

# FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Wednesday 12<sup>th</sup> April, 2023)

TIME : 9 : 00 AM to 12 : 00 NOON

## MATHEMATICS

## TEST PAPER WITH SOLUTION

### SECTION-A

1. The number of five digit numbers, greater than 40000 and divisible by 5, which can be formed using the digits 0, 1, 3, 5, 7 and 9 without repetition, is equal to

- (1) 120  
(2) 132  
(3) 72  
(4) 96

**Official Ans. by NTA (1)**

$$5 \times \times \times 0$$

$$7 \times \times \times 0$$

**Sol.**  $7 \times \times \times 5$

$$9 \times \times \times 0$$

$$9 \times \times \times 5$$

So Required numbers =  $5 \times {}^4P_3 = 120$

2. Let  $\alpha, \beta$  be the roots of the quadratic equation

$$x^2 + \sqrt{6}x + 3 = 0. \text{ Then } \frac{\alpha^{23} + \beta^{23} + \alpha^{14} + \beta^{14}}{\alpha^{15} + \beta^{15} + \alpha^{10} + \beta^{10}} \text{ is}$$

equal to

- (1) 729  
(2) 72  
(3) 81  
(4) 9

**Official Ans. by NTA (3)**

**Sol.**  $\alpha, \beta = \frac{-\sqrt{6} \pm \sqrt{6-12}}{2} = \frac{-\sqrt{6} \pm \sqrt{6} i}{2}$

$$= \sqrt{3} e^{\pm \frac{3\pi i}{4}}$$

Required expression

$$\frac{(\sqrt{3})^{23} \left( 2 \cos \frac{69\pi}{4} \right) + (\sqrt{3})^{14} \left( 2 \cos \frac{42\pi}{4} \right)}{(\sqrt{3})^{15} \left( 2 \cos \frac{45\pi}{4} \right) + (\sqrt{3})^{10} \left( 2 \cos \frac{30\pi}{4} \right)}$$

$$= \frac{(\sqrt{3})^{23} \left( 2 \cos \frac{69\pi}{4} \right) + (\sqrt{3})^{14} \left( 2 \cos \frac{42\pi}{4} \right)}{(\sqrt{3})^{15} \left( 2 \cos \frac{45\pi}{4} \right) + (\sqrt{3})^{10} \left( 2 \cos \frac{30\pi}{4} \right)}$$

$$(\sqrt{3})^8 = 81$$

3. Let  $\langle a_n \rangle$  be a sequence such that

$$a_1 + a_2 + \dots + a_n = \frac{n^2 + 3n}{(n+1)(n+2)}. \text{ If}$$

$$28 \sum_{k=1}^{10} \frac{1}{a_k} = p_1 p_2 p_3 \dots p_m, \text{ where } p_1, p_2, \dots, p_m \text{ are}$$

the first m prime numbers, then m is equal to

- (1) 7  
(2) 6  
(3) 5  
(4) 8

**Official Ans. by NTA (2)**

**Sol.**  $a_n = S_n - S_{n-1} = \frac{n^2 + 3n}{(n+1)(n+2)} - \frac{(n-1)(n+2)}{n(n+1)}$

$$\Rightarrow a_n = \frac{4}{n(n+1)(n+2)}$$

$$\Rightarrow 28 \sum_{k=1}^{10} \frac{1}{a_k} = 28 \sum_{k=1}^{10} \frac{k(k+1)(k+2)}{4}$$

$$= \frac{7}{4} \sum_{k=1}^{10} (k(k+1)(k+2)(k+3) - (k-1)k(k+1)(k+2))$$

$$= \frac{7}{4} \cdot 10 \cdot 11 \cdot 12 \cdot 13 = 2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 \cdot 13$$

So m = 6

4. Let the lines  $l_1 : \frac{x+5}{3} = \frac{y+4}{1} = \frac{z-\alpha}{-2}$  and  $l_2 : 3x +$

$2y + z - 2 = 0 = x - 3y + 2z - 13$  be coplanar. If the point P(a, b, c) on  $l_1$  is nearest to the point Q(-4, -3, 2), then  $|a| + |b| + |c|$  is equal to

- (1) 12  
(2) 14  
(3) 10  
(4) 8

**Official Ans. by NTA (3)**

**Sol.**  $(3x + 2y + z - 2) + \mu(x - 3y + 2z - 13) = 0$   
 $3(3 + \mu) + 1 \cdot (2 - 3\mu) - 2(1 + 2\mu) = 0$   
 $9 - 4\mu = 0$   
 $\mu = \frac{9}{4}$   
 $4(-15 - 8 + \alpha - 2) + 9(-5 + 12 + 2\alpha - 13) = 0$   
 $-100 + 4\alpha - 54 + 18\alpha = 0$   
 $\Rightarrow \alpha = 7$   
 Let  $P \equiv (3\lambda - 5, \lambda - 4, -2\lambda + 7)$   
 Direction ratio of PQ  $(3\lambda - 1, \lambda - 1, -2\lambda + 5)$   
 But  $PQ \perp \ell_1$   
 $\Rightarrow 3(3\lambda - 1) + 1(\lambda - 1) - 2(-2\lambda + 5) = 0$   
 $\Rightarrow \lambda = 1$   
 $P(-2, -3, 5) \Rightarrow |a| + |b| + |c| = 10$

**5.** Let  $P\left(\frac{2\sqrt{3}}{\sqrt{7}}, \frac{6}{\sqrt{7}}\right)$ , Q, R and S be four points on the ellipse  $9x^2 + 4y^2 = 36$ . Let PQ and RS be mutually perpendicular and pass through the origin. If  $\frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{p}{q}$ , where p and q are coprime, then p + q is equal to  
 (1) 143 (2) 137  
 (3) 157 (4) 147

**Official Ans. by NTA (3)**

**Sol.** Let  $R(2\cos\theta, 3\sin\theta)$   
 as  $OP \perp OR$

$$\text{so } \frac{3\sin\theta}{2\cos\theta} \times \frac{\frac{6}{\sqrt{7}}}{\frac{2\sqrt{3}}{\sqrt{7}}} = -1$$

$$\Rightarrow \tan\theta = \frac{-2}{3\sqrt{3}}$$

$$\Rightarrow R\left(\frac{-6\sqrt{3}}{\sqrt{31}}, \frac{6}{\sqrt{31}}\right) \text{ or } R\left(\frac{6\sqrt{3}}{\sqrt{31}}, \frac{-6}{\sqrt{31}}\right)$$

$$\text{Now } = \frac{1}{(PQ)^2} + \frac{1}{(RS)^2} = \frac{1}{4} \left( \frac{1}{(OP)^2} + \frac{1}{(OR)^2} \right)$$

$$= \frac{1}{4} \left( \frac{1}{\frac{48}{7}} + \frac{1}{\frac{144}{31}} \right) = \frac{1}{4} \left( \frac{7}{48} + \frac{31}{144} \right)$$

$$= \frac{13}{144}$$

$$\Rightarrow p + q = 157$$

**6.** Let a, b, c be three distinct real numbers, none equal to one. If the vectors  $a\hat{i} + \hat{j} + \hat{k}, \hat{i} + b\hat{j} + \hat{k}$  and  $\hat{i} + \hat{j} + c\hat{k}$  are coplanar, then

$$\frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} \text{ is equal to}$$

- (1) 1  
 (2) -1  
 (3) -2  
 (4) 2

**Official Ans. by NTA (1)**

**Sol.**  $\begin{vmatrix} a & 1 & 1 \\ 1 & b & 1 \\ 1 & 1 & c \end{vmatrix} = 0$

$$C_2 \rightarrow C_2 - C_1, C_3 \rightarrow C_3 - C_1$$

$$\begin{vmatrix} a & 1-a & 1-a \\ 1 & b-1 & 0 \\ 1 & 0 & c-1 \end{vmatrix} = 0$$

$$a(b-1)(c-1) - (1-a)(c-1) + (1-a)(1-b) = 0$$

$$a(1-b)(1-c) + (1-a)(1-c) + (1-a)(1-b) = 0$$

$$\frac{a}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow -1 + \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 0$$

$$\Rightarrow \frac{1}{1-a} + \frac{1}{1-b} + \frac{1}{1-c} = 1$$

**7.** If the local maximum value of the function

$$f(x) = \left( \frac{\sqrt{3}e}{2\sin x} \right)^{\sin^2 x}, \quad x \in \left( 0, \frac{\pi}{2} \right), \text{ is } \frac{k}{e}, \text{ then}$$

$$\left( \frac{k}{e} \right)^8 + \frac{k^8}{e^5} + k^8 \text{ is equal to}$$

- (1)  $e^5 + e^6 + e^{11}$   
 (2)  $e^3 + e^5 + e^{11}$   
 (3)  $e^3 + e^6 + e^{11}$   
 (4)  $e^3 + e^6 + e^{10}$

**Official Ans. by NTA (3)**

**Sol.** Let  $y = \left( \frac{\sqrt{3e}}{2 \sin x} \right)^{\sin^2 x}$

$$\ln y = \sin^2 x \cdot \ln \left( \frac{\sqrt{3e}}{2 \sin x} \right)$$

$$\frac{1}{y} y' = \ln \left( \frac{\sqrt{3e}}{2 \sin x} \right) 2 \sin x \cos x + \sin^2 x \cdot \frac{2 \sin x \cdot \frac{\sqrt{3e}}{2} (-\operatorname{cosec} x \cot x)}{\sqrt{3e}}$$

$$\frac{dy}{dx} = 0 \Rightarrow \ln \left( \frac{\sqrt{3e}}{2 \sin x} \right) 2 \sin x \cos x - \sin x \cos x = 0$$

$$\Rightarrow \sin x \cos x \left[ 2 \ln \left( \frac{\sqrt{3e}}{2 \sin x} \right) - 1 \right] = 0$$

$$\Rightarrow \ln \left( \frac{3e}{4 \sin^2 x} \right) = 1 \Rightarrow \frac{3e}{4 \sin^2 x} = e \Rightarrow \sin^2 x = \frac{3}{4}$$

$$\Rightarrow \sin x = \frac{\sqrt{3}}{2} \quad \left( \text{as } x \in \left( 0, \frac{\pi}{2} \right) \right)$$

$$\Rightarrow \text{local max value} = \left( \frac{\sqrt{3e}}{\sqrt{3}} \right)^{3/4} = e^{3/8} = \frac{k}{e}$$

$$\Rightarrow k^8 = e^{11}$$

$$\Rightarrow \left( \frac{k}{e} \right)^8 + \frac{k^8}{e^5} + k^8 = e^3 + e^6 + e^{11}$$

8. Let  $D$  be the domain of the function  $f(x) = \sin^{-1} \left( \log_{3x} \left( \frac{6+2 \log_3 x}{-5x} \right) \right)$ . If the range of the function  $g : D \rightarrow \mathbb{R}$  defined by  $g(x) = x - [x]$ , ( $[x]$  is the greatest integer function), is  $(\alpha, \beta)$ , then  $\alpha^2 + \frac{5}{\beta}$  is equal to

- (1) 46
- (2) 135
- (3) 136
- (4) 45

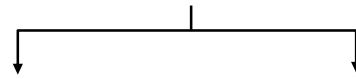
**Official Ans. by NTA (2)**

**Sol.**  $\frac{6+2 \log_3 x}{-5x} > 0$  &  $x > 0$  &  $x \neq \frac{1}{3}$

this gives  $x \in \left( 0, \frac{1}{27} \right) \dots (1)$

$$-1 \leq \log_{3x} \left( \frac{6+2 \log_3 x}{-5x} \right) \leq 1$$

$$3x \leq \frac{6+2 \log_3 x}{-5x} \leq \frac{1}{3x}$$



$$15x^2 + 6 + 2 \log_3 x \geq 0 \quad 6 + 2 \log_3 x + \frac{5}{3} \geq 0$$

$$x \in \left( 0, \frac{1}{27} \right) \dots (2) \quad x \geq 3^{-\frac{23}{6}} \dots (3)$$

from (1), (2) & (3)

$$x \in \left[ 3^{-\frac{23}{6}}, \frac{1}{27} \right)$$

$\therefore \alpha$  is small positive quantity

$$\& \beta = \frac{1}{27}$$

$\therefore \alpha^2 + \frac{5}{\beta}$  is just greater than 135

Ans. (Bonus)

9. Let  $y = y(x)$ ,  $y > 0$ , be a solution curve of the differential equation  $(1+x^2) dy = y(x-y) dx$ .

If  $y(0) = 1$  and  $y(2\sqrt{2}) = \beta$ , then

$$(1) e^{3\beta-1} = e(3+2\sqrt{2})$$

$$(2) e^{\beta-1} = e^{-2}(5+\sqrt{2})$$

$$(3) e^{\beta-1} = e^{-2}(3+2\sqrt{2})$$

$$(4) e^{3\beta-1} = e(5+\sqrt{2})$$

**Official Ans. by NTA (1)**

**Sol.**  $(1+x^2) dy = y(x-y) dx$

$$y(0) = 1. \quad y(2\sqrt{2}) = \beta$$

$$\frac{dy}{dx} = \frac{yx-y^2}{1+x^2}$$

$$\frac{dy}{dx} + y \left( \frac{-x}{1+x^2} \right) = \left( \frac{-1}{1+x^2} \right) y^2$$

$$\frac{1}{y^2} \frac{dy}{dx} + \frac{1}{y} \left( \frac{-x}{1+x^2} \right) = \frac{-1}{1+x^2}$$

$$\text{put } \frac{1}{y} = t \text{ then } \frac{-1}{y^2} \frac{dy}{dx} = \frac{dt}{dx}$$

$$\frac{dt}{dx} + t \frac{x}{1+x^2} = \frac{1}{1+x^2}$$

$$\text{I.F} = e^{\int \frac{x}{1+x^2} dx} = e^{\frac{1}{2} \ln(1+x^2)} = \sqrt{1+x^2}$$

$$t\sqrt{1+x^2} = \int \frac{1}{\sqrt{1+x^2}} dx$$

$$\frac{\sqrt{1+x^2}}{y} = \ln(x + \sqrt{x^2+1}) + c$$

$$y(0) = 1 \Rightarrow c = 1$$

$$\Rightarrow \sqrt{1+x^2} = y \ln(e(x + \sqrt{x^2+1}))$$

$$\beta = \frac{3}{\ln(e(3+2\sqrt{2}))} \Rightarrow \frac{3}{\beta} = \ln(e(3+2\sqrt{2}))$$

$$e^{\frac{3}{\beta}} = e(3+2\sqrt{2})$$

10. Among the two statements

(S1) :  $(p \Rightarrow q) \wedge (q \wedge (\sim q))$  is a contradiction and

(S2) :  $(p \wedge q) \vee ((\sim p) \wedge q) \vee$

$(p \wedge (\sim q)) \vee ((\sim p) \wedge (\sim q))$  is a tautology

(1) only (S2) is true

(2) only (S1) is true

(3) both are false.

(4) both are true

**Official Ans. by NTA (4)**

Sol.  $S_1 : (p \rightarrow q) \wedge (p \wedge (\sim q))$

p	q	$p \rightarrow q$	$p \wedge (\sim q)$	S1
T	T	T	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	F	F

$\Rightarrow S_1$  is Contradiction

$S_2$

p	q	$p \wedge q$	$(\sim p \wedge q)$	$(p \wedge \sim q)$	$(\sim p) \wedge (\sim q)$	$S_2$
T	T	T	F	F	F	T
T	F	F	F	T	F	T
F	T	F	T	F	F	T
F	F	F	F	F	T	T

$S_2$  is tautology

11. Let  $\lambda \in \mathbb{Z}$ ,  $a = \lambda \hat{i} + \hat{j} - \hat{k}$  and  $b = 3\hat{i} - \hat{j} + 2\hat{k}$ . Let  $\vec{c}$  be a vector such that

$$(\vec{a} + \vec{b} + \vec{c}) \times \vec{c} = \vec{0}, \vec{a} \cdot \vec{c} = -17 \quad \text{and} \quad \vec{b} \cdot \vec{c} = -20.$$

Then  $|\vec{c} \times (\lambda \hat{i} + \hat{j} + \hat{k})|^2$  is equal to

(1) 62

(2) 46

(3) 53

(4) 49

**Official Ans. by NTA (2)**

Sol.  $a + b + c \times c = 0$

$$(\vec{a} + \vec{b}) \times \vec{c} = 0$$

$$\vec{c} = \alpha(\vec{a} + \vec{b}) = \alpha(\lambda + 3)\hat{i} + \alpha\hat{k}$$

$$\vec{b} \cdot \vec{c} = -20 \Rightarrow 3\alpha(\lambda + 3) + 2\alpha = -20$$

$$\vec{a} \cdot \vec{c} = -17 \Rightarrow \alpha\lambda(\lambda + 3) - \alpha = -17$$

$$\Rightarrow \alpha(3\lambda + 9 + 2) = -20$$

$$\alpha(\lambda^2 + 3\lambda - 1) = -17$$

$$17(3\lambda + 11) = 20(\lambda^2 + 3\lambda - 1)$$

$$20\lambda^2 + 9\lambda - 207 = 0$$

$$\lambda = 3 \quad (\lambda \in \mathbb{Z})$$

$$\Rightarrow \alpha = -1 \quad \Rightarrow \vec{c} = -(6\hat{i} + \hat{k})$$

$$\vec{v} = \vec{c} \times (3\hat{i} + \hat{j} + \hat{k})$$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -6 & 0 & -1 \\ 3 & 1 & 1 \end{vmatrix} = \hat{i} + 3\hat{j} - 6\hat{k}$$

$$|\vec{v}|^2 = (-1)^2 + 3^2 + 6^2 = 46$$

12. The sum, of the coefficients of the first 50 terms in the binomial expansion of  $(1 - x)^{100}$ , is equal to

(1)  $^{-101}C_{50}$

(2)  $^{99}C_{49}$

(3)  $^{-99}C_{49}$

(4)  $^{101}C_{50}$

**Official Ans. by NTA (3)**

**Sol.**  $(1-x)^{100} = C_0 - C_1x + C_2x^2 - C_3x^3 + \dots - C_{99}x^{99} + C_{100}x^{100}$   
 $\Rightarrow C_0 - C_1 + C_2 - C_3 + \dots - C_{99} + C_{100} = 0$   
 $2(C_0 - C_1 + C_2 + \dots - C_9) + C_{50} = 0$   
 $C_0 - C_1 + C_2 + \dots - C_{99} = -\frac{1}{2} C_{50}$   
 $-\frac{1}{2} \frac{100!}{50!50!} = -\frac{1}{2} \times \frac{100 \times 99!}{50!50!} = -{}^{99}C_{49}$

- 13.** The area of the region enclosed by the curve  $y = x^3$  and its tangent at the point  $(-1, -1)$  is
- (1)  $\frac{27}{4}$
  - (2)  $\frac{19}{4}$
  - (3)  $\frac{23}{4}$
  - (4)  $\frac{31}{4}$

**Official Ans. by NTA (1)**

**Sol.** equation of tangent :  $y + 1 = 3(x + 1)$   
i.e.  $y = 3x + 2$

Point of intersection with curve  $(2, 8)$

So Area =  $\int_{-1}^2 ((3x + 2) - x^3) dx = \frac{27}{4}$

**14.** Let  $A = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix}$ . If  $B = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} A \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$ ,

then the sum of all the elements of the matrix

$\sum_{n=1}^{50} B^n$  is equal to

- (1) 100
- (2) 50
- (3) 75
- (4) 125

**Official Ans. by NTA (1)**

**Sol.** Let  $C = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$ ,  $D = \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$   
 $DC = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$   
 $B = CAD$   
 $B^n = \underbrace{(CAD)(CAD)(CAD)\dots(CAD)}_{n\text{-times}}$

$\Rightarrow B^n = CA^nD \dots(1)$

$A^2 = \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{51} \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{2}{51} \\ 0 & 1 \end{bmatrix}$

$A^3 = \begin{bmatrix} 1 & \frac{3}{51} \\ 0 & 1 \end{bmatrix}$

similarly  $A^n = \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix}$

$B^n = \begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix} \begin{bmatrix} 1 & \frac{n}{51} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1 & \frac{n}{51} + 2 \\ -1 & -\frac{n}{51} - 1 \end{bmatrix} \begin{bmatrix} -1 & -2 \\ 1 & 1 \end{bmatrix}$

$= \begin{bmatrix} \frac{n}{51} + 1 & \frac{n}{51} \\ -\frac{n}{51} & 1 - \frac{n}{51} \end{bmatrix}$

$\sum_{n=1}^{50} B^n = \begin{bmatrix} 25 + 50 & 25 \\ -25 & -25 + 50 \end{bmatrix} = \begin{bmatrix} 75 & 25 \\ -25 & 25 \end{bmatrix}$

Sum of the elements = 100

- 15.** Let the plane  $P : 4x - y + z = 10$  be rotated by an angle  $\frac{\pi}{2}$  about its line of intersection with the plane  $x + y - z = 4$ . If  $\alpha$  is the distance of the point  $(2, 3, -4)$  from the new position of the plane  $P$ , then  $35\alpha$  is

- (1) 90
- (2) 85
- (3) 105
- (4) 126

**Official Ans. by NTA (4)**

**Sol.** Let equation in new position is  
 $(4x - y + z - 10) + \lambda(x + y - z - 4) = 0$   
 $4(4 + \lambda) - 1 \cdot (-1 + \lambda) + 1 \cdot (1 - \lambda) = 0$   
 $\Rightarrow \lambda = -9$

So equation in new position is  
 $-5x - 10y + 10z + 26 = 0$   
 $\Rightarrow \alpha = \frac{54}{15}$

**16.** If  $\frac{1}{n+1} {}^n C_n + \frac{1}{n} {}^n C_{n-1} + \dots + \frac{1}{2} {}^n C_1 + {}^n C_0 = \frac{1023}{10}$  then n is equal to

- (1) 6
- (2) 9
- (3) 8
- (4) 7

**Official Ans. by NTA (2)**

**Sol.**  $\sum_{r=0}^n \frac{{}^n C_r}{r+1} = \frac{1}{n+1} \sum_{r=0}^n {}^{n+1} C_{r+1}$   
 $= \frac{1}{n+1} (2^{n+1} - 1) = \frac{1023}{10}$   
 $n+1 = 10 \Rightarrow n = 9$

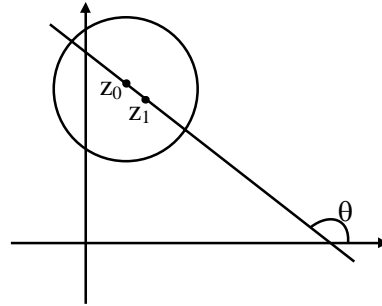
**17.** Let C be the circle in the complex plane with centre  $z_0 = \frac{1}{2}(1 + 3i)$  and radius  $r = 1$ . Let  $z_1 = 1 + i$  and the complex number  $z_2$  be outside the circle C such that  $|z_1 - z_0| = |z_2 - z_0| = 1$ . If  $z_0, z_1$  and  $z_2$  are collinear, then the smaller value of  $|z_2|^2$  is equal to

- (1)  $\frac{13}{2}$
- (2)  $\frac{5}{2}$
- (3)  $\frac{3}{2}$
- (4)  $\frac{7}{2}$

**Official Ans. by NTA (2)**

**Sol.**  $|z_1 - z_0| = \left| \frac{1-i}{2} \right| = \frac{1}{\sqrt{2}}$   
 $\Rightarrow |z_2 - z_0| = \sqrt{2}$  ; centre  $\left( \frac{1}{2}, \frac{3}{2} \right)$

$z_0 \left( \frac{1}{2}, \frac{3}{2} \right)$  and  $z_1 (1, 1)$



$\tan \theta = -1 \Rightarrow \theta = 135^\circ$

$z_2 \left( \frac{1}{2} + \sqrt{2} \cos 135^\circ, \frac{3}{2} + \sqrt{2} \sin 135^\circ \right)$

or

$\left( \frac{1}{2} - \sqrt{2} \cos 135^\circ, \frac{3}{2} - \sqrt{2} \sin 135^\circ \right)$

$\Rightarrow z_2 \left( -\frac{1}{2}, \frac{5}{2} \right)$  or  $z_2 \left( \frac{3}{2}, \frac{1}{2} \right)$

$\Rightarrow |z_2|^2 = \frac{26}{4}, \frac{5}{2}$

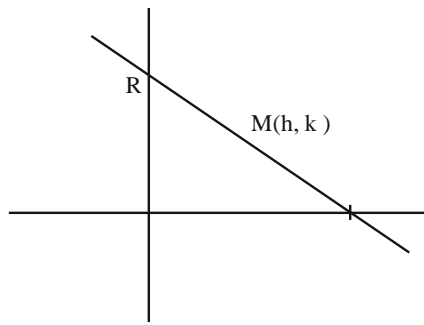
$\Rightarrow |z_2|_{\min}^2 = \frac{5}{2}$

**18.** If the point  $\left( \alpha, \frac{7\sqrt{3}}{3} \right)$  lies on the curve traced by the mid-points of the line segments of the lines  $x \cos \theta + y \sin \theta = 7$ ,  $\theta \in \left( 0, \frac{\pi}{2} \right)$  between the co-ordinates axes, then  $\alpha$  is equal to

- (1) 7
- (2) -7
- (3)  $-7\sqrt{3}$
- (4)  $7\sqrt{3}$

**Official Ans. by NTA (1)**

Sol.  $\text{pt}(\alpha, \frac{7\sqrt{3}}{3})$



$$x \cos \theta + y \sin \theta = 7$$

$$x\text{-intercept} = \frac{7}{\cos \theta}$$

$$y\text{-intercept} = \frac{7}{\sin \theta}$$

$$A: \left( \frac{7}{\cos \theta}, 0 \right) \quad B: \left( 0, \frac{7}{\sin \theta} \right)$$

Locus of mid pt M : (h, k)

$$h = \frac{7}{2 \cos \theta}, k = \frac{7}{2 \sin \theta}$$

$$\frac{7}{2 \sin \theta} = \frac{7\sqrt{3}}{3} \Rightarrow \sin \theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$$

$$\alpha = \frac{7}{2 \cos \theta} = 7$$

19. Two dice A and B are rolled, Let the numbers obtained on A and B be  $\alpha$  and  $\beta$  respectively. If the variance of  $\alpha - \beta$  is  $\frac{p}{q}$ , where p and q are co-

prime, then the sum of the positive divisors of p is equal to

- (1) 36
- (2) 48
- (3) 31
- (4) 72

Official Ans. by NTA (2)

Sol.

$\alpha - \beta$	Case	P
5	(6, 1)	1/36
4	(6, 2) (5, 1)	2/36
3	(6, 3) (5, 2) (4, 1)	3/36
2	(6, 4) (5, 3) (4, 3) (3, 1)	4/36
1	(6, 5) (5, 4) (4, 3) (3, 2) (2, 1)	5/36
0	(6, 6) (5, 5) ..... (1, 1)	6/36
-1	-----	5/36
-2	-----	4/36
-3	-----	3/36
-4	(2, 6) (1, 5)	2/36
-5	(1, 6)	1/36

$$\sum (x^2) = \sum x^2 P(x) = 2 \left[ \frac{25}{36} + \frac{32}{36} + \frac{27}{36} + \frac{16}{36} + \frac{5}{36} \right]$$

$$= \frac{105}{18} = \frac{35}{6}$$

$$\mu = \sum (x) = 0 \text{ as data is symmetric}$$

$$\sigma^2 = \sum (x^2) = \sum x^2 P(x) = \frac{35}{6} \quad P = 35 = 5 \times 7$$

$$\text{Sum of divisors} = (5^0 + 5^1) (7^0 + 7^1) = 6 \times 8 = 48$$

20. In a triangle ABC, if  $\cos A + 2 \cos B + \cos C = 2$  and the lengths of the sides opposite to the angles A and C are 3 and 7 respectively, then  $\cos A - \cos C$  is equal to

- (1)  $\frac{3}{7}$
- (2)  $\frac{9}{7}$
- (3)  $\frac{10}{7}$
- (4)  $\frac{5}{7}$

Official Ans. by NTA (3)

**Sol.**  $\cos A + \cos C = 2(1 - \cos B)$

$$2 \cos \frac{A+C}{2} \cos \frac{A-C}{2} = 4 \sin^2 B / 2$$

$$\text{as } \cos \left( \frac{A+C}{2} \right) = \sin \frac{B}{2}$$

$$\text{so } \cos \frac{A-C}{2} = 2 \sin \frac{B}{2}$$

$$2 \cos B/2 \cos \frac{A-C}{2} = 4 \sin B/2 \cos B/2$$

$$2 \sin \left( \frac{A+C}{2} \right) \cos \left( \frac{A-C}{2} \right) = 4 \sin B / 2 \cos B / 2$$

$$\sin A + \sin C = 2 \sin B$$

$$a + c = 2b \Rightarrow a = 3, c = 7, b = 5$$

$$\cos A - \cos C = \frac{b^2 + c^2 - a^2}{2bc} - \frac{a^2 + b^2 - c^2}{2ab}$$

$$= \frac{25 + 49 - 9}{70} - \frac{9 + 25 - 49}{30}$$

$$= \frac{65}{70} + \frac{1}{2} = \frac{20}{14} = \frac{10}{7}$$

### SECTION-B

- 21.** A fair  $n$  ( $n > 1$ ) faces die is rolled repeatedly until a number less than  $n$  appears. If the mean of the number of tosses required is  $\frac{n}{9}$ , then  $n$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (10.00)**

**Sol.** Mean =  $1 \cdot \frac{n-1}{n} + 2 \cdot \frac{1}{n} \left( \frac{n-1}{n} \right) + 3 \left( \frac{1}{n} \right)^2 \left( \frac{n-1}{n} \right)$

...

$$\frac{n}{9} = \left( \frac{n-1}{n} \right) \left( 1 + 2 \left( \frac{1}{n} \right) + 3 \left( \frac{1}{n} \right)^2 \dots \right)$$

$$\frac{n}{9} = \left( \frac{n-1}{n} \right) \left( 1 - \frac{1}{n} \right)^{-2} = \left( \frac{n-1}{n} \right) \cdot \frac{n^2}{(n-1)^2}$$

$$\frac{n}{9} = \frac{n}{n-1} \Rightarrow n = 10$$

- 22.** Let the digits  $a, b, c$  be in A.P. Nine-digit numbers are to be formed using each of these three digits thrice such that three consecutive digits are in A.P. at least once. How many such numbers can be formed?

**Official Ans. by NTA (1260)**

- Sol.** abc or cba

$$\frac{a \ b \ c}{c \ b \ a}$$

$$\frac{{}^7C_1 \times 2 \times 6!}{2!2!2!} = 1260$$

- 23.** Let  $[x]$  be the greatest integer  $\leq x$ . Then the number of points in the interval  $(-2, 1)$ , where the function  $f(x) = | [x] | + \sqrt{x - [x]}$  is discontinuous is \_\_\_\_\_.

**Official Ans. by NTA (2.00)**

- Sol.** Need to check at doubtful points

discont at  $x \in I$  only

$$\text{at } x = -1 \Rightarrow f(-1^+) = 1 + 0 = 1$$

$$\Rightarrow f(-1^-) = 2 + 1 = 3$$

$$\text{at } x = 0 \Rightarrow f(0^+) = 0 + 0 = 0$$

$$\Rightarrow f(0^-) = 1 + 1 = 2$$

$$\text{at } x = 1 \Rightarrow f(1^+) = 1 + 0 = 1$$

$$\Rightarrow f(1^-) = 0 + 1 = 1$$

discont. at two points

- 24.** Let the plane  $x + 3y - 2z + 6 = 0$  meet the co-ordinate axes at the points  $A, B, C$ . If the orthocentre of the triangle  $ABC$  is  $\left( \alpha, \beta, \frac{6}{7} \right)$ , then  $98(\alpha + \beta)^2$  is equal to \_\_\_\_\_.

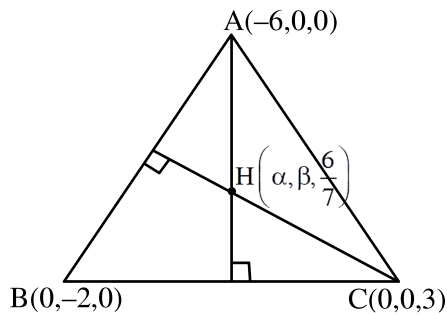
**Official Ans. by NTA (288.00)**



**Sol.** A (-6, 0, 0) B (0, -2, 0) C = (0, 0, 3)

$$\overline{AB} = 6\hat{i} - 2\hat{j}, \quad \overline{BC} = 2\hat{j} + 3\hat{k},$$

$$\overline{AC} = 6\hat{i} + 3\hat{k}$$



$$\overline{AH} \cdot \overline{BC} = 0$$

$$\left(\alpha + 6, \beta, \frac{6}{7}\right) \cdot (0, 2, 3) = 0$$

$$\boxed{\beta = \frac{-9}{7}}$$

$$\overline{CH} \cdot \overline{AB} = 0$$

$$\left(\alpha, \beta, \frac{-15}{7}\right) \cdot (6, -2, 0) = 0$$

$$6\alpha - 2\beta = 0$$

$$\alpha = \frac{-3}{7}$$

$$98(\alpha + \beta)^2 = (98) \frac{(144)}{49} = 288$$

**25.** Let  $I(x) = \int \sqrt{\frac{x+7}{x}} dx$  and  $I(9) = 12 + 7 \log_e 7$ .

If  $I(1) = \alpha + 7 \log_e (1 + 2\sqrt{2})$ , then  $\alpha^4$  is equal to \_\_\_\_\_.

**Official Ans. by NTA (64.00)**

**Sol.**  $\int \sqrt{\frac{x+7}{x}} dx$

Put  $x = t^2$

$dx = 2t dt$

$$\int 2\sqrt{t^2+7} dt = 2 \int \sqrt{t^2 + \sqrt{7}^2} dt$$

$$I(t) = 2 \left[ \frac{t}{2} \sqrt{t^2+7} + \frac{7}{2} \ln |t + \sqrt{t^2+7}| \right] + C$$

$$I(x) = \sqrt{x} \sqrt{x+7} + 7 \ln |\sqrt{x} + \sqrt{x+7}| + C$$

$$I(9) = 12 + 7 \ln 7 = 12 + 7 (\ln(3+4)) + C$$

$$\Rightarrow C = 0$$

$$I(x) = \sqrt{x} \sqrt{x+7} + 7 \ln (\sqrt{x} + \sqrt{x+7})$$

$$I(1) = 1\sqrt{8} + 7 \ln (1 + \sqrt{8})$$

$$I(1) = \sqrt{8} + 7 \ln (1 + 2\sqrt{2})$$

$$\alpha = \sqrt{8}$$

$$\alpha^4 = (8^{1/2})^4$$

$$\alpha^4 = 8^2 = 64$$

**26.** Let  $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$ . If  $\sum_{k=1}^n D_k = 96$ , then n is equal to

$D_k = 96$ , then n is equal to

**Official Ans. by NTA (6.00)**

**Sol.**  $D_k = \begin{vmatrix} 1 & 2k & 2k-1 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix}$

$$\sum_{k=1}^n D_k = 96 \Rightarrow$$

$$\begin{vmatrix} \sum_{k=1}^n 1 & \sum 2k & \sum (2k-1) \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$\Rightarrow \begin{vmatrix} n & n^2+n & n^2 \\ n & n^2+n+2 & n^2 \\ n & n^2+n & n^2+n+2 \end{vmatrix} = 96$$

$$R_2 \rightarrow R_2 - R_1 \text{ and } R_3 \rightarrow R_3 - R_1$$

$$\begin{vmatrix} n & n^2+n & n^2 \\ 0 & 2 & 0 \\ 0 & 0 & n+2 \end{vmatrix} = 96$$

$$\Rightarrow n(2n+4) = 96 \Rightarrow n(n+2) = 48 \Rightarrow n = 6$$

**27.** Let the positive numbers  $a_1, a_2, a_3, a_4$  and  $a_5$  be in a G.P. Let their mean and variance be  $\frac{31}{10}$  and  $\frac{m}{n}$  respectively, where m and n are co-prime. If the mean of their reciprocals is  $\frac{31}{40}$  and  $a_3 + a_4 + a_5 = 14$ , then m + n is equal to \_\_\_\_\_.

**Official Ans. by NTA (211)**

**Sol.** Let  $\frac{a}{r^2}, \frac{a}{r}, a, ar, ar^2$

$$\text{Given } \frac{a}{r^2} + \frac{a}{r} + a + ar + ar^2 = 5 \times \frac{31}{10} \quad \dots(1)$$

$$\text{And } \frac{r^2}{a} + \frac{r}{a} + \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} = 5 \times \frac{31}{40} \quad \dots(2)$$

$$(1) \div (2) \quad a^2 = 4 \Rightarrow a = 2 \quad \therefore r + \frac{1}{r} = 5/2 \quad (a \neq -2)$$

$$\Rightarrow r = 2$$

$$\therefore \text{ Now } \frac{1}{2}, 1, 2, 4, 8$$

$$\therefore \sigma^2 = \frac{\sum x^2}{N} - \left( \frac{\sum x}{N} \right)^2$$

$$= \frac{186}{25} = \frac{M}{N} \Rightarrow 211 = m + n$$

**28.** The number of relations, on the set  $\{1,2,3\}$  containing  $(1,2)$  and  $(2,3)$ , which are reflexive and transitive but not symmetric, is \_\_\_\_\_

**Official Ans. by NTA (4.00)**

**Sol.**  $A = \{1,2,3\}$

For Reflexive  $(1,1), (2,2), (3,3) \in R$

For transitive :  $(1,2)$  and  $(2,3) \in R \Rightarrow (1,3) \in R$

Not symmetric :  $(2,1)$  and  $(3,2) \notin R$

$$R_1 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3)\}$$

$$R_2 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (2,1)\}$$

$$R_3 = \{(1,1), (2,2), (3,3), (1,2), (2,3), (1,3), (3,2)\}$$

**29.** If  $\int_{-0.15}^{0.15} |100x^2 - 1| dx = \frac{k}{3000}$ , then k is equal to \_\_\_\_\_.

**Official Ans. by NTA (575)**

$$\text{Sol. } \int_{-0.15}^{0.15} |100x^2 - 1| dx = 2 \int_0^{0.15} |100x^2 - 1| dx$$

$$\text{Now } 100x^2 - 1 = 0 \Rightarrow x^2 = \frac{1}{100} \Rightarrow x = 0.1$$

$$I = 2 \left[ \int_0^{0.1} (1 - 100x^2) dx + \int_{0.1}^{0.15} (100x^2 - 1) dx \right]$$

$$\begin{aligned} I &= 2 \left[ x - \frac{100}{3} x^3 \right]_0^{0.1} + 2 \left[ \frac{100x^3}{3} - x \right]_{0.1}^{0.15} \\ &= 2 \left[ 0.1 - \frac{0.1}{3} \right] + 2 \left[ \frac{0.3375}{3} - 0.15 - \frac{0.1}{3} + 0.1 \right] \\ &= 2 \left[ 0.2 - \frac{0.2}{3} + 0.1125 - 0.15 \right] \\ &= 2 \left[ \frac{5}{100} - \frac{2}{30} + \frac{1125}{10000} \right] = 2 \left( \frac{1500 - 2000 + 3375}{30000} \right) \\ &= \frac{575}{3000} \Rightarrow k = 575 \end{aligned}$$

**30.** Two circles in the first quadrant of radii  $r_1$  and  $r_2$  touch the coordinate axes. Each of them cuts off an intercept of 2 units with the line  $x + y = 2$ . Then  $r_1^2 + r_2^2 - r_1 r_2$  is equal to \_\_\_\_\_.

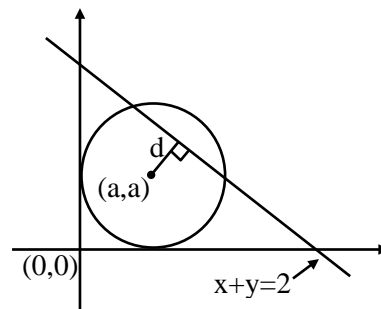
**Official Ans. by NTA (7.00)**

**Sol.** Circle  $(x - a)^2 + (y - a)^2 = a^2$

$$x^2 + y^2 - 2ax - 2ay + a^2 = 0$$

intercept = 2

$$\Rightarrow 2\sqrt{a^2 - d^2} = 2$$



Where  $d$  = perpendicular distance of centre from line  $x + y = 2$

$$\Rightarrow 2\sqrt{a^2 - \left( \frac{a + a - 2}{\sqrt{2}} \right)^2} = 2$$

$$\Rightarrow a^2 - \frac{(2a - 2)^2}{2} = 1 \Rightarrow 2a^2 - 4a^2 + 8a - 4 = 2$$

$$\Rightarrow 2a^2 - 8a + 6 = 0 \Rightarrow a^2 - 4a + 3 = 0$$

$$\therefore r_1 + r_2 = 4 \text{ and } r_1 r_2 = 3$$

$$\therefore r_1^2 + r_2^2 - r_1 r_2 = (r_1 + r_2)^2 - 3r_1 r_2$$

$$= 16 - 9 = 7$$

**PHYSICS****TEST PAPER WITH SOLUTION****SECTION-A**

31. An ice cube has a bubble inside. When viewed from one side the apparent distance of the bubble is 12 cm. when viewed from the opposite side, the apparent distance of the bubble is observed as 4 cm. If the side of the ice cube is 24 cm, the refractive index of the ice cube is

- (1)  $\frac{4}{3}$                                   (2)  $\frac{3}{2}$   
 (3)  $\frac{2}{3}$                                   (4)  $\frac{6}{5}$

**Official Ans. by NTA (2)**

**Sol.**  $d_{\text{apparent}} = \frac{d_{\text{actual}}}{\mu_{\text{rel}}}$

$$12 = \frac{x}{\mu} \quad \dots(1)$$

$$4 = \frac{24 - x}{\mu} \quad \dots(2)$$

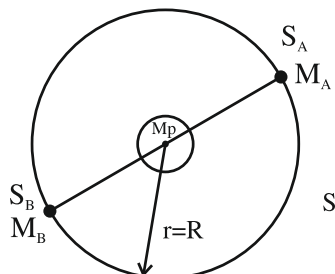
On solving we get  $\mu = 1.5$

32. Two satellites A and B move round the earth in the same orbit. The mass of A is twice the mass of B. The quantity which is same for the two satellites will be :

- (1) Potential energy                  (2) Total energy  
 (3) Kinetic energy                    (4) Speed

**Official Ans. by NTA (4)**

**Sol.**



$$P.E = -\frac{GM_p M_A}{R}$$

$$K.E = +\frac{GM_p M_A}{2R}$$

$$T.E = -\frac{GM_p M_A}{2R}$$

$$\text{Speed} = v = \sqrt{\frac{GM_p}{R}}$$

Speed of satellite is Independent of mass of satellite.

33. The amplitude of  $15 \sin(1000 \pi t)$  is modulated by  $10 \sin(4\pi t)$  signal. The amplitude modulated signal contains frequencies of

1. 500 Hz.                                  2. 2 Hz  
 3. 250 Hz                                  4. 498 Hz  
 5. 502 Hz

Choose the correct answer from the options given below:

- (1) (1) and (3) only  
 (2) (1) and (4) only  
 (3) (1) and (2) only  
 (4) (1), (4) and (5) only

**Official Ans. by NTA (4)**

**Sol.** Equation of Carrier wave

$$c(t) = 15 \sin(1000 \pi t)$$

$$f_c = \frac{\omega_c}{2\pi} = \frac{1000\pi}{2\pi} = 500 \text{ Hz}$$

Equation of modulated wave

$$m(t) = 10 \sin(4 \pi t)$$

$$f_m = \frac{\omega_m}{2\pi} = \frac{4\pi}{2\pi} = 2 \text{ Hz}$$

Frequencies contained in resultant Amplitude modulated wave are  $(500-2)\text{Hz}$ ,  $500 \text{ Hz}$  and  $(500+2) \text{ Hz}$ .

Correct ans is (4)

34. In an n-p-n common emitter (CE) transistor the collector current changes from 5 mA to 16 mA for the change in base current from 100  $\mu$ A and 200  $\mu$ A, respectively. The current gain of transistor is \_\_\_\_\_.

- (1) 110 (2) 0.9  
(3) 210 (4) 9

**Official Ans. by NTA (1)**

**Sol.** Current gain in common emitter transistor

$$\beta = \frac{\Delta I_C}{\Delta I_B} = \frac{16\text{mA} - 5\text{mA}}{200\mu\text{A} - 100\mu\text{A}} = \frac{11\text{mA}}{100\mu\text{A}} = 110$$

35. If the r.m.s. speed of chlorine molecule is 490 m/s at 27° C, the r.m.s. speed of argon molecules at the same temperature will be (Atomic mass of argon = 39.9u, molecular mass of chlorine = 70.9u)

- (1) 751.7 m/s (2) 451.7 m/s  
(3) 651.7 m/s (4) 551.7 m/s

**Official Ans. by NTA (3)**

**Sol.** 
$$V_{\text{rms}} = \sqrt{\frac{3RT}{M}} \quad \frac{v_{\text{Ar}}}{v_{\text{Cl}}} = \sqrt{\frac{M_{\text{Cl}}}{M_{\text{Ar}}}}$$

$$\Rightarrow v_{\text{Ar}} = 1.33 \times 490 = 651.7 \text{ m/s}$$

36. A proton and an  $\alpha$ -particle are accelerated from rest by 2V and 4V potentials, respectively. The ratio of their de-Broglie wavelength is:

- (1) 4:1 (2) 2:1  
(3) 8:1 (4) 16:1

**Official Ans. by NTA (1)**

**Sol.** 
$$\lambda = \frac{h}{mv} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2mq\Delta V}}$$

$$\frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{m_p V_p q_p}{m_{\alpha} V_{\alpha} q_{\alpha}}}$$

$$\Rightarrow \frac{\lambda_{\alpha}}{\lambda_p} = \sqrt{\frac{1 \times 2 \times 1}{4 \times 4 \times 2}} = \frac{1}{4}$$

$$\Rightarrow \lambda_p : \lambda_{\alpha} = 4 : 1$$

37. Given below are two statements:

**Statement I :** The diamagnetic property depends on temperature.

**Statement II :** The included magnetic dipole moment in a diamagnetic sample is always opposite to the magnetizing field.

In the light of given statement, choose the correct answer from the options given below:

- (1) Statement I is incorrect but Statement II is true  
(2) Both Statement I and Statement II are true.  
(3) Both Statement I and Statement II are false.  
(4) Statement I is correct but Statement II is false.

**Official Ans. by NTA (1)**

**Sol. Conceptual**

38. A wire of resistance 160  $\Omega$  is melted and drawn in wire of one-fourth of its length. The new resistance of the wire will be

- (1) 10  $\Omega$  (2) 640  $\Omega$   
(3) 40  $\Omega$  (4) 16  $\Omega$

**Official Ans. by NTA (1)**

**Sol.** Volume = Constant

$$A_1 L_1 = A_2 L_2$$

$$A_1 L = A_2 \frac{L}{4}$$

$$\boxed{4A_1 = A_2}$$

$$R_1 = \frac{\rho L_1}{A_1} \quad R_2 = \frac{\rho L_2}{A_2}$$

$$\frac{R_2}{R_1} = \frac{L_2 A_1}{A_2 L_1} = \frac{L}{4} \frac{A_1}{4 A_1 L}$$

$$R_2 = \frac{1}{16} R_1 = 10 \Omega$$

39. Match List I with List II

List I		List II	
A.	Spring constant	I.	( $T^{-1}$ )
B.	Angular speed	II.	( $MT^{-2}$ )
C.	Angular momentum	III.	( $ML^2$ )
D.	Moment of Inertia	IV.	( $ML^2T^{-1}$ )

Choose the correct answer from the options given below:

- (1) A-II, B-I, C-IV, D-III  
(2) A-IV, B-I, C-III, D-II  
(3) A-II, B-III, C-I, D-IV  
(4) A-I, B-III, C-II, D-IV

**Official Ans. by NTA (1)**

**Sol.** Spring Constant

$$[K] = \frac{[F]}{[x]} = \frac{MLT^{-2}}{L} = MT^{-2}$$

$$[\omega] = \frac{[\theta]}{[t]} = \frac{1}{T} = T^{-1}$$

**40.** Three force  $F_1 = 10N$ ,  $F_2 = 8 N$ ,  $F_3 = 6 N$  are acting on a particle of mass 5 kg. The forces  $F_2$  and  $F_3$  are applied perpendicular so that particle remains at rest. If the force  $F_1$  is removed, then the acceleration of the particle is:

- (1)  $2 \text{ ms}^{-2}$                       (2)  $0.5 \text{ ms}^{-2}$   
(3)  $4.8 \text{ ms}^{-2}$                       (4)  $7 \text{ ms}^{-2}$

**Official Ans. by NTA (1)**

**Sol.** Resultant of  $\vec{F}_2$  and  $\vec{F}_3$  should be opposite to  $\vec{F}_1$

$$a = \frac{10}{5} = 2 \text{ m/s}^2$$

**41.** A body cools from  $80^\circ\text{C}$  to  $60^\circ\text{C}$  in 5 minutes. The temperature of the surrounding is  $20^\circ\text{C}$ . The time it takes to cool from  $60^\circ\text{C}$  to  $40^\circ\text{C}$  is:

- (1) 500 s                      (2)  $\frac{25}{3}$  s  
(3) 450 s                      (4) 420 s

**Official Ans. by NTA (1)**

**Sol.** Rate of cooling  $\propto$  Temperature difference

$$\frac{80 - 60}{5} = k \{70 - 20\} \text{ --- (1)}$$

$$\frac{60 - 40}{t} = k [50 - 20] \text{ ---- (2)}$$

$$\frac{4t}{20} = \frac{50}{30}$$

$$t = \frac{25}{3} \text{ min} = 500 \text{ sec}$$

$$\Rightarrow t = 500 \text{ seconds}$$

**42.** An engine operating between the boiling and freezing points of water will have

1. efficiency more than 27%
2. efficiency less than the efficiency a Carnot engine operating between the same two temperatures.
3. efficiency equal to 27%
4. efficiency less than 27%

- (1) 2, 3 and 4 only              (2) 2 and 3 only  
(3) 2 and 4 only              (4) 1 and 2 only

**Official Ans. by NTA (3)**

**Sol.**  $\eta = \left(1 - \frac{273}{373}\right) \times 100 = 26.8\%$

**43.** Given below are two statements:

**Statement I :** A truck and a car moving with same kinetic energy are brought to rest by applying brakes which provide equal retarding forces. Both come to rest in equal distance.

**Statement II :** A car moving towards east takes a turn and moves towards north, the speed remains unchanged. The acceleration of the car is zero.

In the light of given statements, choose the most appropriate answer from the options given below.

- (1) Statement I is correct but Statement II is incorrect
- (2) Statement I is incorrect but Statement II is correct
- (3) Both Statement I is correct but Statement II are incorrect
- (4) Both Statement I is correct but Statement II are correct

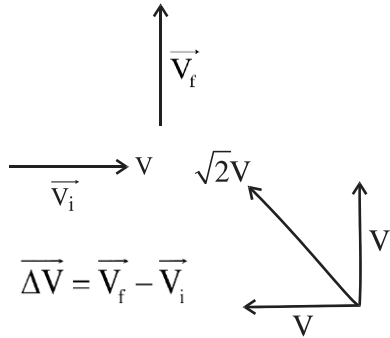
**Official Ans. by NTA (1)**

**Sol.** Work done =  $\Delta KE$   
 Work done =  $-FS = 0 - K$

$$S = \frac{K}{F}$$

Statement 1  $\rightarrow$  correct

Statement 2  $\rightarrow$  incorrect



Velocity is changing  $\Rightarrow \vec{a} \neq 0$

Ans. 1

**44.** A particle is executing Simple Harmonic Motion (SHM). The ratio of potential energy and kinetic energy of the particle when its displacement is half of its amplitude will be:

- (1) 1 : 1                      (2) 2 : 1  
 (3) 1 : 4                      (4) 1 : 3

**Official Ans. by NTA (4)**

**Sol.**  $x = \frac{A}{2}$ ,      P.E. =  $\frac{1}{2}kx^2$

$$K.E. = \frac{1}{2}kA^2 - \frac{1}{2}kx^2$$

$$\frac{P.E.}{K.E.} = \frac{x^2}{A^2 - x^2} = \frac{A^2}{4\left(\frac{3A^2}{4}\right)} = \frac{1}{3}$$

**45.** A ball is thrown vertically upward with an initial velocity of 150 m/s. The ratio of velocity after 3 s and 5 s is  $\frac{x+1}{x}$ . The value of x is \_\_\_\_\_.

Take ( $g = 10 \text{ m/s}^2$ ).

- (1) 6                              (2) 5  
 (3) -5                            (4) 10

**Official Ans. by NTA (2)**

**Sol.**  $\vec{v} = \vec{u} + \vec{a}t$

$$V = 150 - 10t$$

$$V(3) = 150 - 30 = 120$$

$$V(5) = 150 - 50 = 100$$

$$\frac{120}{100} = \frac{x+1}{x} = \frac{6}{5} \Rightarrow x = 5$$

Ans. (2)

**46.** Given below are two statement: one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** If an electric dipole of dipole moment  $30 \times 10^{-5} \text{ Cm}$  is enclosed by a closed surface, the net flux coming out of the surface will be zero.

**Reason R :** Electric dipole consists of two equal and opposite charges.

In the light of above, statements, choose the correct answer from the options given below:

- (1) Both A and R are true and R is the correct explanation of A  
 (2) A is true but R is false  
 (3) Both A and R true but R is NOT the correct explanation of A  
 (4) A is false but R is true

**Official Ans. by NTA (1)**

**Sol.**  $\vec{P} = 30 \times 10^{-5} \text{ Cm}$

Using Gauss law

$$\phi = \frac{Q_{in}}{\epsilon_0} \text{ and } Q_{in} = 0$$

$$\Rightarrow \phi = 0$$

Statement 1 and Statement 2 are correct.

**Ans. (1)**

**47.** Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R.

**Assertion A :** EM waves used for optical communication have longer wavelengths than that of microwave, employed in Radar technology.

**Reason R :** Infrared EM waves are more energetic than microwaves, (used in Radar)

In the light of given statements, choose the correct answer from the options given below:

- (1) A is false but R is true  
 (2) A is true but R is false  
 (3) Both A and R true but R is NOT the correct explanation of A  
 (4) Both A and R true and r is the correct explanation of A

**Official Ans. by NTA (1)**

**Sol.** Optical communication is performed in the frequency range of 1 THz to 1000 THz.

(Microwave to UV)

So, EM waves used for optical communication have shorter wavelength than that of microwaves used in RADAR.

Also,  $\nu_{\text{INFRARED}} > \nu_{\text{MICROWAVE}}$

$\therefore$  Infrared EM waves are more energetic than microwave

**48.** A 12.5 eV electron beam is used to bombard gaseous hydrogen at room temperature. The number of spectral lines emitted will be:

- (1) 2 (2) 1  
(3) 3 (4) 4

**Official Ans. by NTA (3)**

**Sol.** According to Bohr's postulates, an electron makes jump to higher energy orbital if it absorbs a photon of energy equal to difference between the energies of an excited state and the ground state. Assuming that collided electron takes energy equal to 10.2 eV or 12.09 eV from incoming electron beam (some part lost due to collision). The maximum excited state is  $n = 3$ . So, number of spectral lines is

$$\frac{3(3-1)}{2} = 3$$

**49.** The ratio of escape velocity of a planet to the escape velocity of earth will be:

**Given :** Mass of the planet is 16 times mass of earth and radius of the planet is 4 times the radius of earth.

- (1) 4 : 1 (2) 2 : 1  
(3) 1 :  $\sqrt{2}$  (4) 1 : 4

**Official Ans. by NTA (1)**

**Sol.** 
$$V_{\text{escape}} = \sqrt{\frac{2GM}{R}}$$

$$\therefore V_{\text{escape}} \text{ for planet} = \sqrt{\frac{2G(16M_E)}{(4R_E)}} = 2\sqrt{\frac{2GM_E}{R_E}}$$

$$= 2(V_{\text{escape}} \text{ for Earth})$$

**50.** Given below are two statements :

**Statement I :** When the frequency of an a.c. source in a series LCR circuit increases, the current in the circuit first increases, attains a maximum value and then decreases.

**Statement II :** In a series LCR circuit, the value of power factor at resonance is one.

In the light of given statements, choose the most appropriate answer from the options given below:

- (1) Statement I is incorrect but Statement II is true.  
(2) Both Statement I and Statement II are false.  
(3) Statement I is correct but Statement II is false.  
(4) Both Statement I and Statement II are true.

**Official Ans. by NTA (4)**

**Sol.** Both statements are correct. Theory based.

### SECTION-B

**51.** For a certain organ pipe, the first three resonance frequencies are in the ratio of 1:3:5 respectively. If the frequency of fifth harmonic is 405 Hz and the speed of sound in air is  $324 \text{ ms}^{-1}$  the length of the organ pipe is \_\_\_\_\_ m.

**Official Ans. by NTA (1)**

**Sol.** For 5<sup>th</sup> harmonic in closed organ pipe,

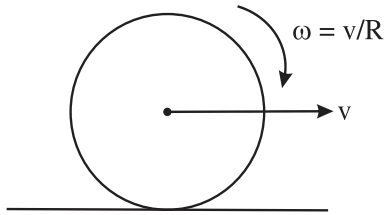
$$f_5 = \frac{5V}{4\ell} \Rightarrow 405 = \frac{5 \times 324}{4\ell}$$

$$\Rightarrow \ell = 1\text{m}$$

**52.** For a rolling spherical shell, the ratio of rotational kinetic energy and total kinetic energy is  $\frac{x}{5}$ . The value of x is \_\_\_\_\_.

**Official Ans. by NTA (2)**

Sol.



$$\frac{K_{\text{rot}}}{K_{\text{Total}}} = \frac{\frac{1}{2} \left( \frac{2}{3} mR^2 \right) \left( \frac{V}{R} \right)^2}{\frac{1}{2} mv^2 + \frac{1}{2} \left( \frac{2}{3} mR^2 \right) \left( \frac{V}{R} \right)^2}$$

$$\Rightarrow \frac{x}{5} = \frac{2}{5} \Rightarrow x = 2$$

53. A compass needle oscillates 20 times per minute at a place where the dip is  $30^\circ$  and 30 times per minute where the dip is  $60^\circ$ . The ratio of total magnetic field due to the earth at two place respectively is  $\frac{4}{\sqrt{x}}$ . The value of x is

**Official Ans. by NTA (243)**

Sol. Period of oscillation  $\propto \frac{1}{\sqrt{B_H}}$

$$T \propto \frac{1}{\sqrt{B \cos \theta}} \Rightarrow \frac{T_1}{T_2} = \sqrt{\frac{B_2 \cos \theta_2}{B_1 \cos \theta_1}}$$

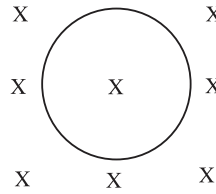
$$\Rightarrow \frac{60/20}{60/30} = \sqrt{\frac{B_2 \cos 60^\circ}{B_1 \cos 30^\circ}} \Rightarrow \frac{3}{2} = \sqrt{\frac{B_2}{\sqrt{3}B_1}}$$

$$\Rightarrow \frac{9}{4} = \frac{B_2}{\sqrt{3}B_1} \Rightarrow \frac{B_1}{B_2} = \frac{4}{9\sqrt{3}} = \frac{4}{\sqrt{243}}$$

54. A conducting circular loop is placed in a uniform magnetic field of 0.4 T with its plane perpendicular to the field. Somehow, the radius of the loop starts expanding at a constant rate of 1 mm/s. The magnitude of induced emf in the loop at an instant when the radius of the loop is 2 cm will be \_\_\_\_\_  $\mu\text{V}$ .

**Official Ans. by NTA (50)**

Sol.



$$\frac{dr}{dt} = 10^{-3} \text{ m/s}$$

$$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$$

$$\varepsilon = \left| \frac{-d\phi}{dt} \right| = \left| \frac{BdA}{dt} \right|$$

$$= 0.4 \times 2 \times \pi \times 2 \times 10^{-2} \times 10^{-3} \text{ V}$$

$$= 16\pi \mu\text{V} = 50.24 \mu\text{V}$$

55. To maintain a speed of 80 km/h by a bus of mass 500 kg on a plane rough road for 4 km distance, the work done by the engine of the bus will be \_\_\_\_\_ KJ. [The coefficient of friction between tyre of bus and road is 0.04].

**Official Ans. by NTA (784)**

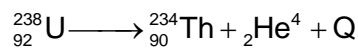
Sol. For constant speed, WD by engine + WD by friction = 0 [by WET]

$$WD_{\text{engine}} = -WD_{\text{friction}} = -[-\mu mgx]$$

$$= 0.04 \times 500 \times 9.8 \times 4 \times 10^3$$

$$= 784 \text{ KJ}$$

56. A common example of alpha decay is



Given :

$${}_{92}^{238}\text{U} = 238.05060\text{u},$$

$${}_{90}^{234}\text{Th} = 234.04360\text{u},$$

$${}_2^4\text{He} = 4.00260\text{u}, \text{ and}$$

$$1\text{u} = 931.5 \frac{\text{MeV}}{c^2}$$

The energy released (Q) during the alpha decay of

$${}_{92}^{238}\text{U} \text{ is } \underline{\hspace{2cm}} \text{ MeV}$$

**Official Ans. by NTA (4)**

Sol. Energy released =  $(\Delta m)_{\text{amu}} \times 931.5 \text{ MeV}$   
 $= (m_{\text{u}} - m_{\text{Th}} - m_{\text{He}})_{\text{amu}} \times 931.5 \text{ MeV}$   
 $= 0.0044 \times 931.5 \text{ MeV} = 4.0986 \text{ MeV}$



57. The current flowing through a conductor connected across a source is 2A and 1.2 A at 0°C and 100°C respectively. The current flowing through the conductor at 50°C will be \_\_\_\_\_ × 10<sup>2</sup> mA.

**Official Ans. by NTA (15)**

**Sol.**  $i_0 R_0 = i_{100} R_{100}$  [For same source]

$$\Rightarrow 2 R_0 = 1.2 R_0 [1 + 100\alpha] \quad \dots (1)$$

$$\Rightarrow 1 + 100\alpha = \frac{5}{3} \Rightarrow 100\alpha = \frac{2}{3}$$

$$\Rightarrow 50 \alpha = \frac{1}{3}$$

$$\therefore i_{50} R_{50} = i_0 R_0$$

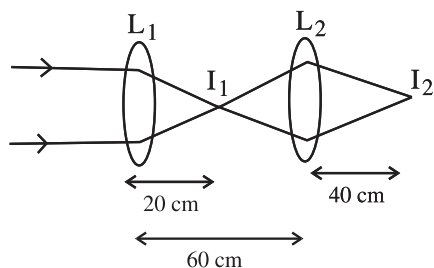
$$\Rightarrow i_{50} = \frac{i_0 R_0}{R_{50}} = \frac{2 \times R_0}{R_0(1 + 50\alpha)} = \frac{2}{1 + \frac{1}{3}} = 1.5A$$

$$= 15 \times 10^2 \text{ mA}$$

58. Two convex lenses of focal length 20 cm each are placed coaxially with a separation of 60 cm between them. The image of the distant object formed by the combination is at \_\_\_\_\_ cm from the first lens.

**Official Ans. by NTA (100)**

**Sol.**  $f_1 = 20 \text{ cm}$   $f_2 = 20 \text{ cm}$



1<sup>st</sup> refraction in  $L_1(I_1)$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{\infty} = \frac{1}{f}$$

$$\therefore v = f$$

2<sup>nd</sup> refraction in  $L_2$

$I_1 \rightarrow$  object

$I_2 \rightarrow$  image

$$u = -40 \text{ cm}$$

$$f = 20 \text{ cm}$$

$$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$$

$$\frac{1}{v} - \frac{1}{(-40)} = \frac{1}{20}$$

$$\frac{1}{v} = \frac{1}{20} - \frac{1}{40} = \frac{6-3}{120}$$

$$\frac{1}{v} = \frac{3}{120} = \frac{1}{40}$$

$$\therefore v = 40 \text{ cm}$$

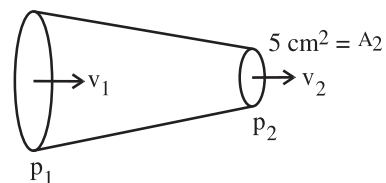
Correct Answer is 100.

59. Glycerine of density  $1.25 \times 10^3 \text{ kg m}^{-3}$  is flowing through the conical section of pipe. The area of cross-section of the pipe at its ends is  $10 \text{ cm}^2$  and  $5 \text{ cm}^2$  and pressure drop across its length is  $3 \text{ Nm}^{-2}$ . The rate of flow of glycerine through the pipe is  $x \times 10^{-5} \text{ m}^3 \text{ s}^{-1}$ . The value of x is \_\_\_\_\_.

**Official Ans. by NTA (4)**

**Sol.**

$$A_1 = 10 \text{ cm}^2$$



$$\Delta P = P_1 - P_2 = 3 \text{ N/m}^2 \text{ (given)}$$

By continuity eq<sup>n</sup>

$$A_1 v_1 = A_2 v_2$$

$$\therefore v_1 = \frac{A_2}{A_1} v_2 \quad \dots (1)$$

By Bernoulli's eq<sup>n</sup>

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2)$$

$$\Delta P = \frac{1}{2} \rho \left( v_2^2 - \frac{A_2^2}{A_1^2} v_2^2 \right)$$

$$\Delta P = \frac{1}{2} \rho \left[ 1 - \left( \frac{A_2}{A_1} \right)^2 \right] v_2^2$$

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \left[ 1 - \left( \frac{5}{10} \right)^2 \right] v_2^2$$

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \left[ 1 - \frac{1}{4} \right] v_2^2$$

$$3 = \frac{1}{2} \times 1.25 \times 10^3 \times \frac{3}{4} v_2^2$$

$$\therefore v_2 = 8 \times 10^{-2} \text{ m/s}$$

So discharge rate =  $A_2 v_2$

$$= 5 \times 10^{-4} \times 8 \times 10^{-2}$$

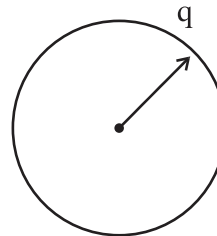
$$= 4 \times 10^{-5} \text{ m}^3/\text{s}$$

**Correct ans is x = 4**

60. 64 identical drops each charged upto potential of 10 mV are combined to form a bigger drop. The potential of the bigger drop will be \_\_\_\_\_ mV.

**Official Ans. by NTA (160)**

**Sol.**



Let  $q$  = charge on each drop

$$V = \frac{Kq}{r} \text{ ---- (1)}$$

Now for combination of 64 drop

$$64 \times \frac{4}{3} \pi r^3 = \frac{4}{3} \pi R^3$$

$$R = 4r$$

And  $Q = 64 q$

Potential of bigger drop

$$= \frac{KQ}{R} = \frac{K64q}{4r} = 16 \frac{Kq}{r}$$

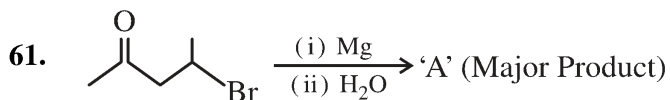
$$= 16 \times 10 \text{ mV} = 160 \text{ mV.}$$

Correct answer is 160.

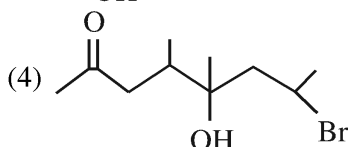
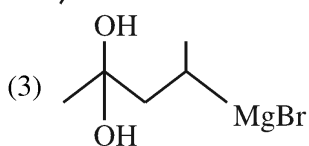
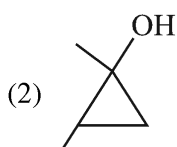
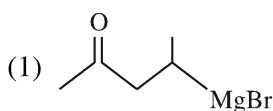
CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

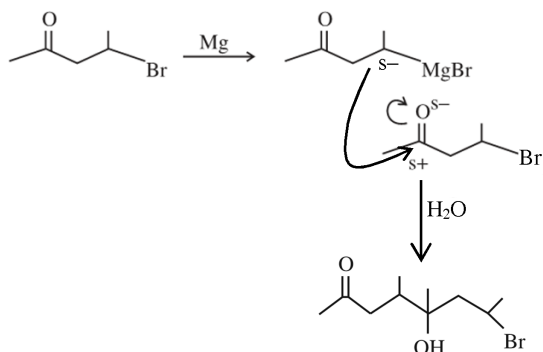


A is



Official Ans. by NTA (4)

Sol.



62. Four gases A, B, C and D have critical temperatures 5.3, 33.2, 126.0 and 154.3K respectively.

For their adsorption on fixed amount of charcoal, the correct order is :

- (1) C > B > D > A
- (2) C > D > B > A
- (3) D > C > A > B
- (4) D > C > B > A

Official Ans. by NTA (4)

Sol. Extent of adsorption ∝ critical temp.

63. Given below are two statements: one is labelled as **Assertion A** and the other is labelled as **Reason R**  
**Assertion A:** 5f electrons can participate in bonding to a far greater extent than 4f electrons

**Reason R:** 5f orbitals are not as buried as 4f orbitals

In the light of the above statements, choose the *correct* answer from the options given below

(1) Both **A** and **R** are true but **R** is **NOT** the correct explanation of **A**

(2) Both **A** and **R** are true and **R** is the correct explanation of **A**

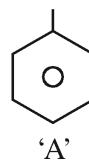
(3) **A** is false but **R** is true

(4) **A** is true but **R** is false

Official Ans. by NTA (2)

Sol. 5f orbital not buried as 4f orbitals so e<sup>-</sup> present in 5f orbital experience less nuclear attraction than e<sup>-</sup> present in 4f orbital. Hence electrons of 5f orbital can take part in bonding to a far greater extent.

64. The incorrect statement regarding the reaction given below is



'A'



(1) The electrophile involved in the reaction is NO<sup>+</sup>

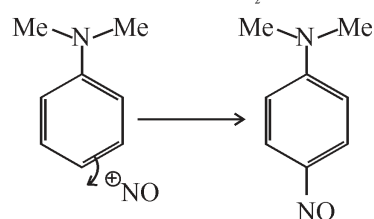
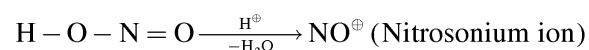
(2) 'B' is N-nitroso ammonium compound

(3) The reaction occurs at low temperature

(4) The product 'B' formed in the above reaction is p-nitroso compound at low temperature

Official Ans. by NTA (2)

Sol. NaNO<sub>2</sub> + HX → HNO<sub>2</sub> + NaX



P - Nitroso product

65. Match List I with List II

LIST I Complex		LIST II CFSE( $\Delta_0$ )	
A.	$[\text{Cu}(\text{NH}_3)_6]^{2+}$	I.	-0.6
B.	$[\text{Ti}(\text{N}_2\text{O})_6]^{3+}$	II.	-2.0
C.	$[\text{Fe}(\text{CN})_6]^{3-}$	III.	-1.2
D.	$[\text{NiF}_6]^{4-}$	IV.	-0.4

Choose the correct answer from the options given below :

- (1) A-I, B-IV, C-II, D-III
- (2) A-II, B-III, C-I, D-IV
- (3) A-I, B-II, C-IV, D-III
- (4) A-III, B-IV, C-I, D-II

**Official Ans. by NTA (1)**

**Sol.** CFSE =  $(-0.4 n_{t_{2g}} + 0.6 n_{e_g}) \Delta_0$   
 $n_{t_{2g}}$  = number of electrons in  $t_{2g}$  orbital  
 $n_{e_g}$  = number of electrons in  $e_g$  orbital

Complex	No. of at electrons	CFSE( $\Delta_0$ )
$[\text{Cu}(\text{NH}_3)_6]^{+2}$	$d^9$ (S.L.) $t_{2g}^{2,2,2} e_g^{2,1}$	-0.6
$[\text{Ti}(\text{H}_2\text{O})_6]^{+3}$	$d^1$ (W.L.) $t_{2g}^{1,0,0} e_g^{0,0}$	-0.4
$[\text{Fe}(\text{CN})_6]^{3-}$	$d^5$ (S.L.) $t_{2g}^{2,2,1} e_g^{0,0}$	-2.0
$[\text{NiF}_6]^{4-}$	$d^8$ (W.L.) $t_{2g}^{2,2,2} e_g^{1,1}$	-1.2

66. Match List I with List II

LIST I (Examples)	LIST I (Examples)
A. 2-Chloro-1, 3 - butadiene	I. Biodegradable polymer
B. Nylon 2-nylon 6	II. Synthetic Rubber
C. Polyacrylonitrile	III. Polyester
D. Dacron	IV. Addition Polymer

Choose the correct answer from the options given below :

- (1) A-IV, B-I, C-III, D-II
- (2) A-IV, B-III, C-I, D-II
- (3) A-II, B-IV, C-I, D-III
- (4) A-II, B-I, C-IV, D-III

**Official Ans. by NTA (4)**

**Sol.** FACT

67. The density of alkali metals is in the order

- (1)  $\text{Na} < \text{K} < \text{Cs} < \text{Rb}$
- (2)  $\text{K} < \text{Na} < \text{Rb} < \text{Cs}$
- (3)  $\text{K} < \text{Cs} < \text{Na} < \text{Rb}$
- (4)  $\text{Na} < \text{Rb} < \text{K} < \text{Cs}$

**Official Ans. by NTA (2)**

**Sol.** In general moving down the group, mass increases more prominently as compared to volume (size) hence density increases for Group I metal. Due to empty 3d subshell in K increase in size is more prominent as compare to mass.

$\text{Li} < \text{K} < \text{Na} < \text{Rb} < \text{Cs}$

68. Given below are two statements :

**Statements :**  $\text{SbCl}_5$  is more covalent than  $\text{SbCl}_3$

**Statements :** The higher oxides of halogens also tend to be more stable than the lower ones.

In the light of the above statements, choose the most appropriate answer from the options given below

- (1) Both statement I and Statement II are correct
- (2) Both statement I and Statement II are incorrect
- (3) Statement I is correct but Statement II is incorrect
- (4) Statement I is incorrect but Statement II is correct

**Official Ans. by NTA (1)**

**Sol. Statement I :** Is correct according to Fajan's rule  $\text{Sb}^{+5}$  more polarising power than  $\text{Sb}^{+3}$ .

**Statement II :** Stability of higher oxides of halogen is primarily due to

- a) Higher oxidation state
- b) More EN halogen
- c) Resonance stabilization

69. A metal chloride contains 55.0% of chlorine by weight. 100 mL vapours of the metal chloride at STP weigh 0.57 g. The molecular formula of the metal chloride is

(Given : Atomic mass of chlorine is 35.5u)

- (1)  $\text{MCl}_2$
- (2)  $\text{MCl}_4$
- (3)  $\text{MCl}_3$
- (4)  $\text{MCl}$

**Official Ans. by NTA (1)**

**Sol.** Molecular. weight of metal chloride

$$= \frac{0.57}{100} \times 22700$$

$$= 129.39$$

$$\text{weight of Cl} = 129.39 \times 0.55$$

$$= 71.1645$$

$$\therefore \text{Mole of Cl} = \frac{71.1645}{35.5} \cong 2$$

Hence  $\text{MCl}_2$

70. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**

**Assertion A :** In the Ellingham diagram, a sharp change in slope of the line is observed for  $\text{Mg} \rightarrow \text{MgO}$  at  $\sim 1120^\circ\text{C}$

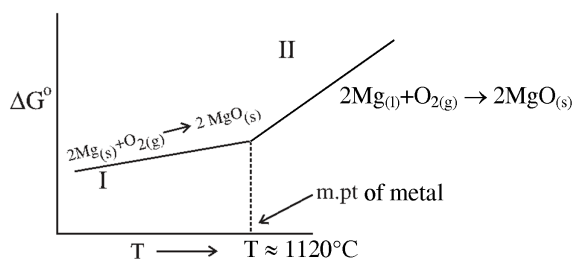
**Reason R :** There is a large change of entropy associated with the change of state

In the light of the above statements, choose the correct answer from the options given below

- (1) Both A and R are true but R is NOT the correct explanation of A
- (2) Both A and R are true and R is the correct explanation of A
- (3) A is false but R is true
- (4) A is true but R is false

**Official Ans. by NTA (2)**

**Sol.**



For line II,  $\Delta S$  is more  $-ve$  than line I. hence higher slope.

$$\text{For I } \Delta S_I = (S_{\text{solid}}) - (S_{\text{solid}} + S_{\text{gas}})$$

$$\text{For II } \Delta S_{II} = (S_{\text{solid}}) - (S_{\text{liq}} + S_{\text{gas}})$$

Hence  $\Delta S_{II}$  more  $-ve$  than  $\Delta S_I$

71. Match List I with List II

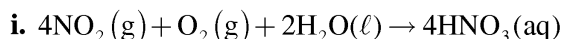
LIST I		LIST II	
A.	Nitrogen oxides in air	I.	Eutrophication
B.	Methane in air	II.	pH of rain water becomes 5.6.
C.	Carbon dioxide	III.	Global warming
D.	Phosphate fertilisers in water	IV.	Acid rain

Choose the correct answer from the options given below :

- (1) A-IV, B-III, C-II, D-I
- (2) A-II, B-III, C-I, D-IV
- (3) A-I, B-II, C-III, D-IV
- (4) A-IV, B-II, C-III, D-I

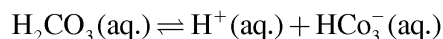
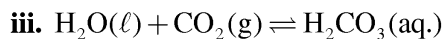
**Official Ans. by NTA (1)**

**Sol.:**



$\text{SO}_2$  &  $\text{NO}_2$  have major contribution in acid rain

ii.  $\text{CO}_2$ ,  $\text{CH}_4$ ,  $\text{O}_3$ , CFC are responsible for global warming



Rain water has pH of 5.6 due to the Presence of  $\text{H}^+$  ions formed by the reaction of rain water with  $\text{CO}_2$

iv. Phosphates present in fertilizers contribution for Eutrophication (Process in which nutrient enriched water bodies support a dense plant population, which kills animal life by depriving it of oxygen and results in subsequent loss of biodiversity is known as Eutrophication.)

72. For lead storage battery pick the correct statements

A. During charging of battery,  $\text{PbSO}_4$  on anode is converted into  $\text{PbO}_2$

B. During charging of battery,  $\text{PbSO}_4$  on cathode is converted into  $\text{PbO}_2$

C. Lead storage battery, consists of grid of lead packed with  $\text{PbO}_2$  as anode

D. Lead storage battery has  $\sim 38\%$  solution of sulphuric acid as an electrolyte

Choose the correct answer from the options given below :

- (1) B, D only
- (2) B, C, D only
- (3) A, B, D only
- (4) B, C only

**Official Ans. by NTA (1)**

**Sol.** Lead storage battery consists of lead anode and a grid of lead packed with lead oxide ( $\text{PbO}_2$ ) as cathode, a 38% solution of  $\text{H}_2\text{SO}_4$  is used as an electrolyte.

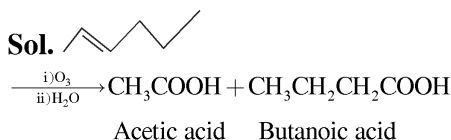
On charging the battery the reaction is reversed and  $\text{PbSO}_4(\text{s})$  on anode and cathode is converted into  $\text{Pb}$  and  $\text{PbO}_2$  respectively.

73. 2-hexene  $\xrightarrow[\text{(ii)H}_2\text{O}]{\text{(i)O}_3}$  Products

The two products formed in above reaction are -

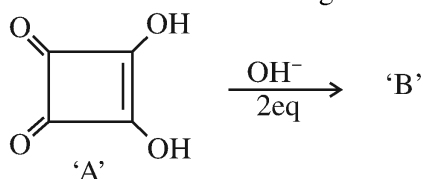
- (1) Butanoic acid and acetic acid
- (2) Butanal and acetic acid
- (3) Butanal and acetaldehyde
- (4) Butanoic acid and acetaldehyde

**Official Ans. by NTA (1)**



it is oxidative ozonolysis.

74. Correct statements for the given reaction are :

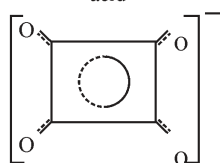
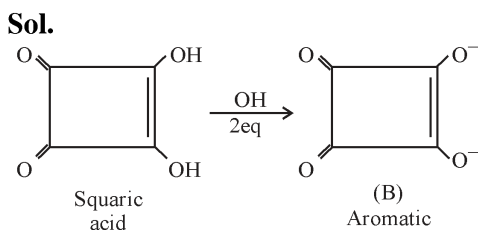


- A. Compound 'B' is aromatic
- B. The completion of above reaction is very slow
- C. 'A' shows tautomerism
- D. The bond lengths C-C in compound B are found to be same

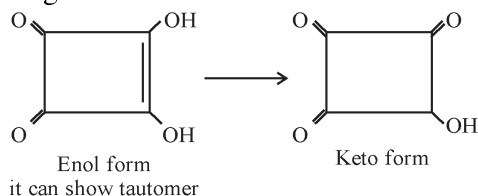
Choose the correct answer from the options given below :

- (1) A, B and D only
- (2) A, B and C only
- (3) B, C and D only
- (4) A, C and D only

**Official Ans. by NTA (4)**



Resonance hybrid of B showing all C-C bond length same



75. The bond order and magnetic property of acetylide ion are same as that of

- (1)  $\text{NO}^+$
- (2)  $\text{O}_2^+$
- (3)  $\text{O}_2^-$
- (4)  $\text{N}_2^+$

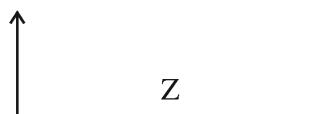
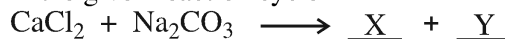
**Official Ans. by NTA (1)**

**Sol.** Acetylide ion  $\rightarrow \text{C}_2^{2-} (\bar{\text{C}} \equiv \bar{\text{C}})$

Bond order = 3 & Diamagnetic

$\text{NO}^+ 14e^- \rightarrow$  Bond order = 3 & Diamagnetic

76. In the given reaction cycle

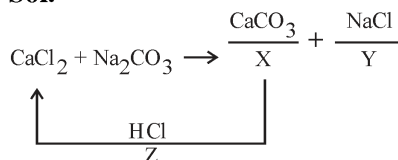


X, Y and Z respectively are

- (1)  $\text{CaO} \quad \text{NaCl} + \text{CO}_2 \quad \text{KCl}$
- (2)  $\text{CaCO}_3 \quad \text{NaCl} \quad \text{KCl}$
- (3)  $\text{CaCO}_3 \quad \text{NaCl} \quad \text{HCl}$
- (4)  $\text{CaO} \quad \text{NaCl} + \text{CO}_2 \quad \text{NaCl}$

**Official Ans. by NTA (3)**

**Sol.**



77. Given below are two statements :

**Statement I :** Boron is extremely hard indicating its high lattice energy

**Statement II :** Boron has highest melting and boiling point compared to its other group members.

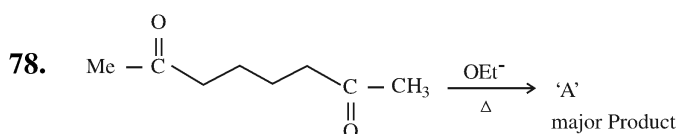
In the light of the above statements, choose the *most appropriate* answer from the options given below

- (1) Statement I is incorrect but Statement II is correct
- (2) Both Statement I and Statement II is correct
- (3) Statement I is correct but Statement II is incorrect
- (4) Both Statement I and Statement II is incorrect

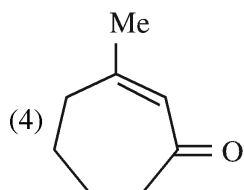
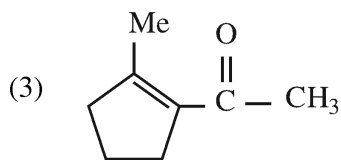
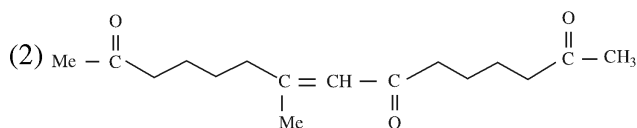
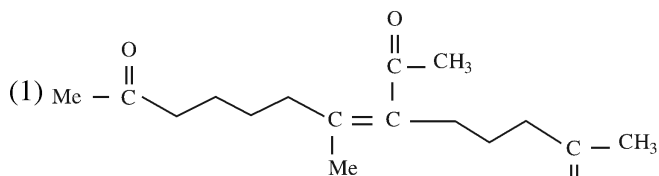
**Official Ans. by NTA (2)**

**Sol.** Boron is non-metallic in nature. It is extremely hard and black coloured solid. It exists in many allotropic forms. Due to very strong crystalline lattice, boron has unusually high melting point and boiling point.

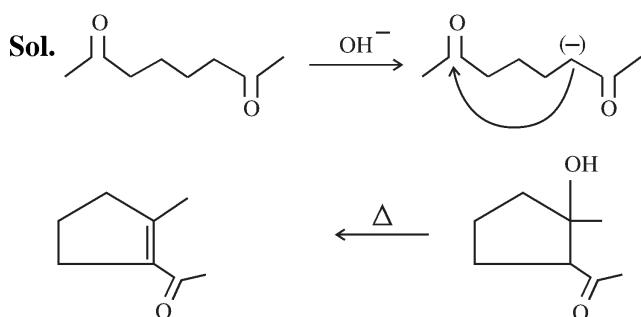
Element					
	B	Al	Ga	In	Tl
Melting point/K	2453	933	303	430	576
Boiling point/K	3923	2740	2676	2353	1730



A in the above reaction is :



**Official Ans. by NTA (3)**



79. Match List I with List II

LIST I		LIST II	
Type of Hydride		Example	
A.	Electron deficient hydride	I.	MgH <sub>2</sub>
B.	Electron rich hydride	II.	HF
C.	Electron precise hydride	III.	B <sub>2</sub> H <sub>6</sub>
D.	Saline hydride	IV.	CH <sub>4</sub>

Choose the correct answer from the options given below :

(1) A-III, B-II, C-IV, D-I

(2) A-II, B-III, C-IV, D-I

(3) A-II, B-III, C-I, D-IV

(4) A-III, B-II, C-I, D-IV

**Official Ans. by NTA (1)**

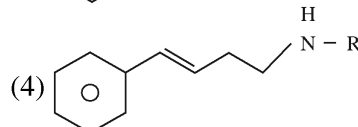
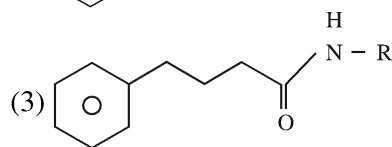
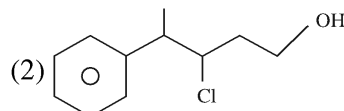
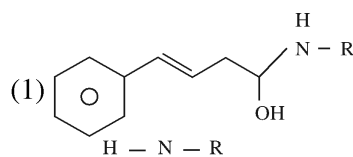
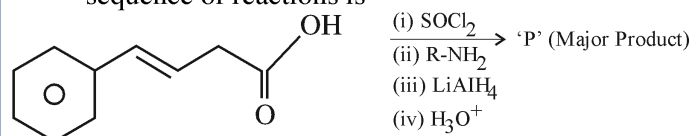
**Sol.** B<sub>2</sub>H<sub>6</sub> ⇒ e<sup>-</sup> deficient hydride

HF ⇒ e<sup>-</sup> rich hydride

CH<sub>4</sub> ⇒ e<sup>-</sup> Precise hydride

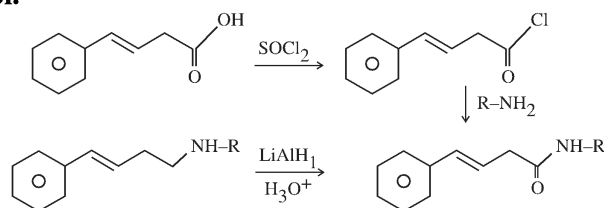
MgH<sub>2</sub> ⇒ Saline hydride

80. The major product 'P' formed in the following sequence of reactions is



**Official Ans. by NTA (4)**

**Sol.**



SECTION-B

81. One mole of an ideal gas at 350K is in a 2.0 L vessel of thermally conducting walls, which are in contact with the surroundings. It undergoes isothermal reversible expansion from 2.0L to 3.0L against a constant pressure of 4 atm. The change in entropy of the surroundings ( $\Delta S$ ) is \_\_\_\_\_ J K<sup>-1</sup> (Nearest integer)

Given : R = 8.314 J K<sup>-1</sup> Mol<sup>-1</sup>.

Official Ans. by NTA (3)

$$\text{Sol. } \Delta S_{\text{System}} = nR \ln \left( \frac{V_2}{V_1} \right)$$

$$= 1 \times 8.314 \ln \left( \frac{3}{2} \right)$$

$$\Delta S_{\text{System}} = 3.37$$

$$\Delta S_{\text{Surr.}} = 3.37$$

Correct Ans : 3

82. The mass of NH<sub>3</sub> produced when 131.8 kg of cyclohexanecarbaldehyde undergoes Tollen's test is \_\_\_\_\_ kg. (Nearest Integer)

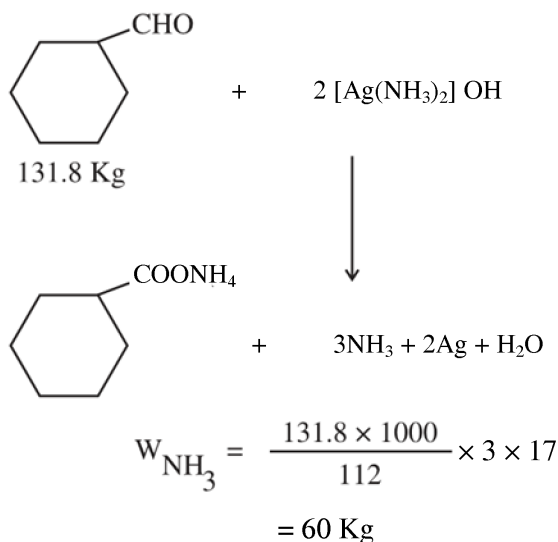
Molar Mass of C = 12g/mol

N = 14g/mol

O = 16g/mol

Official Ans. by NTA (60)

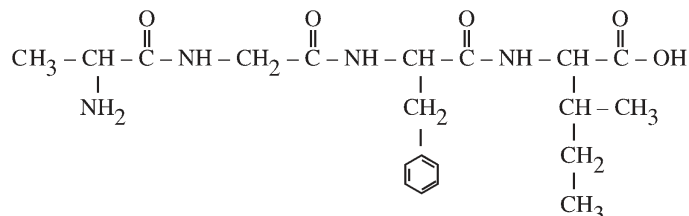
Sol.



83. In an oligopeptide named Alanylglycylphenyl alanyl isoleucine, the number of sp<sup>2</sup> hybridised carbons is \_\_\_\_\_.

Official Ans. by NTA (10)

Sol.



84. An analyst wants to convert. 1L HCl of pH = 1 to a solution of HCl of pH 2. The volume of water needed to do this dilution is \_\_\_\_\_ mL. (Nearest Integer)

Official Ans. by NTA (9000)

Sol.

$$\frac{(M_1 \times V_1)}{10 \times 1} = \frac{(M_2 \times V_2)}{10 \times V_2}$$

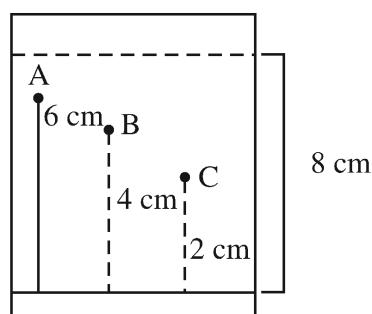
$$V_2 = 10L$$

$$\text{Water added} = 10 - 1$$

$$= 9 \text{ Litre}$$

$$= 9000 \text{ mL}$$

85. Three organic compounds A, B and C were allowed to run in thin layer chromatography using hexane and gave the following result (see figure). The R<sub>f</sub> value of the most polar compound is \_\_\_\_\_ × 10<sup>-2</sup>





**Official Ans. by NTA (25)**

**Sol.** More  $R_f$ , less its polarity

$$R_f = \frac{\text{Distance travelled by compound 'X'}}{\text{Distance travelled by solvent 'Y'}}$$

$$= \frac{2}{8} = 0.25 = 25 \times 10^{-2}$$

- 86.** 80 mole percent of  $\text{MgCl}_2$  is dissociated in aqueous solution. The vapour pressure of 1.0 molal aqueous solution of  $\text{MgCl}_2$  at  $38^\circ\text{C}$  is \_\_\_\_\_ mm Hg. (Nearest integer)

Given : Vapour pressure of water at  $38^\circ\text{C}$  is 50 mm Hg

**Official Ans. by NTA (48)**

**Sol.**



$$1 - \alpha \quad \alpha \quad 2\alpha$$

$$i = 1 + 2\alpha \quad (\alpha = 0.8)$$

$$i = 2.6$$

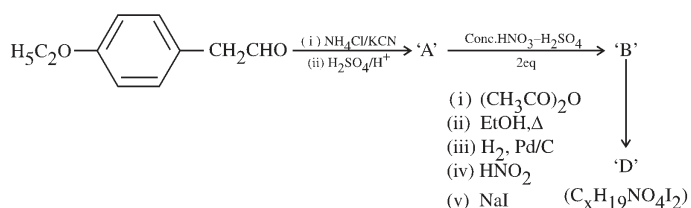
$$\frac{\Delta p}{p^\circ} = \frac{i \times n_2}{n_1}$$

$$\Delta p = 2.34$$

$$p_s = 47.66$$

$$p_s \cong 48$$

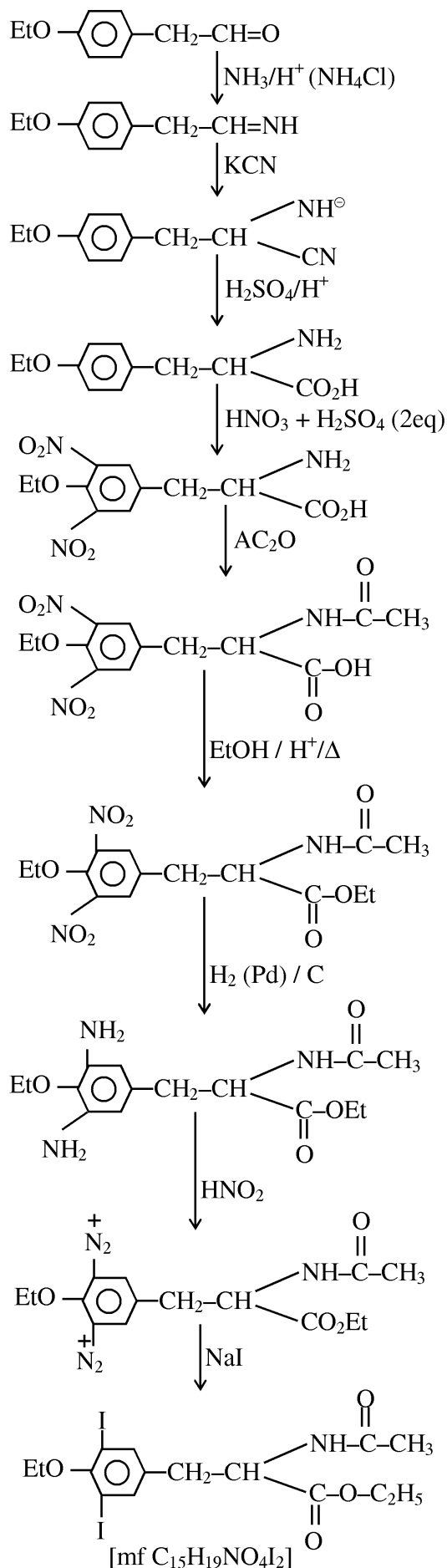
**87.**



The value of x in compound 'D' is \_\_\_\_\_

**Official Ans. by NTA (15)**

**Sol.**



88. At 600K, the root mean square (rms) speed of gas X (molar mass = 40) is equal to the most probable speed of gas Y at 90K. The molar mass of the gas Y is \_\_\_\_\_ g mol<sup>-1</sup>. (Nearest integer)

**Official Ans. by NTA (4)**

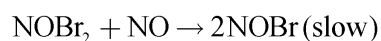
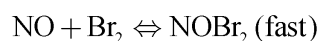
**Sol.**  $(U_{\text{rms}})_{X,600} = (U_{\text{mp}})_{Y,90}$

$$\sqrt{\frac{3 \times R \times 600}{40}} = \sqrt{\frac{2 \times R \times 90}{M}}$$

$$M = 4$$

89. The reaction  $2\text{NO} + \text{Br}_2 \rightarrow 2\text{NOBr}$

takes place through the mechanism given below :



The overall order of the reaction is \_\_\_\_\_.

**Official Ans. by NTA (3)**

**Sol.** RDS :  $\text{NOBr}_2 + \text{NO} \rightarrow 2\text{NOBr}$

$$r = k[\text{NOBr}_2][\text{NO}] \quad \text{--- (i)}$$

$$K_{\text{eq}} = \frac{[\text{NOBr}_2]}{[\text{NO}][\text{Br}_2]} \quad \text{--- (ii)}$$

From (i) & (ii)

$$r = k \cdot K_{\text{eq}} \cdot [\text{NO}] [\text{Br}_2] [\text{NO}]$$

$$r = k' [\text{NO}]^2 [\text{Br}_2]$$

Overall order = 3

Ans. 3

90. Values of work function ( $W_0$ ) for a few metals are given below

Metal	Li	Na	K	Mg	Cu	Ag
$W_0/\text{eV}$	2.42	2.3	2.25	3.7	4.8	4.3

The number of metals which will show photoelectric effect when light of wavelength 400nm falls on it is \_\_\_\_\_

Given :  $h = 6.6 \times 10^{-34} \text{ J s}$

$$c = 3 \times 10^8 \text{ m s}^{-1}$$

$$e = 1.6 \times 10^{-19} \text{ C}$$

**Official Ans. by NTA (3)**

**Sol.**  $E(\text{eV}) = \frac{1240}{400} = 3.1 \text{ eV}$

Mg, Cu, Ag

Ans.3

