JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION
(Exam Date: 20-05-2018)

PART-1 : PHYSICS

1. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?
(A) The force applied on the particle is constant
(B) The speed of the particle is proportional to time
(C) The distance of the particle from the origin increses linerarly with time
(D) The force is conservative

Ans. (A,B,D)

Sol. $\frac{dk}{dt} = \gamma t$ **as** $k = \frac{1}{2}mv^2$

$\therefore \frac{dk}{dt} = mv\frac{dv}{dt} = \gamma t$

$\therefore m\int_0^v vdv = \gamma\int_0^t tdt$

$\frac{mv^2}{2} = \frac{\gamma t^2}{2}$

$v = \sqrt{\frac{\gamma}{m}}t$ .....(i)

$a = \frac{dv}{dt} = \sqrt{\frac{\gamma}{m}}$ = constant

since F = ma

$\therefore F = m\sqrt{\frac{\gamma}{m}} = \sqrt{\gamma m}$ = constant

2. Consider a thin square plate floating on a viscous liquid in a large tank. The height h of the liquid in the tank is much less than the width of the tank. The floating plate is pulled horizontally with a constant velocity u_0. Which of the following statements is (are) true ?

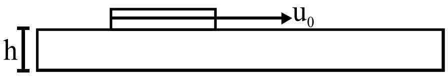
(A) The resistive force of liquid on the plate is inversely proportional to h

(B) The resistive force of liquid on the plate is independent of the area of the plate

(C) The tangential (shear) stress on the floor of the tank increases with u_0.

(D) The tangential (shear) stress on the plate varies linearly with the viscosity η of the liquid.

Ans. (A,C,D)

Sol. 

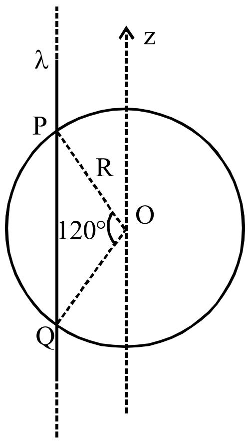
Viscous force is given by F = $-\eta A \frac{dv}{dy}$ since h is very small therefore, magnitude of viscous force is given by

$$F = \eta A \frac{\Delta v}{\Delta y}$$

$\therefore \; F = \frac{\eta A u_0}{h} \Rightarrow F \propto \eta \; \& \; F \propto u_0$; $\qquad F \propto \frac{1}{h}$, $F \propto A$

Since plate is moving with constant velocity, same force must be acting on the floor.

3. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true ?



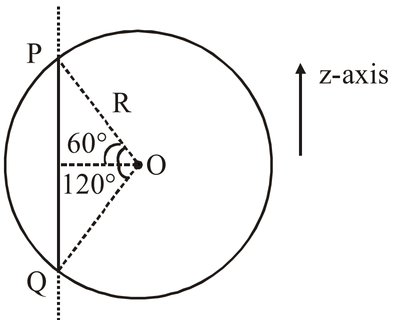
(A) The electric flux through the shell is $\sqrt{3}R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

Ans. (A,B)

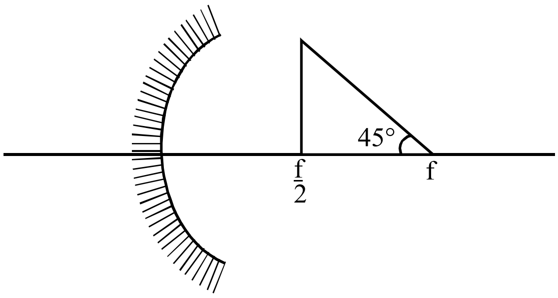
Sol. 

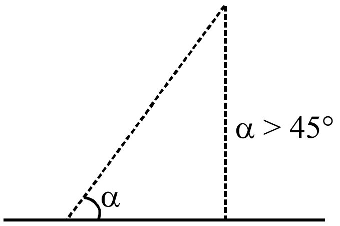
Field due to straight wire is perpendicular to the wire & radially outward. Hence $E_z = 0$

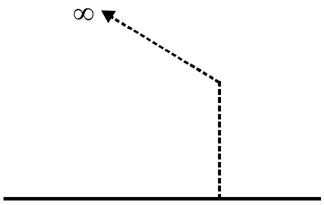
Length, PQ = 2R sin 60 = $\sqrt{3}R$ According to Gauss's law

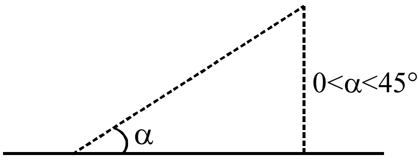
total flux = $\oint \vec{E}.\vec{ds} = \frac{q_{in}}{\epsilon_0} = \frac{\lambda\sqrt{3}R}{\epsilon_0}$

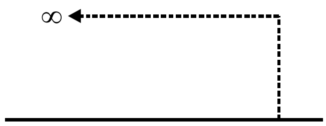
4. A wire is bent in the shape of a right angled triangle and is placed in front of a concave mirror of focal length f, as shown in the figure. Which of the figures shown in the four options qualitatively represent(s) the shape of the image of the bent wire ? (These figures are not to scale.) ?



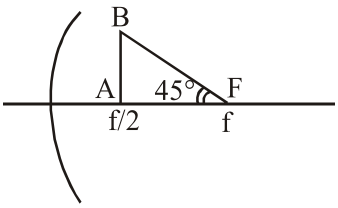
(A) 

(B) 

(C) 

(D) 

Ans. (D)

Sol. 

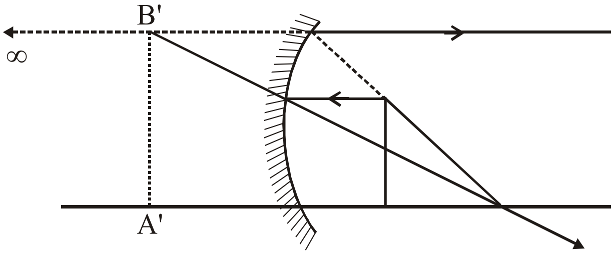
Distance of point A is f/2

Let A' is the image of A from mirror, for this image

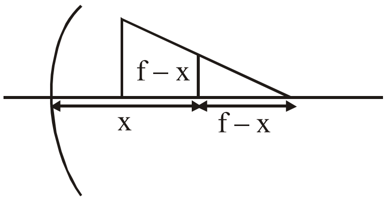
$$\frac{1}{v}+\frac{1}{-f/2}=\frac{1}{-f}$$

$$\frac{1}{v}=\frac{2}{f}-\frac{1}{f}=\frac{1}{f}$$

image of line AB should be perpendicular to the principle axis & image of F will form at infinity, therefor correct image diagram is



OR



$$\frac{f}{f-u}=\frac{h_2}{h_1}$$

$$h_2=\frac{-f(f-x)}{-f+x}$$

h_2 = f

5. In a radioactive decay chain, ${}^{232}_{90}Th$ nucleus decays to ${}^{212}_{82}Pb$ nucleus. Let N_α and N_β be the number of α and β^- particles, respectively, emitted in this decay process. Which of the following statements is (are) true ?

(A) $N_\alpha = 5$ (B) $N_\alpha = 6$ (C) $N_\beta = 2$ (D) $N_\beta = 4$

Ans. (A,C)

Sol. ${}^{232}_{90}Th$ is converting into ${}^{212}_{82}Pb$

Change in mass number (A) = 20

$\therefore$ no of α particle = $\frac{20}{4}=5$

Due to 5 α particle, z will change by 10 unit.

Since given change is 8, therefore no. of β particle is 2

6. In an experment to measure the speed of sound by a resonating air column, a tuning fork of frequency 500 Hz is used. The length of the air column is varied by changing the level of water in the resonance tube. Two successive resonances are heard at air columns of length 50.7 cm and 83.9 cm. Which of the following statements is (are) true ?

(A) The speed of sound determined from this experiment is 332 ms^{-1}

(B) The end correction in this experiment is 0.9 cm

(C) The wavelength of the sound wave is 66.4 cm

(D) The resonance at 50.7 cm corresponds to the fundamental harmonic

Ans. (A,C or A,B,C)

Sol. Let n_1 harmonic is corresponding to 50.7 cm & n_2 harmonic is corresponding 83.9 cm.

since both one consecutive harmonics.

$\therefore$ their difference = $\frac{\lambda}{2}$

$\therefore$ $\frac{\lambda}{2}$ = (83.9 – 50.7) cm

$$\frac{\lambda}{2} = 33.2 \text{ cm.}$$

$$\lambda = 66.4 \text{ cm}$$

$$\therefore \frac{\lambda}{4} = 16.6 \text{ cm}$$

length corresponding to fundamental mode must be close to $\frac{\lambda}{4}$ & 50.7 cm must be closed to an odd multiple of this length as $16.6 \times 3 = 49.8$ cm. therefore 50.7 is 3^{rd} harmonic

If end correction is e, then

$$e + 50.7 = \frac{3\lambda}{4}$$

$$e = 49.8 - 50.7 = -0.9 \text{ cm}$$

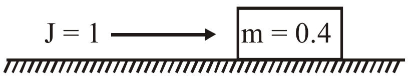
speed of sound, $v = f\lambda$

$$\therefore v = 500 \times 66.4 \text{ cm/sec} = 332.000 \text{ m/s}$$

7. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0e^{-t/\tau}$, where v_0 is a constant and $\tau = 4$ s. The displacement of the block, in metres, at $t = \tau$ is.................. Take $e^{-1} = 0.37$?

Ans. 6.30

Sol.



$$v = v_0e^{-t/\tau}$$

$$v_0 = \frac{J}{m} = 2.5 \text{ m/s}$$

$$v = v_0e^{-t/\tau}$$

$$\frac{dx}{dt} = v_0e^{-t/\tau}$$

$$\int_0^x dx = v_0\int_0^\tau e^{-t/\tau}dt \qquad \int e^{-x}dx = \frac{e^{-x}}{-1}$$

$$x = v_0\left[\frac{e^{-t/\tau}}{-\frac{1}{\tau}}\right]_0^\tau$$

$$x = 2.5\ (-4)\ (e^{-1} - e^{0})$$

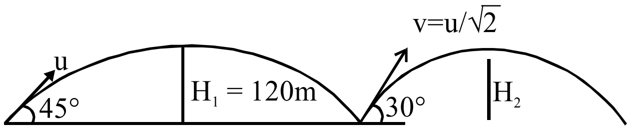
$$x = 25\ (-4)\ (0.37 - 1)$$

$$x = 6.30 \text{ ans.}$$

8. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is................

Ans. 30.00

Sol.



$$H_1 = \frac{u^2 \sin^2 45}{2g} = 120$$

$$\Rightarrow \frac{u^2}{4g} = 120 \quad ....(i)$$

when half of kinetic energy is lost $v = \frac{u}{\sqrt{2}}$

$$H_2 = \frac{\left(\frac{u}{\sqrt{2}}\right)^2 \sin^2 30}{2g} = \frac{u^2}{16g} \quad .....(ii)$$

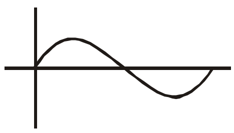
from (i) & (ii)

$H_2 = \frac{H_1}{4}$ = 30 m on 30.00

9. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin \omega t\, \hat{i}$ where E_0 = 1.0 N C^{-1} and $\omega = 10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is..........................

Ans. 2.00

Sol. n = 10^{-3} kg q = 1C t = 0

$E = E_0 \sin \omega t \equiv$ 

Force on particle will be

$F = qE = qE_0 \sin \omega t$

at v_{max}, a, F = 0 $\qquad qE_0 \sin \omega t = 0$

$F = qE_0 \sin\omega t$

$$\frac{dv}{dt} = q\frac{E_0}{m}\sin\omega t$$

$$\int_0^v dv = \int_0^{\pi/\omega}\frac{qE_0}{m}\sin\omega t\, dt$$

$$v - 0 = \frac{qE_0}{m\omega}[-\cos\omega t]_0^{\pi/\omega}$$

$$v - 0 = \frac{qE_0}{m\omega}[(-\cos\pi)-(-\cos 0)]$$

$$v = \frac{1\times 1}{10^{-3}10^{3}}\times 2 = 2 \text{ m/s}$$

Ans. 2. 00

10. A moving coil galvanometer has 50 turns and each turn has an area 2×10^{-4} m^2. The magnetic field porduced by the magnet inside the galvanometer is 0.02 T. The torsional constant of the suspension wire is 10^{-4} N m rad^{-1}. When a current flows through the galvanometer, a full scale deflection occurs if the coil rotates by 0.2 rad. The resistance of the coil of the galvanometer is 50 Ω. This galvanometer is to be converted into an ammeter capable of measuring current in the range 0 – 1.0 A. For this purpose, a shunt resistance is to be added in parallel to the galvanometer. The value of this shunt resistance, in ohms, is...........

Ans. 5.55

Sol. $n = 50$ turns $\quad A = 2 \times 10^{-4}\ m^2$

$B = 0.02$ T $\quad K = 10^{-4}$

$Q_m = 0.2$ rad $\quad R_g = 50\ \Omega$

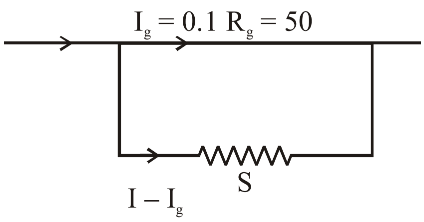
$I_A = 0 - 1.0$ A $\quad \tau = MB = C\theta$, $M = nIA$

$BINA = C\theta$

$0.02 \times 1 \times 50 \times 2 \times 10^{-4} = 10^{-4} \times 0.2\ 10$

$I_g = 0.1$ A

For galvanometer, resistance is to be connected to ammeter in shunt.



$I_g \times R_g = (I - I_g)S$

$0.1 \times 50 = (1 - 0.1)\ S$

$$S = \frac{50}{9} = 5.55$$

11. A steel wire of diameter 0.5 mm and Young's modulus 2×10^{11} N m^{-2} carries a load of mass M. The length of the wire with the load is 1.0 m. A vernier scale with 10 divisions is attached to the end of this wire. Next to the steel wire is a reference wire to which a main scale, of least count 1.0 mm, is attached. The 10 divisions of the vernier scale correspond to 9 divisions of the main scale. Initially, the zero of vernier scale coincides with the zero of main scale. If the load on the steel wire is increased by 1.2kg, the vernier scale division which coincides with a main scale division is....... Take $g = 10$ ms^{-2} and $\pi = 3.2$.

Ans. 3.00

Sol. $d = 0.5$ mm $\quad Y = 2 \times 10^{11} \quad \ell = 1m$

$$\Delta\ell = \frac{F\ell}{Ay} = \frac{mg\ell}{\frac{\pi d^2}{4}y} = \frac{1.2\times10\times1}{\frac{\pi}{4}\times(5\times10^{-4})^2\times2\times10^{11}}$$

$$\Delta\ell = \frac{1.2\times10}{\frac{3.2}{4}\times25\times10^{-8}\times2\times10^{11}}$$

$$= \frac{12}{0.8\times25\times2\times10^3} = \frac{12}{40\times10^3} = 0.3\text{mm}$$

so 3rd division of vernier scale will coincicle with main scale.

12. One mole of a monatomic ideal gas undergoes an adiabatic expansion in which its volume becomes eight times its initial value. If the initial temperature of the gas is 100 K and the universal gas constant $R = 8.0$ J mol^{-1} K^{-1}, the decrease in its internal energy, in Joule, is........... .

Ans. 900

Sol. $v_i = v$

$v_F = 8v$

For adiabatic process $\left\{\gamma = \frac{5}{3}\right.$ for monoatomic process

$$T_1V_1^{\gamma-1} = T_2.V_2^{\gamma-1}$$

$$100(v)^{2/3} = T_2(8v)^{2/3}$$

$T_2 = 25$ k

$$\Delta U = nc_V\Delta T = 1\left(\frac{FR}{2}\right)[100 - 25] = 12 \times 75 = 900 \text{ Joule}$$

13. In a photoelectric experiment a parallel beam of monochromatic light with power of 200 W is incident on a perfectly absorbing cathode of work function 6.25 eV. The frequency of light is just above the threshold frequency so that the photoelectrons are emitted with negligible kinetic energy. Assume that the photoelectron emission efficinecy is 100% A potential difference of 500 V is applied between the cathode and the anode. All the emitted electrons are incident normally on the anode and are absorbed. The anode experiences a force $F = n \times 10^{-4}$ N due to the impact of the electrons. The value of n is........ Mass of the electron $m_e = 9 \times 10^{-31}$ kg and 1.0 eV = 1.6×10^{-19} J.?

Ans. 24

Sol. Power = nhν   n = number of photons per second

Since KE = 0, hν = ϕ

200 = n[6.25 × 1.6 × 10^{-19} Joule]

$$n = \frac{200}{1.6\times10^{-19}\times6.25}$$

As photon is just above threshold frequency KE_{max} is zero and they are accelrated by potential difference of 500V.

$KE_f = q\Delta V$

$$\frac{P^2}{2m} = q\Delta V \Rightarrow P = \sqrt{2mq\Delta V}$$

Since efficiency is 100%, number of electrons = number of photons per second

As photon is completely absorbed force exerted = nmv

$$= \frac{200}{6.25\times1.6\times10^{-19}}\times\sqrt{2\left(9\times10^{-31}\right)\times1.6\times10^{-19}\times500}$$

$$= \frac{3\times200\times10^{-25}\times\sqrt{1600}}{6.25\times1.6\times10^{-19}} = \frac{2\times40}{6.25\times1.6}\times10^{-4}\times3 = 24$$

14. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is............. .

Ans. 3

Sol. $$\Delta E_{2-1} = 13.6\times z^2\left[1-\frac{1}{4}\right] = 13.6\times z^2\left[\frac{3}{4}\right]$$

$$\Delta E_{3-2} = 13.6\times z^2\left[\frac{1}{4}-\frac{1}{9}\right] = 13.6\times z^2\left[\frac{5}{36}\right]$$

$\Delta E_{2-1} = \Delta E_{3-2} + 74.8$

$$13.6\times z^2\left[\frac{3}{4}\right] = 13.6\times z^2\left[\frac{5}{36}\right]+74.8$$

$$13.6\times z^2\left[\frac{3}{4}-\frac{5}{36}\right] = 74.8$$

$z^2 = 9$

z = +3 ans

15. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

List–I	**List–II**
P. E is indpendent of d	**1.** A point charge Q at the origin
Q. $E \propto \frac{1}{d}$	2. A small dipole with point charges Q at $(0,0,\ell)$ and – Q at $(0,0,-\ell)$. Take $2\ell << d$
R. $E \propto \frac{1}{d^2}$	3. An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S. $E \propto \frac{1}{d^3}$	4. Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -\ell)$ has a charge density $-\lambda$. Take $2\ell << d$
	5. Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) P → 5 ; Q → 3, 4 ; R → 1 ; S → 2
(B) P → 5 ; Q → 3, ; R → 1,4 ; S → 2
(C) P → 5 ; Q → 3, ; R → 1,2 ; S → 4
(D) P → 4 ; Q → 2, 3 ; R → 1 ; S → 5

Ans. **(B)**

Sol. (i) $E = \frac{KQ}{d^2} \Rightarrow E \propto \frac{1}{d^2}$

(ii) Dipole

$$E = \frac{2kp}{d^3}\sqrt{1+3\cos^2\theta}$$

$E \propto \frac{1}{d^3}$ for dipole

(iii) For line charge

$$E = \frac{2k\lambda}{d}$$

$$E \propto \frac{1}{d}$$

(iv) $E = \frac{2K\lambda}{d-\ell} - \frac{2K\lambda}{d+\ell}$

$$= 2K\lambda\left[\frac{d+\ell-d+\ell}{d^2-\ell^2}\right]$$

$$E = \frac{2K\lambda(2\ell)}{d^2\left[1-\frac{\ell^2}{d^2}\right]}$$

$$E \propto \frac{1}{d^2}$$

(v) Electric field due to sheet

$$\in = \frac{\sigma}{2\in_0}$$

$\in$ = v is independent of r

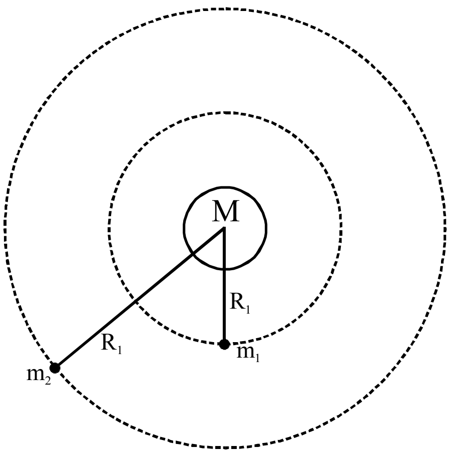
16. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II.

List–I	List–II
P. $\frac{v_1}{v_2}$	1. $\frac{1}{8}$
Q. $\frac{L_1}{L_2}$	2. 1
R. $\frac{K_1}{K_2}$	3. 2
S. $\frac{T_1}{T_2}$	4. 8

(A) P $\rightarrow$ 4 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 1 ; S $\rightarrow$ 3
(B) P $\rightarrow$ 3 ; Q $\rightarrow$ 2 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1
(C) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 1 ; S $\rightarrow$ 4
(D) P $\rightarrow$ 2 ; Q $\rightarrow$ 3 ; R $\rightarrow$ 4 ; S $\rightarrow$ 1

Ans. (B)

Sol.



$$\left.\begin{aligned}\frac{m_1}{m_2} = 2\\ \frac{R_1}{R_2} = \frac{1}{4}\end{aligned}\right\}\text{given}$$

$$\frac{GMm_1}{R_1^2} = \frac{m_1 v_1^2}{R_1}$$

$$v_1^2 = \frac{GM}{R_1}, \quad v_2^2 = \frac{GM}{R_2}$$

$$\frac{v_1^2}{v_2^2} = \frac{R_2}{R_1} = 4$$

(P) $\frac{v_1}{v_2} = 2$

(Q) $L = mvR$

$$\frac{L_1}{L_2} = \frac{m_1 v_1 R_1}{m_2 v_2 R_2} = 2 \times 2 \times \frac{1}{4} = 1$$

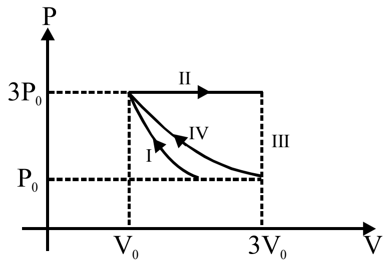
(R) $K = \frac{1}{2} mv^2$

$$\frac{K_1}{K_2} = \frac{m_1 v_1^2}{m_2 v_2^2} = 2 \times (2)^2 = 8$$

(S) $T = 2\pi R/V$

$$\frac{T_1}{T_2} = \frac{R_1}{v_1} \times \frac{v_2}{R_2} = \frac{R_1}{R_2} \times \frac{v_2}{v_1} = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

17. One mole of a monatomic ideal gas undergoes four thermodynamic processes as shown schematically in the PV-diagram below. Among these four processes, one is isobaric, one is isochoric, one is isothermal and one is adiabatic. Match the processes mentioned in List-I with the corresponding statements in List–II.



List–I		**List–II**
P.	In process I	1. Work done by the gas is zero
Q.	In process II	2. Temperature of the gas remains unchanged
R.	In process III	3. No heat is exchanged between the gas and its surroundings
S.	In process IV	4. Work done by the gas is 6 P_0V_0

(A) $P \to 4$; $Q \to 3$; $R \to 1$; $S \to 2$

(B) $P \to 1$; $Q \to 3$; $R \to 2$; $S \to 4$

(C) $P \to 3$; $Q \to 4$; $R \to 1$; $S \to 2$

(D) $P \to 3$; $Q \to 4$; $R \to 2$; $S \to 1$

Ans. **(C)**

Sol. Process – I is an adiabatic process

$\Delta Q = \Delta U + W$ $\qquad\qquad \Delta Q = 0$

$W = -\Delta U$

Volume of gas is decreasing $\Rightarrow$ $W < 0$

$\Delta U > 0$

$\Rightarrow$ Temperatuer of gas increases.

$\Rightarrow$ No heat is exchanged between the gas and surrounding.

Process – II is an isobaric process

(Pressure remain constant)

$W = P\,\Delta V = 3P_0[3V_0 - V_0] = 6P_0V_0$

Process - III is an isochoric process

(Volume remain constant)

$\Delta Q = \Delta U + W$

$W = 0$

$\Delta Q = \Delta U$

Process – IV is an isothermal process

(Temperature remains constant)

$\Delta Q = \Delta U + W$

$\Delta U = 0$

18. In the List-I below, four different paths of a particle are given as functions of time. In these functions, α and β are positive constants of appropriate dimensions and $\alpha \neq B$. In each case, the force acting on the particle is either zero or conservative. In List–II, five physical quantities of the particle are mentioned; $\vec{p}$ is the linear momentum $\vec{L}$ is the angular momentum about the origin, K is the kinetic energy, U is the potential energy and E is the total energy. Match each path in List–I with those quantities in List–II, which are conserved for that path

	List–I		**List–II**
P.	$\vec{r}(t) = \alpha t\,\hat{i} + \beta t\,\hat{j}$	1.	$\vec{p}$
Q.	$\vec{r}(t) = \alpha\cos\omega t\,\hat{i} + \beta\sin\omega t\,\hat{j}$	2.	$\vec{L}$
R.	$\vec{r}(t) = \alpha(\cos\omega t\,\hat{i} + \sin\omega t\,\hat{j})$	3.	K
S	$\vec{r}(t) = \alpha t\,\hat{i} + \frac{\beta}{2}t^2\,\hat{j}$	4.	U
		5.	E

(A) P $\rightarrow$ 1,2,3,4,5 ; Q $\rightarrow$ 2,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 5

(B) P $\rightarrow$ 1,2,3,4,5 ; Q $\rightarrow$ 3,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 2,5

(C) P $\rightarrow$ 2,3,4 ; Q $\rightarrow$ 5; R $\rightarrow$ 1,2,4 ; S $\rightarrow$ 2,5

(D) P $\rightarrow$ 1,2,3,5 ; Q $\rightarrow$ 2,5 ; R $\rightarrow$ 2,3,4,5 ; S $\rightarrow$ 2,5

Ans. **(A)**

Sol. (P) $\vec{r}(t) = \alpha t\hat{i} + \beta t\hat{j}$

$$\vec{v} = \frac{d\vec{r}(t)}{dt} = \alpha\hat{i} + \beta\hat{j} \ \{\text{constant}\}$$

$$\vec{a} = \frac{\overrightarrow{dv}}{dt} = 0$$

$\vec{P} = m\vec{v}$ (remain constant)

$$k = \frac{1}{2}mv^2 \ \{\text{remain constant}\}$$

$$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial y}\hat{i}\right] = 0$$

$\Rightarrow$ U $\rightarrow$ constant

E = K + U

$$\frac{d\vec{L}}{dt} = \vec{\tau} = \vec{r} \times \vec{F} = 0$$

$\vec{L}$ = constant

(Q) $\vec{r} = \alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}$

$$\vec{v} = \frac{d\vec{r}}{dt} = -\alpha\omega\sin(\omega t)\hat{i} + \beta\omega\cos(\omega t)\hat{j}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = -\alpha\omega^2\cos(\omega t)\hat{i} - \beta\omega^2\sin(\omega t)\hat{j}$$

$$= -\omega^2\left[\alpha\cos(\omega t)\hat{i} + \beta\sin(\omega t)\hat{j}\right]$$

$$\vec{a} = -\omega^2\vec{r}$$

$\vec{\tau} = \vec{r} \times \vec{F} = 0$ $\{\vec{r}$ and $\vec{F}$ are parallel$\}$

$$\Delta U = -\int \vec{F}.dr = +\int_0^r m\omega^2.r.dr$$

$$\Delta U = m\omega^2\left[\frac{r^2}{2}\right]$$

$U \propto r^2$

$r = \sqrt{\alpha^2\cos^2(\omega t) + \beta^2\sin^2(\omega t)}$

r is a function of time (t)

U depends on r hence it will change with time

Total energy remain constant because force is central.

(R) $\vec{r}(t) = \alpha\left(\cos\omega t\hat{i} + \sin(\omega t)\hat{j}\right)$

$$\vec{v}(t) = \frac{d\vec{r}(t)}{dt} = \alpha\left[-\omega\sin(\omega t)\hat{i} + \omega\cos(\omega t)\hat{j}\right]$$

$|\vec{v}| = \alpha\omega$ (Speed remains constant)

$$\vec{a}(t) = \frac{d\vec{v}(t)}{dt} = \alpha\left[-\omega^2\cos(\omega t)\hat{i} - \omega^2\sin(\omega t)\hat{j}\right]$$

$$= -\alpha\omega^2\left[\cos(\omega t)\hat{i} + \sin(\omega t)\hat{j}\right]$$

$$\vec{a}(t) = -\omega^2(\vec{r})$$

$$\vec{\tau} = \vec{F}\times\vec{r} = 0$$

$|\vec{r}| = \alpha$ (remain constant)

Force is central in nature and distance from fixed point is constant.

Potential energy remains constant

Kinetic energy is also constant (speed is constant)

(S) $\vec{r} = \alpha t\hat{i} + \frac{\beta}{2}t^2\hat{j}$

$\vec{v} = \frac{d\vec{r}}{dt} = \alpha t\,\hat{i} + \beta t\,\hat{j}$ (speed of particle depends on 't')

$\vec{a} = \frac{d\vec{v}}{dt} = \beta\hat{j}$ {constant}

$\vec{F} = m\vec{a}$ {constant}

$$\Delta U = -\int \vec{F}.d\vec{r} = -m\int_0^t \beta\hat{j}.\left(\alpha\hat{i} + \beta t\hat{j}\right)dt$$

$$U = \frac{-m\beta^2 t^2}{2}$$

$$k = \frac{1}{2}mv^2 = \frac{1}{2}m\left(\alpha^2 + \beta^2 t^2\right)$$

$E = k + U = \frac{1}{2}m\alpha^2$ [remain constant]

JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-1 : MATHEMATICS

SECTION 1

1. For any positive integer n, define $f_n : (0, \infty) \to \mathbb{R}$ as

$$f_n(x) = \sum_{j=1}^{n} \tan^{-1}\left(\frac{1}{1+(x+j)(x+j-1)}\right) \text{ for all } x \in (0, \infty).$$

(Here, the inverse trigonometric function $\tan^{-1}x$ assume values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.)

Then, which of the following statement(s) is (are) TRUE ?

(A) $\sum_{j=1}^{5} \tan^2\left(f_j(0)\right) = 55$

(B) $\sum_{j=1}^{10} \left(1+f_j'(0)\right)\sec^2\left(f_j(0)\right) = 10$

(C) For any fixed positive integer n, $\lim\limits_{x\to\infty} \tan\left(f_n(x)\right) = \frac{1}{n}$

(D) For any fixed positive integer n, $\lim\limits_{x\to\infty} \sec^2\left(f_n(x)\right) = 1$

Ans. (D)

Sol. $f_n(x) = \sum\limits_{j=1}^{n} \tan^{-1}\left(\frac{(x+j)-(x+j-1)}{1+(x+j)(x+j-1)}\right)$

$$f_n(x) = \sum_{j=1}^{n} [\tan^{-1}(x+j) - \tan^{-1}(x+j-1)]$$

$f_n(x) = \tan^{-1}(x + n) - \tan^{-1}x$

$\therefore\ \tan(f_n(x)) = \tan[\tan^{-1}(x + n) - \tan^{-1}x]$

$$\tan(f_n(x)) = \frac{(x+n)-x}{1+x(x+n)}$$

$$\tan(f_n(x)) = \frac{n}{1+x^2+nx}$$

$\therefore\ \sec^2(f_n(x)) = 1 + \tan^2(f_n(x))$

$$\sec^2(f_n(x)) = 1+\left(\frac{n}{1+x^2+nx}\right)^2$$

$$\lim_{x\to\infty}\sec^2(f_n(x)) = \lim_{x\to\infty}1+\left(\frac{n}{1+x^2+nx}\right)^2 = 1$$

2. Let T be the line passing through the points P(–2, 7) and Q(2, –5). Let F_1 be the set of all pairs of circles (S_1, S_2) such that T is tangents to S_1 at P and tangent to S_2 at Q, and also such that S_1 and S_2 touch each other at a point, say, M. Let E_1 be the set representing the locus of M as the pair (S_1, S_2) varies in F_1. Let the set of all straight line segments joining a pair of distinct points of E_1 and passing through the point R(1, 1) be F_2. Let E_2 be the set of the mid-points of the line segments in the set F_2. Then, which of the following statement(s) is (are) TRUE ?

(A) The point (–2, 7) lies in E_1

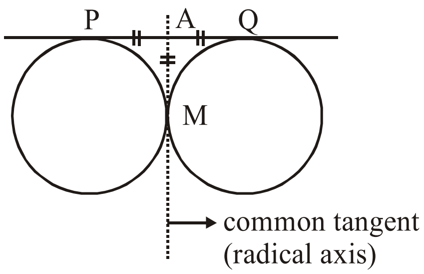
(B) The point $\left(\frac{4}{5},\frac{7}{5}\right)$ does **NOT** lie in E_2

(C) The point $\left(\frac{1}{2},1\right)$ lies in E_2

(D) The point $\left(0,\frac{3}{2}\right)$ does **NOT** lie in E_1

Ans. (D)

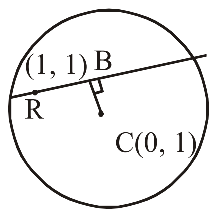
Sol.



AP = AQ = AM

Locus of M is a circle having PQ as its diameter

Hence, $E_1 : (x - 2)(x + 2) + (y - 7)(y + 5) = 0$ and $x \neq \pm 2$



Locus of B (midpoint)

is a circle having RC as its diameter

$E_2 : x(x - 1) + (y - 1)^2 = 0$

Now, after checking the options, we get (D)

3. Let S be the of all column matrices $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$ such that $b_1, b_2, b_3 \in \mathbb{R}$ and the system of equations (in real variables)

$$-x + 2y + 5z = b_1$$
$$2x - 4y + 3z = b_2$$
$$x - 2y + 2z = b_3$$

has at least one solution. Then, which of the following system(s) (in real variables) has (have) at least one solution of each $\begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \in S$?

(A) $x + 2y + 3z = b_1$, $4y + 5z = b_2$ and $x + 2y + 6z = b_3$

(B) $x + y + 3z = b_1$, $5x + 2y + 6z = b_2$ and $-2x - y - 3z = b_3$

(C) $-x + 2y - 5z = b_1$, $2x - 4y + 10z = b_2$ and $x - 2y + 5z = b_3$

(D) $x + 2y + 5z = b_1$, $2x + 3z = b_2$ and $x + 4y - 5z = b_3$

Ans. (A,D)

Sol. We find D = 0 & since no pair of planes are parallel, so there are infinite number of solutions.

Let $\alpha P_1 + \lambda P_2 = P_3$

$\Rightarrow$ $P_1 + 7P_2 = 13P_3$

$\Rightarrow$ $b_1 + 7b_2 = 13b_3$

(A) $D \neq 0$ $\Rightarrow$ unique solution for any b_1, b_2, b_3

(B) $D = 0$ but $P_1 + 7P_2 \neq 13P_3$

(C) As planes are parallel and there exist infinite ordered triplet for which they will be non coincident although satisfying $b_1 + 7b_2 = 13b_3$.

$\therefore$ rejected.

(D) $D \neq 0$

4. Consider two straight lines, each of which is tangent to both the circle $x^2 + y^2 = \frac{1}{2}$ and the parabola $y^2 = 4x$. Let these lines intersect at the point Q. Consider the ellipse whose center is at the origin O(0, 0) and whose semi-major axis is OQ. If the length of the minor axis of this ellipse is $\sqrt{2}$, then the which of the following statement(s) is (are) TRUE ?

(A) For the ellipse, the eccentricity is $\frac{1}{\sqrt{2}}$ and the length of the latus rectum is 1

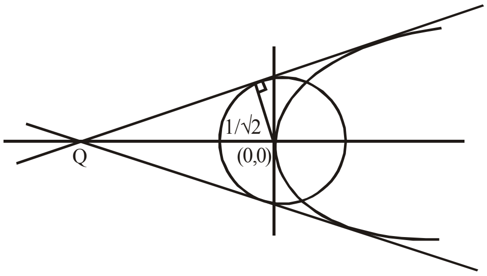
(B) For the ellipse, the eccentricity is $\frac{1}{2}$ and the length of the latus rectum is $\frac{1}{2}$

(C) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{4\sqrt{2}}(\pi - 2)$

(D) The area of the region bounded by the ellipse between the lines $x = \frac{1}{\sqrt{2}}$ and $x = 1$ is $\frac{1}{16}(\pi - 2)$

Ans. (A,C)

Sol.



Let equation of common tangent is $y = mx + \frac{1}{m}$

$$\therefore \quad \left|\frac{0+0+\frac{1}{m}}{\sqrt{1+m^2}}\right| = \frac{1}{\sqrt{2}} \Rightarrow m^4 + m^2 - 2 = 0 \Rightarrow m = \pm 1$$

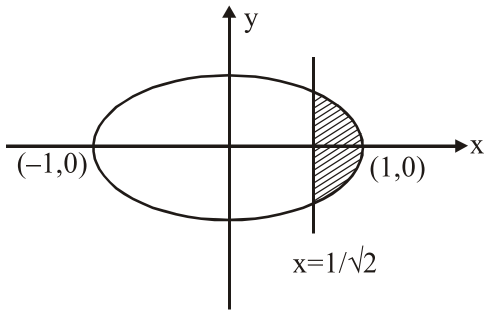
Equation of common tangents are $y = x + 1$ and $y = -x - 1$

point Q is $(-1, 0)$

$\therefore$ Equation of ellipse is $\frac{x^2}{1} + \frac{y^2}{1/2} = 1$

(A) $\quad e = \sqrt{1-\frac{1}{2}} = \frac{1}{\sqrt{2}}$ and $LR = \frac{2b^2}{a} = 1$

(C)



$$\text{Area} \quad 2.\int_{1/\sqrt{2}}^{1} \frac{1}{\sqrt{2}}.\sqrt{1-x^2}\,dx = \sqrt{2}\left[\frac{x}{2}\sqrt{1-x^2} + \frac{1}{2}\sin^{-1}x\right]_{1/\sqrt{2}}^{1}$$

$$= \sqrt{2}\left[\frac{\pi}{4} - \left(\frac{1}{4}+\frac{\pi}{8}\right)\right] = \sqrt{2}\left(\frac{\pi}{8}-\frac{1}{4}\right) = \frac{\pi-2}{4\sqrt{2}}$$

correct answer are (A) and (D)

5. Let s, t, r be the non-zero complex numbers and L be the set of solutions $z = x + iy \left(x, y \in \mathbb{R}, i = \sqrt{-1}\right)$ of the equation $sz + t\overline{z} + r = 0$, where $\overline{z} = x - iy$. Then, which of the following statement(s) is (are) TRUE ?

(A) If L has exactly one element, then $|s| \neq |t|$

(B) If $|s| = |t|$, then L has infinitely many elements

(C) The number of elements in $L \cap \{z : |z - 1 + i| = 5\}$ is at most 2

(D) If L has more than one element, then L has infinitely many elements

Ans. **(A,C,D)**

Sol. Given

$$sz + t\overline{z} + r = 0 \qquad (1)$$

$\overline{z} = x - iy$ (Conjugate of z)

Taking conjugate throughout $\overline{s}\overline{z} + \overline{t}z + \overline{r} = 0 \qquad (2)$

Adding (1) and (2)

$$\left(s + \overline{t}\right)z + \left(\overline{s} + t\right)\overline{z} + \left(r + \overline{r}\right) = 0$$

And Subtracting (1) and (2)

$$\left(s - \overline{t}\right)z + \left(t - \overline{s}\right)\overline{z} + \left(r - \overline{r}\right) = 0$$

For unique solution

$$\frac{t + \overline{s}}{t - s} \neq \frac{s + \overline{t}}{s - \overline{t}}$$

On further simplification $\Rightarrow |t| \neq |s|$

Hence option A proved.

If the lines coincide, then

$$\frac{t + \overline{s}}{t - \overline{s}} = \frac{\overline{t} + s}{s - \overline{t}} = \frac{r + \overline{r}}{r - \overline{r}}$$

On comparing

$$\frac{t + \overline{s}}{t - \overline{s}} = \frac{r + \overline{r}}{r - \overline{r}}$$

and simplification, we get $\Rightarrow |s| = |t|$

The lines can be parallel or coincidental.

Since, no concrete outcome.

Hence, option B is not correct.

Clearly L is either a single or represents a line and $|z-1+i|=5$ represents a circle.

$\therefore$ Intersection of L and $\{|z-1+i|=5\}$ is ATMOST 2.

Hence, option C is correct.

Let $s=\alpha_1+i\beta_1; t=\alpha_2+i\beta_2$ *and* $r=\alpha_3+i\beta_3$

Then $sz+t\bar{z}+r=0$

$$\Rightarrow (\alpha_1+\alpha_2)x+(\beta_2-\beta_1)y+\alpha_3=0$$
$$\textit{and} \quad (\beta_1+\beta_2)x+(\alpha_1-\alpha_2)y+\beta_3=0$$

If L has more than 1 element then it implies L will have $\propto$ elements.

As L represents linear equation in x and y.

Hence, option D is correct.

6. Let $f:(0,\pi)\to\mathbb{R}$ be a twice differentiable function such that

$$\lim_{t\to x}\frac{f(x)\sin t-f(t)\sin x}{t-x}=\sin^2 x \text{ for all } x\in(0,\pi).$$

If $f\left(\frac{\pi}{6}\right)=-\frac{\pi}{12}$, then which of the following statement(s) is (are) TRUE ?

(A) $f\left(\frac{\pi}{4}\right)=\frac{\pi}{4\sqrt{2}}$

(B) $f(x)<\frac{x^4}{6}-x^2$ for all $x\in(0,\pi)$

(C) There exists $\alpha\in(0,\pi)$ such that $f'(\alpha)=0$

(D) $f''\left(\frac{\pi}{2}\right)+f\left(\frac{\pi}{2}\right)=0$

Ans. (B,C,D)

Sol. $\lim\limits_{t\to x}\dfrac{f(x)\sin t - f(t)\sin x}{t-x} = \sin^2 x$

by using L'Hopital

$\lim\limits_{t\to x}\dfrac{f(x)\cos t - f'(t)\sin x}{1} = \sin^2 x$

$\Rightarrow$ f(x)cosx –f'(x)sinx = sin^{2}x

$\Rightarrow \quad -\left(\dfrac{f'(x)\sin x - f(x)\cos x}{\sin^2 x}\right) = 1$

$\Rightarrow \quad -d\left(\dfrac{f(x)}{\sin x}\right) = 1$

$\Rightarrow \quad \dfrac{f(x)}{\sin x}$=-x+c

Put $x = \dfrac{\pi}{6}$ & $f\left(\dfrac{\pi}{6}\right) = -\dfrac{\pi}{12}$

$\therefore$ c = 0 $\Rightarrow$ f(x) = –xsinx

(A) $f\left(\dfrac{\pi}{4}\right) = \dfrac{-\pi}{4}\dfrac{1}{\sqrt{2}}$

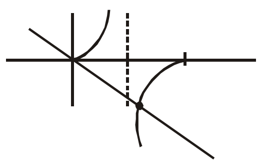
(B) f(x) = –x sinx

as sinx > $x - \dfrac{x^3}{6}$, –x sinx < – x^2 + $\dfrac{x^4}{6}$

$\therefore$ f(x) < – x^2 + $\dfrac{x^4}{6}$ $\forall$ x $\in$ (0, π)

(C) f'(x) = –sinx – x cosx

f'(x) = 0 $\Rightarrow$ tanx = –x $\Rightarrow$ there exist $\alpha \in (0, \pi)$ for which f'(α) = 0



(D) f''(x) = –2cosx + xsinx

$f''\left(\dfrac{\pi}{2}\right) = \dfrac{\pi}{2}$, $f\left(\dfrac{\pi}{2}\right) = -\dfrac{\pi}{2}$

$f''\left(\dfrac{\pi}{2}\right) + f\left(\dfrac{\pi}{2}\right) = 0$

SECTION 2

7. The value of the integral

$$\int_0^{\frac{1}{2}} \frac{1+\sqrt{3}}{\left((x+1)^2(1-x)^6\right)^{\frac{1}{4}}} dx$$

is ______ .

Ans. (2)

Sol. $\displaystyle\int_0^{\frac{1}{2}} \frac{\left(1+\sqrt{3}\right)dx}{\left[(1+x)^2(1-x)^6\right]^{1/4}}$

$$\int_0^{\frac{1}{2}} \frac{\left(1+\sqrt{3}\right)dx}{(1+x)^2\left[\frac{(1-x)^6}{(1+x)^6}\right]^{1/4}}$$

Put $\frac{1-x}{1+x} = t \Rightarrow \frac{-2dx}{(1+x)^2} = dt$

$$I = \int_1^{1/3} \frac{\left(1+\sqrt{3}\right)dt}{-2t^{6/4}} = \frac{-(1+\sqrt{3})}{2} \times \left|\frac{-2}{\sqrt{t}}\right|_1^{1/3} = \left(1+\sqrt{3}\right)\left(\sqrt{3}-1\right) = 2$$

8. Let P be a matrix of order 3×3 such that all the entries in P are from the set $\{-1, 0, 1\}$. Then, the maximum possible value of the determinant of P is ______ .

Ans. (4)

Sol. $\Delta = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = \underbrace{(a_1b_2c_3 + a_2b_3c_1 + a_3b_1c_2)}_{x} - \underbrace{(a_3b_2c_1 + a_2b_1c_3 + a_1b_3c_2)}_{y}$

Now if $x \le 3$ and $y \ge -3$

the Δ can be maximum 6

But it is not possible

as $x = 3 \Rightarrow$ each term of $x = 1$

and $y = 3 \Rightarrow$ each term of $y = -1$

$\Rightarrow \prod_{i=1}^{3} a_ib_ic_i = 1$ and $\prod_{i=1}^{3} a_ib_ic_i = -1$

which is contradiction

so now next possibility is 4

which is obtained as $\begin{vmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 1 & -1 & 1 \end{vmatrix} = 1(1+1) - 1(-1-1) + 1(1-1) = 4$

9. Let X be a set with exactly 5 elements and Y be a set with exactly 7 elements. If α is the number of one-one functions from X to Y and β is the number of onto functions from Y to X, then the value of $\frac{1}{5!}(\beta - \alpha)$ is _____ .

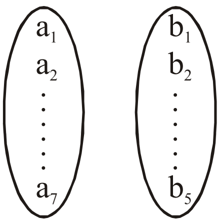
Ans. (119)

Sol. $n(X) = 5$

$n(Y) = 7$

$\alpha \rightarrow$ Number of one-one function $= {}^7C_5 \times 5!$

$\beta \rightarrow$ Number of onto function Y to X



1, 1, 1, 1, 3   1, 1, 1, 2, 2

$$\frac{7!}{3!4!} \times 5! + \frac{7!}{(2!)^3 3!} \times 5! = \left({}^7C_3 + 3.{}^7C_3\right)5! = 4 \times {}^7C_3 \times 5!$$

$$\frac{\beta - \alpha}{5!} = 4 \times {}^7C_3 - {}^7C_5 = 4 \times 35 - 21 = 119$$

10. Let $f : \mathbb{R} \rightarrow \mathbb{R}$ be a differentiable function with $f(0) = 0$. If $y = f(x)$ satisfies the differential equation

$$\frac{dy}{dx} = (2+5y)(5y-2),$$

then the value of $\lim\limits_{x \to -\infty} f(x)$ is _____ .

Ans. (0.4)

Sol. $\frac{dy}{dx} = 25y^2 - 4$

So, $\frac{dy}{25y^2 - 4} = dx$

Integrating, $\frac{1}{25} \times \frac{1}{2 \times \frac{2}{5}} \ln \left| \frac{y - \frac{2}{5}}{y + \frac{2}{5}} \right| = x + c$

$$\Rightarrow \quad \ln\left|\frac{5y-2}{5y+2}\right| = 20(x+c)$$

Now, c = 0 as f(0) = 0

Hence $\left|\frac{5y-2}{5y+2}\right| = e^{(20x)}$

$$\underset{x\to-\infty}{\text{let}}\left|\frac{5f(x)-2}{5f(x)+2}\right| = \underset{x\to-\infty}{\text{let}}\ e^{(20x)}$$

Now, RHS = 0 $\Rightarrow \underset{x\to-\infty}{\text{let}}(5f(x)-2)=0$

$$\Rightarrow \quad \underset{x\to-\infty}{\text{let}}\ f(x) = \frac{2}{5}$$

11. Let $f:\mathbb{R}\to\mathbb{R}$ be a differentiable function with $f(0) = 1$ and satisfying the equation

$$f(x+y) = f(x)f'(y) + f'(x)f(y) \text{ for all } x, y \in \mathbb{R}.$$

Then, then value of $\log_e(f(4))$ is _____ .

Ans. (2)

Sol. P(x, y) : $f(x + y) = f(x)f'(y) + f'(x) f(y)\ \forall\ x, y \in R$

P(0, 0) : $f(0) = f(0)f'(0) + f'(0) f(0)$

$\Rightarrow$ 1 = 2f'(0)

$$\Rightarrow \quad f'(0) = \frac{1}{2}$$

P(x, 0) : $f(x) = f(x).\ f'(0) + f'(x).f(0)$

$$\Rightarrow \quad f(x) = \frac{1}{2}f(x) + f'(x)$$

$$\Rightarrow \quad f'(x) = \frac{1}{2}f(x)$$

$$\Rightarrow \quad f(x) = e^{\frac{1}{2}x}$$

$\Rightarrow$ ln(f(4)) = 2

12. Let P be a point in the first octant, whose image Q in the plane x + y = 3 (that is, the line segment PQ is perpendicular to the plane x + y = 3 and the mid-point of PQ lies in the plane x + y = 3) lies on the z-axis. Let the distance of P from the x-axis be 5. If R is the image of P in the xy-plane, then the length of PR is _____ .

Ans. (8)

Sol. Let $P(\alpha, \beta, \gamma)$

$Q(0, 0, \gamma)$ &

$R(\alpha, \beta, -\gamma)$

Now, $\overline{PQ} \parallel \hat{i}+\hat{j} \Rightarrow \left(\alpha\hat{i}+\beta\hat{j}\right) \parallel \left(\hat{i}+\hat{j}\right)$

$\Rightarrow \quad \alpha = \beta$

Also, mid point of PQ lies on the plane $\Rightarrow \dfrac{\alpha}{2}+\dfrac{\beta}{2}=3 \Rightarrow \alpha + \beta = 6 \Rightarrow \alpha = 3$

Now, distance of point P from X-axis is $\sqrt{\beta^2+\gamma^2}=5$

$\Rightarrow \quad \beta^2+\gamma^2=25 \;\Rightarrow\; \gamma^2=16$

as $\beta = \alpha = 3$

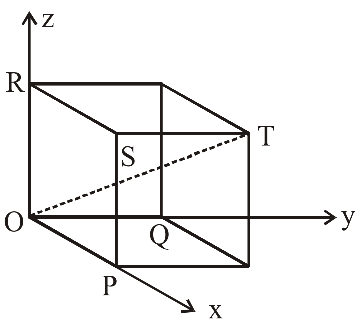
as $\gamma = 4$

Hence, $PR = 2\gamma = 8$

13. Consider the cube in the first octant with sides OP, OQ and OR of length 1, along the x-axis, y-axis and z-axis, respectively, where O(0, 0, 0) is the origin. Let $S\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)$ be the centre of the cube and T be the vertex of the cube opposite to the origin O such that S lies on the diagonal OT. If $\vec{p}=\overrightarrow{SP}$, $\vec{q}=\overrightarrow{SQ}$, $\vec{r}=\overrightarrow{SR}$ and $\vec{t}=\overrightarrow{ST}$, then the value of $\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|$ is _____ .

Ans. (0.5)

Sol.



$\vec{p}=\overrightarrow{SP}=\left(\frac{1}{2},-\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}-\hat{j}-\hat{k}\right)$

$$\vec{q}=\overrightarrow{SQ}=\left(-\frac{1}{2},\frac{1}{2},-\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}+\hat{j}-\hat{k}\right)$$

$$\vec{r}=\overrightarrow{SR}=\left(-\frac{1}{2},-\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(-\hat{i}-\hat{j}+\hat{k}\right)$$

$$\vec{t}=\overrightarrow{ST}=\left(\frac{1}{2},\frac{1}{2},\frac{1}{2}\right)=\frac{1}{2}\left(\hat{i}+\hat{j}+\hat{k}\right)$$

$$\left|(\vec{p}\times\vec{q})\times(\vec{r}\times\vec{t})\right|=\frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 1 & -1 & -1\\ -1 & 1 & -1\end{vmatrix}\times\frac{1}{4}\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & -1 & 1\\ 1 & 1 & 1\end{vmatrix}$$

$$=\frac{1}{16}\left|\left(2\hat{i}+2\hat{j}\right)\times\left(-2\hat{i}+2\hat{j}\right)\right|=\left|\frac{\hat{k}}{2}\right|=\frac{1}{2}$$

14. Let $X=\left({}^{10}C_1\right)^2+2\left({}^{10}C_2\right)^2+3\left({}^{10}C_3\right)^2+\ldots+10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r$, $r\in\{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is _____ .

Ans. (646)

Sol. $X=\sum_{r=0}^{n} r.\left({}^{n}C_r\right)^2; n=10$

$$X=n.\sum_{r=0}^{n}{}^{n}C_r.{}^{n-1}C_{r-1}$$

$$X=n.\sum_{r=1}^{n}{}^{n}C_{n-r}.{}^{n-1}C_{r-1}$$

$$X=n.{}^{2n-1}C_{n-1};\ n=10$$

$$X=10.{}^{19}C_9$$

$$\frac{X}{1430}=\frac{1}{143}.{}^{19}C_9$$

$= 646$

SECTION 3

15. Let $E_1 = \left\{x \in \mathbb{R} : x \neq 1 \text{ and } \frac{x}{x-1} > 0\right\}$

and $E_2 = \left\{x \in E_1 : \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right) \text{ is a real number}\right\}$.

$\left(\text{Here, the inverse trigonometric function } \sin^{-1}x \text{ assumes values in } \left[-\frac{\pi}{2}, \frac{\pi}{2}\right].\right)$

Let $f : E_1 \to \mathbb{R}$ be the function defined by $f(x) = \log_e\left(\frac{x}{x-1}\right)$

and $g : E_2 \to \mathbb{R}$ be the function defined by $g(x) = \sin^{-1}\left(\log_e\left(\frac{x}{x-1}\right)\right)$.

LIST-I	**LIST-II**
P. The range of f is	**1.** $\left(-\infty, \frac{1}{1-e}\right] \cup \left[\frac{e}{e-1}, \infty\right)$
Q. The range of g contains	**2.** $(0, 1)$
R. The domain of f contains	**3.** $\left[-\frac{1}{2}, \frac{1}{2}\right]$
S. The domain of g is	**4.** $(-\infty, 0) \cup (0, \infty)$
	5. $\left(-\infty, \frac{e}{e-1}\right]$
	6. $(-\infty, 0) \cup \left(\frac{1}{2}, \frac{e}{e-1}\right]$

The correct option is :

(A) **P → 4; Q → 2; R → 1; S → 1**

(B) **P → 3; Q → 3; R → 6; S → 5**

(C) **P → 4; Q → 2; R → 1; S → 6**

(D) **P → 4; Q → 3; R → 6; S → 5**

Ans. (A)

Sol. $E_1 : \dfrac{x}{x-1} > 0$

\+ − +

0 1

$\Rightarrow \quad E_1 : x \in (-\infty, 0) \cup (1, \infty)$

$E_2 : -1 \le \ell n\left(\dfrac{x}{x+1}\right) \le 1$

$\dfrac{1}{e} \le \dfrac{x}{x-1} \le e$

Now $\dfrac{x}{x-1} - \dfrac{1}{e} \ge 0$

$\Rightarrow \quad \dfrac{(e-1)x+1}{e(x-1)} \ge 0$

\+ − +

–1/(e–1) 1

$\Rightarrow \quad x \in \left(-\infty, \dfrac{1}{1-e}\right] \cup (1, \infty)$

also $\dfrac{x}{x-1} - e \le 0$

$\dfrac{(e-1)x-e}{x-1} \ge 0$

\+ − +

1 e/(e–1)

$\Rightarrow \quad x \in (-\infty, 1) \cup \left[\dfrac{e}{e-1}, \infty\right]$

So $E_2 : \left(-\infty, \dfrac{1}{1-e}\right) \cup \left[\dfrac{e}{e-1}, \infty\right]$

as Range of $\dfrac{x}{x-1}$ is $R^+ - \{1\}$

$\Rightarrow$ Range of f is $R - \{0\}$ or $(-\infty, 0) \cup (0, \infty)$

Range of g is $\left[\frac{-\pi}{2},\frac{\pi}{2}\right] \setminus \{0\}$ or $\left[-\frac{\pi}{2},0\right)\cup\left(0,\frac{\pi}{2}\right]$

Now $P \to 4$, $Q \to 2$, $R \to 1$, $S \to 1$

Hence A is correct

16. In a high school, a committee has to be formed from a group of 6 boys M_1, M_2, M_3, M_4, M_5, M_6 and 5 girls G_1, G_2, G_3, G_4, G_5.

(i) Let α_1 be the total number of ways in which the committee can be formed such that the committee has 5 members, having exactly 3 body and 2 girls.

(ii) Let α_2 be the total number of ways in which the committe can be formed such that the committee has at least 2 members, and having an equal number of boys and girls.

(iii) Let α_3 be the total number of ways in which the committe can be formed such that the committee has 5 members, at least 2 of them being girls.

(iv) Let α_4 be the total number of ways in which the committee can be formed such that the commitee has 4 members, having at least 2 girls and such that both M_1 and G_1 are **NOT** in the committee together.

LIST-I		**LIST-II**	
P.	The value of α_1 is	**1.**	136
Q.	The value of α_2 is	**2.**	189
R.	The value of α_3 is	**3.**	192
S.	The value of α_4 is	**4.**	200
		5.	381
		6.	461

The correct option is :-

(A) $\mathbf{P \to 4;\ Q \to 6,\ R \to 2;\ S \to 1}$

(B) $\mathbf{P \to 1;\ Q \to 4;\ R \to 2;\ S \to 3}$

(C) $\mathbf{P \to 4;\ Q \to 6,\ R \to 5;\ S \to 2}$

(D) $\mathbf{P \to 4;\ Q \to 2;\ R \to 3;\ S \to 1}$

Ans. (C)

Sol. (1) $\alpha_1 = \binom{6}{3}\binom{5}{2} = 200$

So $P \to 4$

(2) $\alpha_2 = \binom{6}{1}\binom{5}{1} + \binom{6}{2}\binom{5}{2} + \binom{6}{3}\binom{5}{3} + \binom{6}{4}\binom{5}{4} + \binom{6}{5}\binom{5}{5}$

$= \binom{11}{5} - 1$

$= 46!$

So $Q \to 6$

(3) $\alpha_3 = \binom{5}{2}\binom{6}{3} + \binom{5}{3}\binom{6}{2} + \binom{5}{4}\binom{6}{1} + \binom{5}{5}\binom{6}{0}$

$= \binom{11}{5} - \binom{5}{0}\binom{6}{5} - \binom{5}{1}\binom{6}{4}$

$= 381$

So $R \to 5$

(4) $\alpha_2 = \binom{5}{2}\binom{6}{2} - \binom{4}{1}\binom{5}{1} + \binom{5}{3}\binom{6}{1} - \binom{4}{2}\binom{1}{1} + \binom{5}{4} = 189$

So $S \to 2$

17. Let $H : \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, where $a > b > 0$, be a hyperbola in the xy-plane whose conjugate axis LM subtends an angle of 60° at one of its vertices N. Let the area of the triangle LMN be $4\sqrt{3}$.

LIST-I	**LIST-II**
P. The length of the conjugate axis of H is	**1.** 8
Q. The eccentricity of H is	**2.** $\frac{4}{\sqrt{3}}$
R. The distance between the foci of H is	**3.** $\frac{2}{\sqrt{3}}$
S. The length of the latus rectum of H is	**4.** 4

The correct option is :

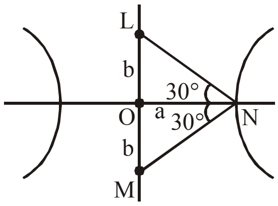
(A) **P → 4; Q → 2, R → 1; S → 3**

(B) **P → 4; Q → 3; R → 1; S → 2**

(C) **P → 4; Q → 1, R → 3; S → 2**

(D) **P → 3; Q → 4; R → 2; S → 1**

Ans. (B)

Sol. 

$$\tan 30° = \frac{b}{a}$$

$\Rightarrow \quad a = b\sqrt{3}$

Now area of $\Delta LMN = \frac{1}{2}.2b.b\sqrt{3}$

$4\sqrt{3} = \sqrt{3}b^2$

$\Rightarrow \quad b = 2 \quad \& \quad a = 2\sqrt{3}$

$\Rightarrow \quad e = \sqrt{1+\frac{b^2}{a^2}} = \frac{2}{\sqrt{3}}$

P. Length of conjugate axis = 2b = 4

So $P \rightarrow 4$

Q. Eccentricity $e = \frac{2}{\sqrt{3}}$

So $Q \rightarrow 3$

R. Distance between foci = 2ae

$= 2\left(2\sqrt{3}\right)\left(\frac{2}{\sqrt{3}}\right) = 8$

So $R \rightarrow 1$

S. Length of latus rectum $= \frac{2b^2}{a} = \frac{2(2)^2}{2\sqrt{3}} = \frac{4}{\sqrt{3}}$

So $S \rightarrow 2$

18. Let $f_1 : \mathbb{R} \to \mathbb{R}$, $f_2 : \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \to \mathbb{R}$, $f_3 : \left(-1, e^{\frac{\pi}{2}} - 2\right) \to \mathbb{R}$ and $f_4 : \mathbb{R} \to \mathbb{R}$ be functions defined by

(i) $f_1(x) = \sin\left(\sqrt{1 - e^{-x^2}}\right)$

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$, where the inverse trigonometric function $\tan^{-1}x$ assumes values in $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$,

(iii) $f_3(x) = [\sin(\log_e(x + 2))]$, where for $t \in \mathbb{R}$, $[t]$ denotes the greatest integer less than or equal to t,

(iv) $f_4(x) = \begin{cases} x^2 \sin\left(\dfrac{1}{x}\right) & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$

List-I	List-II
P. the function f_1 is	**1.** **NOT** continuous at $x = 0$
Q. The function f_2 is	**2.** continuous at $x = 0$ and **NOT** differentiable at $x = 0$
R. The function f_3 is	**3.** differentiable at $x = 0$ and its derivative is **NOT** continuous at $x = 0$
S. The function f_4 is	**4.** differentiable at $x = 0$ and its derivative is continuous at $x = 0$

The correct option is :

(A) **P → 2; Q → 3, R → 1; S → 4**

(B) **P → 4; Q → 1; R → 2; S → 3**

(C) **P → 4; Q → 2, R → 1; S → 3**

(D) **P → 2; Q → 1; R → 4; S → 3**

Ans. (D)

Sol. (i) $f(x) = \sin\sqrt{1-e^{-x^2}}$

$$f_1'(x) = \cos\sqrt{1-e^{-x^2}} \cdot \frac{1}{2\sqrt{1-e^{-x^2}}}\left(0-e^{-x^2}.(-2x)\right)$$

at x = 0 $\quad f_1'(x)$ does not exist

So. $P \to 2$

(ii) $f_2(x) = \begin{cases} \dfrac{|\sin x|}{\tan^{-1} x}, & x \neq 0 \\ 0 & x = 0 \end{cases}$

$$\lim_{x\to 0^+} \frac{\sin x}{x} \frac{x}{\tan^{-1} x} = 1$$

$\Rightarrow$ $f_2(x)$ does not continuous at x = 0

So $Q \to 1$

(iii) $f_3(x) = \left[\sin \ell n(x+2)\right] = 0$

$1 < x + 2 < e^{\pi/2}$

$\Rightarrow \quad 0 < \ell n(x + 2) < \dfrac{\pi}{2}$

$\Rightarrow \quad 0 < \sin(\ell n(x + 2) < 1$

$\Rightarrow \quad f_3(x) = 0$

So $R \to 4$

(iv) $f_4(x) = \begin{cases} x^2 \sin\dfrac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$

So $S \to 3$

JEE(Advanced) – 2018 TEST PAPER - 2 WITH SOLUTION

(Exam Date: 20-05-2018)

PART-1 : CHEMISTRY

1. The correct option(s) regarding the complex $[Co(en)\ (NH_3)_3(H_2O)]^{3+}$:-

(en = $H_2NCH_2CH_2NH_2$) is (are)

(A) It has two geometrical isomers

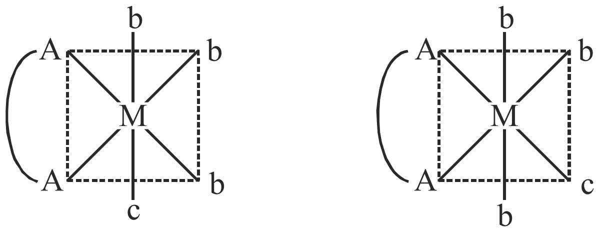
(B) It will have three geometrical isomers if bidentate 'en' is replaced by two cyanide ligands

(C) It is paramagnetic

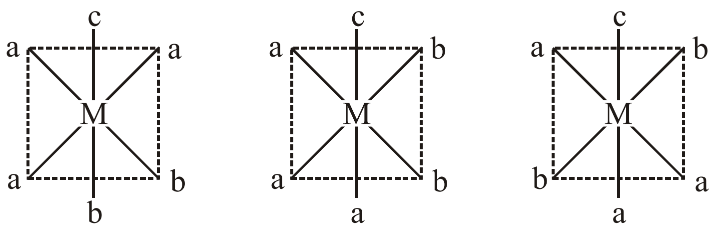
(D) It absorbs light at longer wavelength as compared to $[Co(en)\ (NH_3)_4]^{3+}$

Ans. (A,B,D)

Sol. (A) $[Co(en)(NH_3)_3(H_2O)]^{+3}$ complex is type of $[M(AA)b_3c]$ have two G.I.



(B) If (en) is replaced by two cynide ligand, complex will be type of $[Ma_3b_2c]$ and have 3 G.I.



(C) $[Co(en)(NH_3)_3(H_2O)]^{3+}$ have d^6 configuration (t_{2g}^6) on central metal with SFL therefore it is dimagnetic in nature.

(D) Complex $[Co(en)(NH_3)_3(H_2O)]^{3+}$ have lesser CFSE (Δ_O) value than $[Co(en)(NH_3)_4]^{3+}$ therefore complex $[Co(en)(NH_3)_3(H_2O)]^{+}$ absorbs longer wavelength for d–d transition.

2. The correct option(s) to distinguish nitrate salts of Mn^{2+} and Cu^{2+} taken separately is (are) :-

(A) Mn^{2+} shows the characteristic green colour in the flame test

(B) Only Cu^{2+} shows the formation of precipitate by passing H_2S in acidic medium

(C) Only Mn^{2+} shows the formation of precipitate by passing H_2S in faintly basic medium

(D) Cu^{2+}/Cu has higher reduction potential than Mn^{2+}/Mn (measured under similar conditions)

Ans. (B,D)

Sol. (A) Cu^{+2} and Mn^{+2} both gives green colour in flame test and cannot distinguished.

(B) Cu^{+2} belongs to group-II of cationic radical will gives ppt. of CuS in acidic medium.

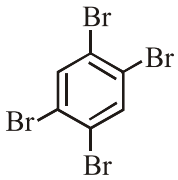
(C) Cu^{+2} and Mn^{+2} both form ppt. in basic medium.

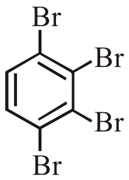
(D) $Cu^{+2}/Cu = +0.34$ V (SRP)

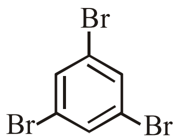
$Mn^{+2}/Mn = -\ 1.18$ V (SRP)

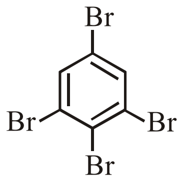
3. Aniline reacts with mixed acid (conc. HNO_3 and conc. H_2SO_4) at 288 K to give P (51%), Q (47%) and R (2%). The major product(s) the following reaction sequence is (are) :-

$$R \xrightarrow[\substack{3)\ H_3O^+ \\ 4)\ NaNO_2,\ HCl/273\text{-}278K \\ 5)\ EtOH,\ \Delta}]{\substack{1)\ Ac_2O,\ pyridine \\ 2)\ Br_2,\ CH_3CO_2H}} S \xrightarrow[\substack{3)\ NaNO_2,\ HCl/273\text{-}278K \\ 4)\ H_3PO_2}]{\substack{1)\ Sn/HCl \\ 2)\ Br_2/H_2O\ (excess)}} \text{major product(s)}$$

(A) 

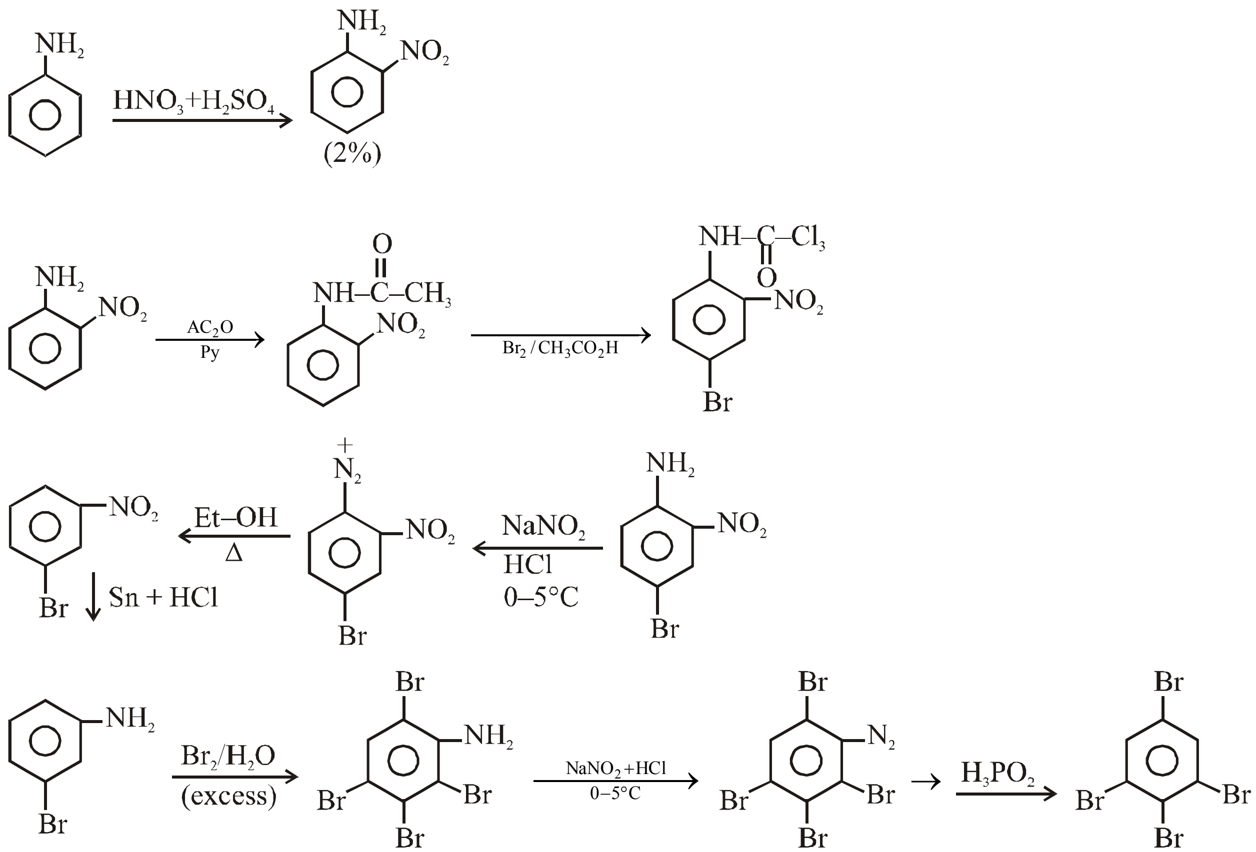
(B) 

(C) 

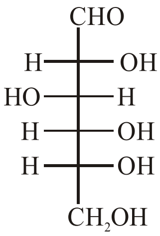
(D) 

Ans. (D)

Sol.

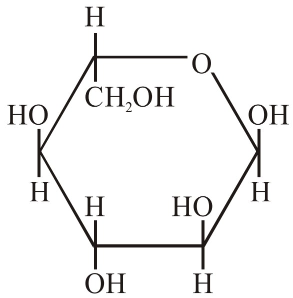


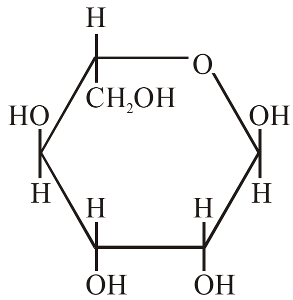
4. The Fischer presentation of D-glucose is given below.

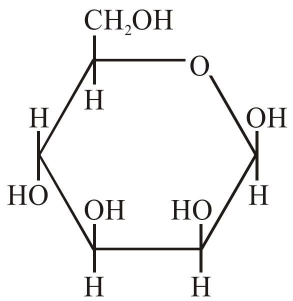


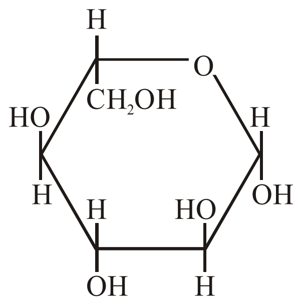
D-glucose

The correct structure(s) of β-L-glucopyranose is (are) :-

(A) 

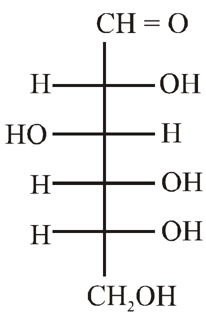
(B) 

(C) 

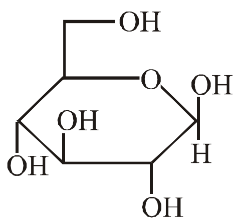
(D) 

Ans. (D)

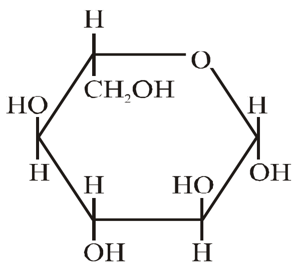
Sol.



D-glucose



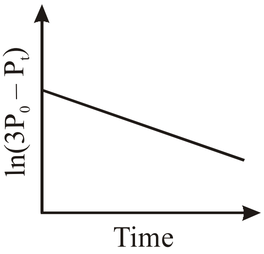
β-D-glucopyranose

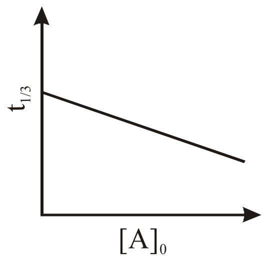


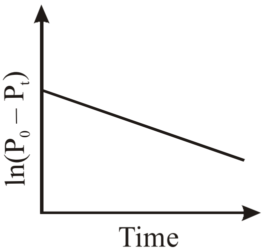
β-L-glucopyranose

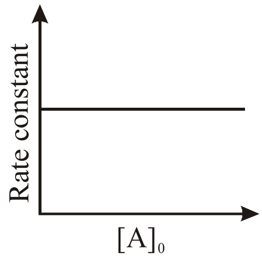
5. For a first order reaction A(g) $\rightarrow$ 2B(g) + C(g) at constant volume and 300 K, the total pressure at the beginning (t = 0) and at time t are P_0 and P_t, respectively. Initially, only A is present with concentration $[A]_0$, and $t_{1/3}$ is the time required for the partial pressure of A to reach $1/3^{rd}$ of its initial value. The correct option(s) is (are) :-

(Assume that all these gases behave as ideal gases)

(A) 

(B) 

(C) 

(D) 

Ans. **(A,D)**

Sol.

$$
\begin{array}{llll}
 & A \longrightarrow & 2B & + \; C \\
t = 0 & P_0 & - & - \\
t = t & P_0 - P & 2P & P
\end{array}
$$

$P_0 + 2P = P_t$

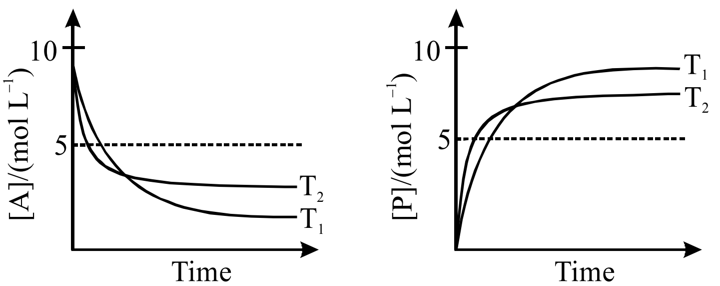
$$K = \frac{1}{t}\ln\frac{P_0}{P_0 - P} = \frac{1}{t}\ln\frac{P_0}{P_0 - \frac{(P_t - P_0)}{2}}$$

$$K = \frac{1}{t}\ln\frac{2P_0}{3P_0 - P_t} \Rightarrow -Kt + \ln 2P_0 = \ln(3P_0 - P_t)$$

and $t_{1/3} = \frac{1}{K}\ln\frac{P_0}{P_0/3} = \frac{1}{K}\ln 3 = \text{constant}$

Rate constant does not depends on concentration

6. For a reaction, A $\rightleftharpoons$ P, the plots of [A] and [P] with time at temperatures T_1 and T_2 are given below.



If $T_2 > T_1$, the correct statement(s) is (are)

(Assume ΔH^θ and ΔS^θ are independent of temperature and ratio of lnK at T_1 to lnK at T_2 is greater than $^{T_2}/_{T_1}$. Here H, S, G and K are enthalpy, entropy, Gibbs energy and equilibrium constant, respectively.)

(A) $\Delta H^{\theta} < 0$, $\Delta S^{\theta} < 0$ (B) $\Delta G^{\theta} < 0$, $\Delta H^{\theta} > 0$

(C) $\Delta G^{\theta} < 0$, $\Delta S^{\theta} < 0$ (D) $\Delta G^{\theta} < 0$, $\Delta S^{\theta} > 0$

Ans. **(A,C)**

Sol. $A \rightleftharpoons P$

given $T_2 > T_1$

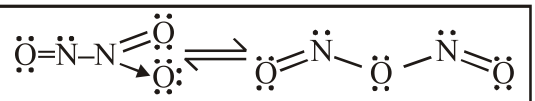
$$\frac{\ln K_1}{\ln K_2} > \frac{T_2}{T_1}$$

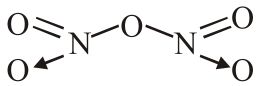
$\Rightarrow T_1 \ln k_1 > T_2 \ln k_2$
$\Rightarrow -\Delta G^{\circ}_1 > -\Delta G^{\circ}_2$
$\Rightarrow (-\Delta H^{\circ} + T_1\Delta S^{\circ}) > (-\Delta H^{\circ} + T_2\Delta S^{\circ})$
$\Rightarrow T_1\Delta S^{\circ} > T_2\Delta S^{\circ}$
$\Rightarrow \Delta S^{\circ} < 0$

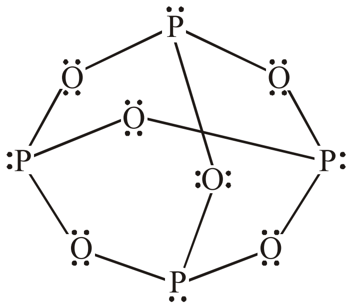
7. The total number of compounds having at least one bridging oxo group among the molecules given below is______.

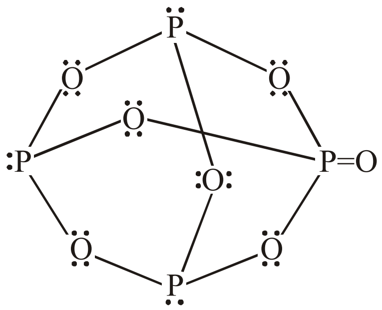
N_2O_3, N_2O_5, P_4O_6, P_4O_7, $H_4P_2O_5$, $H_5P_3O_{10}$, $H_2S_2O_3$, $H_2S_2O_5$

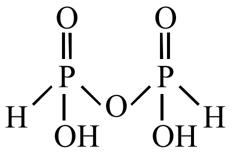
Ans. **(5 or 6)**

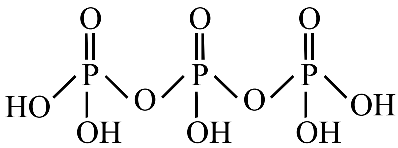
Sol. $N_2O_3 =$ 

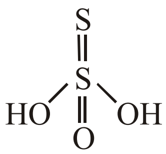
$N_2O_5 =$ 

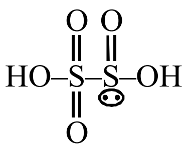
$P_4O_6 =$ 

$P_4O_7 =$ 

$H_4P_2O_5$ = 

$H_5P_3O_{10}$ = 

$H_2S_2O_3$ = 

$H_2S_2O_5$ = 

8. Galena (an ore) is partially oxidized by passing air through it at high temperature. After some time, the passage of air is stopped, but the heating is continued in a closed furnance such that the contents undergo self-reduction. The weight (in kg) of Pb produced per kg of O_2 consumed is ______ .

(Atomic weights in g mol^{-1} : O = 16, S = 32, Pb = 207)

Ans. (6.47)

Sol. $PbS + O_2 \longrightarrow Pb + SO_2$

$$\frac{1000}{32}\text{mol} \qquad \frac{1000}{32}\times 207\text{ gm}$$

mol of Pb = mol of O_2

$$= \frac{1000}{32}\text{mol}$$

$$\therefore \text{ mass of Pb} = \frac{1000}{32}\times 207\text{g}$$

$$= \frac{207}{32}\text{kg} = 6.47\text{ kg}$$

9. To measure the quantity of $MnCl_2$ dissolved in an aqueous solution, it was completely converted to $KMnO_4$ using the reaction,

$MnCl_2 + K_2S_2O_8 + H_2O \rightarrow KMnO_4 + H_2SO_4 + HCl$ (equation not balanced).

Few drops of concentrated HCl were added to this solution and gently warmed. Further, oxalic acid (225 g) was added in portions till the colour of the permanganate ion disappeard. The quantity of $MnCl_2$ (in mg) present in the initial solution is _____.

(Atomic weights in g mol^{-1} : Mn = 55, Cl = 35.5)

Ans. (126)

Sol. $\underset{\text{a mole}}{MnCl_2} + K_2S_2O_8 + H_2O \rightarrow \underset{\text{a mole}}{KMnO_4} + H_2SO_4 + HCl$

$C_2O_4^{--} + MnO_4^- \xrightarrow{H^+} CO_2$

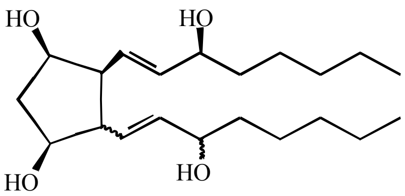
m_{eq} of $C_2O_4^{--}$ = m_{eq} of MnO_4^-

$2 \times 0.225/90 = a \times 5$

$a = 1 \times [55 + 71]$

$= 126$ mg

10. For the given compound X, the total number of optically active stereoisomers is____.



X

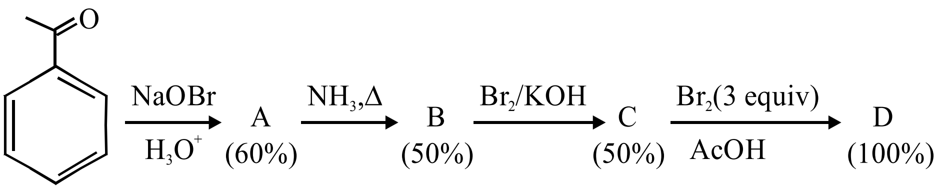
This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is fixed

This type of bond indicates that the configuration at the specific carbon and the geometry of the double bond is NOT fixed

Ans. (7)

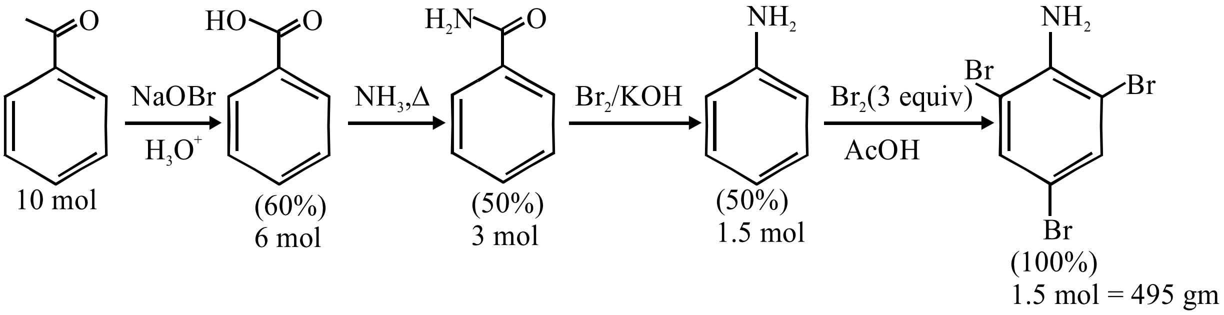
11. In the following reaction sequence, the amount of D (in g) formed from 10 moles of acetophenone is____.

(Atomic weight in g mol^{-1}: H = 1, C = 12, N = 14, O = 16, Br = 80. The yield (%) corresponding to the product in each step is given in the parenthesis)



Ans. (495)

Sol.



12. The surface of copper gets tarnished by the formation of copper oxide. N_2 gas was passed to prevent the oxide formation during heating of copper at 1250 K. However, the N_2 gas contains 1 mole % of water vapour as impurity. The water vapour oxidises copper as per the reaction given below :

$2Cu(s) + H_2O(g) \rightarrow Cu_2O(s) + H_2(g)$

p_{H_2} is the minimum partial pressure of H_2 (in bar) needed to prevent the oxidation at 1250 K. The value of $\ln(p_{H_2})$ is ___.

(Given : total pressure = 1 bar, R (universal gas constant) = 8 $JK^{-1}mol^{-1}$, ln(10) = 2.3. Cu(s) and $Cu_2O(s)$ are mutually immiscible.

At 1250 K : $2Cu(s) + 1/2O_2(g) \rightarrow Cu_2O(s)$; $\Delta G^\theta = -78{,}000$ J mol^{-1}

$H_2(g) + 1/2O_2(g) \rightarrow H_2O(g)$; $\Delta G^\theta = -1{,}78{,}000$ J mol^{-1}; G is the Gibbs energy)

Ans. (–14.6)

Sol. $2Cu(s) + \frac{1}{4}O_2(g) \rightarrow 1Cu_2O(s)$ $\qquad \Delta G° = -78$ kJ

$[H_2(g) + \frac{1}{2}O_2 \rightarrow H_2O(g)$ $\qquad \Delta G° = -178$ kJ$] \times (-1)$

Hence, $2Cu(s) + H_2O(g) \rightarrow Cu_2O + H_2(g)$ $\qquad \Delta G° = +100$ kJ

$\Delta G = \Delta G° + RT \ln Q$

$$0 = +100 + \frac{8}{1000} \times 1250 \ln \frac{p_{H_2}}{p_{H_2O}}$$

$$-\frac{100 \times 1000}{8} = 1250 \ln \frac{p_{H_2}}{\left(\frac{1}{100} \times 1\right)}$$

$\ln p_{H_2} = -14.6$

13. Consider the following reversible reaction,

$$A(g) + B(g) \rightleftharpoons AB(g).$$

The activition energy of the backward reaction exceeds that of the forward reaction by 2RT (in J mol^{-1}). If the pre-exponential factor of the forward reaction is 4 times that of the reverse reaction, the absolute value of ΔG^θ (in J mol^{-1}) for the reaction at 300 K is____.

(Given ; ln (2) = 0.7, RT = 2500 J mol^{-1} at 300 K and G is the Gibbs energy)

Ans. (8500)

Sol. $A_{(g)} + B_{(g)} \rightleftharpoons AB_{(g)}$

$E_{ab} - E_{af} = 2RT \qquad \Rightarrow \Delta H = -2RT \qquad$ and $\frac{A_f}{A_b} = 4$

$$K_{eq} = \left(\frac{K_f}{K_b}\right) = \frac{A_f e^{-E_{af}/RT}}{A_b e^{-E_{ab}/RT}} = 4(e^2)$$

$\Delta G° = -RT \ln K = -2500 \times \ln (4 \times e^2) = -8500$ J/mol

$\therefore$ Absolute value of $\Delta G° = 8500$ J/mol

14. Consider an electrochemical cell: A(s) | A^{n+} (aq, 2M) || B^{2n+} (aq, 1M) | B(s). The value of ΔH^θ for the cell reaction is twice that of ΔG^θ at 300 K. If the emf of the cell is zero, the ΔS^θ (in JK^{-1} mol^{-1}) of the cell reaction per mole of B formed at 300 K is___.

(Given : ln (2) = 0.7, R (universal gas constant) = 8.3 J K^{-1} mol^{-1}. H, S and G are enthalpy, entropy and Gibbs energy, respectively.)

Ans. (–11.62)

Sol. A(s) | A^{+n} (aq, 2M) || B^{+2n} (aq, 1M) | B(s)

$\Delta H^\circ = 2\Delta G^\circ_0$ $\qquad E_{cell} = 0$

Cell Rx $\quad A \rightarrow A^{+n} + ne^- \;] \times 2$

$B^{+2n} + 2n\, e^- \rightarrow B(s)$

$$2A(s) + \underset{1M}{B^{+2n}}(aq) \rightarrow 2\underset{2M}{A^{+n}}(aq) + B(s)$$

$$\Delta G = \Delta G^\circ + RT \ln \frac{[A^{+n}]^2}{[B^{+2n}]}$$

$$\Delta G^\circ = -RT \ln \frac{[A^{+n}]^2}{[B^{+2n}]} = -RT.\ln \frac{2^2}{1} = -RT.\ln 4$$

$\Delta G^\circ = \Delta H^\circ - T\Delta S^\circ$

$\Delta G^\circ = 2\Delta G^\circ - T\Delta S^\circ$

$$\Delta S^\circ = \frac{\Delta G^\circ}{T} = -\frac{RT \ln 4}{T}$$

$= -8.3 \times 2 \times 0.7 = -11.62$ J/K.mol

15. Match each set of hybrid orbitals from LIST-I with complex (es) given in LIST-II.

LIST-I	**LIST-II**
P. dsp^2	1. $[FeF_6]^{4-}$
Q. sp^3	2. $[Ti(H_2O)_3Cl_3]$
R. sp^3d^2	3. $[Cr(NH_3)_6]^{3+}$
S. d^2sp^3	4. $[FeCl_4]^{2-}$
	5. $Ni(CO)_4$
	6. $[Ni(CN)_4]^{2-}$

The correct option is

(A) P $\rightarrow$ 5; Q $\rightarrow$ 4,6; R $\rightarrow$ 2,3; S $\rightarrow$ 1

(B) P $\rightarrow$ 5,6; Q $\rightarrow$ 4; R $\rightarrow$ 3; S $\rightarrow$ 1,2

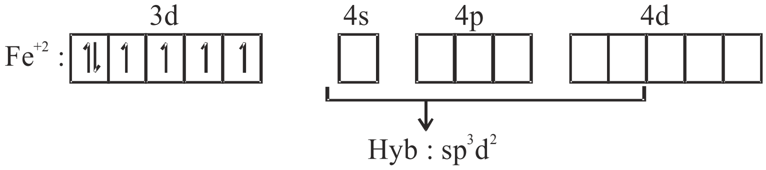
(C) P $\rightarrow$ 6; Q $\rightarrow$ 4,5; R $\rightarrow$ 1; S $\rightarrow$ 2,3

(D) P $\rightarrow$ 4,6; Q $\rightarrow$ 5,6; R $\rightarrow$ 1,2; S $\rightarrow$ 3

Ans. (C)

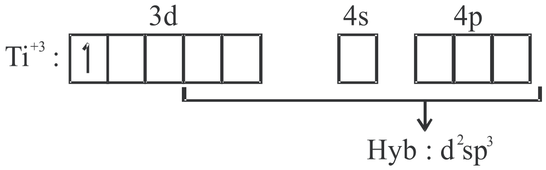
Sol. [1] $[FeF_6]^{4-}$

$Fe^{2+} = [Ar]3d^6$,



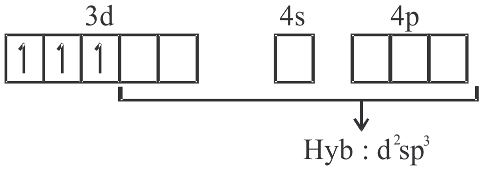
[2] $[Ti(H_2O)_3Cl_3]$

$Ti^{+3} = [Ar]3d^1$,



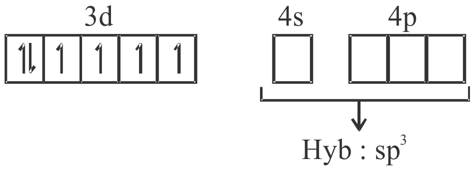
[3] $[Cr(NH_3)_6]^{3+}$

$Cr^{+3} = [Ar]3d^3$, Cr^{+3} :



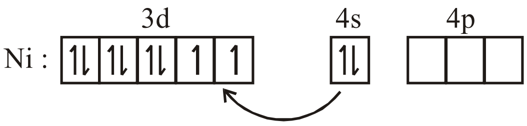
[4] $[FeCl_4]^{2-}$

$Fe^{2+} = [Ar]3d^6$, Fe^{+2} :

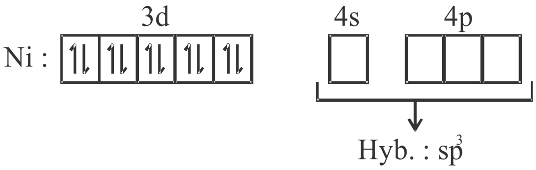


[5] $[Ni\ (CO)_4]$

Ni : $3d^8\ 4s^2$



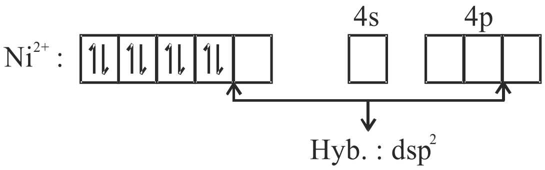
Back pairing of electrons due to presence of strong field ligand



[6] $[Ni\ (CN)_4]^{2-}$

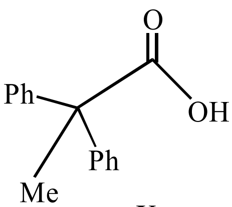
Ni^{2+} : $3d^8$

Electron pairing take place due to presence of S.F.L.



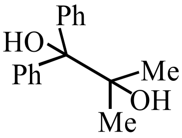
16. The desired product X can be prepared by reacting the major product of the reactions in LIST-I with one or more appropriate reagents in LIST-II.

(given, order of migratory aptitude: aryl > alkyl > hydrogen)

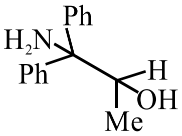


X

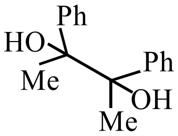
LIST-I | **LIST-II**

P. 

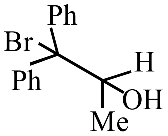
$+ H_2SO_4$

Q. 

$+ HNO_2$

R. 

$+ H_2SO_4$

S. 

$+ AgNO_3$

1. I_2, NaOH
2. $[Ag(NH_3)_2]OH$
3. Fehling solution
4. HCHO, NaOH
5. NaOBr

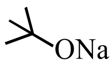
The correct option is

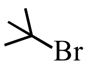
(A) P → 1; Q → 2,3; R → 1,4; S → 2,4
(B) P → 1,5; Q → 3,4; R → 4,5; S → 3
(C) P → 1,5; Q → 3,4; R → 5; S → 2,4
(D) P → 1,5; Q → 2,3; R → 1,5; S → 2,3

Ans. (D)

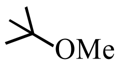
17. LIST-I contains reactions and LIST-II contains major products.

LIST-I | **LIST-II**

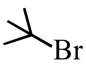
P. 

+ 

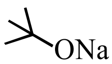
$\longrightarrow$

Q. 

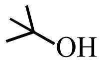
+ HBr $\longrightarrow$

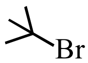
R. 

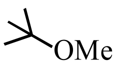
+ NaOMe $\longrightarrow$

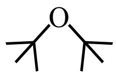
S. 

+ MeBr $\longrightarrow$

1. 

2. 

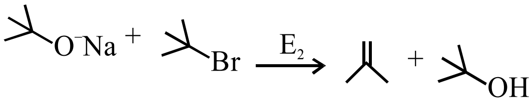
3. 

4. 
5. 

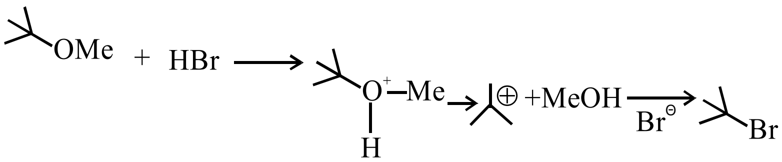
Match each reaction in LIST-I with one or more product in LIST-II and choose the correct option.

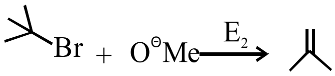
(A) P → 1,5; Q → 2; R → 3; S → 4
(B) P → 1,4; Q → 2; R → 4; S → 3
(C) P → 1,4; Q → 1,2; R → 3,4; S → 4
(D) P → 4,5; Q → 4; R → 4; S → 3,4

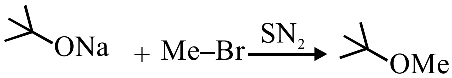
Ans. (B)

Sol. P. 

(Elimination product)

Q. 

R. 

S. 

18. Dilution process of different aqueous solutions; with water, are given in LIST-I. The effects of dilution of the solutions on $[H^+]$ are given in LIST-II.

(Note : Degree of dissociation (α) of weak acid and weak base is << 1; degree of hydrolysis of salt <<1; $[H^+]$ represents the concentration of H^+ ions)

LIST-I	**List-II**
P. (10 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 60 mL	1. the vale of $[H^+]$ does not change on dilution
Q. (20 mL of 0.1 M NaOH + 20 mL of 0.1 M acetic acid) diluted to 80 mL	2. the value of $[H^+]$ change to half of its initial value on dilution
R. (20 mL of 0.1 M HCl + 20 mL of 0.1 M ammonia solution) diluted to 80 mL	3. the value of $[H^+]$ changes to two times of its initial value on dilution
S. 10 mL saturated solution of $Ni(OH)_2$ in equilibrium with excess solid $Ni(OH)_2$ is diluted to 20 mL (solid $Ni(OH)_2$ is still present after dilution).	4. the value of $[H^+]$ changes to $\frac{1}{\sqrt{2}}$ times of its initial value on dilution
	5. the value of $[H^+]$ changes to $\sqrt{2}$ times of its initial value on dilution

Match each process given in LIST-I with one or more effect(s) in LIST-II. The correct option is

(A) $P \to 4$; $Q \to 2$; $R \to 3$; $S \to 1$

(B) $P \to 4$; $Q \to 3$; $R \to 2$; $S \to 3$

(C) $P \to 1$; $Q \to 4$; $R \to 5$; $S \to 3$

(D) $P \to 1$; $Q \to 5$; $R \to 4$; $S \to 1$

Ans. (D)

Sol. P. $\underset{0.1M,\,20ml}{CH_3COOH} + \underset{0.1M,10ml}{NaOH} \rightarrow CH_3COONa + H_2O$

pH = pKa $\Rightarrow$ $[H^+]$ will not change on dilution

correct match : P-1

Q. $\underset{0.1M,\,20ml}{CH_3COOH} + \underset{0.1M,20ml}{NaOH} \rightarrow CH_3COONa + H_2O$

$$\text{—} \qquad \text{—} \qquad 0.05\ M$$

$$[OH^-] = \sqrt{K_H C} = \sqrt{\left(\frac{k_w}{k_a}C\right)}$$

$$[H^+]_1 = \sqrt{\frac{k_w k_a}{C}}$$

$$\frac{[H^+]_2}{[H^+]_1} = \sqrt{\frac{C_1}{C_2}} = \sqrt{\frac{0.05}{0.025}} = \sqrt{2}$$

correct match : Q-5

R. $\underset{0.1M,20ml}{NH_4OH} + \underset{0.1M,20ml}{HCl} \rightarrow \underset{0.05M}{NH_4Cl}$

$$[H^+] = \sqrt{K_H C}$$

$$\frac{[H^+]_2}{[H^+]_1} = \sqrt{\frac{C_2}{C_1}} = \frac{1}{\sqrt{2}}$$

correct match : R-4

S. Because of dilution solubility does not change so $[H^+]$ = constant