FINAL JEE–MAIN EXAMINATION – APRIL, 2023

(Held On Saturday 15th April, 2023) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | **TEST PAPER WITH SOLUTION**

SECTION-A

1. The total number of three-digit numbers, divisible by 3, which can be formed using the digits 1, 3, 5, 8, if repetition of digits is allowed, is:

(1) 22 (2) 18

(3) 21 (4) 20

Official Ans. by NTA (1)

Sol. (1,1,1) (3,3,3) (5,5,5) (8,8,8)

(5,5,8) (8,8,5) (1,3,5) (1,3,8)

Total number $= 1+1+1+1+\frac{3!}{2!}+\frac{3!}{2!}+3!+3! = 22$

2. Let S be the set of all values of λ, for which the shortest distance between the lines $\frac{x-\lambda}{0}=\frac{y-3}{4}=\frac{z+6}{1}$ and $\frac{x+\lambda}{3}=\frac{y}{-4}=\frac{z-6}{0}$ is 13.

Then $8\left|\sum_{\lambda\in S}\lambda\right|$ is equal to

(1) 304 (2) 308

(3) 306 (4) 302

Official Ans. by NTA (3)

Sol. Shor test distance $= \dfrac{\left|\begin{vmatrix} 0 & 4 & 1 \\ 3 & -4 & 0 \\ 2\lambda & 3 & -12 \end{vmatrix}\right|}{\left|\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 4 & 1 \\ 3 & -4 & 0 \end{vmatrix}\right|}$

$13=\frac{|153+8\lambda|}{|4\hat{i}+3\hat{j}-12\hat{k}|}$

$=\frac{|153+8\lambda|}{13}$

$|153 + 8\lambda| = 169$

$153 + 8\lambda = 169, -169$

$\lambda=\frac{16}{8},\frac{-322}{8}$

$8\left|\sum_{\lambda\in S}\lambda\right|=306$

3. The mean and standard deviation of 10 observations are 20 and 8 respectively. Later on, it was observed that one observation was recorded as 50 instead of 40. Then the correct variance is:

(1) 14 (2) 13

(3) 12 (4) 11

Official Ans. by NTA (2)

Sol. $\mu = 20, \sigma = 8$

$\mu_{\text{Corrected}} = \frac{200-50+40}{10}=19$

$\sigma^2=\frac{1}{10}\sum x_i^2-20^2$

$(64 + 400)\, 10 = \sum x_i^2$

$\sigma^2_{\text{Corrected}}=\frac{1}{10}\left[(64+400)10-2500+1600\right]-19^2$

$= 374 - 361$

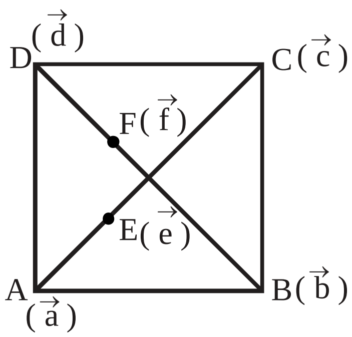
$= 13$

4. Let ABCD be a quadrilateral. If E and F are the mid points of the diagonals AC and BD respectively and $\left(\overrightarrow{AB}-\overrightarrow{BC}\right)+\left(\overrightarrow{AD}-\overrightarrow{DC}\right)=k\ \overrightarrow{FE}$, then k is equal to

(1) 2 (2) –2

(3) –4 (4) 4

Official Ans. by NTA (3)



Sol.

$\overrightarrow{AB}-\overrightarrow{BC}+\overrightarrow{AB}-\overrightarrow{DC}=k\overrightarrow{FE}$

$\left(\vec{b}-\vec{a}\right)-\left(\vec{c}-\vec{b}\right)+\left(\vec{d}-\vec{a}\right)-\left(\vec{c}-\vec{d}\right)=k\overrightarrow{FE}$

$2\left(\vec{b}+\vec{d}\right)-2\left(\vec{a}-\vec{c}\right)=k\overrightarrow{FE}$

$2(2f - 2(2e)) = kFE$

$4(\vec{f} - \vec{e}) = k\overrightarrow{FE}$

$-4\overrightarrow{FE} = k\overrightarrow{FE}$

k = –4

5. Let x = x (y) be the solution of the differential equation $2(y + 2)\log_e (y + 2)dx + (x + 4 - 2\log_e (y + 2))dy = 0$, $y > -1$ with $x(e^4 - 2) = 1$. Then $x(e^9 - 2)$ is equal to

(1) $\frac{4}{9}$ (2) $\frac{10}{3}$

(3) $\frac{32}{9}$ (4) 3

Official Ans. by NTA (3)

Sol. $2(y+2)\ln(y+2)dx + (x + 4 - 2\ln(y + 2))dy = 0$

$2\ln(y+2) + (x + 4 - 2\ln(y + 2))\frac{1}{y+2}.\frac{dy}{dx} = 0$

let, ln(y+2) = t

$\frac{1}{y+2}.\frac{dy}{dx} = \frac{dt}{dx}$

$2t + (x + 4 - 2t).\frac{dt}{dx} = 0$

$(x + 4 - 2t)\frac{dt}{dx} = -2t$

$\frac{dx}{dt} = \frac{2t - 4 - x}{2t}$

$\frac{dx}{dt} + \frac{x}{2t} = \frac{2t - 4}{2t}$

$x.t^{1/2} = \int \frac{2t-4}{2t}.t^{1/2}.dt$

$x.t^{1/2} = \int\left(t^{1/2} - \frac{2}{t^{1/2}}\right).dt$

$= \frac{t^{\frac{3}{2}}}{\frac{3}{2}} - 2.\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + C$

$x.t^{\frac{1}{2}} = \frac{2t^{\frac{3}{2}}}{3} - 4t^{\frac{1}{2}} + C$

$x = \frac{2}{3}.t - 4 + C.t^{\frac{-1}{2}}$

$x = \frac{2}{3}\ln(y+2) - 4 + C.(\ln(y+2))^{\frac{-1}{2}}$

Put $y = e^4 - 2$, x = 1

$1 = \frac{2}{3} \times 4 - 4 + C \times \frac{1}{2}$

$\frac{C}{2} = 5 - \frac{8}{3} = \frac{7}{3}$

$C = \frac{14}{3}$

$x = \frac{2}{3} \times 9 - 4 + \frac{14}{3} \times \frac{1}{3}$

$= 2 + \frac{14}{9}$

$= \frac{32}{9}$

6. Let [x] denote the greatest integer function and f(x) = max {1 + x + [x], 2 + x, x + 2[x]}, $0 \le x \le 2$. Let *m* be the number of points in [0,2], where *f* is not continuous and *n* be the number of points in (0,2), where *f* is not differentiable. Then $(m + n)^2 + 2$ is equal to:

(1) 11 (2) 2

(3) 6 (4) 3

Official Ans. by NTA (4)

Sol. Let $g(x) = 1 + x + [x] = \begin{cases} 1+x; & x \in [0,1) \\ 2+x; & x \in [1,2) \\ 5; & x = 2 \end{cases}$

$\lambda(x) = x + 2[x] = \begin{cases} x; & x \in [0,1) \\ x+2; & x \in [1,2) \\ 6; & x = 2 \end{cases}$

r(x) = 2 + x

$f(x) = \begin{cases} 2+x; & x \in [0,2) \\ 6; & x = 2 \end{cases}$

f(x) is discontinuous only at x = 2 $\Rightarrow$ m = 1

f(x) is differentiable in (0,2) $\Rightarrow$ n = 0

$(m + n)^2 + 2 = 3$

7. The number of real roots of the equation $x|x| - 5|x+2| + 6 = 0$, is

(1) 5 (2) 3

(3) 6 (4) 4

Official Ans. by NTA (2)

Sol. $x|x| - 5|x+2| + 6 = 0$

C–1 :- $x \in [0,\infty]$

$x^2 - 5x - 4 = 0$

$$x=\frac{5\pm\sqrt{25+16}}{2}=\frac{5+\sqrt{41}}{2}$$

$$x=\frac{5\pm\sqrt{41}}{2}$$

C–2 :- :- $x\in[-2, 0)$

$-x^2-5x-4=0$

$x^2+5x+4=0$

$x=-1, -4$

$x=-1$

C–3 : $x\in[-\infty, -2)$

$-x^2+5x+16=0$

$x^2-5x-16=0$

$$x=\frac{5\pm\sqrt{25+64}}{2}$$

$$\frac{5\pm\sqrt{89}}{2}$$

$$x=\frac{5-\sqrt{89}}{2}$$

8. Let $(a+bx+cx^2)^{10}=\sum_{i=0}^{20}p_i x^i$, $a, b, c\in\mathbb{N}$. If $p_1 = 20$ and $p_2 = 210$, then $2(a + b + c)$ is equal to

(1) 8 (2) 12

(3) 15 (4) 6

Official Ans. by NTA (2)

Sol. $(a+bx+cx^2)^{10}=\sum_{i=0}^{20}p_i x^i$

Coefficient of $x^1 = 20$

$$20=\frac{10!}{9!1!}\times a^9\times b^1$$

$a^9. b = 2$

$a = 1, b = 2$

Coefficient of $x^2 = 210$

$$210=\frac{10!}{9!1!}\times a^9\times c^1+\frac{10!}{8!2!}\times a^8b^2$$

$210 = 10.c + 45 \times 4$

$10c = 30$

$c = 3$

$2(a + b = c) = 12$

9. Let the determinant of a square matrix A of order m be m – n, where m and n satisfy 4m + n = 22 and 17m + 4n = 93. If det (n adj(adj(mA))) = $3^a5^b6^c$. then a + b + c is equal to:

(1) 96 (2) 101

(3) 109 (4) 84

Official Ans. by NTA (1)

Sol. $|A| = m - n$

$4m + n = 22$

$17m + 4n = 93$

$m = 5, n = 2$

$|A| = 3$

$|2\text{ adj (adj }5A))| = 2^5\,|5A|^{16}$

$= 2^5.\,5^{80}\,|A|^{16}$

$= 2^5.\,5^{80}.\,3^{16}$

$= 3^{11}.\,5^{80}.\,6^5$

$a + b + c = 96$

10. Let A_1 and A_2 be two arithmetic means and G_1, G_2, G_3 be three geometric means of two distinct positive numbers. The $G_1^4+G_2^4+G_3^4+G_1^2G_3^2$ is equal to

(1) $2(A_1+A_2)G_1G_3$

(2) $(A_1+A_2)^2G_1G_3$

(3) $(A_1+A_2)G_1^2G_3^2$

(4) $2(A_1+A_2)G_1^2G_3^2$

Official Ans. by NTA (2)

Sol. a, A_1, A_2, b are in A.P.

$$d=\frac{b-a}{3};\quad A_1=a+\frac{b-a}{3}=\frac{2a+b}{3}$$

$$A_2=\frac{a+2b}{3}$$

$A_1 + A_2 = a + b$

a,G_1,G_2,G_3,b are in G.P.

$$r=\left(\frac{b}{a}\right)^{\frac{1}{4}}$$

$$G_1=\left(a^3b\right)^{\frac{1}{4}}$$

$$G_2=\left(a^2b^2\right)^{\frac{1}{4}}$$

$$G_3=\left(ab^3\right)^{\frac{1}{4}}$$

$$G_1^4+G_2^4+G_3^4+G_1^2G_3^2=$$

$$a^3b+a^2b^2+ab^3+\left(a^3b\right)^{\frac{1}{2}}.\left(ab^3\right)^{\frac{1}{2}}$$

$= a^3b + a^2b^2 + ab^3 + a^2.b^2$

$= ab(a^2 + 2ab + b^2)$

$= ab(a + b)^2$

$= G_1.G_3.(A_1 + A_2)^2$

11. If the set $\left\{\text{Re}\left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right): z \in \mathbb{C}, \text{Re}(z)=3\right\}$ is equal to the interval $(\alpha,\beta]$, then $24(\beta-\alpha)$ is equal to

(1) 36 (2) 42

(3) 27 (4) 30

Official Ans. by NTA (4)

Sol. Let $z_1 = \left(\frac{z-\bar{z}+z\bar{z}}{2-3z+5\bar{z}}\right)$

Let $z = 3 + iy$

$\bar{z} = 3 - iy$

$$z_1 = \frac{2iy + (9+y^2)}{2-3(3+iy)+5(3-iy)}$$

$$= \frac{9+y^2+i(2y)}{8-8iy}$$

$$= \frac{(9+y^2)+i(2y)}{8(1-iy)}$$

$$\text{Re}(z_1) = \frac{(9+y^2)-2y^2}{8(1+y^2)}$$

$$= \frac{9-y^2}{8(1+y^2)}$$

$$= \frac{1}{8}\left[\frac{10-(1+y^2)}{(1+y^2)}\right]$$

$$= \frac{1}{8}\left[\frac{10}{1+y^2}-1\right]$$

$1 + y^2 \in [1, \infty]$

$$\frac{1}{1+y^2} \in (0,1]$$

$$\frac{10}{1+y^2} \in (0,10]$$

$$\frac{10}{1+y^2} - 1 \in (-1,9]$$

$$\text{Re}(z_1) \in \left(\frac{-1}{8}, \frac{9}{8}\right]$$

$$\alpha = \frac{-1}{8}, \beta = \frac{9}{8}$$

$$24(\beta-\alpha) = 24\left(\frac{9}{8}+\frac{1}{8}\right) = 30$$

12. The number of common tangents, to the circles $x^2 + y^2 - 18x - 15y + 131 = 0$ and $x^2 + y^2 - 6x - 6y - 7 = 0$, is :

(1) 3 (2) 2

(3) 1 (4) 4

Official Ans. by NTA (1)

Sol. $C_1\left(9, \frac{15}{2}\right)$ $r_1 = \sqrt{81+\frac{225}{4}-131} = \frac{5}{2}$

C_2 (3,3) $r_2 = 5$

$$C_1C_2 = \sqrt{6^2+\frac{81}{4}} = \frac{15}{2}$$

$$r_1 + r_2 = \frac{15}{2}$$

$C_1C_2 = r_1 + r_2$

Number of common tangents = 3

13. Negation of $p \wedge (q \wedge \sim (p \wedge q))$ is

(1) $\sim(p \vee q)$ (2) $p \vee q$

(3) $(\sim(p \wedge q)) \wedge q$ (4) $(\sim(p \wedge q)) \vee p$

Official Ans. by NTA (4)

Sol. $\sim\left[p \wedge (q \wedge \sim(p \wedge q))\right]$

$\sim p \vee (\sim q \vee (p \wedge q))$

$\sim p \vee ((\sim q \vee p) \wedge (\sim q \vee q))$

$\sim p \vee (\sim q \vee p)$

$\sim (p \wedge q) \vee p$

14. Let the system of linear equations

$-x + 2y - 9z = 7$

$-x + 3y + 7z = 9$

$-2x + y + 5z = 8$

$-3x + y + 13z = \lambda$

has a unique solution $x = \alpha$, $y = \beta$, $z = \gamma$. Then the distance of the point (α, β, γ) from the plane $2x - 2y + z = \lambda$ is

(1) 9 (2) 11

(3) 13 (4) 7

Official Ans. by NTA (4)

Sol. $-x + 2y - 9z = 7$ –(1)

$-x + 3y - 7z = 9$ –(2)

$-2x + y + 5z = 8$ –(3)

(2) – (1)

$y + 16z = 2$ (4)

(3) – 2 × (1)

$-3y + 23z = -6$ – (5)

3 × (4) + (5)

$71z = 0 \Rightarrow z = 0$

$y = 2$

$x = -3$

$(-3, 2, 0) \to (\alpha, \beta, \gamma)$

Put in $-3x + y + 13z = \lambda$

$\lambda = 9 + 2 = 11$

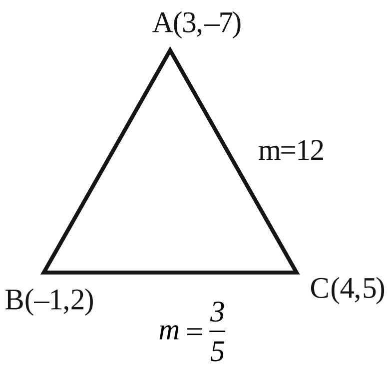
$$d = \left|\frac{-6-4-11}{3}\right| = 7$$

15. If (α, β) is the orthocentre of the triangle ABC with vertices A(3, –7), B(–1, 2) and C(4, 5), then $9\alpha - 6\beta + 60$ is equal to:

(1) 30 (2) 25

(3) 40 (4) 35

Official Ans. by NTA (2)



Sol.

Altitude of BC: $y + 7 = \frac{-5}{3}(x-3)$

$3y + 21 = -5x + 15$

$5x + 3y + 6 = 0$

Altitude of AC: $y - 2 = \frac{-1}{12}(x + 1)$

$12y - 24 = -x - 1$

$x + 12y = 23$

$$\alpha = \frac{-47}{19}, \quad \beta = \frac{121}{57}$$

$9\alpha - 6\beta + 60 = 25$

16. Let the foot of perpendicular of the point P(3, –2,–9) on the plane passing through the points (–1, –2, –3), (9, 3, 4), (9, –2, 1) be Q (α, β, γ). Then the distance of Q from the origin is:

(1) $\sqrt{29}$ (2) $\sqrt{35}$

(3) $\sqrt{42}$ (4) $\sqrt{38}$

Official Ans. by NTA (3)

Sol. P(3, –2, –9)

Equation of plane through A,B,C.

$$\begin{vmatrix} x+1 & y+2 & z+3 \\ 10 & 5 & 7 \\ 10 & 0 & 4 \end{vmatrix} = 0$$

$2x + 3y - 5z - 7 = 0$

Foot of I^r of P (3, –2, –9) is

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -\frac{(\cancel{6} - \cancel{6} + 45 - 7)}{4+9+25}$$

$$\frac{x-3}{2} = \frac{y+2}{3} = \frac{z+9}{-5} = -1$$

$Q(1, -5, -4) \equiv (\alpha, \beta, \gamma)$

$$OQ = \sqrt{\alpha^2 + \beta^2 + \gamma^2} = \sqrt{42}$$

17. A bag contains 6 white and 4 black balls. A die is rolled once and the number of balls equal to the number obtained on the die are drawn from the bag at random. The probability that all the balls drawn are white is:

(1) $\frac{1}{4}$ (2) $\frac{9}{50}$

(3) $\frac{1}{5}$ (4) $\frac{11}{50}$

Official Ans. by NTA (3)

Sol.

6	W
4	R

$$\frac{1}{6} \times \left[\frac{^6C_1}{^{10}C_1} + \frac{^6C_2}{^{10}C_2} + \frac{^6C_3}{^{10}C_3} + \frac{^6C_4}{^{10}C_4} + \frac{^6C_5}{^{10}C_5} + \frac{^6C_6}{^{10}C_6}\right]$$

$$= \frac{1}{6}\left(\frac{126+70+35+15+5+1}{210}\right) = \frac{42}{210} = \frac{1}{5}$$

18. If

$$\int_0^1 \frac{1}{\left(5+2x-2x^2\right)\left(1+e^{(2-4x)}\right)}dx = \frac{1}{\alpha}\log_e\left(\frac{\alpha+1}{\beta}\right),$$

$\alpha, \beta > 0$, then $\alpha^4 - \beta^4$ is equal to:

(1) 21 (2) 0

(3) 19 (4) –21

Official Ans. by NTA (1)

Sol. $I = \int_0^1 \frac{dx}{(5+2x-2x^2)(1+e^{2-4x})}$ …(i)

$x \to 1-x$

$I = \int_0^1 \frac{e^{2-4x}dx}{\left(5+2x-2x^2\right)\left(1+e^{2-4x}\right)}$ …(ii)

Add (i) and (ii)

$$2I = \int_0^1 \frac{dx}{5+2x-2x^2} = \int_0^1 \frac{dx}{2\left(\frac{11}{4}-\left(x-\frac{1}{2}\right)^2\right)}$$

$I = \frac{1}{\sqrt{11}}\ln\left(\frac{\sqrt{11}+1}{\sqrt{10}}\right)$ $\alpha = \sqrt{11}$

$\beta = \sqrt{10}$

$\alpha^4 - \beta^4 = 121 - 100 = 21$

19. Let S be the set of all (λ, μ) for which the vectors $\lambda\hat{i} - \hat{j} + \hat{k}$, $\hat{i} + 2\hat{j} + \mu\hat{k}$ and $3\hat{i} - 4\hat{j} + 5\hat{k}$, where $\lambda - \mu = 5$, are coplanar, then $\sum_{(\lambda,\mu)\in S} 80(\lambda^2 + \mu^2)$ is equal to :

(1) 2370 (2) 2130

(3) 2290 (4) 2210

Official Ans. by NTA (3)

Sol. $\begin{vmatrix} \lambda & -1 & 1 \\ 1 & 2 & \mu \\ 3 & -4 & 5 \end{vmatrix} = 0$ & $\lambda - \mu = 5$

$\lambda(10 + 4\mu) + (5 - 3\mu) + (-10) = 0$

$(\mu + 5)(4\mu + 10) + 5 - 3\mu - 10 = 0$

$\mu = -15;\ \lambda = 5/4$

$\mu = -3;\ \lambda = 2$

Hence $\sum_{(\lambda,\mu)\in S} 80(\lambda^2 + \mu^2)$

$= 80\left(\frac{250}{16} + 13\right)$

$= 1250 + 1040$

$= 2290$

20. If the domain of the function $f(x) = \log_e\left(4x^2 + 11x + 6\right) + \sin^{-1}\left(4x+3\right) + \cos^{-1}\left(\frac{10x+6}{3}\right)$ is $(\alpha, \beta]$,

Then $36\,|\alpha + \beta|$ is equal to :

(1) 63 (2) 45

(3) 72 (4) 54

Official Ans. by NTA (2)

Sol. $f(x) = \ln(4x^2 + 11x + 6) + \sin^{-1}(4x + 3)$

$+\cos^{-1}\left(\frac{10x+6}{3}\right)$

(i) $4x^2 + 11x + 6 > 0$

$4x^2 + 8x + 3x + 6 > 0$

$(4x + 3)(x + 2) > 0$

$x \in (-\infty, -2) \cup \left(-\frac{3}{4}, \infty\right)$

(ii) $4x + 3 \in [-1, 1]$

$x \in [-1, -1/2]$

(iii) $\frac{10x+6}{3} \in [-1, 1]$

$x \in \left[-\frac{9}{10}, -\frac{3}{10}\right]$

$x \in \left(-\frac{3}{4}, -\frac{1}{2}\right]$ $\alpha = -\frac{3}{4}, \beta = -\frac{1}{2}$

$\alpha + \beta = -\frac{5}{4}$

$36\,|\alpha + \beta| = 45$

SECTION-B

21. If the sum of the series

$$\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{2^2}-\frac{1}{2.3}+\frac{1}{3^2}\right)+\left(\frac{1}{2^3}-\frac{1}{2^2.3}+\frac{1}{2.3^2}-\frac{1}{3^3}\right)+\left(\frac{1}{2^4}-\frac{1}{2^3.3}+\frac{1}{2^2.3^2}-\frac{1}{2.3^3}+\frac{1}{3^4}\right)+\ldots$$

is $\frac{\alpha}{\beta}$, where α and β are co-prime, then $\alpha + 3\beta$ is equal to….

Official Ans. by NTA (7)

Sol. $P = \left(\frac{1}{2} - \frac{1}{3}\right) + \left(\frac{1}{2^2} - \frac{1}{2.3} + \frac{1}{3^2}\right) +$

$\left(\frac{1}{2^3} - \frac{1}{2^2.3} + \frac{1}{2.3^2} - \frac{1}{3^3}\right) + ...$

$P\left(\frac{1}{2} + \frac{1}{3}\right) = \left(\frac{1}{2^2} - \frac{1}{3^2}\right) + \left(\frac{1}{2^3} + \frac{1}{3^3}\right) + \left(\frac{1}{2^4} - \frac{1}{3^4}\right) + ...$

$$\frac{5P}{6} = \frac{\frac{1}{4}}{1 - \frac{1}{2}} - \frac{\frac{1}{9}}{1 + \frac{1}{3}}$$

$$\frac{5P}{6} = \frac{1}{2} - \frac{1}{12} = \frac{5}{12}$$

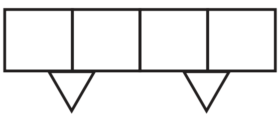
$\therefore P = \frac{1}{2} = \frac{\alpha}{\beta}$ $\therefore \alpha = 1, \beta = 2$

$\alpha + 3\beta = 7$

22. A person forgets his 4-digit ATM pin code. But he remembers that in the code all the digits are different, the greatest digit is 7 and the sum of the first two digits is equal to the sum of the last two digits. Then the maximum number of trials necessary to obtain the correct code is____

Official Ans. by NTA (72)

Sol.



Sum of first two digits

Sum of last two digits = α

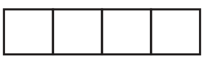
Case-I : $\alpha = 7$

$2 \times 12 = 24$ ways.

7 0 | 0 7 — 1 6, 2 5, 3 4, 4 3, 5 2, 6 1

Case-II : $\alpha = 8$

17, 71 — 26, 62, 35, 53

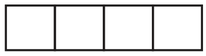


2×8 ways

= 16 ways

Case-III : $\alpha = 9$

27, 72 — 36, 63, 45, 54



2×8 ways

= 16 ways

Case-IV : $\alpha = 10$

37, 73 — 46, 64

2×4 ways

8 ways

Case-V : $\alpha = 11$

47, 74 — 56, 65

2×4 ways

= 8 ways

Ans. 24 + 16 + 16 + 8 + 8 = 72

23. Let the plane P contain the line $2x + y - z - 3 = 0 = 5x - 3y + 4z + 9$ and be parallel to the line $\frac{x+2}{2} = \frac{3-y}{-4} = \frac{z-7}{5}$. Then the distance of the point A(8, –1, –19) from the plane P measured parallel to the line $\frac{x}{-3} = \frac{y-5}{4} = \frac{2-z}{-12}$ is equal to____

Official Ans. by NTA (26)

Sol. Plane $P \equiv P_1 + \lambda P_2 = 0$

$(2x + y - z - 3) + \lambda(5x - 3y) + 4z + 9) = 0$

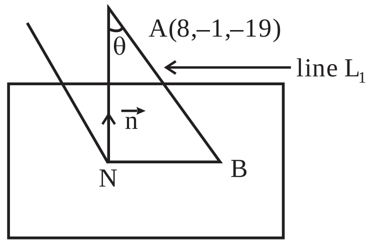
$(5\lambda + 2)x + (1 - 3\lambda)y + (4\lambda - 1)z + 9\lambda - 3 = 0$

$\vec{n}.\vec{b} = 0$ where $\vec{b}$ (2, 4, 5)

$2(5\lambda + 2) + 4(1 - 3\lambda) + 5(4\lambda - 1) = 0$

$\lambda = -\frac{1}{6}$

Plane $7x + 9y - 10z - 27 = 0$



Equation of line AB is

$$\frac{x-8}{-3}=\frac{y+1}{4}=\frac{z+19}{12}=\lambda$$

Let B = (8–3λ,–1+4λ, –19 + 12λ) lies on plane P

∴ 7 (8 – 3λ) + 9 (4λ – 1) – 10 (12λ – 19) = 27

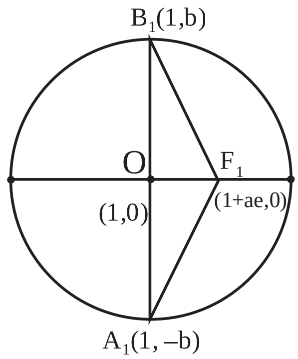
λ = 2

∴ Point B = (2, 7, 5)

$$AB=\sqrt{6^2+8^2+24^2}=26$$

24. Let an ellipse with centre (1, 0) and latus rectum of length $\frac{1}{2}$ have its major axis along x-axis. If its minor axis subtends an angle 60° at the foci, then the square of the sum of the lengths of its minor and major axes is equal to___

Official Ans. by NTA (9)



Sol.

L.R. = $\frac{2b^2}{a}=\frac{1}{2}$

$4b^2 = a$ …(i)

Ellipse $\frac{(x-1)^2}{a^2}+\frac{y^2}{b^2}=1$

$$m_{B_2F_1}=\frac{1}{\sqrt{3}}$$

$$\frac{b}{ae}=\frac{1}{\sqrt{3}}$$

$3b^2 = a^2e^2 = a^2 – b^2$

$4b^2 = a^2$ …(ii)

From (i) and (ii)

$a = a^2$

∴ a = 1

$b^2=\frac{1}{4}$

$((2a) + (2b))^2 = 9$

25. Let A = {1, 2, 3, 4} and R be a relation on the set A × A defined by $R=\{((a,b),(c,d)):2a+3b=4c+5d\}$. Then the number of elements in R is:

Official Ans. by NTA (6)

Sol. A = {1, 2, 3, 4}

R = {(a, b), (c, d)}

2a + 3b = 4c + 5d = α let

2a = {2, 4, 6, 8} 4c = {4, 8, 12, 16}

3b = {3, 6, 9, 12} 5d = {5, 10, 15, 20}

$$2a+3b=\begin{Bmatrix}5,8,11,14\\7,10,13,16\\9,12,15,18\\11,14,17,20\end{Bmatrix}\quad 4c+5d\begin{Bmatrix}9,14,19,24\\13,18....\\17,22....\\21,26....\end{Bmatrix}$$

Possible value of α = 9, 13, 14, 14, 17, 18

Pairs of {(a, b), (c, d)} = 6

26. The number of elements in the set

$\{n\in\mathbb{N}:10\le n\le 100$ and $3^n – 3$ is a multiple of 7}

is____

Official Ans. by NTA (15)

Sol. n∈[10, 100]

3^n – 3 is multiple of 7

$3^n = 7\lambda + 3$

n = 1, 7, 13, 20,……97

Number of possible values of n = 15

27. If the line x = y = z intersects the line x sin A + y sin B + z sin C – 18 = 0 = x sin 2A + y sin 2B + z sin 2C – 9, where A, B, C are the angles of a triangle ABC, then $80\left(\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}\right)$ is equal to_____

Official Ans. by NTA (5)

Sol. $\sin A + \sin B + \sin C = \frac{18}{x}$

$\text{Sin}2A + \sin 2B + \sin 2C = \frac{9}{x}$

$\therefore \sin A + \sin B + \sin C = 2(\sin 2A + \sin 2B + \sin 2C)$

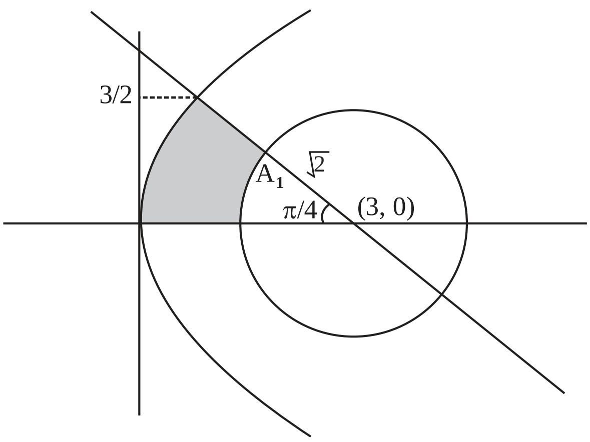
$4\cos A/2 \cos B/2 \cos C/2 = 2(4\sin A \sin B \sin C)$

$16\sin A/2 \sin B/2 \sin C/2 = 1$

Hence Ans. = 5.

28. If the area bounded by the curve $2y^2 = 3x$, lines $x + y = 3$, $y = 0$ and outside the circle $(x-3)^2 + y^2 = 2$ is A, then $4(\pi + 4A)$ is equal to___

Official Ans. by NTA (42)



Sol.

$y^2 = \frac{3x}{2},\ x + y = 3,\ y = 0$

$2y^2 = 3(3 - y)$

$2y^2 + 3y - 9 = 0$

$2y^2 - 3y + 6y - 9 = 0$

$(2y - 3)(y + 2) = 0;\ y = 3/2$

$$\text{Area}\left(\int_0^{\frac{3}{2}} (x_R - x_2)dy\right) - A_1$$

$$= \int_0^{\frac{3}{2}} \left((3 - y) - \frac{2y^2}{3}\right)dy - \frac{\pi}{8}(2)$$

$$A = \left(3y - \frac{y^2}{2} - \frac{2y^3}{9}\right)_0^{\frac{3}{2}} - \frac{\pi}{4}$$

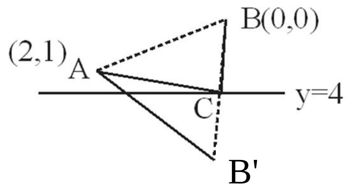
$$4A + \pi = 4\left[\frac{9}{2} - \frac{9}{8} - \frac{3}{4}\right] = \frac{21}{2} = 10.50$$

$\therefore 4(4A + \pi) = 42$

29. Consider the triangles with vertices A(2, 1) B (0, 0) and C (t, 4), $t \in [0, 4]$. It the maximum and the minimum perimeters of such triangles are obtained at $t = \alpha$ and $t = \beta$ respectively, then $6\alpha + 21\beta$ is equal to____

Official Ans. by NTA (48)

Sol. A (2,1), B (0,0), C (t, 4) : $t \in [0,4]$



$B_1(0,8) \equiv$ image of B w.r.t. y = 4

for AC+BC+AB to be minimum.

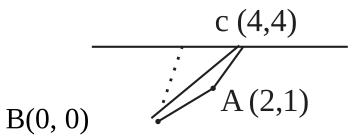
$$m_{AB'} = \frac{-7}{2}$$

line $AB_1 \equiv 7x + 2y = 16$

$$C\left(\frac{8}{7}, 4\right)$$

$$\beta = \frac{8}{7}$$

For max. perimeter



$\alpha = 4$

$AB = \sqrt{5} : BC = 4\sqrt{2},\ AC = \sqrt{13}$

$6\alpha + 21\beta = 24 + 24 = 48$

30. Let $f(x)=\int \frac{dx}{\left(3+4x^2\right)\sqrt{4-3x^2}}, |x|<\frac{2}{\sqrt{3}}$. If f(0) = 0 and $f(1)=\frac{1}{\alpha\beta}\tan^{-1}\left(\frac{\alpha}{\beta}\right)$, α, $\beta > 0$, then $\alpha^2 + \beta^2$ is equal to____

Official Ans. by NTA (28)

Sol. $f(x)=\int \frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

$x=\frac{1}{t}$

$$=\int \frac{\frac{-1}{t^2}dt}{\frac{(3t^2+4)}{t^2}\frac{\sqrt{4t^2-3}}{t}}$$

$$=\int \frac{-dt.t}{(3t^2+4)\sqrt{4t^2-3}} \ : \text{ Put } 4t^2-3=z^2$$

$$=-\frac{1}{4}\int \frac{z\ dz}{\left(3\left(\frac{z^2+3}{4}\right)+4\right)z}$$

$$=\int \frac{-dz}{3z^2+25}=-\frac{1}{3}\int \frac{dz}{z^2+\left(\frac{5}{\sqrt{3}}\right)^2}$$

$$=-\frac{1}{3}\frac{\sqrt{3}}{5}\tan^{-1}\left(\frac{\sqrt{3}z}{5}\right)+C$$

$$=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{4t^2-3}\right)+C$$

$$f(x)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\sqrt{\frac{4-3x^2}{x^2}}\right)+C$$

$$\because f(0)=0 \because c=\frac{\pi}{10\sqrt{3}}$$

$$f(1)=-\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{\sqrt{3}}{5}\right)+\frac{\pi}{10\sqrt{3}}$$

$$f(1)=\frac{1}{5\sqrt{3}}\cot^{-1}\left(\frac{\sqrt{3}}{5}\right)=\frac{1}{5\sqrt{3}}\tan^{-1}\left(\frac{5}{\sqrt{3}}\right)$$

$$\alpha=5:\beta=\sqrt{3} \therefore \alpha^2+\beta^2=28$$

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

31. The electric field due to a short electric dipole at a large distance (r) from center of dipole on the equatorial plane varies with distance as :

(1) r (2) $\frac{1}{r}$

(3) $\frac{1}{r^3}$ (4) $\frac{1}{r^2}$

Official Ans. by NTA (3)

Sol. Electric field due to a dipole at point on its axis

$$E = \frac{2kp}{r^3}$$

32. In a linear simple harmonic motion (SHM)

(A) Restoring force is directly proportional to the displacement.

(B) The acceleration and displacement are opposite in direction.

(C) The velocity is maximum at mean position.

(D) The acceleration is minimum at extreme points.

Choose the correct answer from the options given below :

(1) (A), (B) and (C) only

(2) (C) and (D) only

(3) (A), (B) and (D) only

(4) (A), (C) and (D) only

Official Ans. by NTA (1)

Sol. $F = -kx$ A true

$a = -\omega^2 x$ B true

Velocity is maximum at mean position C true

Acceleration is maximum at extreme points D false

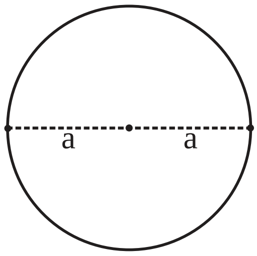
33. Two identical particles each of mass 'm' go round a circle of radius a under the action of their mutual gravitational attraction. The angular speed of each particle will be :

(1) $\sqrt{\frac{Gm}{2a^3}}$ (2) $\sqrt{\frac{Gm}{8a^3}}$

(3) $\sqrt{\frac{Gm}{4a^3}}$ (4) $\sqrt{\frac{Gm}{a^3}}$

Official Ans. by NTA (3)

Sol.



$F = m\omega^2 r$

$$\Rightarrow \frac{Gmm}{(2a)^2} = m\omega^2 a$$

$$\Rightarrow \omega = \sqrt{\frac{Gm}{4a^3}}$$

34. The height of transmitting antenna is 180 m and the height of the receiving antenna is 245 m. The maximum distance between them for satisfactory communication in line of sight will be :

(given R = 6400 km)

(1) 48 km (2) 56 km

(3) 96 km (4) 104 km

Official Ans. by NTA (4)

Sol. $d_{max} = \sqrt{2Rh_t} + \sqrt{2Rh_r}$

$= \sqrt{2\times 64\times 10^5 \times 180} + \sqrt{2\times 64\times 10^5 \times 245}$

$= \{(8 \times 6 \times 10^3) + (8 \times 7 \times 10^3)\}$ m

$= (48 + 56)$ km

$= 104$ km

35. The half-life of a radioactive nucleus is 5 years, The fraction of the original sample that would decay in 15 years is :

(1) $\frac{1}{8}$ (2) $\frac{1}{4}$

(3) $\frac{7}{8}$ (4) $\frac{3}{4}$

Official Ans. by NTA (3)

Sol. 15 year = 3 half lives

Number of active nuclei = $\frac{N_0}{8}$

Number of decay = $\frac{7N_0}{8}$

36. The de Broglie wavelength of an electron having kinetic energy E is λ. If the kinetic energy of electron becomes $\frac{E}{4}$, then its de-Broglie wavelength will be :

(1) $\frac{\lambda}{\sqrt{2}}$ (2) $\frac{\lambda}{2}$

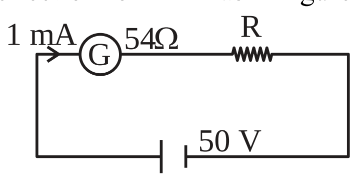
(3) 2λ (4) $\sqrt{2}\lambda$

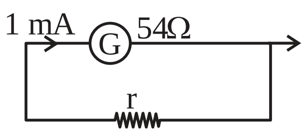
Official Ans. by NTA (3)

Sol. $\lambda = \frac{h}{\sqrt{2mE}}$

$\lambda' = \frac{h}{\sqrt{2m\left(\frac{E}{4}\right)}} = \frac{2h}{\sqrt{2mE}} = 2\lambda$

37. For designing a voltmeter of range 50 V and an ammeter of range 10 mA using a galvanometer which has a coil of resistance 54 Ω showing a full scale deflection for 1 mA as in figure.





(A) for voltmeter R ≈ 50 kΩ

(B) for ammeter r ≈ 0.2 Ω

(C) for ammeter r ≈ 6 Ω

(D) for voltmeter R ≈ 5 kΩ

(E) for voltmeter R ≈ 500 Ω

Choose the correct answer from the options given below :

(1) (C) and (E) (2) (C) and (D)

(3) (A) and (C) (4) (A) and (B)

Official Ans. by NTA (3)

Sol. For voltmeter

$R = \frac{V}{I_g} - G$

$= \frac{50}{10^{-3}} - 54 \approx 50k\Omega$ (A)

For ammeter

$S = \frac{I_g G}{I - I_g} = \frac{10^{-3} \times 54}{(10-1)\times 10^{-3}} = 6\Omega$ (C)

38. (A flask contains Hydrogen and Argon in the ratio 2:1 by mass. The temperature of the mixture is 30°C. The ratio of average kinetic energy per molecule of the two gases (K argon/K hydrogen) is: (Given: Atomic Weight of Ar = 39.9)

(1) 1 (2) 2

(3) $\frac{39.9}{2}$ (4) 39.9

Official Ans. by NTA (1)

Sol. Average KE per molecule = $\frac{3}{2}kT$

$\frac{K_{Ar}}{K_H} = \frac{1}{1}$

39. Given below are two statements:

Statement I : The equivalent resistance of resistors in a series combination is smaller than least resistance used in the combination.

Statement II : The resistivity of the material is independent of temperature.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but Statement II is true

(2) Both Statement I and Statement II are false

(3) Statement I is true but Statement II is false

(4) Both Statement I and Statement II are true

Official Ans. by NTA (2)

Sol. $R_{eq} = R_1 + R_2 + R_3$ So St–1 False

Resistivity depends on temperature. St–2 False

40. A body is released from a height equal to the radius (R) of the earth. The velocity of the body when it strikes the surface of the earth will be :

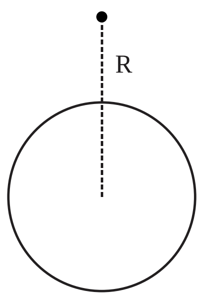
(Given g = acceleration due to gravity on the earth.)

(1) $\sqrt{gR}$ (2) $\sqrt{4gR}$

(3) $\sqrt{2gR}$ (4) $\sqrt{\frac{gR}{2}}$

Official Ans. by NTA (1)

Sol.



By conservation of mechanical energy

$U_i + K_i = U_f + K_i$

$$-\frac{GMm}{2R} + 0 = -\frac{GMm}{R} + \frac{1}{2}mv^2$$

$$\frac{GMm}{2R} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{GM}{R}} = \sqrt{gR}$$

41. A 12 V battery connected to a coil of resistance 6 Ω through a switch, drives a constant current in the circuit. The switch is opened in 1 ms. The emf induced across the coil is 20 V. The inductance of the coil is :

(1) 5 mH (2) 12 mH

(3) 8 mH (4) 10 mH

Official Ans. by NTA (D)

Sol. Induced emf = $-L\frac{dI}{dt}$

$$\Rightarrow 20 = -L\frac{(0-2)}{10^{-3}}$$

$\Rightarrow$ L = 10 mH

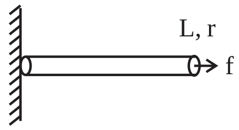
42. A wire of length 'L' and radius 'r' is clamped rigidly at one end. When the other end of the wire is pulled by a force f, its length increases by 'ℓ'. Another wire of same material of length '2L' and radius '2r' is pulled by a force '2f'. Then the increase in its length will be :

(1) 2ℓ (2) ℓ

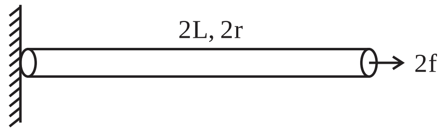
(3) 4ℓ (4) $\ell/2$

Official Ans. by NTA (2)

Sol.



$$\frac{f}{\pi r^2} = Y\frac{\ell}{L}$$



$$\frac{2f}{\pi(2r)^2} = Y\frac{\ell'}{2L}$$

$$\Rightarrow \frac{2}{1} = \frac{2\ell'}{\ell} \Rightarrow \ell' = \ell$$

43. The position of a particle related to time is given by x = $(5t^2 - 4t + 5)$m. The magnitude of velocity of the particle at t = 2s will be :

(1) 10 ms^{-1} (2) 14 ms^{-1}

(3) 16 ms^{-1} (4) 06 ms^{-1}

Official Ans. by NTA (3)

Sol. $x = 5t^2 - 4t + 5$

$\mathbf{v} = 10t - 4$

At t = 2s v = 16m/s

44. The position vector of a particle related to time t is given by

$$\vec{r} = \left(10t\hat{i} + 15t^2\hat{j} + 7\hat{k}\right)m$$

The direction of net force experienced by the particle is :

(1) Positive y-axis (2) Positive x-axis

(3) Positive z-axis (4) In x-y plane

Official Ans. by NTA (1)

Sol. $\vec{r} = 10t\hat{i} + 15t^2\hat{j} + 7\hat{k}$

$\vec{v} = 10\hat{i} + 30t\,\hat{j}$

$\vec{a} = 30\hat{j}$

So Net force is along +y direction

45. Match List I with List II of Electromagnetic waves with corresponding wavelength range :

List I	List II
(A) Microwave	(I) 400 nm to 1 nm
(B) Ultraviolet	(II) 1 nm to 10^{-3} nm
(C) X-Ray	(III) 1 mn to 700 nm
(D) Infra-red	(IV) 0.1 m to 1mm

Choose the correct answer from the options given below :

(1) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

(2) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(3) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

(4) (A)-(IV), (B)-(I), (C)-(III), (D)-(II)

Official Ans. by NTA (2)

Sol. Increasing order of wave length

X-ray	1 nm to 10^{-3} nm
Ultra Violet	400 nm to 1 nm
Intra red	1 mm to 700 nm
Micro wave	0.1 m to 1mm

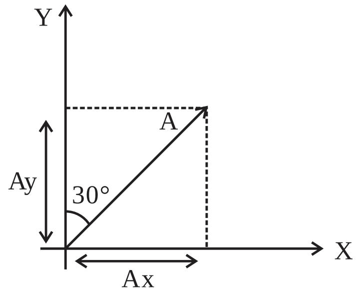
46. A vector in x-y plane makes an angle of 30° with y-axis The magnitude of y-component of vector is $2\sqrt{3}$. The magnitude of x-component of the vector will be :

(1) $\frac{1}{\sqrt{3}}$ (2) 6

(3) $\sqrt{3}$ (4) 2

Official Ans. by NTA (4)

Sol.



$A_y = A\cos 30° = 2\sqrt{3}$

$\Rightarrow A\frac{\sqrt{3}}{2} = 2\sqrt{3}$

$\Rightarrow A = 4$

Now $A_x = A\sin 30° = 4 \times \frac{1}{2} = 2$

47. The speed of a wave produced in water is given by $\upsilon = \lambda^a g^b \rho^c$. Where λ, g and ρ are wavelength of wave, acceleration due to gravity and density of water respectively. The values of a, b and c respectively, are :

(1) $\frac{1}{2}, \frac{1}{2}, 0$ (2) 1, 1, 0

(3) 1, –1, 0 (4) $\frac{1}{2}, 0, \frac{1}{2}$

Official Ans. by NTA (1)

Sol. $\nu = \lambda^a g^b \rho^c$

using dimension formula

$\Rightarrow [M^0L^1T^{-1}] = [L^1]^a [L^1T^{-2}]^b [M^1L^{-3}]^c$

$\Rightarrow [M^0L^1T^{-1}] = [M^c L^{a+b-3c} T^{-2b}]$

$\therefore c = 0, a + b - 3c = 1, -2b = -1 \Rightarrow b = \frac{1}{2}$

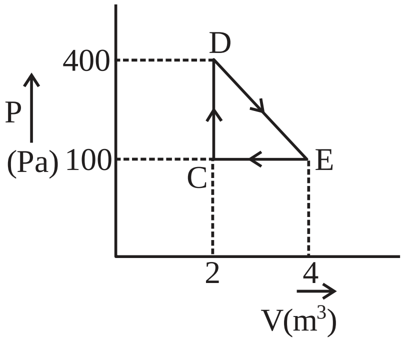
Now a + b – 3c = 1

$\Rightarrow a + \frac{1}{2} - 0 = 1$

$\Rightarrow a = \frac{1}{2}$

$\therefore a = \frac{1}{2}, b = \frac{1}{2}, c = 0$

48. A thermodynamic system is taken through cyclic process. The total work done in the process is :



(1) 100 J (2) 300 J
(3) Zero (4) 200 J

Official Ans. by NTA (2)

Sol. On P–V scale area of loop = work done

$\Rightarrow W = +\frac{1}{2}(2) \times 300$

W = 300J

49. A single slit of width *a* is illuminated by a monochromatic light of wavelength 600 nm. The value of 'a' for which first minimum appears at $\theta = 30°$ on the screen will be :

(1) 0.6 μ m (2) 1.2 μ m
(3) 1.8 μ m (4) 3 μ m

Official Ans. by NTA (2)

Sol. As for first minima

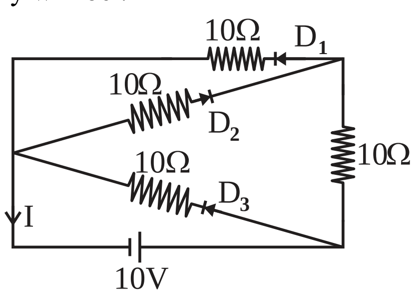
$a \sin\theta = \lambda$

$\Rightarrow a \sin 30° = 600 \times 10^{-9}$

$\Rightarrow a = 1200 \times 10^{-9}$ m

$\Rightarrow a = 1.2$ μm

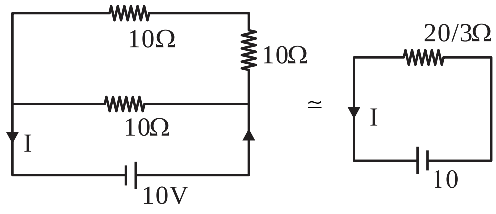
50. In the given circuit, the current (I) through the battery will be :



(1) 1.5 A (2) 1 A
(3) 2.5 A (4) 2 A

Official Ans. by NTA (1)

Sol. In the circuit D_1 and D_3 are forward biased and D_2 is reverse biased.



$$\therefore I = \frac{10}{20/3} = \frac{3}{2} A = 1.5A$$

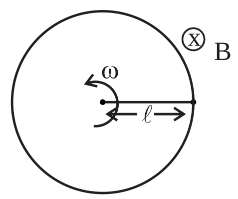
SECTION-B

51. A 20 cm long metallic rod is rotated with 210 rpm about an axis normal to the rod passing through its one end. The order end of the rod is in contact with a circular metallic ring. A constant and uniform magnetic field 0.2T parallel to the axis exists everywhere. The emf developed between the centre and the ring is ________ mV.

Take $\pi = \frac{22}{7}$

Official Ans. by NTA (88)

Sol.



Here ω = 210 rpm

$= 210 \times \frac{2\pi}{60}$ rad/s

$\Rightarrow \omega = 7\pi$ rad/s

& $\ell = 0.2$m

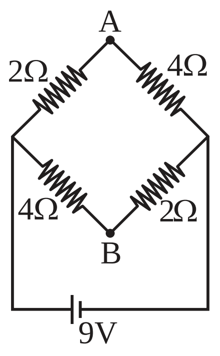
& B = 0.2T

emf developed across rod is = $\frac{1}{2}B\omega\ell^2$

$\frac{1}{2} \times 0.2 \times 7\pi \times (0.2)^2$

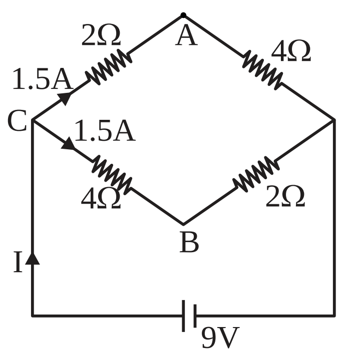
= 88 mV

52. A network of four resistances is connected to 9V battery, as shown in figure. The magnitude of voltage difference between the points A and B is __________V.



Official Ans. by NTA (3)

Sol.



In the circuit $I = \frac{9}{3} = 3A$

$V_C - V_A = 2 \times 1.5 = 3$ ....(I)

$V_C - V_B = 4 \times 1.5 = 6$ ....(I)

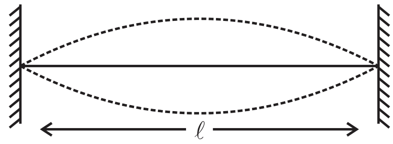
Eq^n (II) – Eq^n (I)

$V_A - V_B = 6 - 3 = 3$ Volt

53. The fundamental frequency of vibration of a string stretched between two rigid support is 50 Hz. The mass of the string is 18 g and its linear mass density is 20 g/m. The speed of the transverse waves so produced in the string is_________ ms^{-1}.

Official Ans. by NTA (90)

Sol.



Fundamental frequency = 50 Hz

mass/length = 20g/m

mass = 18g

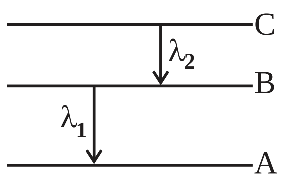
length of string = $\frac{18}{20}$ m = $\frac{9}{10}$ m

from diagram $\frac{\lambda}{2} = \ell$

$\Rightarrow \lambda = 2\ell = \frac{9}{5}$ m

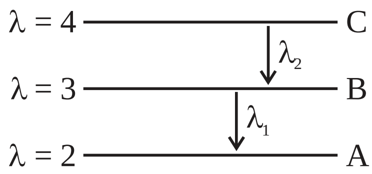
again speed $v = f\lambda = 50 \times \frac{9}{5} = 90$ m/s

54. As per given figure A, B and C are the first, second and third excited energy level of hydrogen atom respectively. If the ratio of the two wavelengths $\left(\text{i.e.} \frac{\lambda_1}{\lambda_2}\right)$ is $\frac{7}{4n}$, then the value of n will be ___________ .



Official Ans. by NTA (5)

Sol.



As $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

$$\frac{1}{\lambda_1} = R(1)^2\left[\frac{1}{(2)^2} - \frac{1}{(3)^2}\right] = R\left(\frac{5}{36}\right) \quad \text{(i)}$$

$$\& \ \frac{1}{\lambda_2} = R(1)^2\left[\frac{1}{(3)^2} - \frac{1}{(4)^2}\right] = R\left(\frac{7}{144}\right) \quad \text{(ii)}$$

(ii) ÷ (i) gives

$$\frac{\lambda_1}{\lambda_2} = \frac{7/144}{5/36} = \frac{7}{20} = \frac{7}{4 \times 5}$$

$\therefore n = 5$

55. A solid sphere and a solid cylinder of same mass and radius are rolling on a horizontal surface without slipping. The ratio of their radius of gyrations respectively (k_{sph} : k_{cyl}) is $2:\sqrt{x}$, then value of x is __________ .

Official Ans. by NTA (5)

Sol. For solid sphere $\frac{2}{5}mR^2 = mk_{sph}^2$

$$k_{sph} = \sqrt{\frac{2}{5}}R$$

For solid cylinder $\frac{mR^2}{2} = mk_{cyl}^2$

$$\Rightarrow k_{cyl} = \frac{R}{\sqrt{2}}$$

$$\frac{k_{sph}}{k_{cyl}} = \frac{\sqrt{\frac{2}{5}}}{\frac{1}{\sqrt{2}}} = \frac{2}{\sqrt{5}} = \frac{2}{\sqrt{x}}$$

$\therefore$ x = 5

56. The refractive index of a transparent liquid filled in an equilateral hollow prism is $\sqrt{2}$. The angle of minimum deviation for the liquid will be __________°.

Official Ans. by NTA (30)

Sol. As $\mu = \frac{\sin\left(\frac{D_{min} + A}{2}\right)}{\sin\left(\frac{A}{2}\right)}$

$$\sqrt{2} = \frac{\frac{\sin(D_{min} + 60)}{2}}{\sin\left(\frac{60}{2}\right)}$$

$$\Rightarrow \frac{1}{\sqrt{2}} = \sin\left(\frac{D_{min} + 60}{2}\right)$$

$$\Rightarrow \frac{D_{min} + 60}{2} = 45$$

$\Rightarrow D_{min} = 30$

57. An electron in a hydrogen atom revolves around its nucleus with a speed of 6.76×10^6 ms^{-1} in an orbit of radius 0.52 A°. The magnetic field produced at the nucleus of the hydrogen atom is __________T.

Official Ans. by NTA (40)

Sol. Magnetic field due to moving charge

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\,v\sin\theta}{r^2}$$

$$B = \frac{\mu_0}{4\pi}\frac{ev\sin(\pi/2)}{r^2}$$

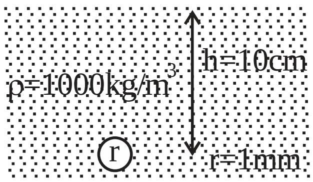
$$B = \frac{10^{-7} \times 1.6 \times 10^{-19} \times 6.76 \times 10^6}{0.52 \times 0.52 \times 10^{-20}}$$

B = 40 T

58. There is an air bubble of radius 1.0 mm in a liquid of surface tension 0.075 Nm^{-1} and density 1000 kg m^{-3} at a depth of 10 cm below the free surface. The amount by which the pressure inside the bubble is greater than the atmospheric pressure is__________ Pa (g = 10 ms^{-2})

Official Ans. by NTA (1150)

Sol.



Pressure inside the bubble

$$P = P_0 + h\rho g + \frac{2T}{r}$$

$$P - P_0 = h\rho g + \frac{2T}{r}$$

$$= 0.1 \times 1000 \times 10 + \frac{2 \times .075}{10^{-3}}$$

= 1000 + (0.15) (1000)

= 1150 Pa

59. A block of mass 10 kg is moving along x-axis under the action of force F = 5x N. The work done by the force in moving the block from x = 2m to 4m will be _________J.

Official Ans. by NTA (30)

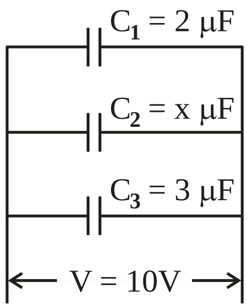
Sol. Work done = $\int F dx$

$$\int_2^4 5x dx = 5\left[\frac{x^2}{2}\right]_2^4$$

$$= \frac{5}{2}[16-4]$$

= 30 J

60. In the given figure the total charge stored in the combination of capacitors is 100 μC. The value of 'x' is____________ .



Official Ans. by NTA (5)

Sol. Charge on C_1 is $Q_1 = 2 \times 10 = 20\mu C$ (i)

Charge on C_2 is $Q_2 = x \times 10 = 10x\mu C$ (ii)

Charge on C_3 is $Q_3 = 3 \times 10 = 30\mu C$ (iii)

Total charge 20 + 10x + 30 = 100

$\Rightarrow x = 5$

SECTION-A

61. Match List I with List II:

List I-(Monomer)	**List II-(Polymer)**
(A) Tetrafluoroethene	(i) Orlon
(B) Acrylonitrile	(ii) Natural rubber
(C) Caprolactam	(iii) Teflon
(D) Isoprene	(IV) Nylon-6

Choose the correct answer from the options given below :

(1) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

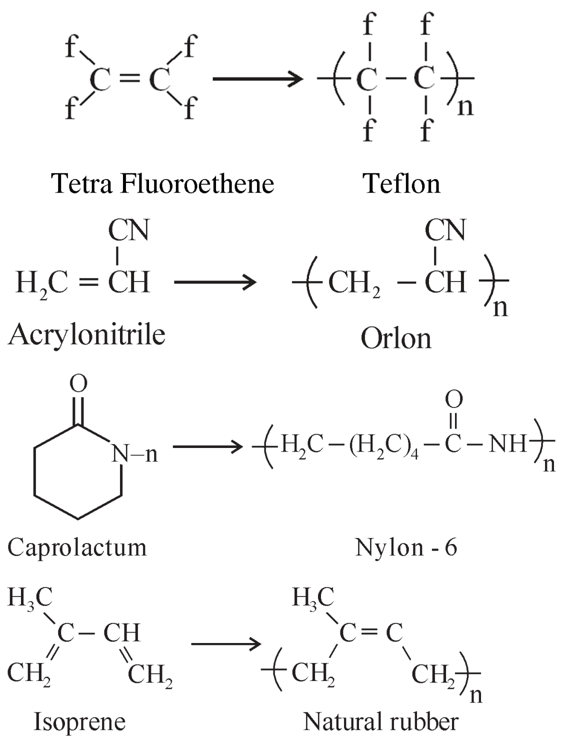
(2) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(3) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(4) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

Official Ans. by NTA (1)

Sol.



62. The product formed in the following multistep reaction is:

$$CH_3-CH=CH_2 \xrightarrow[\text{iii) PCC} \quad \text{iv) } CH_3MgBr]{\text{i) } B_2H_6 \quad \text{ii) } H_2O_2, NaOH}$$

(1) $CH_3-CH_2-CH(OH)-CH_3$

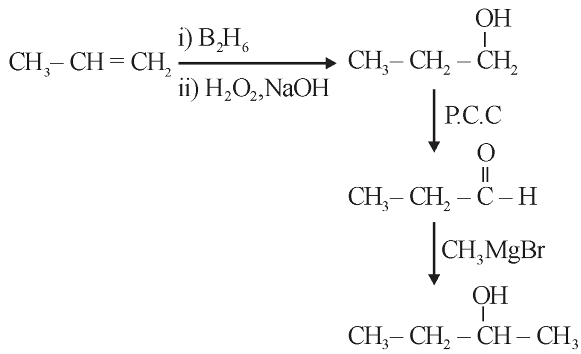
(2) $CH_3-CH_2-CH_2-CH_2-OH$

(3) $CH_3-CH_2-C(=O)-OCH_3$

(4) $CH_3-C(OH)(CH_3)-CH_3$

Official Ans. by NTA (1)

Sol.



63. The possibility of photochemical smog formation will be minimum at

(1) Kolkata in October

(2) Mumbai in May

(3) New-Delhi in August (Summer)

(4) Srinagar, Jammu and Kashmir in January

Official Ans. by NTA (4)

Sol. Photochemical smog occurs in warm, dry and sunny climate.

64. Which one of the following is not an example of calcination ?

(1) $Fe_2O_3 \cdot xH_2O \xrightarrow{\Delta} Fe_2O_3 + xH_2O$

(2) $CaCO_3 \xrightarrow{\Delta} CaO + CO_2$

(3) $CaCO_3 \cdot MgCO_3 \xrightarrow{\Delta} CaO + MgO + 2CO_2$

(4) $2PbS + 3O_2 \xrightarrow{\Delta} 2PbO + 2SO_2$

Official Ans. by NTA (4)

Sol. $2\ PbS + 3\ O_2(g) \xrightarrow{\Delta} 2\ PbO + 2\ SO_2(g)$

It is a roasting reaction.

65. Consider the following statements:

(A) NF_3 molecule has a trigonal planar structure.

(B) Bond length of N_2 is shorter than O_2.

(C) Isoelectronic molecules or ions have identical bond order.

(D) Dipole moment of H_2S is higher than that of water molecule.

Choose the correct answer from the option below:

(1) (A) and (D) are correct

(2) (C) and (D) are correct

(3) (A) and (B) are correct

(4) (B) and (C) are correct

Official Ans. by NTA (4)

Sol. (A) NF_3 has trigonal pyramidal shape.

(B) Bond order $\Rightarrow N_2 > O_2$

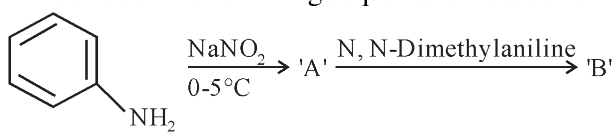
Bond length $\Rightarrow N_2 < O_2$

$\Rightarrow$ (C)

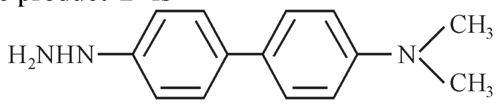
(D) Dipole moment $H_2O > H_2S$

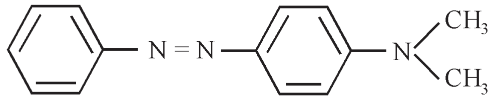
Due to Electronegativity difference.

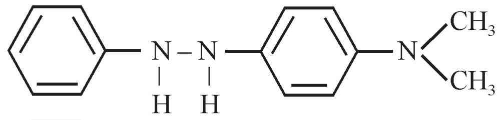
66. Consider the following sequence of reactions:

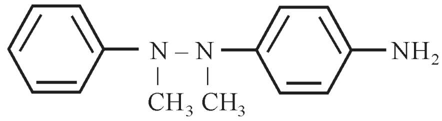


The product 'B' is

(1) 

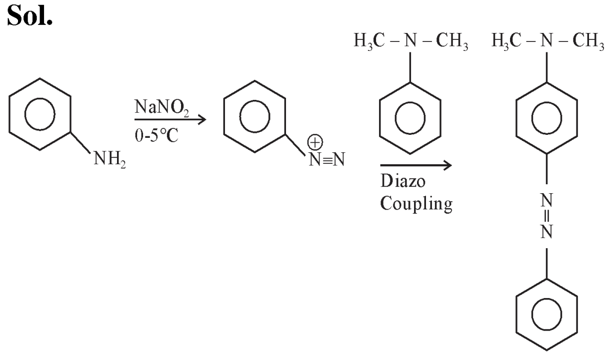
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



67. The number of P – O – P bonds in $H_4P_2O_7$, $(HPO_3)_3$ and P_4O_{10} are respectively.

(1) 1, 3, 6

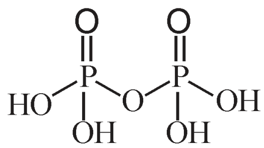
(2) 0, 3, 6

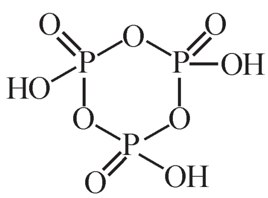
(3) 0, 3, 4

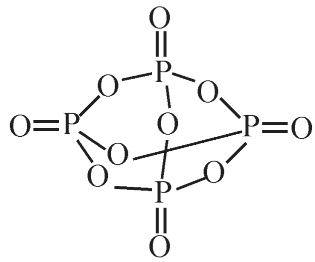
(4) 1, 2, 4

Official Ans. by NTA (1)

Sol.







Molecule	Number of P-O-P Bond
$H_4P_2O_7$	1
$(HPO_3)_3$	3
P_4O_{10}	6

68. Given below are two statements:

Statement I: According to Bohr's model of hydrogen atom, the angular momentum of an electron in a given stationary state is quantised.

Statement II : The concept of electron in Bohr's orbit, violates the Heisenberg uncertainty principle.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both Statement I and Statement II are correct.

(2) Statement I is correct but Statement II is incorrect.

(3) Statement I is incorrect but Statement II is correct

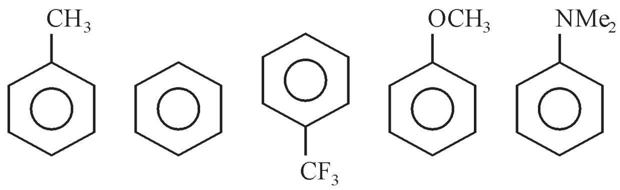
(4) Both Statement I and Statement II are incorrect.

Official Ans. by NTA (1)

Sol. According to Bohr's model the angular momentum is quantised and equal to $\frac{nh}{2\pi}$.

Heisenberg uncertainty principle explains orbital concept, which is based on probability of finding electron.

69. Decreasing order of reactivity towards electrophilie substitution for the following compounds is :



(a) (b) (c) (d) (e)

(1) c > b > a > d > e

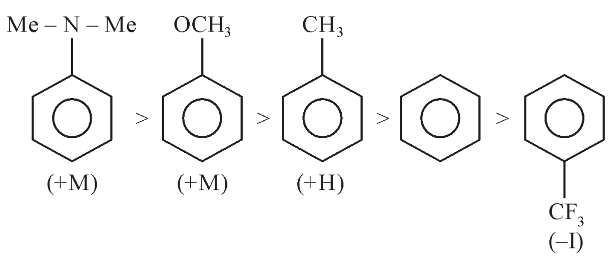
(2) e > d > a > b > c

(3) a > d > e > b > c

(4) d > a > e > c > b

Official Ans. by NTA (2)

Sol. Higher the electron density on Benzene Ring, Higher its Reactivity towards electrophilic substitution Reaction



70. Which of the following statement(s) is/are correct?

(A) The pH of 1×10^{-8} M HCl solution is 8.

(B) The conjugate base $H_2PO_4^-$ is HPO_4^{2-}.

(C) K_w increases with increase in temperature.

(D) When a solution of weak monoprotic acid is titrated against a strong base at half neutralisation point, $pH = \frac{1}{2}pK_a$

Choose the correct answer from the option given below.

(1) (B), (C), (D)

(2) (A), (D)

(3) (A), (B), (C)

(4) (B), (C)

Official Ans. by NTA (4)

Sol. (A) pH of 10^{-8} M HCl is in acidic range (6.98).

(B) Conjugate Base of $H_2PO_4^-$ is HPO_4^{2-}

(C) K_w increases with increasing Temperature, as the temperature increases, the dissociation of water increases.

(D) At half neutralisation point, half of the acid is present in the form of salt.

$$pH = Pk_a + \log\frac{1}{1} = Pk_a$$

71. Given below are two statements : One is labelled as Assertion A and the other is labelled as Reason R :

Assertion (A) : $BeCl_2$ and $MgCl_2$ produce characteristic flame.

Reason (R) : The excitation energy is high in $BeCl_2$ and $MgCl_2$

In the light of the above statements, choose the correct answer from the option given below :

(1) Both (A) and (R) are true but (R) is NOT the correct explanation of (A)

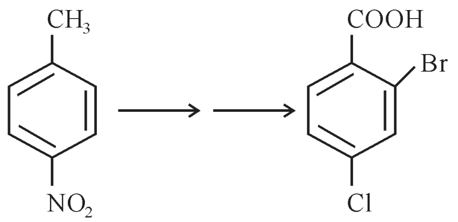
(2) (A) is false but (R) is true

(3) Both (A) and (R) are true and (R) is the correct explanation of (A)

(4) (A) is true but (R) is false.

Official Ans. by NTA (2)

Sol. Be, Mg do not give colour to flame due to high excitation energy.

72. 

In the above conversion the correct sequence of reagents to be added is

(1) (i) Fe/H^+, (ii) HONO, (iii) CuCl, (iv) $KMnO_4$, (v) Br_2

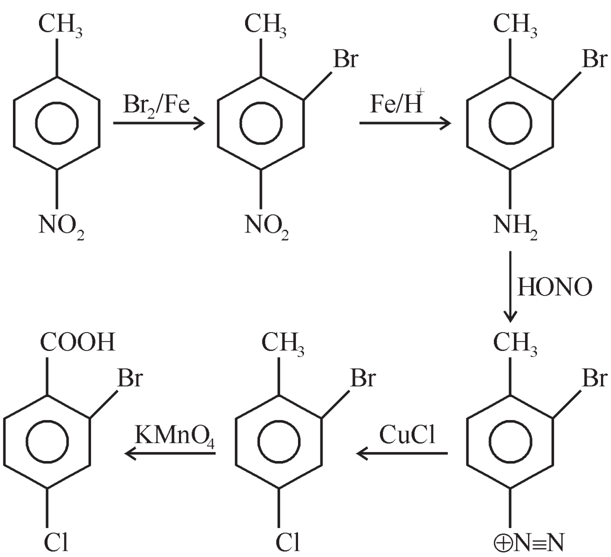
(2) (i) $KMnO_4$, (ii) Br_2/Fe, (iii) Fe/H^+, (iv) Cl_2

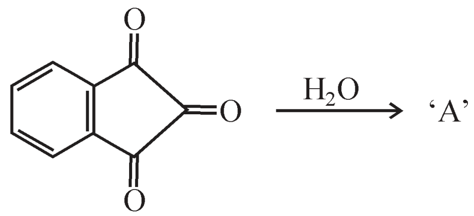
(3) (i) Br_2/Fe, (ii) Fe/H^+, (iii) HONO, (iv) CuCl, (v) $KMnO_4$

(4) Br_2/Fe, (ii) Fe/H^+, (iii) $KMnO_4$, (iv) Cl_2

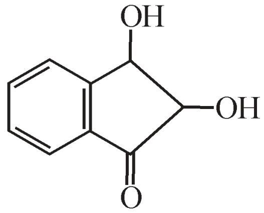
Official Ans. by NTA (3)

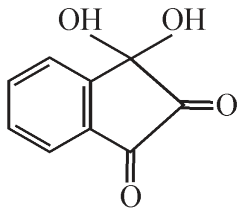
Sol.

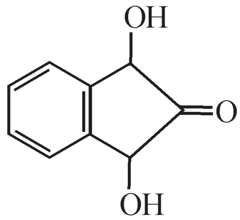


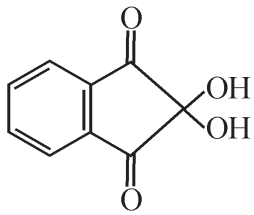
73. 

major product 'A' formed in the above reaction is

(1) 

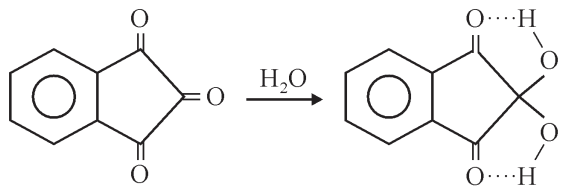
(2) 

(3) 

(4) 

Official Ans. by NTA (4)

Sol.



74. Which is not true for arginine?

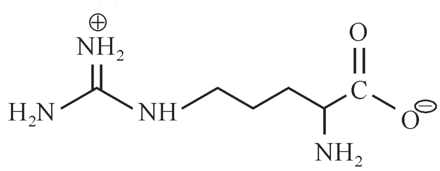
(1) It is a crystalline solid.

(2) It is associated with more than one pK_a values.

(3) It has a fairly high melting point.

(4) It has high solubility in benzene.

Official Ans. by NTA (4)

Sol. 

Arginine exist is zwitterion, so solid in nature and soluble in polar solvent.

75. During water-gas shift reaction

(1) carbon monoxide is oxidized to carbon dioxide.

(2) carbon is oxidized to carbon monoxide.

(3) carbon dioxide is reduced to carbon monoxide.

(4) water is evaporated in presence of catalyst.

Official Ans. by NTA (1)

Sol. Water gas shift reaction

$$\underbrace{CO(g) + H_2}_{\text{water gas}} + \underset{\text{Steam}}{H_2O} \xrightarrow[\text{Catalyst}]{\text{Iron Chromate}} CO_2(g) + 2H_2(g)$$

76. For a good quality cement, the ratio of silica to alumina is found to be

(1) 3

(2) 4.5

(3) 2

(4) 1.5

Official Ans. by NTA (1)

Sol. For good quality cement, the ratio of silica (SlO_2) to Alumina (Al_2O_3) should be between 2.5 to 4.

77. Which of the following statement is correct for paper chromatography ?

(1) Water present in the mobile phase gets absorbed by the paper which then forms the stationary phase.

(2) Water present in the pores of the paper forms the stationary phase.

(3) Paper sheet forms the stationary phase.

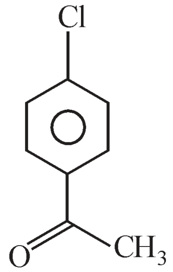
(4) Paper and water present in its pores together form the stationary phase.

Official Ans. by NTA (2)

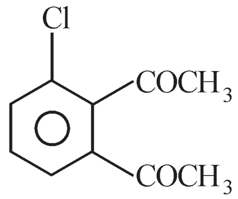
Sol. In paper chromatography, a special quality paper known as chromatography paper is used. Paper contains water trapped in it, which acts as the stationary phase.

78. The major product formed in the Friedel-Craft acylation of chlorobenzene is .

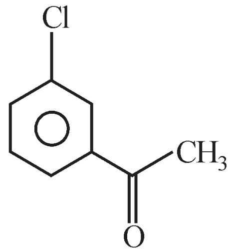
(1)



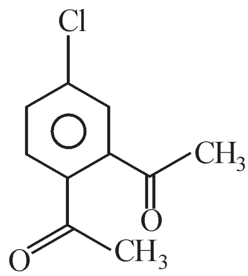
(2)



(3)

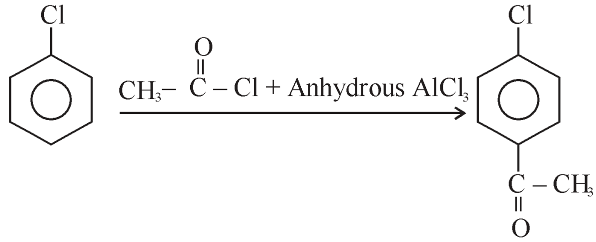


(4)



Official Ans. by NTA (1)

Sol.



Chlorine is ortho/para directing, para is major.

79. The complex with highest magnitude of crystal field splitting energy (Δ_o) is

(1) $[Cr(OH_2)_6]^{3+}$

(2) $[Ti(OH_2)_6]^{3+}$

(3) $[Fe(OH_2)_6]^{3+}$

(4) $[Mn(OH_2)_6]^{3+}$

Official Ans. by NTA (1)

Sol. Ti^{+3} = 67 pm radius

Cr^{3+} = 62 pm radius

Mn^{+3} = 65 pm radius

Fe^{+3} = 65 pm radius

So, Cr^{3+} has highest tendency to attract ligand.

80. Which of the following expressions is correct in case of a CsCl unit cell (edge length'a')?

(1) $r_{Cs^+} + r_{Cl^-} = \frac{a}{\sqrt{2}}$

(2) $r_{Cs^+} + r_{Cl^-} = a$

(3) $r_{Cs^+} + r_{Cl^-} = \frac{\sqrt{3}}{2}a$

(4) $r_{Cs^+} + r_{Cl^-} = \frac{a}{2}$

Official Ans. by NTA (3)

Sol. For CsCl, $Cs^{\oplus}$ is present at Body centre and $Cl^{\ominus}$ at all corner. $\frac{\sqrt{3}a}{2} = r_{cs^{\oplus}} + r_{Cl^{\ominus}}$

SECTION-B

81. The homoleptic and octahedral complex of Co^{2+} and H_2O has_______unpaired electron(s) in the t_{2g} set of orbitals.

Official Ans. by NTA (1)

Sol. Co^{2+} : $3d^7$ configuration

$t_{2g}^{221}\ e_g^{11}$

82. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 20 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$ as silver chloride is______

Official Ans. by NTA (4)

Sol. $[Cr(H_2O)_5Cl]Cl_2 + 2AgNO_3 \rightarrow$

0.01M, 20mL 0.1M

For 0.2 milimole $AgNO_3$ required

= 0.4 milimole

$0.4 = 0.1 \times V(ml)$

V = 4mL

83. The total change in the oxidation state of manganese involved in the reaction of $KMnO_4$ and potassium iodide in the acidic medium is________

Official Ans. by NTA (5)

Sol. $KMnO_4 \rightarrow Mn^{2+}$

↓ ↓

+7 **+2**

Change in oxidation state of Mn= 5

84. In Chromyl chloride, the oxidation state of chromium is (+)_________

Official Ans. by NTA (6)

Sol. CrO_2Cl_2 $x - 4 - 2 = 0$

Oxidation State = +6

85. The total number of isoelectronic species from the given set is _______

O^{2-}, F^-, Al, Mg^{2+}, Na^+, O^+, Mg, Al^{3+}, F

Official Ans. by NTA (5)

Sol. Isoelectronic species $O^{2\ominus}, F^{\ominus}, Mg^{2+}, Na^{\oplus}, Al^{3+}$

86. The vapour pressure of 30% (w/v) aqueous solution of glucose is_______mm Hg at 25°C.

[Given : The density of 30% (w/v), aqueous solution of glucose is 1.2 g cm^{-3} and vapour pressure of pure water is 24 mm Hg.]

(Molar mass of glucose is 180 g mol^{-1}.)

Official Ans. by NTA (23)

Sol. $\frac{24-P_s}{P_s}=\frac{m\times18}{1000}$

wt of solute = 30 gm

Volume of solution = 100 mL

wt. of solution = 1.2 × 100 = 120 gm

wt. of solvent = 120 – 30 = 90 gm

$m=\frac{30\times1000}{180\times90}=1.85$

$\frac{24-P_s}{P_s}=\frac{1.85\times18}{1000}$

$24-P_s = 0.0333\ P_s$

$P_s(1.033) = 24$

$P_s = 23.22$

87. 20 mL of 0.5 M NaCl is required to coagulate 200 mL of As_2S_3 solution in 2 hours. The coagulating value of NaCl is ________

Official Ans. by NTA (50)

Sol. Coagulating value is required milimole of electrolyte needed to coagulate 1 L sol in 2 hours.

Coagulating value $=\frac{20\times0.5}{200}\times1000=50$

88. For a reversible reaction $A \rightleftharpoons B$, the $\Delta H_{\text{forward reaction}}$ $=20$ kJ mol^{-1}. The activation energy of the uncatalysed forward reaction is 300 kJ mol^{-1}. When the reaction is catalysed keeping the reactant concentration same, the rate of the catalysed forward reaction at 27°C is found to be same as that of the uncatalysed reaction at 327°C. The activation energy of the cataysed backward reactoion is _______kJ mol^{-1}.

Official Ans. by NTA (130)

Sol. $E_a = 300$ kJ mol^{-1}

$\frac{E_a}{T}=\frac{E_a'}{T'}$

(Since rate of catalysed and uncatalysed reaction is same)

$\frac{300}{600}=\frac{E_{a,f}'}{300}$

$E_{a,f}'=150$

$20=150-E_{a,b}'$

$E_{a,b}'=130$

89. The number of correct statements from the following is ________

(A) Conductivity always decreases with decrease in concentration for both strong and weak electrolytes.

(B) The number of ions per unit volume that carry current in a solution increases on dilution.

(C) Molar conductivity increases with decrease in concentration.

(D) The variation in molar conductivity is different for strong and weak electrolytes.

(E) For weak electrolytes, the change in molar conductivity with dilution is due to decrease in degree of dissociation.

Official Ans. by NTA (3)

Sol.

(A) Conductivity decreases with dilution for strong electrolyte as well as weak electrolyte.

(B) On dilution, The number of ions per unit volume that carry current in a solution decreases.

(C) Molar conductivity increases with dilution.

(D) Molar conductivity of strong electrolyte follows DHO equation but it is not applicable for weak electrolyte.

(E) On dilution degree of dissociation of weak electrolyte increases.

So answer is (A), (C) & (D).

90. 30.4 kJ of heat is required to melt one mole of sodium chloride and the entropy change at the melting point is 28.4 J K^{-1} mol^{-1} at 1 atm. The melting point of sodium chloride is _______K (Nearest Integer)

Official Ans. by NTA (1070)

Sol. $\Delta S=\frac{\Delta H}{T_{mp}}$

$28.4=\frac{30.4\times1000}{T_{mp}}$

$T_{mp}=1070.422$ K.