FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Tuesday 11th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The value of the integral $\int_{-\log_e 2}^{\log_e 2} e^x\left(\log_e\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$ is equal to

(1) $\log_e\left(\frac{2(2+\sqrt{5})}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

(2) $\log_e\left(\frac{\sqrt{2}(3-\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

(3) $\log_e\left(\frac{(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)+\frac{\sqrt{5}}{2}$

(4) $\log_e\left(\frac{\sqrt{2}(2+\sqrt{5})^2}{\sqrt{1+\sqrt{5}}}\right)-\frac{\sqrt{5}}{2}$

Official Ans. by NTA (4)

Sol. $I=\int_{-\ln 2}^{\ln 2} e^x\left(\ln\left(e^x+\sqrt{1+e^{2x}}\right)\right)dx$

Put $e^x=t \Rightarrow e^x dx=dt$

$I=\int_{1/2}^{2}\ln\left(t+\sqrt{1+t^2}\right)dt$

Applying integration by parts.

$=\left[t\ln\left(t+\sqrt{1+t^2}\right)\right]_{\frac{1}{2}}^{2}-\int_{1/2}^{2}\frac{t}{t+\sqrt{1+t^2}}\left(1+\frac{2t}{2\sqrt{1+t^2}}\right)dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\int_{1/2}^{2}\frac{t}{\sqrt{1+t^2}}dt$

$=2\ln\left(2+\sqrt{5}\right)-\frac{1}{2}\ln\left(\frac{1+\sqrt{5}}{2}\right)-\frac{\sqrt{5}}{2}$

$=\ln\left(\frac{\left(2+\sqrt{5}\right)^2}{\left(\frac{\sqrt{5}+1}{2}\right)^{\frac{1}{2}}}\right)-\frac{\sqrt{5}}{2}$

2. If equation of the plane that contains the point (–2,3,5) and is perpendicular to each of the planes $2x+4y+5z=8$ and $3x-2y+3z=5$ is $\alpha x+\beta y+\gamma z+97=0$ then $\alpha+\beta+\gamma=$

(1) 18

(2) 17

(3) 16

(4) 15

Official Ans. by NTA (4)

Sol. The equation of plane through (–2,3,5) is

a(x+2) + b(y-3) + c (z-5) = 0

it is perpendicular to 2x+4y+5z=8 & 3x-2y+3z=5

$\therefore$ $2a+4b+5c=0$

$3a-2b+3c=0$

$\therefore$ $\frac{a}{\begin{vmatrix}4 & 5\\ -2 & 3\end{vmatrix}}=\frac{-b}{\begin{vmatrix}2 & 5\\ 3 & 3\end{vmatrix}}=\frac{c}{\begin{vmatrix}2 & 4\\ 3 & -2\end{vmatrix}}$

$\Rightarrow$ $\frac{a}{22}=\frac{b}{9}=\frac{c}{-16}$

$\therefore$ Equation of Plane is

$22(x+2)+9(y-3)-16(z-5)=0$

$\Rightarrow$ $22x+9y-16z+97=0$

Comparing with $\alpha x+\beta y+\gamma x+97=0$

We get $\alpha+\beta+\gamma=22+9-16=15$

3. Let R be a rectangle given by the lines x = 0, x = 2, y = 0 and y = 5. Let $A(\alpha,0)$ and $B(0,\beta), \alpha \in [0,2]$ and $\beta \in [0,5]$, be such that the line segment AB divides the area of the rectangle R in the ratio 4:1. Then, the mid-point of AB lies on a

(1) parabola

(2) hyberbola

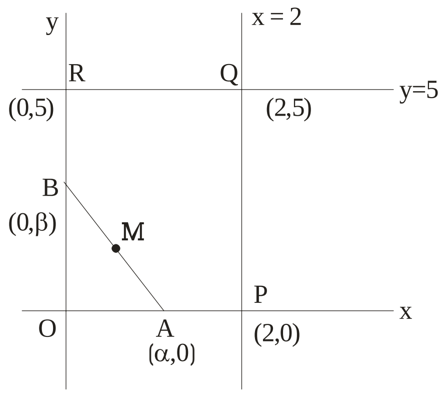
(3) straight line

(4) circle

Official Ans. by NTA (2)

Sol. $\frac{ar(OPQR)}{or(OAB)} = \frac{4}{1}$

Let M be the mid-point of AB.



$M(h,k) \equiv \left(\frac{\alpha}{2}, \frac{\beta}{2}\right)$

$\Rightarrow \quad \frac{10 - \frac{1}{2}\alpha\beta}{\frac{1}{2}\alpha\beta} = 4$

$\Rightarrow \quad \frac{5}{2}\alpha\beta = 10 \Rightarrow \alpha\beta = 4$

$\Rightarrow \quad (2h)(2K) = 4$

$\therefore$ Locus of M is xy = 1

Which is a hyperbola.

4. Let sets A and B have 5 elements each. Let the mean of the elements in sets A and B be 5 and 8 respectively and the variance of the elements in sets A and B be 12 and 20 respectively A new set C of 10 elements is formed by subtracting 3 from each element of A and adding 2 to each element of B. Then the sum of the mean and variance of the elements of C is ________.

(1) 32

(2) 38

(3) 40

(4) 36

Official Ans. by NTA (2)

Sol. $\omega \quad A = \{a_1, a_2, a_3, a_4, a_5\}$

$B = \{b_1, b_2, b_3, b_4, b_5\}$

Given, $\sum_{i=1}^{5} a i = 25, \sum_{i=1}^{5} b i = 40$

$\frac{\sum_{i=1}^{5} a_i^2}{5} - \left(\frac{\sum_{i=1}^{5} a_i}{5}\right)^2 = 12, \frac{\sum_{i=1}^{5} b_i^2}{5} - \left(\frac{\sum_{i=1}^{5} b_i}{5}\right)^2 = 20$

$\Rightarrow \quad \sum_{i=1}^{5} a_i^2 = 185 \quad , \quad \sum_{i=1}^{5} b_i^2 = 420$

Now, $C = \{C_1, C_2, ...C_{10}\}$

s.f. $C_i = \underset{\text{First five elements}}{a_i = 3}$ or $\underset{\text{Last five elements}}{b_i + 2}$

$\therefore$ Mean of C, $\bar{C} = \frac{\left(\sum a_i - 15\right) + \left(\sum b_i + 10\right)}{10}$

$\bar{C} = \frac{10 + 50}{10} = 6$

$\therefore \quad \sigma^2 = \frac{\sum_{i=1}^{10} C_i^2}{10} = \left(\bar{C}\right)^2$

$= \frac{\sum (a_i - 3)^2 + \sum (b_i + 2)^2}{10} - (6)^2$

$= \frac{\sum a_i^2 + \sum b_i^2 - 6\sum a_i + 4\sum b_i + 65}{10} - 36$

$= \frac{185 + 420 - 150 + 160 + 65}{10} - 36$

= 32

$\therefore$ Mean + Variance $= \bar{C} + \sigma^2 = 6 + 32 = 38$

5. Let $f(x) = x^2 - x + |-x+[x]|$, where $x \in \mathbb{R}$ and [t] denotes the greatest integer less than or equal to t. Then, f is

(1) continuous at x = 0, but not continuous at x = 1

(2) continuous at x = 0 and x = 1

(3) not continuous at x = 0 and x = 1

(4) continuous at x = 1, but not continuous at x = 0

Official Ans. by NTA (4)

Sol. Here $f(x) = [x(x-1)] + \{x\}$

$f(o^+) = -1+0 = -1$ $f(o) = 0$	$f(1^+) = 0+0 = 0$ $f(1) = 0$ $f(1^-) = -1+1 = 0$

$\therefore$ f(x) is continuous at x = 1, discontinuous at x = 0

6. The number of triplets (x, y, z). where x, y, z are distinct non negative integers satisfying x + y + z = 15, is

(1) 80

(2) 114

(3) 92

(4) 136

Official Ans. by NTA (2)

Sol. x + y + z = 15

Total no. of solution = ${}^{15+3-1}C_{3-1} = 136$ …(1)

Let $x = y \neq z$

$2x + z = 15 \Rightarrow z = 15 - 2t$

$\Rightarrow r \in \{0,1,2,...7\} - \{5\}$

$\therefore$ 7 solutions

$\therefore$ there are 21 solutions in which exactly

Two of x_1 y_1 z are equal …(2)

There is one solution in which x=y=z …(3)

Required answer = 136–21–1 = 114

7. For any vector $a = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$, with $10|a_i| < 1, i = 1,2,3,$ consider the following statements:

(A) : max $\{|a_1|,|a_2|,|a_3|\} \leq |\vec{a}|$

(B) : $|\vec{a}| \leq 3\max\{|a_1|,|a_2|,|a_3|\}$

(1) Only (B) is true

(2) Only (A) is true

(3) Neither (A) nor (B) is true

(4) Both (A) and (B) are true

Official Ans. by NTA (4)

Sol. Without loss of generality

Let $|a_1| \leq |a_2| \leq |a_3|$

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \geq (a_3)^2$

$\Rightarrow$ $|\vec{a}| \geq |a_3| = \max\{|a_1|,|a_2|,|a_3|\}$

A is true

$|\vec{a}|^2 = |a_1|^2 + |a_2|^2 + |a_3|^2 \leq |a_3|^2 + |a_3|^2 + |a_3|^2$

$\Rightarrow$ $|\vec{a}|^2 \leq 3|a_3|^2$

$\Rightarrow$ $|\vec{a}| \leq \sqrt{3}|a_3| = \sqrt{3}\max\{|a_1|,|a_2|,|a_3|\}$

$\leq 3\max\{|a_1|,|a_2|,|a_3|\}$

(2) is true

8. Let w_1 be the point obtained by the rotation of $z_1 = 5 + 4i$ about the origin through a right angle in the anticlockwise direction, and w_2 be the point obtained by the rotation of $z_2 = 3 + 5i$ about the origin through a right angle in the clockwise direction. Then the principal argument of $w_1 - w_2$ is equal to

(1) $-\pi + \tan^{-1}\frac{33}{5}$

(2) $-\pi - \tan^{-1}\frac{33}{5}$

(3) $-\pi + \tan^{-1}\frac{8}{9}$

(4) $\pi - \tan^{-1}\frac{8}{9}$

Official Ans. by NTA (4)

Sol. $W_1 = z_i i = (5+4i)i = -4+5i$ ...(i)

$W_2 = z_2(-i) = (3+5i)(-i) = 5-3i$ ...(2)

$W_1 - W_2 = -9+8i$

Principal argument = $\pi - \tan^{-1}\left(\frac{8}{9}\right)$

9. An organization awarded 48 medals in event 'A', 25 in event 'B' and 18 in event 'C'. If these medals went to total 60 men and only five men got medals in all the three events, then, how many received medals in exactly two of three events?

(1) 10

(2) 9

(3) 21

(4) 15

Official Ans. by NTA (3)

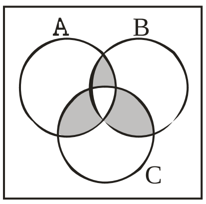
Sol. $|A| = 48$

$|B| = 25$

$|C| = 18$

$|A \cup B \cup C| = 60$ [Total]

$|A \cap B \cap C| = 5$



$|A \cup B \cup C| = \sum|A| - \sum|A \cap B| + |A \cap B \cap C|$

$\Rightarrow \sum|A \cap B| = 48+25+18+5-60$

= 36

No. of men who received exactly 2 medals

$= \sum|A \cap B| - 3|A \cap B \cap C|$

= 36 – 15

= 21

10. Let $S = \{M = [a_{ij}],\ a_{ij} \in \{0,1,2\},\ 1 \le i, j \le 2\}$ be a sample space and $A = \{M \in S : M \text{ is invertible}\}$ be an event. Then P(A) is equal to

(1) $\frac{50}{81}$

(2) $\frac{47}{81}$

(3) $\frac{49}{81}$

(4) $\frac{16}{27}$

Official Ans. by NTA (1)

Sol. $M\begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where a, b, c, d, $\in \{0,1,2\}$

n(s) = 3^4 = 81

we first bound $p(\bar{A})$

$|m| = 0 \Rightarrow ad = bc$

$ad = bc = 0 \Rightarrow$ no. of (a,b,c,d) = $(3^2–2^2)^2$ = 25

$ad = bc = 1 \Rightarrow$ no. of (a,b,c,d) = 1^2 = 1

$ad = bc = 2 \Rightarrow$ no. of (a,b,c,d) = 2^2 = 4

$ad = bc = 4 \Rightarrow$ no. of (a,b,c,d) = 1^2 = 1

: $P(\bar{A}) = \frac{31}{81} \Rightarrow p(A) = \frac{50}{81}$

11. Consider ellipses $E_k : kx^2 + k^2y^2 = 1, k$ = 1, 2, ..., 20. Let C_k be the circle which touches the four chords joining the end points (one on minor axis and another on major axis) of the ellipse E_k, If r_k is the radius of the circle C_k, then the value of $\sum_{k=1}^{20} \frac{1}{r_k^2}$ is

(1) 3080

(2) 3210

(3) 3320

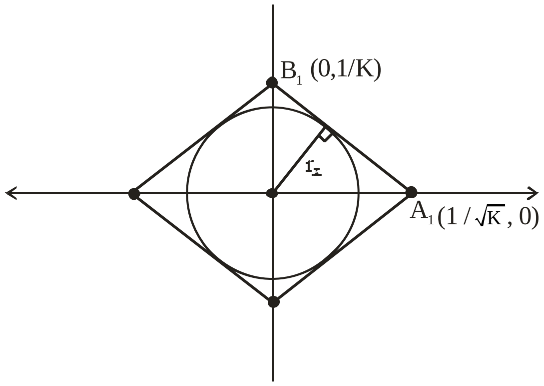
(4) 2870

Official Ans. by NTA (1)

Sol. $Kx^2+K^2y^2=1$

$$\frac{x^2}{1/K}+\frac{y^2}{1/K^2}=1$$

Now



Equation of

$$A_1B_2;\frac{x}{1/\sqrt{K}}+\frac{y}{1/K}=1\Rightarrow\sqrt{K}x+Ky=1$$

$r_K=\perp r$ distance of (0,0) from line A_1B_1

$$r_k=\left|\frac{(0+0-1)}{\sqrt{K+K^2}}\right|=\frac{1}{\sqrt{K+K^2}}$$

$$\frac{1}{r_K^2}=K+K^2\Rightarrow\sum_{k=1}^{20}\frac{1}{r_K^2}=\sum_{K=1}^{20}\left(K+K^2\right)$$

$$=\sum_{K=1}^{20}K+\sum_{K=1}^{20}K^2$$

$$=\frac{20\times21}{2}+\frac{20.21.41}{6}$$

$$=210+10\times7\times41$$

$$=210+2870$$

$$=3080$$

12. The number of integral solutions x of $\log_{\left(x+\frac{7}{2}\right)}\left(\frac{x-7}{2x-3}\right)^2\ge0$ is

(1) 6 (2) 8

(3) 5 (4) 7

Official Ans. by NTA (1)

Sol. $\log_{x+\frac{7}{2}}\left(\frac{x-7}{2x-3}\right)^2\ge0$

Feasible region : $x+\frac{7}{2}>0\Rightarrow x>-\frac{7}{2}$

And $x+\frac{7}{2}\ne1\Rightarrow x\ne-\frac{5}{2}$

And $\frac{x-7}{2x-3}\ne0$ and $2x-3\ne0$

$\Downarrow$ $\Downarrow$

$x\ne7$ $x\ne\frac{3}{2}$

Taking intersection : $x\in\left(\frac{-7}{2},\infty\right)-\left\{-\frac{5}{2},\frac{3}{2},7\right\}$

Now $\log_a b\ge0$ if $a>1$ and $b\ge1$

Or

$a\in(0,1)$ and $b\in(0,1)$

C – I; $x+\frac{7}{2}>1$ and $\left(\frac{x-7}{2x-3}\right)^2\ge1$

$x>-\frac{5}{2}$ $(2x-3)^2-(x-7)^2\le0$

$(2x-3+n-7)(2x-3-x+7)\le0$

$(3x-10)(x+4)\le0$

$x\in\left[-4,\frac{10}{3}\right]$

Intersection : $x\in\left(\frac{-5}{2},\frac{10}{3}\right]$

C-II $x+\frac{7}{2}\in(0,1)$ and $\left(\frac{x-7}{2x-3}\right)^2\in(0,1)$

$0<x+\frac{7}{2}<1$ $\left(\frac{x-7}{2x-3}\right)^2<1$

$-\frac{7}{2}<x<\frac{-5}{2}$ $(x-7)^2<(2x-3)^2$

$$x\in(-\infty,-4)\cup\left(\frac{10}{3},\infty\right)$$

No common values of x.

Hence intersection with feasible region

We get $x\in\left(\frac{-5}{2},\frac{10}{3}\right]-\left\{\frac{3}{2}\right\}$

Integral value of x are {–2,–1,0,1,2,3}

No. of integral values = 6

13. Area of the region $\{(x,y): x^2 + (y-2)^2 \le 4, x^2 \ge 2y\}$ is

(1) $2\pi - \frac{16}{3}$ (2) $\pi - \frac{8}{3}$

(3) $\pi + \frac{8}{3}$ (4) $2\pi + \frac{16}{3}$

Official Ans. by NTA (1)

Sol. $x^2 + (y-2)^2 \le 2^2$ and $x^2 \ge 2y$

Solving circle and parabola simultaneously :

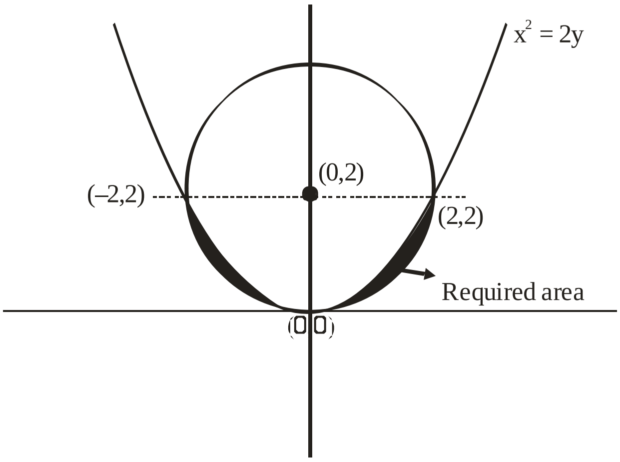
$2y + y^2 - 4y + 4 = 4$

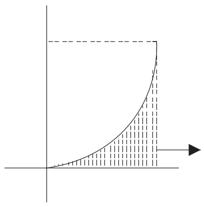
$y^2 - 2y = 0$

$y = 0, 2$

Put y = 2 in $x^2 = 2y \rightarrow x = \pm 2$

$\Rightarrow$ (2,2) and (–2,2)





$$= 2\times 2 - \frac{1}{4}\cdot \pi \cdot 2^2 = 4 - \pi$$

Required area $= 2\left[\int_0^2 \frac{x^2}{2}dx - (4-\pi)\right]$

$$= 2\left[\left.\frac{x^3}{6}\right|_0^2 - 4 + \pi\right]$$

$$= 2\left[\frac{4}{3} + \pi - 4\right]$$

$$= 2\left[\pi - \frac{8}{3}\right]$$

$$= 2\pi - \frac{16}{6}$$

14. Let $f:[2,4] \rightarrow \mathbb{R}$ be a differentiable function such that $(x\log_e x)f'(x) + (\log_e x)f(x) + f(x) \ge 1$, $x \in [2,4]$ with $f(2) = \frac{1}{2}$ and $f(4) = \frac{1}{4}$.

Consider the following two statements:

(A) : $f(x) \le 1$, for all $x \in [2,4]$

(B) : $f(x) \ge \frac{1}{8}$, for all $x \in [2,4]$

Then,

(1) Only statement (B) is true

(2) Neither statement (A) nor statement (B) is true

(3) Both the statement (A) and (B) are true

(4) Only statement (A) is true

Official Ans. by NTA (3)

Sol. $x\,\ell nxf'(x) + \ln x\, f(x) + f(x) \ge I, x \in [2,4]$

And $f(2) = \frac{1}{2}, f(4) = \frac{1}{4}$

Now $x\ln x\frac{dy}{dx} + (\ln + 1)y \ge 1$

$$\frac{d}{dx}(y\cdot x\ln x) \ge 1$$

$$\frac{d}{dx}(f(x).x\ln x) \ge 1$$

$\Rightarrow \frac{d}{dx}(x\ln x\, f(x) - x) \ge 0,\ x \in [2,4]$

$\Rightarrow$ The function g(x) = $x\ln x\, f(x) - x$ is increasing in [2.4]

And $g(2) = 2\ln 2 f(2) - 2 = \ln 2 - 2$

$g(4) = 4\ln 4 f(4) - 4 = \ln 4 - 4$

$$= 2(\ln 2 - 2)$$

Now $g(2) \le g(x) \le g(4)$

$\ln 2 - 2 \le x\ln x\, f(x) - x \le 2(\ln 2 - 2)$

$$\frac{\ln 2 - 2}{x\ln x} + \frac{1}{\ln x} \le f(x) \le \frac{2(\ln 2 - 2)}{x\ln x} + \frac{1}{\ln x}$$

Now for $x \in \; 2, 4$

$$\frac{2(\ell n2-2)}{x\ln x}+\frac{1}{\ell n x}<\frac{2(\ln 2-2)}{2\ln 2}+\frac{1}{\ln 2}=1-\frac{1}{\ln 2}<1$$

$\Rightarrow\; f(x)\le 1$ for $x\in[2,4]$

Also for $x \in [2,4]$:

$$\frac{\ln 2-2}{x\ln x}+\frac{1}{\ln x}\ge\frac{\ln 2-2}{4\ln 4}+\frac{1}{\ln 4}=\frac{1}{8}+\frac{1}{2\ln 2}>\frac{1}{8}$$

$\Rightarrow f(x)\ge\frac{1}{8}$ for $x\in[2,4]$

Hence both A and B are true.

LMVT on (yx (lnx)) not satisfied.

Hence no such function exists.

Therefore it should be bonus.

15. Let y = y(x) be a solution curve of the differential equation, $(1-x^2y^2)dx = ydx + xdy$.

If the line x = 1 intersects the curve y = y(x) at y = 2 and the line x = 2 intersects the curve y = y(x) at $y=\alpha$, then a value of α is

(1) $\frac{3e^2}{2(3e^2-1)}$

(2) $\frac{3e^2}{2(3e^2+1)}$

(3) $\frac{1-3e^2}{2(3e^2+1)}$

(4) $\frac{1+3e^2}{2(3e^2-1)}$

Official Ans. by NTA (4)

Sol. $(1-x^2y^2)dx = ydx + x\,dy, y(1)=2$

$$y(2)=\propto=?$$

$$dx=\frac{d(xy)}{1-(xy)^2}$$

$$\int dx=\int\frac{d\;\,xy}{1-(xy)^2}$$

$$x=\frac{1}{2}\ln\left|\frac{1+xy}{1-xy}\right|+C$$

Put x = 1 and y = 2 :

$$1=\frac{1}{2}\ln\left|\frac{1+2}{1-2}\right|+C$$

$$C=1-\frac{1}{2}\ln 3$$

Now put x = 2 :

$$2=\frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right|+1-\frac{1}{2}\ln 3$$

$$1+\frac{1}{2}\ln 3=\frac{1}{2}\ln\left|\frac{1+2\alpha}{1-2\alpha}\right|$$

$$2+\ln 3=\ln\left(\frac{1+2\alpha}{1-2\alpha}\right)$$

$$\left|\frac{1+2\alpha}{1-2\alpha}\right|=3e^2$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^2\;,\;-3e^2$$

$$\frac{1+2\alpha}{1-2\alpha}=3e^2\Rightarrow\alpha=\frac{3e^2-1}{2(3e^2+1)}$$

And $\frac{1+2\alpha}{1-2\alpha}=-3e^2\Rightarrow\alpha=\frac{3e^2+1}{2(3e^2-1)}$

16. Let A be a 2 × 2 matrix with real entries such that $A'=\alpha A+I$, where $\alpha\in\mathbb{R}-\{-1,1\}$. If det $(A^2 - A) = 4$, then the sum of all possible values of α is equal to

(1) 0

(2) $\frac{3}{2}$

(3) $\frac{5}{2}$

(4) 2

Official Ans. by NTA (3)

Sol. $A^T = \alpha A + I$

$A = \alpha A^T + I$

$A = \alpha(\alpha A + I) + I$

$A = \alpha^2 A + (\alpha + 1)I$

$A(1-\alpha^2) = (\alpha+1)I$

$A = \frac{I}{1-\alpha}$ …(1)

$|A| = \frac{1}{(1-\alpha)^2}$ ...(2)

$|A^2 - A| = |A||A - I|$ …(3)

$A - I = \frac{I}{1-\alpha} - I = \frac{\alpha}{1-\alpha} I$

$|A - I| = \left(\frac{\alpha}{1-\alpha}\right)^2$ …(4)

Now $|A^2 - A| = 4$

$|A||A - I| = 4$

$\Rightarrow \frac{1}{(1-\alpha)^2} \frac{\alpha^2}{(1-\alpha)^2} = 4$

$\Rightarrow \frac{\alpha}{(1-\alpha)^2} = \pm 2$

$\Rightarrow 2(1-\alpha)^2 = \pm\alpha$

(C_1) $2(1-\alpha)^2 = \alpha$ $2\alpha^2 - 5\alpha + 2 = 0 <^{\alpha_1}_{\alpha_2}$ $\alpha_1 + \alpha_2 = \frac{5}{2}$	(C_2) $2(1-\alpha)^3 = -\alpha$ $2\alpha^2 - 3\alpha + 2 = 0$ $\alpha \notin R$

Sum of value of $\alpha = \frac{5}{2}$

17. Let (α, β, γ) be the image of the point P(2, 3, 5) in the plane 2x + y – 3z = 6. Then $\alpha + \beta + \gamma$ is equal to

(1) 10

(2) 5

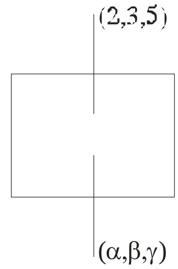
(3) 12

(4) 9

Official Ans. by NTA (1)

Sol. $\frac{\alpha - 2}{2} = \frac{\beta - 3}{1} = \frac{\gamma - 5}{-3} = -2\left(\frac{2x2 + 3 - 3\times5 - 6}{2^2 + 1^2 + 1 - 3^2}\right) = 2$

$\frac{\alpha-2}{2} = 2$ $\alpha = 6$	$\beta - 3 = 2$ $\beta = 5$	$\gamma - 5 = -6$ $\gamma = -1$



$\alpha + \beta + \gamma = 10$

18. Let $\vec{a}$ be a non-zero vector parallel to the line of intersection of the two planes described by $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ *and* $\hat{i} - \hat{j}, \hat{j} - \hat{k}$. If θ is the angle between the vector $\vec{a}$ and the vector $\vec{b} = 2\hat{i} - 2\hat{j} + \hat{k}$ and $\vec{a}\cdot\vec{b} = 6$ then the ordered pair $\left(\theta, |\vec{a}\times\vec{b}|\right)$ is equal to

(1) $\left(\frac{\pi}{4}, 3\sqrt{6}\right)$

(2) $\left(\frac{\pi}{3}, 3\sqrt{6}\right)$

(3) $\left(\frac{\pi}{3}, 6\right)$

(4) $\left(\frac{\pi}{4}, 6\right)$

Official Ans. by NTA (4)

Sol. n_1 and n_2 are normal vector to the plane $\hat{i} + \hat{j}, \hat{i} + \hat{k}$ and $\hat{i} - \hat{j}; \hat{j} - \hat{k}$ respectively

$$\vec{n}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{vmatrix} = \hat{i} - \hat{j} - \hat{k}$$

$$\vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{j} \\ 1 & -1 & 0 \\ 1 & 0 & -1 \end{vmatrix} = \hat{i} + \hat{j} + \hat{k}$$

$\vec{a} = \lambda|\vec{n}_2 \times \vec{n}_2|$

$$= \lambda \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & -1 \\ 1 & 1 & 1 \end{vmatrix} = \lambda\left(-2\hat{j}+2\hat{k}\right)$$

$\vec{a}\cdot\vec{b} = \lambda\left|0+4+2\right| = 6$

$\Rightarrow \lambda = 1$

$\vec{\alpha} = -2\hat{j}+2\hat{k}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{|a||b|}$

$\cos\theta = \dfrac{6}{2\sqrt{2}\times 3} = \dfrac{1}{\sqrt{2}}$

$\theta = \dfrac{\pi}{4}$

Now $\left|\vec{a}.\vec{b}\right|^2 + \left|\vec{a}\times\vec{b}\right|^2 = |a|^2|b|^2$

$36 + \left|\vec{a}\times b^2\right| = 8\times 9 = 72$

$\left|\vec{a}\times b\right|^2 = 36$

$\left|\vec{a}\times\vec{b}\right| = 6$

19. The number of elements in the set $S = \left\{\theta \in [0,2\pi]: 3\cos^4\theta - 5\cos^2\theta - 2\sin^2\theta + 2 = 0\right\}$ is

(1) 10 (2) 8

(3) 9 (4) 12

Official Ans. by NTA (3)

Sol. $3\cos^4\theta - 5\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow$ $3\cos^4\theta - 3\cos^2\theta - 2\cos^2\theta - 2\sin^6\theta + 2 = 0$

$\Rightarrow$ $3\cos^4\theta - 3\cos^2\theta + 2\sin^2\theta - 2\sin^6\theta = 0$

$\Rightarrow$ $3\cos^2\theta\left(\cos^2\theta - 1\right) + 2\sin^2\theta\left(\sin^4\theta - 1\right) = 0$

$\Rightarrow$ $-3\cos^2\theta\sin^2\theta + 2\sin^2\theta\left(1+\sin^2\theta\right)\cos^2\theta - 1$

$\Rightarrow$ $\sin^2\theta\cos^2\theta\left(2+2\sin^2\theta - 3\right) = 0$

$\Rightarrow$ $\sin^2\theta\cos^2\theta\left(2\sin^2\theta - 1\right) = 0$

(C1) $\sin^2\theta = 0 \;\rightarrow$ 3 solution ; $\theta = \{0, \pi, 2\pi\}$

(C2) $\cos^2\theta = 0 \;\rightarrow$ 2 solution ; $\theta = \left\{\dfrac{\pi}{2}, \dfrac{3\pi}{2}\right\}$

(C3) $\sin^2\theta = \dfrac{1}{2} \rightarrow$ 4 solution ; $\theta = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$

No. of solution = 9

20. Let x_1, x_2 …., x_{100} be in an arithmetic progression, with x_1 = 2 and their mean equal to 200. If $y_i = i\left(x_i - i\right), 1 \le i \le 100$, then the mean of y_1, y_2, ….,, y_{100} is .

(1) 10101.50

(2) 10051.50

(3) 10049.50

(4) 10100

Official Ans. by NTA (3)

Sol. Mean = 200

$\Rightarrow \dfrac{\frac{100}{2}\left(2\times 2 + 99d\right)}{100} = 200$

$\Rightarrow 4 + 99d = 400$

$\Rightarrow d = 4$

$y_i = i\left(xi - i\right)$

$= i\left(2 + \left(i-1\right)4 - i\right) = 3i^2 - 2i$

Mean = $\dfrac{\sum y_i}{100}$

$= \dfrac{1}{100}\sum_{i=1}^{100} 3i^2 - 2i$

$= \dfrac{1}{100}\left\{\dfrac{3\times 100\times 101\times 201}{6} - \dfrac{2\times 100\times 101}{2}\right\}$

$= 101\left\{\dfrac{201}{2} - 1\right\} = 101\times 99.5$

$= 10049\cdot 50$

SECTION-B

21. The mean of the coefficients of $x, x^2, ...x^7$ in the binomial expansion of $(2 + x)^9$ is _______ .

Official Ans. by NTA (2736)

Sol. Coefficient of $x = {}^9C_1 2^8$

Of $x^2 = {}^9C_2 2^7$

Of $x^7 = {}^9C_7 \cdot 2^2$

$$\text{Mean} = \frac{{}^9C_1 \cdot 2^8 + {}^9C_2 \cdot 2^7 ..... + {}^9C_7 \cdot 2^2}{7}$$

$$= \frac{(1+2)^9 - {}^9C_0 \cdot 2^9 - {}^9C_8 \cdot 2^1 - {}^9C_9}{7}$$

$$= \frac{3^9 - 2^9 - 18 - 1}{7}$$

$$= \frac{19152}{7} = 2736$$

22. Let $S = 109 + \frac{108}{5} + \frac{107}{5^2} + ... + \frac{2}{5^{107}} + \frac{1}{5^{108}}$. Then the value of $(16S - (25)^{-54})$ is equal to _______ .

Official Ans. by NTA (2175)

Sol. $S = 109 + \frac{108}{5} + \frac{107}{5^2} ..... + \frac{1}{5^{108}}$

$$\frac{S}{5} = \frac{109}{5} + \frac{108}{5^2} ...... + \frac{2}{5^{108}} + \frac{1}{5^{109}}$$

$$\frac{4S}{5} = 109 - \frac{1}{5} - \frac{1}{5^2} ...... - \frac{1}{5^{108}} - \frac{1}{5^{109}}$$

$$= 109 - \left(\frac{1}{5} \frac{\left(1 - \frac{1}{5^{109}}\right)}{\left(1 - \frac{1}{5}\right)} \right)$$

$$= 109 - \frac{1}{4}\left(1 - \frac{1}{5^{109}}\right)$$

$$= 109 - \frac{1}{4} + \frac{1}{4} \times \frac{1}{5^{109}}$$

$$s = \frac{5}{4}\left(109 - \frac{1}{4} + \frac{1}{4.5^{109}}\right)$$

$$16S = 20 \times 109 - 5 + \frac{1}{5^{108}}$$

$$16S - (25)^{-54} = 2180 - 5 = 2175$$

23. For m, n > 0, let $\alpha(m,n) = \int_0^2 t^m (1 + 3t)^n \, dt$. If $11\alpha(10,6) + 18\alpha(11,5) = p(14)^6$, then p is equal to _______ .

Official Ans. by NTA (32)

Sol. $\alpha(m,n) = \int_0^2 t^m (1+3t)^n \, dt$

If $11\alpha(10,6) + 18\alpha(11,5) = p(14)^6$ then P

$$= 11\int_0^2 \underset{\text{II}}{t^{10}} \underset{\text{I}}{(1+3t)^6} + 10\int^2 t^{11}(1+3t)^5 \, dt$$

$$= 11\left[(1+3t)^6 \cdot \frac{t^{11}}{11} - \int 6(1+3t)^5 \cdot 3\frac{t^{11}}{11}\right]_0^2 + 18\int_0^2 t^{11}(1+3t)^5 \, dt$$

$$= \left(t^{11}(1+3t)^6\right)_0^2$$

$$= 2^{11}(7)^6$$

$$= 2^5(14)^6$$

$$= 32(14)^6$$

24. In an examination, 5 students have been allotted their seats as per their roll numbers. The number of ways, in which none of the students sits on the allotted seat, is _______ .

Official Ans. by NTA (44)

Sol. Derangement of 5 students

$$D_5 = 5!\left(1 - \frac{1}{1!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \frac{1}{5!}\right)$$

$$= 120\left(\frac{1}{2} - \frac{1}{6} + \frac{1}{24} - \frac{1}{120}\right)$$

$$= 60 - 20 + 5 - 1$$

$$= 40 + 4$$

$$= 44$$

25. Let a line l pass through the origin and be perpendicular to the lines

$l_1 : \vec{r} = \left(\hat{i} - 11\hat{j} - 7\hat{k}\right) + \lambda\left(\hat{i} + 2\hat{j} + 3\hat{k}\right), \lambda \in \mathbb{R}$

and $l_2 : \vec{r} = \left(-\hat{i} + \hat{k}\right) + \mu\left(2\hat{i} + 2\hat{j} + \hat{k}\right), \mu \in \mathbb{R}$.

If P is the point of intersection of l and l_1, and Q(α, β, γ) is the foot of perpendicular from P on l_2, then $9(\alpha+\beta+\gamma)$ is equal to_______ .

Official Ans. by NTA (5)

Sol. Let $\ell = \left(0\hat{i} + 0\hat{j} + 0\hat{k}\right) + \gamma\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$

$= \gamma\left(a\hat{i} + b\hat{j} + c\hat{k}\right)$

$$a\hat{i} + b\hat{j} + c\hat{k} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ 2 & 2 & 1 \end{vmatrix}$$

$= \hat{i}(2-6) - \hat{j}(1-6) + \hat{k}(2-4)$

$= -4\hat{i} - 5\hat{j} - 2\hat{k}$

$\ell = \gamma\left(-4\hat{i} + 5\hat{j} - 2\hat{k}\right)$

P is intersection of ℓ and ℓ_1

$-4\gamma = 1 + \lambda, 5\gamma = -11 + 2\lambda, -2\gamma = -7 + 3\lambda$

By solving there equation $\gamma = -1$, P (4,–5,2)

Let $Q(-1+2\mu, 2\mu, 1+\mu)$

$\overrightarrow{PQ} \cdot \left(2\hat{i} + 2\hat{j} + \hat{k}\right) = 0$

$-2 + 4\mu + 4\mu + 1 + \mu = 0$

$9\mu = 1$

$\mu = \frac{1}{9}$

$Q\left(\frac{-7}{9}, \frac{2}{9}, \frac{10}{9}\right)$

$9(\alpha + \beta + \gamma) = 9\left(\frac{-7}{9} + \frac{2}{9} + \frac{10}{9}\right)$

$= 5$

26. The number of integral terms in the expansion of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

Official Ans. by NTA (171)

Sol. The number of integral term in the expression of $\left(3^{\frac{1}{2}} + 5^{\frac{1}{4}}\right)^{680}$ is equal to

General term $= {}^{680}C_r \left(3^{\frac{1}{2}}\right)^{680-r} \left(5^{\frac{1}{4}}\right)^{r}$

$= {}^{680}C_r 3^{\frac{680-r}{2}} 5^{\frac{r}{4}}$

Value's of r, where $\frac{r}{4}$ goes to integer

r = 0, 4, 8, 12, ……….680

All value of r are accepted for $\frac{680-r}{2}$ as well so

No of integral terms = 171.

27. The number of ordered triplets of the truth values of p, q and r such that the truth value of the statement $(p \vee q) \wedge (p \vee r) \Rightarrow (q \vee r)$ is True, is equal to _______ .

Official Ans. by NTA (7)

Sol.

p	**q**	**r**	**Pvq**	**Pvr**	**(pvq) ∧ (pvr)**	**qvr**	**(pvq) ∧ (pvr) → qvr**
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	F	F
F	T	T	T	T	T	T	T
F	T	F	T	F	F	T	T
F	F	T	F	T	F	T	T
F	F	F	F	F	F	F	T

Hence total no of ordered triplets are 7

28. Let $H_n = \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1, n \in N$. Let k be the smallest even value of n such that the eccentricity of H_k is a rational number. If l is length of the latus return of H_k, then $21l$ is equal to ________

Official Ans. by NTA (306)

Sol. $Hn \Rightarrow \frac{x^2}{1+n} - \frac{y^2}{3+n} = 1$

$e = \sqrt{1+\frac{b^2}{a^2}} = \sqrt{1+\frac{3+n}{1+n}} = \sqrt{\frac{2n+4}{n+1}}$

$e = \sqrt{\frac{2n+4}{n+1}}$

$n = 48$ (smallest even value for which $e \in Q$)

$\boxed{e = \frac{10}{7}}$

$a^2 = n+1 = 49$, $b^2 = n+3 = 51$

1 = length of LR $= \frac{2b^2}{a}$

$L = 2 \cdot \frac{51}{7}$

$1 = \frac{102}{7}$

$\boxed{21\ell = 306}$

29. If a and b are the roots of equation $x^2 - 7x - 1 = 0$, then the value of $\frac{a^{21} + b^{21} + a^{17} + b^{17}}{a^{19} + b^{19}}$ is equal to ________ .

Official Ans. by NTA (51)

Sol. $x^2 - 7x - 1 = 0 <_b^a$

By newton's theorem

$S_{n+2} - 7S_{n+1} - S_n = 0$

$S_{21} - 7S_{20} - S_{19} = 0$

$S_{20} - 7S_{19} - S_{18} = 0$

$S_{19} - 7S_{18} - S_{17} = 0$

$\frac{S_{21} + S_{17}}{S_{19}} = \frac{S_{21} + (S_{19} - 7S_{18})}{S_{19}}$

$= \frac{S_{21} + S_{19} - 7(S_{20} - 7S_{19})}{S_{19}}$

$= \frac{50S_{19} + (S_{21} - 7S_{20})}{S_{19}}$

$= 51 \cdot \frac{S_{19}}{S_{19}} = \boxed{51}$

30. Let $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$, where $a, c \in R$. If $A^3 = A$ and the positive value of a belongs to the interval $(n-1, n]$, where $n \in N$, then n is equal to ________ .

Official Ans. by NTA (2)

Sol. $A = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^3 = A$

$A^2 = \begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^2 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & 1 & 2+3c \end{bmatrix}$

$A^3 = \begin{bmatrix} a+2 & 2c & 3 \\ 3 & a+3c & 2a \\ ac & a & 2+3c \end{bmatrix}\begin{bmatrix} 0 & 1 & 2 \\ a & 0 & 3 \\ 1 & c & 0 \end{bmatrix}$

$A^3 = \begin{bmatrix} 2ac+3 & a+2+3c & 2a+4+6c \\ a(a+3c)+2a & 3+2ac & 6+3a+9c \\ a+2+3c & ac+c(2+3c) & 2ac+3 \end{bmatrix}$

Given $A^3 = A$

$2ac + 3 = 0$ …(1) and $a+2+3c = 1$

$a + 1 + 3c = 0$

$a + 1 - \frac{9}{2a} = 0$

$2a^2 + 2a - 9 = 0$

$f(1) < 0, f(2) > 0$

$a \in (1, 2]$ $\boxed{n = 2}$

SECTION-A

31. The electric field in an electromagnetic wave is given as $\vec{E} = 20\sin\omega\left(t - \frac{x}{c}\right)\vec{j}NC^{-1}$

Where ω and c are angular frequency and velocity of electromagnetic wave respectively. The energy contained in a volume of $5\times10^{-4}m^3$ will be (Given $\varepsilon_0 = 8.85\times10^{-12}C^2/Nm^2$)

(1) $28.5\times10^{-13}J$ (2) $17.7\times10^{-13}J$

(3) $8.85\times10^{-13}J$ (4) $88.5\times10^{-13}J$

Official Ans. by NTA (3)

Sol. $\vec{E} = 20\sin\omega\left(t - \frac{x}{C}\right)\hat{j}N/C$

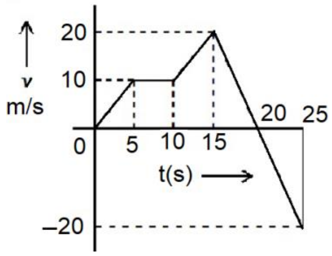
Average energy density of an em wave = $\frac{1}{2}\epsilon_0 E_0^2$

Energy stored = $\left(\frac{1}{2}\epsilon_0 E_0^2\right)(\text{volume})$

$= \frac{1}{2}\times8.85\times10^{-12}\times(20)^2\times(5\times10^{-4})J$

$= 8.85\times10^{-13}J$

32. From the v - t graph shown. the ratio of distance to displacement in 25 s of motion



(1) $\frac{3}{5}$ (2) $\frac{1}{2}$

(3) $\frac{5}{3}$ (4) 1

Official Ans. by NTA (3)

Sol. Area under the graph from t = 0 to t = 20 sec = 200 m

Area under the graph from t = 20 to t = 25 sec = 50 m

So distance covered = (200 + 50)m = 250 m

Displacement = (200 – 50)m = 150 m

$\frac{250}{150} = \frac{5}{3}$

33. The radii of two planets 'A' and 'B' are 'R' and '4R' and their densities are ρ and $\rho/3$ respectively. The ratio of acceleration due to gravity at their surfaces ($g_A : g_B$) will be :

(1) 1 : 16 (2) 3 : 16

(3) 3 : 4 (4) 4 : 3

Official Ans. by NTA (3)

Sol. $g = \frac{GM}{R^2} = \frac{G}{R^2}\times\rho\times\frac{4\pi}{3}R^3 = \left(\frac{4\pi}{3}G\right)\rho R$

$\frac{g_A}{g_B} = \frac{R\times\rho}{4R\times\frac{\rho}{3}} = \frac{3}{4}$

34. A coin placed on a rotating table just slips when it is placed at a distance of 1 cm from the center. If the angular velocity of the table in halved, it will just slip when placed at a distance of_____ from the centre:

(1) 2 cm

(2) 1 cm

(3) 8 cm

(4) 4 cm

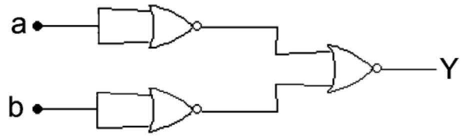
Official Ans. by NTA (4)

Sol. $f_{s\,max} = \mu\,mg = m\omega^2R \Rightarrow R = \frac{\mu g}{\omega^2}$

So if ω becomes $\frac{\omega}{2}$, R will become 4R.

So distance from the center will be 4 cm.

35. The logic performed by the circuit shown in figure is equivalent to :

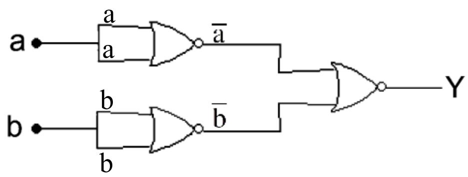


(1) AND (2) NAND

(3) OR (4) NOR

Official Ans. by NTA (1)

Sol.



$Y = \overline{\overline{a} + \overline{b}} = a \cdot b$

The truth table for the given circuit will be

a	b	output
0	0	0
0	1	0
1	0	0
1	1	1

Hence it will be equivalent to AND gate.

36. A parallel plate capacitor of capacitance 2 F is charged to a potential V. The energy stored in the capacitor is E_1. The capacitor is now connected to another uncharged identical capacitor in parallel combination. The energy stored in the combination is E_2. The ratio E_2/E_1 is :

(1) 2 : 1 (2) 1 : 2

(3) 1 : 4 (4) 2 : 3

Official Ans. by NTA (2)

Sol. Initially

$Q_1 = CV = (2)\ V$

$E_1 = 1/2\ CV^2 = 1/2\ (2)V^2 = V^2$

Finally

Charge on each capacitor, $Q_2 = \frac{Q_1}{2} = \frac{2V}{2} = V$

$E_2 = 2\left(\frac{1}{2}\frac{Q_2^2}{C}\right) = \frac{V^2}{2} \qquad \therefore \frac{E_2}{E_1} = \frac{1}{2}$

37. Two identical heater filaments are connected first in parallel and then in series. At the same applied voltage, the ratio of heat produced in same time for parallel to series will be:

(1) 4 : 1 (2) 2 : 1 (3) 1 : 2 (4) 1 : 4

Official Ans. by NTA (1)

Sol. Parallel combination

$$H_p = \left[\frac{V^2}{\left(\frac{R}{2}\right)}\right]t = \frac{2V^2t}{R}$$

Series combination

$$H_s = \left(\frac{V^2}{2R}\right)t \qquad \therefore \frac{H_p}{H_s} = 4$$

38. A transmitting antenna is kept on the surface of the earth. The minimum height of receiving antenna required to receive the signal in line of sight at 4 km distance from it is x × 10^{-2} m. The value of x is (Let. radius of earth R = 6400 km)

(1) 125 (2) 12.5 (3) 1.25 (4) 1250

Official Ans. by NTA (1)

Sol. $d_r = \sqrt{2h_rR} \quad \therefore h_r = \frac{d_r^2}{2R}$

$$= \frac{(4\text{km})^2}{2(6400\text{ km})} = \left(\frac{1}{800}\right)\text{km} = 1.25\text{ m}$$

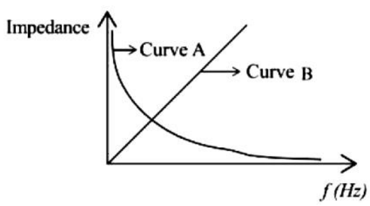
39. As per the given graph choose the correct representation for curve A and curve B.

{Where X_C = reactance of pure capacitive circuit connected with A.C. source

X_L = reactance of pure inductive circuit connected with A.C. source

R = impedance of pure resistive circuit connected with A.C. source

Z = Impedance of the LCR series circuit}



(1) A = X_C, B = R (2) A = X_L, B = Z

(3) A = X_C, B = X_L (4) A = X_L, B = R

Official Ans. by NTA (3)

Sol. $X_C = \frac{1}{\omega C} = \frac{1}{(2\pi f)C}$

$\therefore X_C \propto \frac{1}{f}$

$\therefore$ Curve A

$X_L = \omega L = (2\pi f)L$

$\therefore X_L \propto f$

$\therefore$ Curve B

40. 1 kg of water at 100°C is converted into steam at 100°C by boiling at atmospheric pressure. The volume of water changes from 1.00×10^{-3} m^3 as a liquid to 1.671 m^3 as steam. The change in internal energy of the system during the process will be (Given latent heat of vaporisaiton = 2257 kJ/kg. Atmospheric pressure = 1×10^5 Pa)

(1) + 2090 kJ (2) – 2090 kJ

(3) – 2426 kJ (4) + 2476 kJ

Official Ans. by NTA (1)

Sol. $\Delta Q = \Delta U + \Delta W$

$\therefore \Delta U = \Delta Q - \Delta W$

$= mL_V - P\Delta V$

$= (1Kg)(2257 \times 10^3 J/kg)$

$-(1\times 10^5 Pa)(1.671m^3 - 1\times 10^{-3} m^3)$

$= 2257 \times 10^3 J - 167 \times 10^3 J$

$= 2090$ KJ

41. The critical angle for a denser-rarer interface is 45°. The speed of light in rarer medium is 3×10^8 ms. The speed of light in the denser medium is:

(1) 5×10^7 m/s (2) 2.12×10^8 m/s

(3) 3.12×10^7 m/s (4) $\sqrt{2} \times 10^8$ m/s

Official Ans. by NTA (2)

Sol. $i_C =$ Critical angle

$\frac{\nu}{C} = \frac{1}{\mu} = \sin i_C = \sin 45° = \frac{1}{\sqrt{2}}$

$\Rightarrow \nu = \frac{C}{\sqrt{2}} = \frac{3\times 10^8}{\sqrt{2}} m/s = 2.12 \times 10^8 m/s$

42. A metallic surface is illuminated with radiation of wavelength λ, the stopping potential is V_o. If the same surface is illuminated with radiation of wavelength 2λ, the stopping potential becomes $\frac{V_o}{4}$. The threshold wavelength for this metallic surface will be -

(1) $\frac{\lambda}{4}$ (2) 4λ

(3) $\frac{3}{2}\lambda$ (4) 3λ

Official Ans. by NTA (4)

Sol. From the equation of photoelectric effect

$eV_0 = \frac{hc}{\lambda} - \phi_0 = \frac{hc}{\lambda} - \frac{hc}{\lambda_0}$

$\& \frac{eV_0}{4} = \frac{hc}{2\lambda} = \frac{hc}{\lambda_0}$

$\Rightarrow \frac{1}{4}\left(\frac{hc}{\lambda} - \frac{hc}{\lambda_0}\right) = \frac{hc}{2\lambda} - \frac{hc}{\lambda_0}$

$\frac{1}{\lambda_0} - \frac{1}{4\lambda_0} = \frac{1}{2\lambda} - \frac{1}{4\lambda}$

$\frac{3}{4\lambda_0} = \frac{1}{4\lambda}$

$\Rightarrow \lambda_0 = 3\lambda$

43. The free space inside a current carrying toroid is filled with a material of susceptibility 2×10^{-2}. The percentage increase in the value of magnetic field inside the toroid will be

(1) 2% (2) 0.2%

(3) 0.1% (4) 1%

Official Ans. by NTA (1)

Sol. As $X_m = 2\times 10^{-2}$

$\mu_r = 1 + X_m = 1.02$

$\Rightarrow B = \mu_r B_0 = 1.02 B_0$

So percentage increase in magnetic field

$= \frac{B - B_0}{B_0} \times no\% = 2\%$

44. The current sensitivity of moving coil galvanometer is increased by 25%. This increase is achieved only by changing in the number of turns of coils and area of cross section of the wire while keeping the resistance of galvanometer coil constant. The percentage change in the voltage sensitivity will be:

(1) +25% (2) – 50%

(3) Zero (4) – 25%

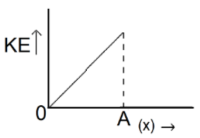
Official Ans. by NTA (1)

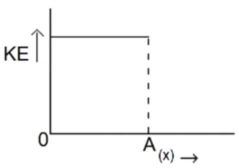
Sol. $I_S = \frac{NBA}{C}$ & $V_S = \frac{NBA}{CG}$

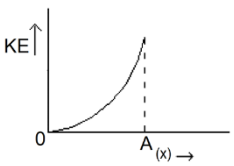
$\Rightarrow V_S = \frac{I_S}{G}$, If G (galvanometer resistance) is constant, then $V_S \propto I_S$

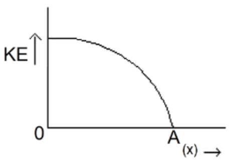
so percentage change in V_S is also 25%.

45. The variation of kinetic energy (KE) of a particle executing simple harmonic motion with the displacement (x) starting from mean position to extreme position (A) is given by

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol. For a particle executing SHM

$$KE = \frac{1}{2}m\omega^2\left(A^2 - x^2\right)$$

When x = 0, KE is maximum & when x = A, KE is zero and KE V/S x graph is parabola.

46. On a temperature scale 'X'. The boiling point of water is 65° X and the freezing point is –15°X. Assume that the X scale is linear. The equivalent temperature corresponding to –95° X on the Farenheit scale would be:

(1) –63°F (2) –112°F

(3) –48°F (4) –148°F

Official Ans. by NTA (4)

Sol. $\frac{X - X_{freez}}{X_{boil} - X_{freez}} = \frac{t-32}{212-32}$

$\frac{-95-(-15)}{65-(-15)} = \frac{t-32}{180}$

$\frac{-80}{80} = \frac{t-32}{180}$

$t = -180 + 32$

$t = -148°f$

47. Given below are two statements :

Statements I : Astronomical unit (Au). Parsec (Pc) and Light year (ly) are units for measuring astronomical distances.

Statements II: Au < Parsec (Pc) < ly

In the light of the above statements. choose the most appropriate answer from the options given below:

(1) Both Statements I and Statements II are correct.

(2) Statements I is correct but Statements II is incorrect.

(3) Both Statements I and Statements II are incorrect.

(4) Statements I is incorrect but statements II is correct.

Official Ans. by NTA (2)

Sol. $1AU = 1.496 \times 10^{11}$ m

$1 \text{ par sec} = 3.08 \times 10^{16}$ m

$1 \text{ light year} = 9.46 \times 10^{15}$ m

So, $Au < ly < Per\,sec$

48. Three vessels of equal volume contain gases at the same temperature and pressure. The first vessel contains neon (monoatomic), the second contains chlorine (diatomic) and third contains uranium hexafloride (polyatomic). Arrange these on the basis of their root mean square speed (v_{rms}) and choose the correct answer from the options given below:

(1) $v_{rms}(mono) = v_{rms}(dia) = v_{rms}(poly)$

(2) $v_{rms}(mono) > v_{rms}(dia) > v_{rms}(poly)$

(3) $v_{rms}(dia) < v_{rms}(poly) < v_{rms}(mono)$

(4) $v_{rms}(mono) < v_{rms}(dia) < v_{rms}(poly)$

Official Ans. by NTA (2)

Sol. $v_{rms}(mono) = \sqrt{\frac{3RT}{4\times10^{-3}}}$

$v_{rms}(dia) = \sqrt{\frac{3RT}{71\times10^{-3}}}$

$v_{rms}(ply) = \sqrt{\frac{3RT}{146\times10^{-3}}}$

So correct relation is

$v_{rms}(mono) > v_{rms}(dia) > v_{rms}(poly)$

49. An average force of 125 N is applied on a machine gun firing bullets each of mass 10 g at the speed of 250 m/s to keep it in position. The number of bullets fired per second by the machine gun is :

(1) 5 (2) 50

(3) 100 (4) 25

Official Ans. by NTA (2)

Sol. $F = n\,m\,v$

where n = number of bullets fired per second

$n = \frac{f}{mv} = \frac{125}{10\times10^{-3}\times250} = 50$

50. Two radioactive elements A and B initially have same number of atoms. The half life of A is same as the average life of B. If λ_A and λ_B are decay constants of A and B respectively, then choose the correct relation from the given options.

(1) $\lambda_A = \lambda_B$ (2) $\lambda_A = 2\lambda_B$

(3) $\lambda_A = \lambda_B \ln 2$ (4) $\lambda_A \ln 2 = \lambda_B$

Official Ans. by NTA (3)

Sol. $T_{1/2}(A) = T_{av}(B)$

$\frac{\ell n2}{\lambda_A} = \frac{1}{\lambda_B}$

$\lambda_A = \lambda_B \ell 2$

SECTION-B

51. A monochromatic light is incident on a hydrogen sample in ground state. Hydrogen atoms absorb a fraction of light and subsequently emit radiation of six different wavelengths. The frequency of incident light is x × 10^{15} Hz. The value of x is ________. (Given h = 4.25 × 10^{-15} eVs)

Official Ans. by NTA (3)

Sol. $6 = {}^4C_2 \quad \Rightarrow n_2 = 4$

$h\nu = E_4 - E_1$

$\therefore \nu = 13.6\left(\frac{1}{1^2} - \frac{1}{4^2}\right)\times\frac{1}{4.25\times10^{-15}}$

$= 3\times10^{15}\,Hz$

52. The radius of curvature of each surface of a convex lens having refractive index 1.8 is 20 cm. The lens is now immersed in a liquid of refractive index 1.5. The ratio of power of lens in air to its power in the liquid will be x : 1. The value of x is _____.

Official Ans. by NTA (4)

Sol. $P = (1.8-1)\left(\frac{1}{20}+\frac{1}{20}\right)$ by lens maker's formula

$P' = \left(\frac{1.8}{1.5}-1\right)\left(\frac{1}{20}+\frac{1}{20}\right)$

Dividing $\frac{P}{P'} = \frac{0.8}{1.2-1} = 4$

53. The equation of wave is given by

$$Y = 10^{-2}\sin 2\pi\left(160t - 0.5x + \frac{\pi}{4}\right)$$

Where x and Y are in m and t in s. The speed of the wave is ____ km h^{-1}

Official Ans. by NTA (1152)

Sol. $V = \frac{\omega}{k} = \frac{2\pi \times 60}{2\pi \times 0.5} = \frac{160}{0.5}\, m/s$

$= \frac{160}{0.5} \times \frac{18}{5}\, km/h$

$= 1152\, km$

54. A force $\vec{F} = (2 + 3x)\hat{i}$ acts on a particle in the x direction where F is in newton and x is in meter. The work done by this force during a displacement from x = 0 to x = 4 m, is ____J.

Official Ans. by NTA (32)

Sol. $W = \int_0^4 (2 + 3x)dx$

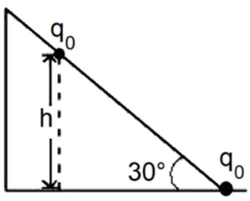
$= \left[2x + \frac{3x^2}{2}\right]_0^4$

$= 8 + 3 \times 8$

$= 32\, J$

55. As shown in the figure. a configuration of two equal point charges (q_0 = +2μ C) is placed on an inclined plane. Mass of each point charge is 20 g. Assume that there is no friction between charge and plane. For the system of two point charges to be in equilibrium (at rest) the height h = x × 10^{-3} m The value of x is ____.

(Take $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9\, N\, m^2 C^{-2}, g = 10\, ms^{-1}$)



Official Ans. by NTA (300)

Sol. For equilibrium along the plane

$$mg\sin\theta = \frac{1}{4\pi\epsilon_0} \times \frac{q_0^2}{(h\ \text{cosec}\ 30°)^2}$$

$$\therefore h^2 = \frac{1}{4\pi\epsilon_o} \times \frac{q_0^2}{mg\ \text{cosec}\ 30°}$$

$$= 9 \times 10^9 \times \frac{(2 \times 10^{-6})^2}{0.02 \times 10 \times 2}$$

$$\therefore h = 3 \times 10^4 \times \frac{2 \times 10^{-6}}{0.2}$$

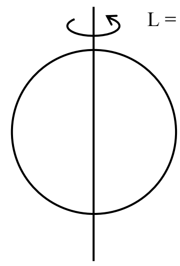
$= 0.3\, m$

$= 300\, mm$

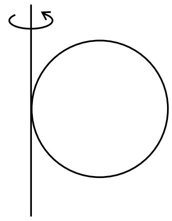
56. A solid sphere of mass 500 g and radius 5 cm is rotated about one of its diameter with angular speed of 10 rad s^{-1}. If the moment of inertia of the sphere about its tangent is x × 10^{-2} times its angular momentum about the diameter. Then the value of x will be _____.

Official Ans. by NTA (35)

Sol.



moment of inertia about tangent = I_T



$I_t = x \times 10^{-2} L$

$\frac{7}{5}mR^2 = x \times 10^{-2}\frac{2}{5}mR^2\omega$

$\frac{7}{2\omega} = x \times 10^{-2} = \frac{7}{2 \times 10}$

57. The length of wire becomes l_1 and l_2 when 100N and 120 N tensions are applied respectively. If $10\, l_2 = 11\, l_1$, the natural length of wire will be $\frac{1}{x} l_1$. Here the value of x is ____.

Official Ans. by NTA (2)

Sol. Let the original length be 'ℓ_0'

When $T_1 = 100$ N, Extension $= \ell_1 - \ell_0$

When $T_2 = 120$ N, Extension $= \ell_2 - \ell_0$

Then $100 = K(\ell_1 - \ell_0)$ ...(1)

And $120 = K(\ell_2 - \ell_0)$ ...(2)

$\frac{1}{2} \Rightarrow \frac{5}{6} = \frac{\ell_1 - \ell_0}{\ell_2 - \ell_0}$

$5\ell_2 - 5\ell_0 = 6\ell_1 - 6\ell_0$

$\ell_0 = 6\ell_1 - 5\ell_2$

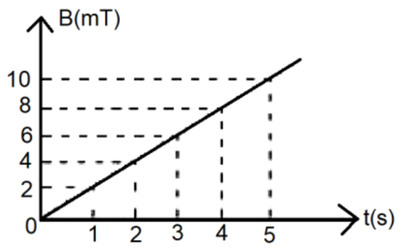
$\ell_0 = 6\ell_1 - 5\left(\frac{11\ell_1}{10}\right)$

$\ell_0 = 6\ell_1 - \frac{11\ell_1}{2}$

$\ell_0 = \frac{\ell_1}{2}$

$\therefore x = 2$

58. The magnetic field B crossing normally a square metallic plate of area 4 m^2 is changing with time as shown in figure. The magnitude of induced emf in the plate during t = 2s to t = 4s, is _______ mV

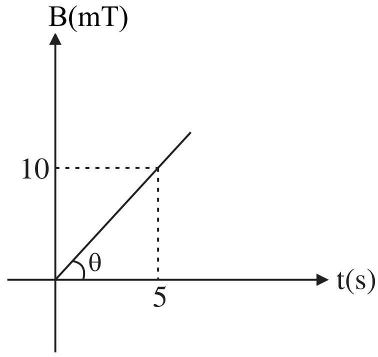


Official Ans. by NTA (8)

Sol. $m = \tan\theta = \frac{10}{5} = 2$

$B = mt$

$B = 2t$



$\varepsilon = \left|\frac{d\phi}{dt}\right| = \frac{d(BA)}{dt} = \frac{AdB}{dt}$

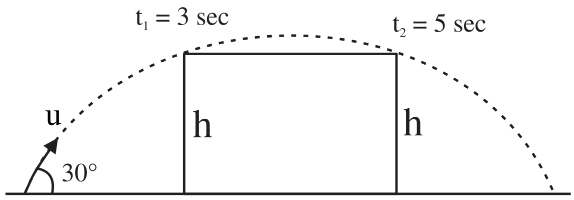
$\varepsilon = \frac{4d(2t)}{dt} = 4 \times 2 = 8$ mVolt

59. A projectile fired at 30° to the ground is observed to be at same height at time 3s and 5s after projection, during its flight. The speed of projection of the projectile is _______ ms^{-1}

(Given g = 10 m s^{-2})

Official Ans. by NTA (80)

Sol. Time of flight $t_1 + t_2 = 3 + 5 = 8$ sec

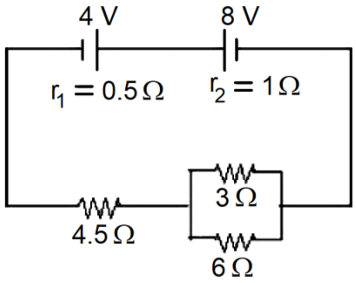


$T = \frac{2u\sin 30°}{g}$

$8 = \frac{2u\sin(30°)}{10}$

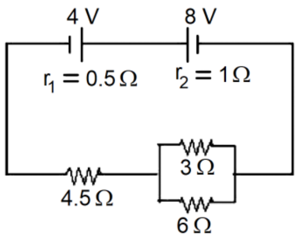
$u = 80$ m/s

60. In the circuit diagram shown in figure given below, the current flowing through resistance $3\,\Omega$ is $\frac{x}{3}$A. The value of x is ____.

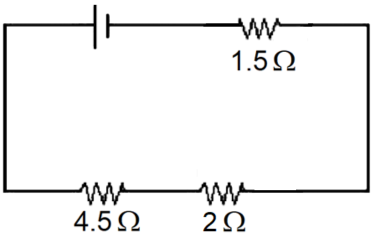


Official Ans. by NTA (1)

Sol.

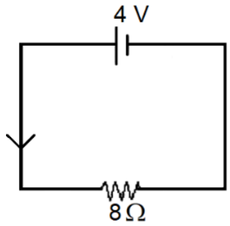


$E_2 - E_1 = 8 - 4 = 4V$

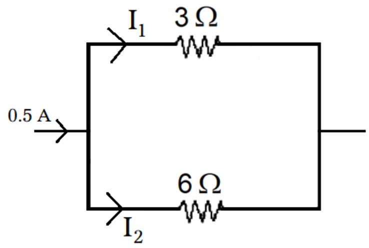


$$\frac{1}{3} + \frac{1}{6} = \frac{1}{2} = \frac{1}{R}$$

$R = 2\,\Omega$



$$I = \frac{4}{8} = 0.5A$$



$$I_1 = \left(\frac{6}{3+6}\right) \times 0.5$$

$$I_1 = \frac{2}{3} \times 0.5 = \frac{1}{3}A$$

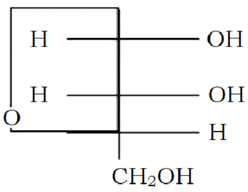
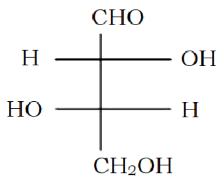
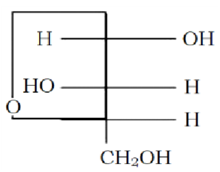
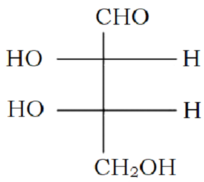
$$I_1 = \frac{x}{3} = \frac{1}{3} \therefore x = 1$$

SECTION-A

61. L –isomer of tetrose X ($C_4H_8O_4$) gives positive Schiff's test and has two chiral carbons. On acetylation. 'X' yields triacetate. 'X' also undergoes following reactions

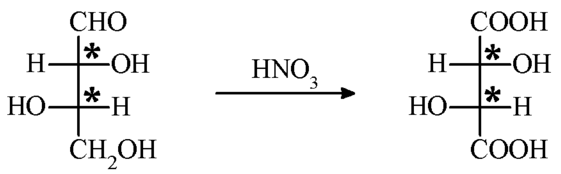
$$'A' \xleftarrow{HNO_3} 'X' \xrightarrow{NaBH_4} \underset{\text{Chiral compound}}{'B'}$$

'X' is

(1) 
(2) 
(3) 
(4) 

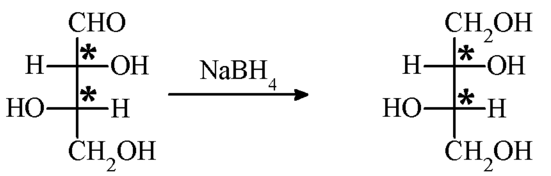
Official Ans. by NTA (2)

Sol.



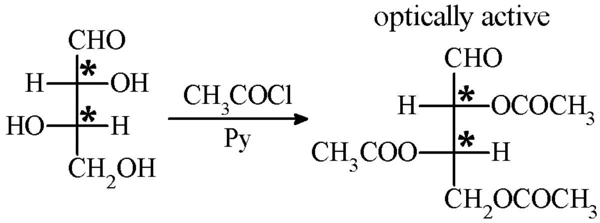
(x) (A)

L-tetrose with two chiral centre



(B)

optically active



(x) gives positive schiff's test due –CHO group

(x) is L–tetrose.

62. The polymer X – consists of linear molecules and is closely packed. It is prepared in the presence of triethylaluminium and titanium tetrachloride under low pressure. The polymer X is –

(1) Polyacrylonitrile

(2) Low density polythene

(3) Polytetrafluoroethane

(4) High density polythene

Official Ans. by NTA (4)

Sol. Ethene undergoes addition polymerisation to high density polythene in the presence of catalyst such as $AlEt_3$ and $TiCl_4$ (Ziegler – Natta catalyst) at a temperature of 333 K to 343 K and under a pressure of 6–7 atmosphere.

63. When a solution of mixture having two inorganic salts was treated with freshly prepared ferrous sulphate in acidic medium, a dark brown ring was formed whereas on treatment with neutral $FeCl_3$, it gave deep red colour which disappeared on boiling and a brown red ppt was formed. The mixture contains

(1) CH_3COO^- & NO_3^-

(2) $C_2O_4^{2-}$ & NO_3^-

(3) SO_3^{2-} & CH_3COO^-

(4) SO_3^{2-} & $C_2O_4^{2-}$

Official Ans. by NTA (1)

Sol. $CH_3COO^- + FeCl_3 \rightarrow Fe(CH_3COO)_3$ or

$$[Fe_3(OH)_2(CH_3COO)_6]^+$$

Blood red colour

$\downarrow \Delta$

$$Fe(OH)_2(CH_3COO)\downarrow$$

Red–brown precipitate

$$2NO_3^- + 4H_2SO_4 + 6Fe^{2+} \rightarrow 6Fe^{3+} + 2NO\uparrow + 4SO_4^{2-} + 4H_2O$$

$$[Fe(H_2O)_6]^{2+} + NO \rightarrow \underset{\text{Brown}}{[Fe(H_2O)_5(NO)]^{2+}} + H_2O$$

64. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : In the photoelectric effect, the electrons are ejected from the metal surface as soon as the beam of light of frequency greater than threshold frequency strikes the surface.

Reason R : When the photon of any energy strikes an electron in the atom, transfer of energy from the photon to the electron takes place.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are correct but R is NOT the correct explanation of A

(2) A is correct but R is not correct

(3) Both A and R are correct and R is the correct explanation of A

(4) A is not correct but R is correct

Official Ans. by NTA (2)

Sol. There is a characteristic minimum frequency, or "threshold frequency," for each metal below which the photoelectric effect is not seen. The ejected electrons leave with a specific amount of kinetic energy at a frequency $\nu > \nu_0$ with an increase in light frequency of these electron kinetic energies also rise.

65. 25 mL of silver nitrate solution (1 M) is added dropwise to 25 mL of potassium iodide (1.05 M) solution. The ion(s) present in very small quantity in the solution is/are

(1) NO_3^- only

(2) K^+ only

(3) Ag^+ and I^- both

(4) I^- only

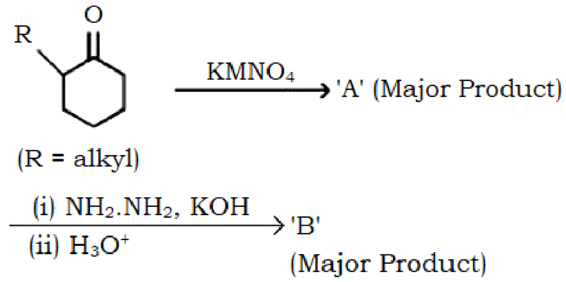
Official Ans. by NTA (3)

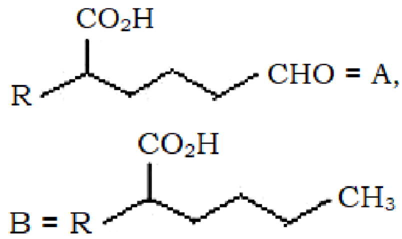
Sol. $AgNO_3 + KI \rightarrow AgI\downarrow + KNO_3$

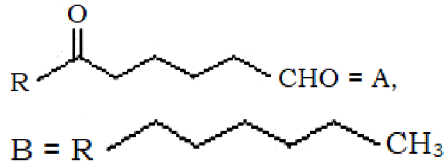
$$AgI \rightarrow \underset{S}{Ag^+} + \underset{S+0.625}{I^-}$$

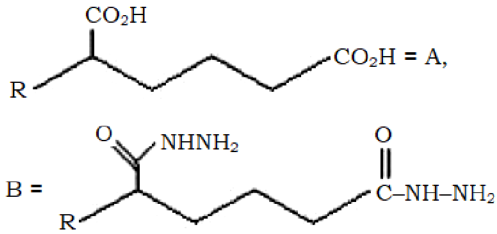
AgI is a insoluble salt so concentration Ag^+ and I^- will be negligible.

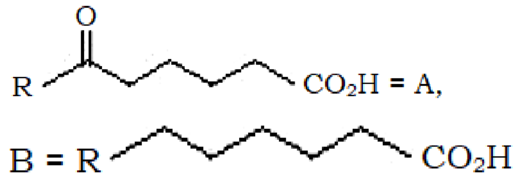
66. 'A' and 'B' in the below reactions are :



(1) 

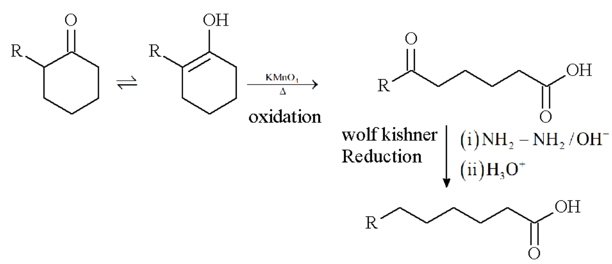
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



67. The set which does not have ambidentate ligand(s) is

(1) $C_2O_4^{2-}$, ethylene diammine, H_2O

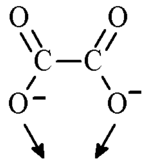
(2) $EDTA^{4-}$, NCS^-, $C_2O_4^{2-}$

(3) NO_2^-, $C_2O_4^{2-}$, $EDTA^{4-}$

(4) $C_2O_4^{2-}$, NO_2^-, NCS^-

Official Ans. by NTA (1)

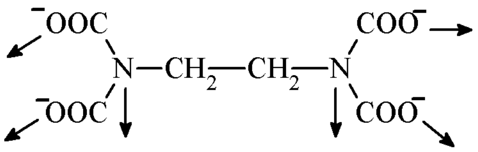
Sol. NO_2^-, NCS^- are ambidentate ligand

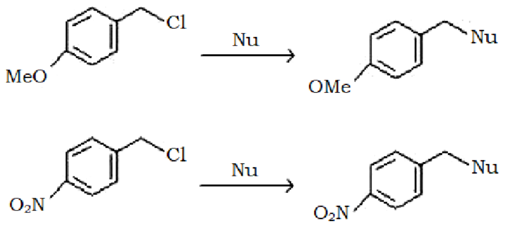
$C_2O_4^{--}$ 

Ethylene diammine

$H_2N-CH_2-CH_2-NH_2$

EDTA Ethylene diamine tetra acetate



68. 

— (I)

— (II)

Where Nu = Nucleophile

Find out the correct statement from the options given below for the above 2 reactions.

(1) Reaction (I) is of 2[nd] order and reaction (II) is of 1[st] order

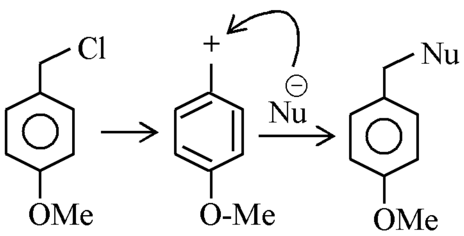
(2) Reaction (I) and (II) both are of 2[nd] order

(3) Reaction (I) is of 1[st] order and reaction (II) is of 2[nd] order

(4) Reactions (I) and (II) both are of 1[st] order

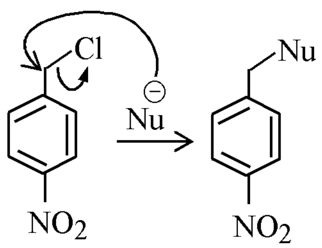
Official Ans. by NTA (3)

Sol.



Electron Donating group

S_N1 Mech. : I[st] order



Electron withdrawing group

S_N2 Mech : 2[nd] order

69. For elements B, C, N, Li, Be, O and F the correct order of first ionization enthalpy is

(1) Li < Be < B < C < N < O < F

(2) B > Li > Be > C > N > O > F

(3) Li < B < Be < C < O < N < F

(4) Li < Be < B < C < O < N < F

Official Ans. by NTA (3)

Sol. First I.E.

F > N > O > C > Be > B > Li

Li – 520 kJ/mol

Be – 899 kJ/mol

B – 801 kJ/mol

C – 1086 kJ/mol

N – 1402 kJ/mol

O – 1314 kJ/mol

F – 1681 kJ/mol

70. Match List-I with List-II :

List-I Species	List-II Geometry/Shape
A. H_3O^+	I. Tetrahedral
B. Acetylide anion	II. Linear
C. NH_4^+	III. Pyramidal
D. ClO_2^-	IV. Bent

Choose the correct answer from the options given below :

(1) A-III, B-II, C-I, D-IV

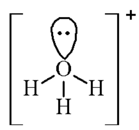
(2) A-III, B-I, C-II, D-IV

(3) A-III, B-IV, C-I, D-II

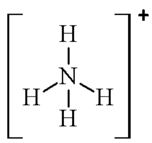
(4) A-III, B-IV, C-II, D-I

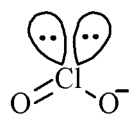
Official Ans. by NTA (1)

Sol. Molecule/Ion Hybridisation Shape

H_3O^+ sp^3 Pyramidal 

Acelylide sp linear $\bar{C}\equiv\bar{C}$

NH_4^+ sp^3 tetrahedral 

ClO_2^- sp^3 Bent 

71. For compound having the formula $GaAlCl_4$, the correct option from the following is

(1) Ga is more electronegative than Al and is present as a cationic part of the salt $GaAlCl_4$

(2) Oxidation state of Ga in the salt $GaAlCl_4$ is +3.

(3) Cl forms bond with both Al and Ga in $GaAlCl_4$

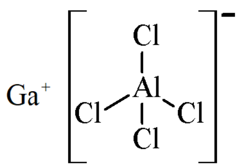
(4) Ga is coordinated with Cl in $GaAlCl_4$

Official Ans. by NTA (1)

Sol. Gallous tetrachloro aluminate $Ga^+AlCl_4^-$

$$2Ga + Ga^+GaCl_4^- + 2Al_2Cl_6 \xrightarrow{190^\circ} 4Ga^+AlCl_4^-$$

Structure of $\overset{+}{G}a\overset{-}{A}lCl_4$



Ga is cationic part of salt $GaAlCl_4$.

72. In the extraction process of copper, the product obtained after carrying out the reactions

(i) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(ii) $2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$ is called

(1) Blister copper

(2) Copper scrap

(3) Reduced copper

(4) Copper matte

Official Ans. by NTA (1)

Sol. $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 3SO_2$

$2Cu_2O + Cu_2S \rightarrow 6Cu + SO_2$

Blister copper

Due to evolution of SO_2, the solidified copper formed has a blistered look and is referred to as blister copper.

73. Match List-I with List-II :

List-I	**List-II**
A. K	I. Thermonuclear reactions
B. KCl	II. Fertilizer
C. KOH	III. Sodium potassium pump
D. Li	IV. Absorbent of CO_2

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-IV, B-I, C-III, D-II

(3) A-IV, B-III, C-I, D-II

(4) A-III, B-IV, C-II, D-I

Official Ans. by NTA (1)

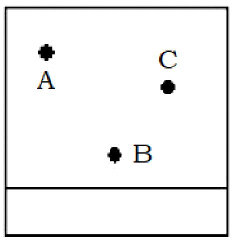
Sol. K^+ – Sodium – Potassium Pump

KCl – Fertiliser

KOH – absorber of CO_2

Li – used in thermonuclear reactions

74. Thin layer chromatography of a mixture shows the following observation :



The correct order of elution in the silica gel column chromatography is

(1) A, C, B

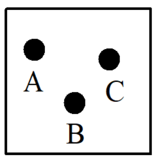
(2) B, C, A

(3) C, A, B

(4) B, A, C

Official Ans. by NTA (1)

Sol.



According to the observation, A is more mobile and interacts with the mobile phase more than C, and C is more drawn to the mobile phase than B.

Hence, the correct order of elution in the silico gel column chromatography is – $B < C < A$

75. Which of the following complex has a possibility to exist as meridional isomer?

(1) $[Co(NH_3)_3(NO_2)_3]$

(2) $[Co(en)_3]$

(3) $[Co(en)_2Cl_2]$

(4) $[Pt(NH_3)_2Cl_2]$

Official Ans. by NTA (1)

Sol. $[MA_3B_3]$ type of compound exists as facial and meridonial isomer.

76. Given below are two statements :

Statement-I : Methane and steam passed over a heated Ni catalyst produces hydrogen gas.

Statement-II : Sodium nitrite reacts with NH_4Cl to give H_2O, N_2 and NaCl.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both the statements I and II are correct

(2) Both the statements I and II are incorrect

(3) Statement I is incorrect but Statement II is correct

(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (1)

Sol. $CH_4(g) + \underset{\text{Steam}}{H_2O(g)} \xrightarrow[1270\,K]{Ni} CO(g) + 3H_2(g)$

$NaNO_2(aq) + NH_4Cl(aq) \rightarrow N_2(g) + NaCl(aq) + 2H_2O(\ell)$

77. Given below are two statements :

Statement I : If BOD is 4 ppm and dissolved oxygen is 8 ppm, then it is a good quality water.

Statement II : If the concentration of zinc and nitrate salts are 5 ppm each, then it can be a good quality water.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both the statements I and II are incorrect

(2) Statement I is incorrect but Statement II is correct

(3) Both the statements I and II are correct

(4) Statement I is correct but Statement II is incorrect

Official Ans. by NTA (3)

Sol. Clean water would have BOD value of less than 5 ppm.

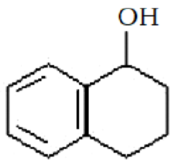
Maximum limit of Zn in clean water

$= 5.0$ ppm or mg dm^{-3}

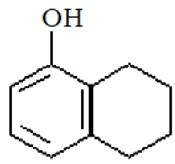
Maximum limit of NO_3^- in clean water

$= 50$ ppm or mg dm^{-3}

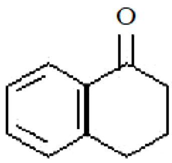
78. Arrange the following compounds in increasing order of rate of aromatic electrophilic substitution reaction



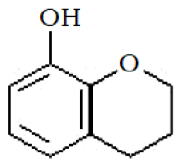
(a)



(b)



(c)



(d)

(1) d, b, c, a

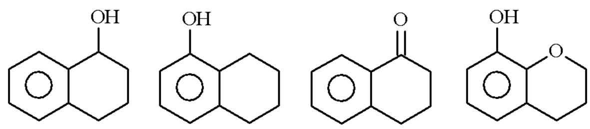
(2) b, c, a, d

(3) c, a, b, d

(4) d, b, a, c

Official Ans. by NTA (3)

Sol. Benzene becomes more reactive towards EAS when any substituent raises the electron density.



a $-CH_2$ has +H effect

b -OH has +R while $-CH_2-$ group has +H effect.

c $-\overset{O}{\overset{||}{C}}-$ has -R effect

d -OH and -O- both show +R effect

Correct order

$c < a < b < d$

79. The complex that dissolves in water is

(1) $Fe_4[Fe(CN)_6]_3$

(2) $[Fe_3(OH)_2(OAc)_6]Cl$

(3) $K_3[Co(NO_2)_6]$

(4) $(NH_4)_3[As(Mo_3O_{10})_4]$

Official Ans. by NTA (2)

Allen Ans. (2)

Sol. $Fe_4[Fe(CN)_6]_3$ Prussian Blue–water insoluble

$K_3[Co(NO_2)_6]$ very poorly water soluble

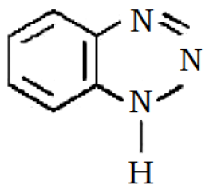
$(NH_4)_3$ [As $(MO_3O_{10})_4$] water insoluble

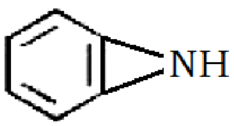
ammonium arseno molybdate

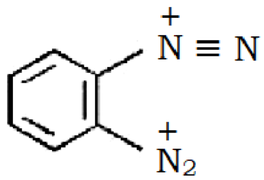
$[Fe_3(OH)_2(OAc)_6]Cl$ is water soluble.

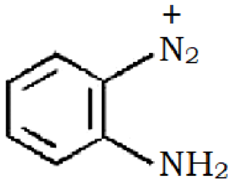
80. o-Phenylenediamine $\xrightarrow{HNO_2}$ 'X'
Major Product

'X' is

(1) 

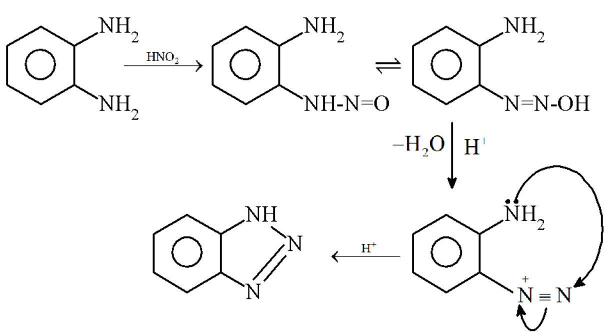
(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. Orthophenyl amine.



SECTION-B

81. A mixture of 1 mole of H_2O and 1 mole of CO is taken in a 10 litre container and heated to 725 K. At equilibrium 40% of water by mass reacts with carbon monoxide according to the equation :

$CO(g) + H_2O(g) \rightleftharpoons CO_2(g) + H_2(g)$.

The equilibrium constant $K_C \times 10^2$ for the reaction is ____________. (Nearest integer)

Official Ans. by NTA (44)

Sol. $CO_{(g)} + H_2O_{(g)} \rightleftharpoons CO_{2(g)} + H_{2(g)}$

t = 0 1 mol 1 mol 0 0

at equ. 1–x 1–x x x

at equilibrium 40% by mass water reacts with CO

x = 0.4 1 – x = 0.6

$$K_C = \frac{[CO_2][H_2]}{[CO][H_2O]} = \frac{0.4 \times 0.4}{0.6 \times 0.6} = 0.44$$

$$K_C \times 10^2 = 44$$

82. The ratio of spin-only magnetic moment values $\mu_{eff}[Cr(CN)_6]^{3-} / \mu_{eff}[Cr(H_2O)_6]^{3+}$ is _______.

Official Ans. by NTA (1)

Sol. Spin magnetic moment of $[Cr(CN)_6]^{3-} (t_{2g}^3 e_g^0)$

$$\mu_1 = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}$$

Spin magnetic moment of $[Cr(H_2O)_6]^{3+} (t_{2g}^3 e_g^0)$

$$\mu_2 = \sqrt{3(3+2)} = \sqrt{15} \text{ BM}$$

$$\frac{\mu_1}{\mu_2} = \frac{\sqrt{15}}{\sqrt{15}} = 1$$

83. An atomic substance A of molar mass 12 g mol^{-1} has a cubic crystal structure with edge length of 300 pm. The no. of atoms present in one unit cell of A is ________. (Nearest integer)

Given the density of A is 3.0 g mL^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1}

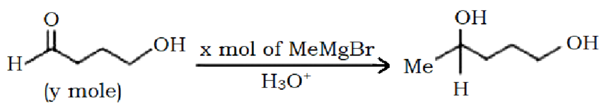
Official Ans. by NTA (4)

Sol. d = 3 g/cc M = 12 g/mol

a = 300 pm = 3×10^{-8} cm

$$Z = \frac{d \times N_A \times a^3}{M} = \frac{3 \times 6.02 \times 10^{23} \times (3 \times 10^{-8})^3}{12}$$

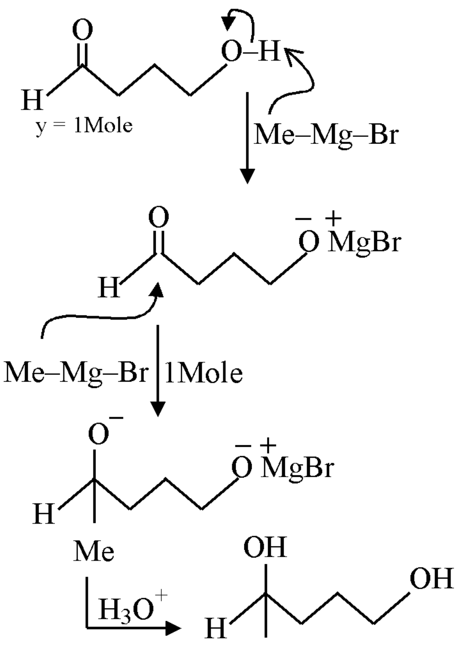
$= 4.06 \approx 4$

84. 

The ratio x/y on completion of the above reaction is __________.

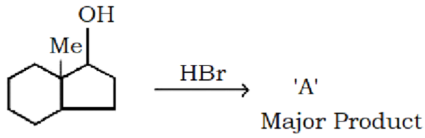
Official Ans. by NTA (2)

Sol.



$\therefore$ x = 2 mole

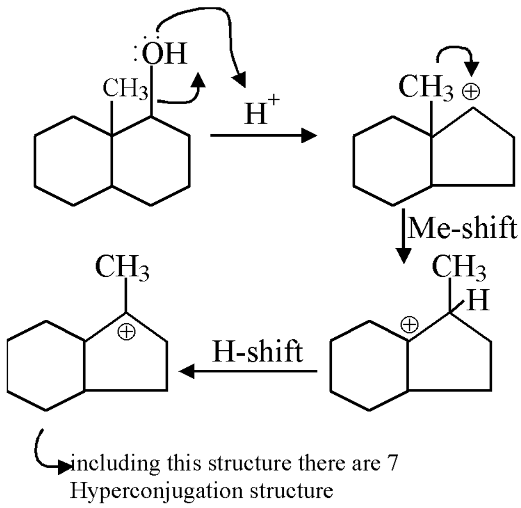
$$\frac{x}{y} = \frac{2}{1} = 2$$

85. 

The number of hyperconjugation structures involved to stabilize carbocation formed in the above reaction is ______________.

Official Ans. by NTA (7)

Sol.



86. Solid fuel used in rocket is a mixture of Fe_2O_3 and Al (in ratio 1 : 2). The heat evolved (kJ) per gram of the mixture is ___________ (Neatest integer)

Given : $\Delta H_f^\theta(Al_2O_3) = -1700\,kJ\,mol^{-1}$

$\Delta H_f^\theta(Fe_2O_3) = -840\,kJ\,mol^{-1}$

Molar mass of Fe, Al and O are 56, 27 and 16 g mol^{-1} respectively.

Official Ans. by NTA (4)

Sol. $Fe_2O_3 + 2Al \rightarrow Al_2O_3 + 2Fe$

Molar mass 160g 27g

$$(\Delta H_f^0)_{reaction} = \left[(\Delta H_f^0)_{Al_2O_3} + 2(\Delta H_f^0)_{Fe}\right] - \left[(\Delta H_f^0)_{Fe_2O_3} + 2(\Delta H_f^0)_{Al}\right]$$

= [– 1700 + 0] – [– 840 + 0]
= – 860 kJ/mol
Total mass of mixture = Fe_2O_3 + Al (1 : 2 molar ratio)
= 160 + 2 × 27
= 214 g/mol

Heat evolved per gram $= \frac{860}{214} = 4\,kJ / g$

87. A solution of sugar is obtained by mixing 200 g of its 25% solution and 500 g of its 40% solution (both by mass). The mass percentage of the resulting sugar solution is ____________. (Nearest integer)

Official Ans. by NTA (36)

Sol. Total mass of sugar in mixture of 25% of 200 and 40% of 500 g

Sugar solution = 0.25 × 200 + 0.40 × 500

= 50 + 200 = 250 g

Total mass of solution = 200 + 500 = 700 g

Mass of sugar in solution = $\frac{250}{700} \times 100 = 35.7\%$

$\approx 36\%$

88. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow$

$KCl + 3Fe_2(SO_4)_3 + 3H_2O$

The above reaction was studied at 300 K by monitoring the concentration of $FeSO_4$ in which initial concentration was 10 M and after half an hour became 8.8 M. The rate of production of $Fe_2(SO_4)_3$ is ________ $\times 10^{-6}$ mol L^{-1} s^{-1}.

(Nearest integer)

Official Ans. by NTA (333)

Sol. $KClO_3 + 6FeSO_4 + 3H_2SO_4 \rightarrow KCl + 3Fe_2(SO_4)_3 + 3H_2O$

$$ROR = -\frac{\Delta[KClO_3]}{\Delta t} = \frac{-1}{6}\frac{\Delta[FeSO_4]}{\Delta t}$$

$$= \frac{+1}{3}\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t}$$

$$\frac{\Delta[Fe_2(SO_4)_3]}{\Delta t} = \frac{1}{2}\frac{-\Delta[FeSO_4]}{\Delta t}$$

$$= \frac{1}{2}\frac{(10-8.8)}{30 \times 60}$$

$$= 0.333 \times 10^{-3}$$

$$= 333 \times 10^{-6} \text{ mol litre}^{-1} \text{ sec}^{-1}$$

89. 0.004 M K_2SO_4 solution is isotonic with 0.01 M glucose solution. Percentage dissociation of K_2SO_4 is ___________ (Nearest integer)

Official Ans. by NTA (75)

Sol. Isotonic solutions,

$\pi_{K_2SO_4} = \pi_{Glucose}$

$i \times 0.004 \times RT = 0.01 \times RT$

$i = 2.5$

For K_2SO_4 {for dissociation $i = 1 + (n - 1)\alpha$}

$$DOD(\alpha) = \frac{i-1}{n-1} = \frac{2.5-1}{3-1} = 0.75$$

% dissociation = 75

90. In an electrochemical reaction of lead, at standard temperature, if $E^0_{(Pb^{2+}/Pb)} = m$ Volt and $E^0_{(Pb^{4+}/Pb)} = n$ Volt, then the value of $E^0_{(Pb^{2+}/Pb^{4+})}$ is given by m – xn. The value of x is _________.

(Nearest integer)

Official Ans. by NTA (2)

Sol. $Pb^{2+} + 2e^- \rightarrow Pb \qquad \Delta G^0_1 = -2FE^0_1$

$Pb^{4+} + 4e^- \rightarrow Pb \qquad \Delta G^0_2 = -4FE^0_2$

$Pb^{2+} \rightarrow Pb^{4+} + 2e^- \qquad \Delta G^0_3 = -2FE^0_3$

$\Delta G^0_3 = \Delta G^0_1 - \Delta G^0_2$

$-2FE^0_3 = 2F(2n - m)$

$E^0_3 = m - 2n = m - xn$

Hence x = 2