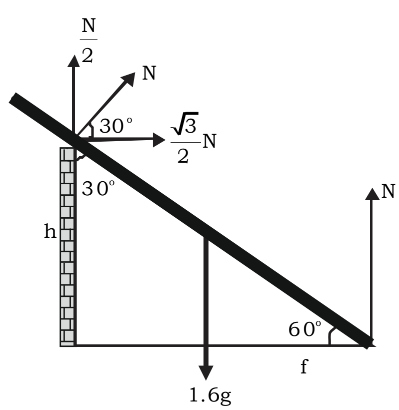
(B) $\frac{h}{l} = \frac{3}{16}, f = \frac{16\sqrt{3}}{3} N$

(C) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{8\sqrt{3}}{3} N$

(D) $\frac{h}{l} = \frac{3\sqrt{3}}{16}, f = \frac{16\sqrt{3}}{3} N$

Ans. (D)

Sol: $\Sigma F_x = 0 \Rightarrow f = \frac{\sqrt{3}}{2} N \quad \ldots(1)$



$N + \frac{N}{2} = 1.6g$

$\Sigma F_y = 0 \Rightarrow \frac{3N}{2} = 1.6g \qquad \ldots(2)$

From equation (1) & (2)

$f = \frac{\sqrt{3}}{2} . \frac{1.6g \times 2}{3} = \frac{16}{\sqrt{3}} = \frac{16\sqrt{3}}{3} N$

$$\Sigma\tau = 0 \Rightarrow 1.6g\frac{l}{2}\cos 60^\circ = \frac{\sqrt{3}}{2}Nh + \frac{N}{2}\times\frac{h}{\tan 60^\circ}$$

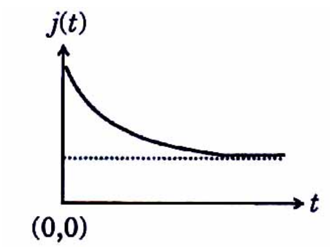
$$\Rightarrow 1.6g\times\frac{l}{4} = \frac{\sqrt{3}}{2}\times\frac{1.6g\times 2}{3}h + \frac{1}{2}\frac{1.6g\times 2}{3}\times\frac{h}{\sqrt{3}}$$

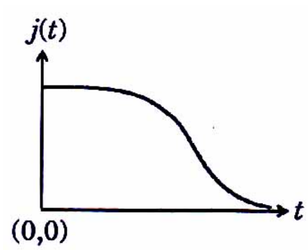
$$\Rightarrow \frac{l}{4} = \frac{h}{\sqrt{3}} + \frac{h}{3\sqrt{3}}$$

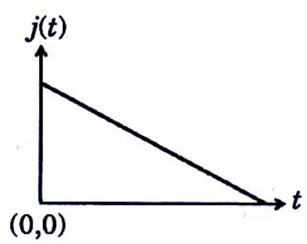
$$\Rightarrow \frac{h}{l} = \frac{3\sqrt{3}}{16}$$

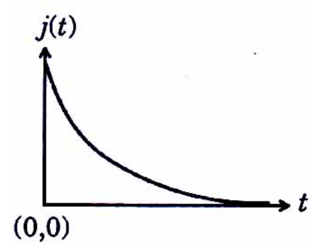
$\therefore$ answer is (D)

5. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material?

(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol: The situation is similar to discharging of R-C circuit. Hence

$i = i_o e^{\frac{-t}{\tau}}$ $\therefore$ answer is (D)

Section 2 (Maximum Marks : 32)

- This section contains **EIGHT** questions
- Each question has **FOUR** options (A), (B), (C) and (D) ONE OR MORE THAN ONE of these four option(s) is (are) correct.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +4 if only the bubble(s) corresponding to all the correct option(s) is (are) darkened.

  Zero Marks : 0 if none of the bubbles is darkened.

  Negative Marks : -2 in all other cases
- For example, if (A), (C) and (D) are all the correct options for a question, darkening all these three will result in +4 marks; darkening only (A) and (D) will result in +2 marks; and darkening (A) and (B) will result in -2 marks, as a wrong option is also darkened.

6. A plano-convex lens is made of a material of refractive index n. When a small object is placed 30 cm away in front of the curved surface of the lens, an image of double the size of the object is produced. Due to reflection from the convex surface of the lens, another faint image is observed at a distance of 10 cm away from the lens. Which of the following statement(s) is(are) true?

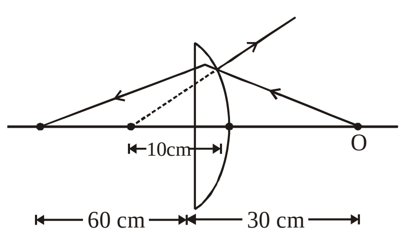
(A) The refractive index of the lens is 2.5

(B) The radius of curvature of the convex surface is 45 cm

(C) The faint image is erect and real

(D) The focal length of the lens is 20 cm

Sol. (A, D)



Due to refraction, real image of double size is formed

$\therefore \quad m = -2$

$$m = \frac{v}{u} \Rightarrow -2 = \frac{v}{-30} \Rightarrow v = 60$$

For refraction on spherical surface

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f} \Rightarrow \frac{1}{60} - \frac{1}{-30} = \frac{1}{f}$$

$\Rightarrow f = 20$ cm

For reflection on convex surface

$$\frac{1}{v}+\frac{1}{u} = \frac{1}{f_m} \Rightarrow \frac{1}{10}-\frac{1}{30} = \frac{1}{f_m}$$

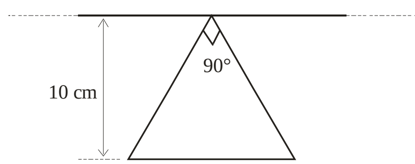
f_m = 15 cm $\Rightarrow$ R = 30 cm = Radius of curvature of mirror.

Now $\frac{1}{f} = (n-1)\left(\frac{1}{R_1}-\frac{1}{R_2}\right)$

$$\frac{1}{20} = (n-1)\left(\frac{1}{30}-\frac{1}{\infty}\right) \Rightarrow n = 2.5$$

Hence, option (A) and (D) are correct.

7. A conducting loop in the shape of a right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at a constant rate of 10 As^{-1}. Which of the following statement(s) is(are) true?



(A) There is a repulsive force between the wire and the loop

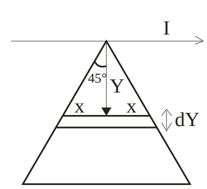
(B) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

(C) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt

(D) The induced current in the wire is in opposite direction to the current along the hypotenuse

Sol. (A, C)

Using the concept of mutual induction, if we take the reverse case i.e. if current is passed in the conducting infinite wire. Magnetic flux with the triangular loop can be calculate as



Here, $\tan 45 = \frac{X}{Y} = 1 \Rightarrow Y = X; dY = dX$

The flux through the elemental strip

$$d\phi = B.dA = \frac{\mu_0 I}{2\pi x}\times 2xdY = \frac{\mu_0 I}{\pi}dx$$

$\therefore \qquad \phi = \int d\phi = \frac{\mu_0 I}{\pi}\int_0^{Y=0.1} dx = \frac{\mu_0 I}{\pi}\times .1 = \frac{\mu_0 I}{10\pi}$

$\therefore \qquad$ induced emf in the loop $= \frac{d\phi}{dt} = \frac{\mu_0}{10\pi}\frac{di}{dt}$

$= \frac{\mu_0}{10\pi}\times 10 = \frac{\mu_0}{\pi}$

Therefore induced emf in the wire will be same and hence option C is correct.

As per Lenz's law, if current is increased in the wire towards right, then the induced current in the triangular loop is anticlockwise.

Hence option (D) is incorrect.

As per Lenz's law if current is increased in the straight wire the loop will have a tendency to move from stranger to weaker field.

$\therefore \qquad$ option (A) is correct.

When the loop is rotated about the wire the flux linked through the triangular loop does not change due to the current in the straight wire. So emf induced is zero.

So, option B is incorrect.

8. The position vector $\vec{r}$ of a particle of mass m is given by the following equation

$\vec{r}(t) = \alpha t^3\hat{i} + \beta t^2\hat{j}$

where $\alpha = 10/3\ \text{ms}^{-3}$, $\beta = 5\ \text{ms}^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle?

(A) The velocity $\vec{v}$ is given by $\vec{v} = (10\hat{i} + 10\hat{j})\text{ms}^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L} = -(5/3)\hat{k}$ Nms

(C) The force $\vec{F}$ is given by $\vec{F} = (\hat{i} + 2\hat{j})$N

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau} = -(20/3)\hat{k}$ N m

Sol. (A, B, D)

$\vec{r} = \alpha t^3\hat{i} + \beta t^2\hat{j}$

$\vec{v} = 3\alpha t^2\hat{i} + 2\beta t\hat{j}$

$\vec{a} = 6\alpha t\hat{i} + 2\beta\hat{j}$

Now,

$\vec{P} = m\vec{v} = 0.1(3\alpha t^2\hat{i} + 2\beta t\hat{j})$

$\vec{F} = m\vec{a} = 0.1(6\alpha t\hat{i} + 2\beta\hat{j})$

Now,

At t = 1s

$\vec{r} = \alpha\hat{i} + \beta\hat{j} = \frac{10}{3}\hat{i} + 5\hat{j}$

$\vec{v} = 3\alpha\hat{i} + 2\beta\hat{j} = 10\hat{i} + 10\hat{j}$

$\vec{a} = 6\alpha\hat{i} + 2\beta\hat{j} = 20\hat{i} + 10\hat{j}$

$\vec{P} = (\hat{i} + \hat{j})$

$\vec{F} = 2\hat{i} + \hat{j}$

As $\vec{L} = \vec{r} \times \vec{P}$

$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times (\hat{i} + \hat{j}) = \frac{10}{3}\hat{k} - 10\hat{k} = -\frac{20}{3}\hat{k}$

$\vec{\tau} = \vec{r} \times \vec{F}$

$= \left(\frac{10}{3}\hat{i} + 5\hat{j}\right) \times (2\hat{i} + \hat{j})$

$= \frac{10}{3}\hat{k} - 10\hat{k} = -\frac{20}{3}\hat{k}$ N.m.

Hence, (A), (B) and (D) are correct.

9. A length-scale (l) depends on the permittivity (ε) of a dielectric material, Boltzmann constant (k_B), the absolute temperature (T), the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles. Which of the following expression(s) for l is (are) dimensionally correct?

(A) $l = \sqrt{\left(\frac{nq^2}{\varepsilon k_B T}\right)}$ (B) $l = \sqrt{\left(\frac{\varepsilon k_B T}{nq^2}\right)}$ (C) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $l = \sqrt{\left(\frac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

Ans. (B, D)

Potential energy $U = \frac{q^2}{4\pi\varepsilon_0 L} \Rightarrow \frac{q^2}{\varepsilon_0} = \frac{1}{4\pi UL}$

$\left[\frac{q^2}{\varepsilon_0}\right] = UL = JL(J \to \text{Joule})$

Average KE / Molecular / degree of freedom = $\frac{1}{2}k_b T$

$[k_b T] = J(\text{Joule})$

$[n] = L^{-3}$

(A) $l = \sqrt{(n)\left(\frac{q^2}{\varepsilon}\right)\left(\frac{1}{k_b P}\right)} = \frac{1}{L}$

(B) $l = \sqrt{\left(\frac{\varepsilon}{q^2}\right)(k_b T)\left(\frac{1}{n}\right)} = L$

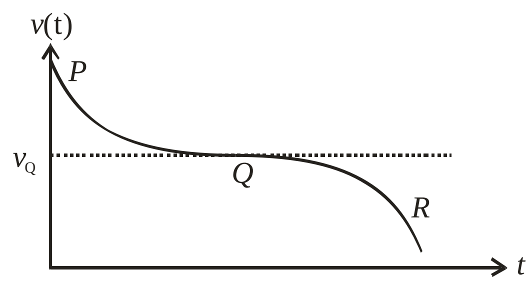
(C) $\sqrt{\left[\frac{q^2}{\varepsilon}\right]\left[\frac{1}{k_bT}\right]\left[\frac{1}{n^2h}\right]} = \sqrt{L}$

(D) $\sqrt{\left[\frac{q^2}{\varepsilon}\right]\left[\frac{1}{k_bT}\right]\left[\frac{1}{n^{1/3}}\right]} = L$

Hence (B) and (D) are correct.

10. Two loudspeakers M and N are located 20 m apart and emit sound at frequencies 118 Hz and 121 Hz, respectively. A car is initially at a point P, 1800 m away from the midpoint Q of the line MN and moves towards Q constantly at 60 km/hr along the perpendicular bisector of MN. It crosses Q and eventually reaches a point R, 1800 m away from Q. Let $v(t)$ represent the beat frequency measured by a person sitting in the car at time t. Let v_P, v_Q and v_R be the beat frequencies measured at locations P, Q and R, respectively. The speed of sound in air is 330 ms^{-1}. Which of the following statement(s) is (are) true regarding the sound heard by the person?

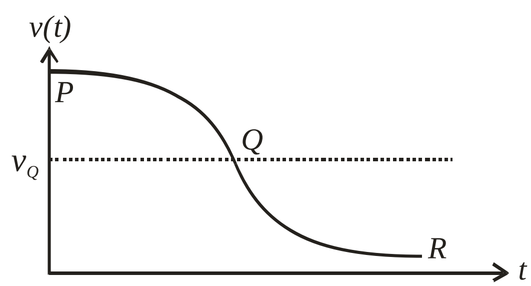
(A) The plot below represents schematically the variation of beat frequency with time



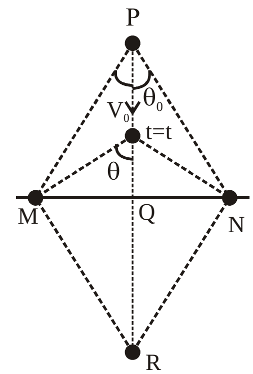
(B) The rate of change in beat frequency is maximum when the car passes through Q

(C) $v_P + v_R = 2v_Q$

(D) The plot below represents schematically the variation of beat frequency with time



Ans. (B, C, D)



Sol. At position of point P. Beat frequency is given by

$$v_P = v_N\left[\frac{v_S + v_0\cos\theta_0}{v_S - 0}\right] - v_M\left[\frac{v_S + v_0\cos\theta_0}{v_S - 0}\right] \quad \text{........(i)}$$

at point of R the beat frequency is given by

$$v_R = v_N\left[\frac{v_S - v_0\cos\theta_0}{v_S - 0}\right] - v_M\left[\frac{v_S - v_0\cos\theta_0}{v_S - 0}\right] \quad \text{........(ii)}$$

At position of Q the beat frequency is given by

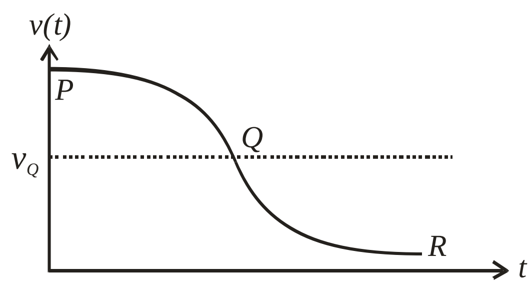
$v_Q = \left|v_N - v_M\right|$ ....(iii)

After adding both equations (i) and (ii) we get the result

$v_P + v_R = 2v_Q$

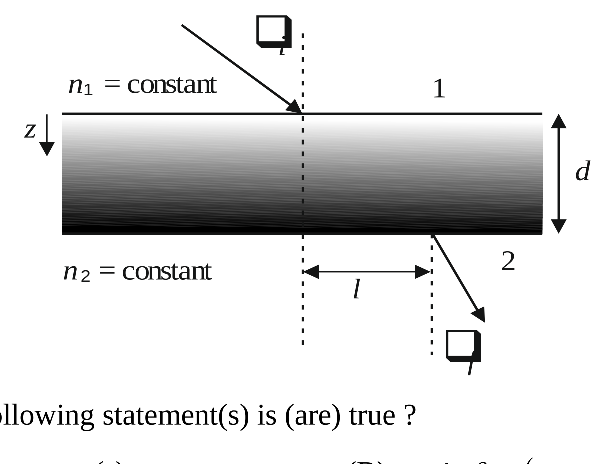
and at any time moment the beat frequency is given by

$v\,|_{t=t} = \left|v_N - v_M\right| + 50\cos\theta.$ So, the graph will be



and in this above graph the rate of change of frequency is maximum at position Q.

11. A transparent slab of thickness *d* has a refractive index $n(z)$ that increases with z. Here z is the vertical distance inside the slab, measured from the top. The slab is placed between two media with uniform refractive indices n_1 and n_2 (> n_1), as shown in the figure A ray of light is incident with angle θ_i from medium 1 and emerges in medium 2 with refraction angle θ_f with a lateral displacement *l*.



Which of the following statement(s) is (are) true ?

(A) *l* is dependent on $n(z)$

(B) $n_1\sin\theta_i = (n_2 - n_1)\sin\theta_f$

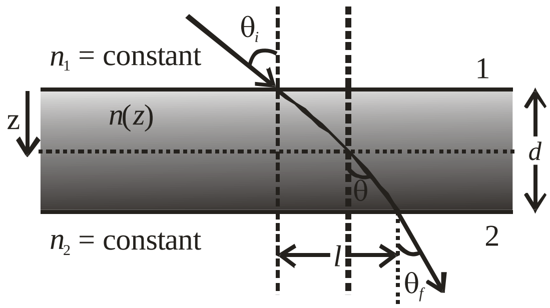
(C) $n_1\sin\theta_i = n_2\sin\theta_f$

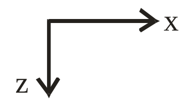
(D) *l* is independent of n_2

Ans. (A, C, D)

Sol. $n_1 \sin\theta_i = n_2 \sin\theta_f$

(C) is correct





$$n_1 \sin\theta_i = n(z).\sin\theta$$

$$\sin\theta = \frac{n_1 \sin\theta_i}{n(z)}$$

$$\cot\theta = \frac{\sqrt{n(z)^2 - n_1^2 \sin^2\theta_i}}{n_1 \sin\theta_i}$$

$$\frac{dz}{dx} = \frac{\sqrt{n(z)^2 - n_1^2 \sin^2\theta_i}}{n_1 \sin\theta_i}$$

$$\int_0^d \frac{dz}{\sqrt{n(z)^2 - n_1^2 \sin^2\theta_i}} = \int_0^l \frac{dx}{n_1 \sin\theta_i}$$

From this equation

l is dependent of $n(z)$

l is independent of n_2

12. Highly excited states for hydrogen-like atoms (also called Rybderg states) with nuclear charge Ze are defined by their principal quantum number n, where $n >> 1$. Which of the following statement(s) is (are) true?

(A) Relative change in the radii of two consecutive orbitals does not depend on Z

(B) Relative change in the radii of two consecutive orbitals varies as $1/n$

(C) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(D) Relative change in the angular momenta of two consecutive orbitals varies as $1/n$

Ans. (A, B, D)

Sol. $r = 0.529 \dfrac{n^2}{z}$

$$\frac{\Delta r}{r} \propto \frac{1}{n} \text{ (independent of z)}$$

B is correct, A is also correct

$$E = -13.6\frac{z^2}{n^2}$$

$$\Delta E = -13.6z^2\left(-2n^{-3}.\Delta n\right)$$

$$\frac{\Delta E}{E} = \frac{13.6z^2.2\Delta n}{n^3.\left(-13.6\frac{z^2}{n^2}\right)}$$

$$\frac{\Delta E}{E} \propto \frac{1}{n}$$

(C) is not correct

$$L = n\frac{h}{2\pi}$$

$$\frac{\Delta L}{L} = \frac{\Delta n}{n}$$

$$\frac{\Delta L}{L} \propto \frac{1}{n}$$

(D) is correct

13. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is (are) true?

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

Ans. (C, D)

Sol. Since there is non uniform evaporation therefore heat dissipation (production i.e., i^2R) is non uniform, it implies resistance of different part is different. So temperature can't be same everywhere, also length of filament increases with time so resistance increases. As temperature T rises with time so $\lambda_m T = b$, this implies λ decreases and frequency increases. Before breakup, resistance is maximum so $P = \frac{V^2}{R}$, this implies at the end of life P will decrease.

Section 3 (Maximum Marks : 15)

- This section contains **FIVE** questions
- The answer to each question is a SINGLE DIGIT INTEGER ranging from 0 to 9, both inclusive.
- For each question, darken the bubble corresponding to the correct integer in the ORS.
- For each question, marks will be awarded in one of the following categories :

  Full Marks : +3 if only the bubble corresponding to all the correct answer is darkened.

  Zero Marks : 0 in all other cases

14. A hydrogen atom in its ground state is irradiated by light of wavelength 970Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, The number of lines present in the emission spectrum is

Ans. (6)

Sol. $\Delta E = \frac{hc}{e\lambda}$

$= 12.752\ eV$

Q $\Delta E = E_1 - E_2$

$\Rightarrow$ E_1 – (-13.6)

$E_1 = -0.85$

$\frac{-13.6}{n^2} = -0.85 \quad \Rightarrow \quad n = 4$

No. Of emission spectrum $= \frac{n(n-1)}{2} = \frac{4\times 3}{2} = 6$

15. The isotope ${}^{12}_{5}\mathrm{B}$ having a mass 12.014 u undergoes $\beta-$decay to ${}^{12}_{6}\mathrm{C}$. ${}^{12}_{6}\mathrm{C}$ has an excited state of the nucleus $\left({}^{12}_{6}\mathrm{C}^*\right)$ at 4.041 MeV above its ground state. If ${}^{12}_{5}\mathrm{B}$ decays to ${}^{12}_{6}\mathrm{C}^*$, the maximum kinetic energy of the β-particle in units of MeV is (1u = 931.5 MeV/c^2, where c is the speed of light in vacuum).

Ans. (9)

Sol. K=(12.0144–12) × 931.5 – 4.041

K=13.041 – 4.041 = 9

16. Consider two solid spheres P and Q each of density 8 gm cm^{-3} and diameter 1 cm and 0.5 cm, respectively. Sphere P is dropped into a liquid of density 0.8 gm cm^{-3} and viscosity $\eta = 3$ poiseulles. Sphere Q is dropped into a liquid of density 1.6 gm cm^{-3} and viscosity $\eta = 2$ poiseulles. The ratio of the terminal velocities of P and Q is

Ans. (3)

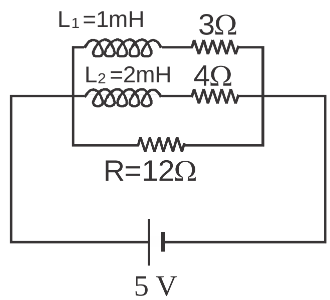
Sol. $v_t = \frac{2r^2 g(\sigma - \rho)}{9\eta}$

$$\frac{V_p}{V_Q} = \frac{r_P^2}{r_Q^2} \times \frac{(\sigma_P - \rho_P)}{(\sigma_Q - \rho_\rho)} \times \frac{\eta_Q}{\eta_P} = (2)^2 \left(\frac{8-0.8}{8-1.6}\right)\frac{2}{3} = 3$$

17. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. the ratio of the maximum to the minimum current (I_{max} / I_{min}) drawn from the battery is

Ans. (8)

Sol.



$$i_{min} = \frac{5}{12}$$

$$I_{max} = \frac{5}{12} + \frac{5}{3} + \frac{5}{4} = 5\left(\frac{1+3+4}{12}\right) = 8 \times \frac{5}{12}$$

$$\frac{I_{max}}{I_{min}} = \frac{8 \times 5}{12} \times \frac{12}{5} = 8$$

18. A metal is heated in a furnace where a sensor is kept above the metal surface to read the power radiated (P) by the metal. The sensor has a scale that displays $\log_2 (P/P_0)$, where P_0 is a constant. When the metal surface is at a temperature of 487^{o}C, the sensor shows a value 1. Assume that the emissivity of the metallic surface remains constant. What is the value displayed by the sensor when the temperature of the metal surface is raised to 2767^{o}C?

Ans. (9)

Sol. $R = \log_2(P/P_0)$ $(R \to \text{reading})$

$T = 487°C \to 2767°C$

$$R_1 = 4\log_2\left(\frac{T_1}{P_0}\right) = 1 \ldots \text{(i)}$$

$$R_2 = 4\log_2\left(\frac{T_2}{P_0}\right) = x \text{ (say)} \qquad \ldots \text{(ii)}$$

equation (i) substrated from equation (ii), we get

$$4\left\{\log_2\left(\frac{T_2}{T_1}\right)\right\} = x - 1$$

$$4\log_2\left(\frac{3040}{760}\right) = x - 1$$

x=1+4 $\log_2 (4)$ = 9