PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. One main scale division of a vernier callipers is 'a' cm and n^{th} division of the vernier scale coincide with $(n-1)^{th}$ division of the main scale. The least count of the callipers in mm is :

(1) $\frac{10na}{(n-1)}$ (2) $\frac{10a}{(n-1)}$

(3) $\left(\frac{n-1}{10n}\right)a$ (4) $\frac{10a}{n}$

Official Ans. by NTA (4)

Sol. $(n-1)a = n(a')$

$a' = \frac{(n-1)a}{n}$

$\therefore$ L.C. = 1 MSD – 1 VSD

$= (a - a')$cm

$= a - \frac{(n-1)a}{n}$

$= \frac{na - na + a}{n} = \frac{a}{n}$ cm

$= \left(\frac{10a}{n}\right)$mm

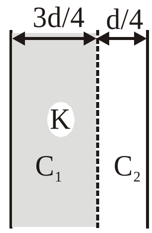
2. For changing the capacitance of a given parallel plate capacitor, a dielectric material of dielectric constant K is used, which has the same area as the plates of the capacitor. The thickness of the dielectric slab is $\frac{3}{4}$d, where 'd' is the separation between the plates of parallel plate capacitor. The new capacitance (C') in terms of original capacitance (C_0) is given by the following relation :

(1) $C' = \frac{3+K}{4K}C_0$ (2) $C' = \frac{4+K}{3}C_0$

(3) $C' = \frac{4K}{K+3}C_0$ (4) $C' = \frac{4}{3+K}C_0$

Official Ans. by NTA (3)

Sol.



$C_0 = \frac{\epsilon_0 A}{d}$

$C' = C_1$ and C_2 in series.

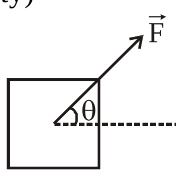
i.e. $\frac{1}{C'} = \frac{1}{C_1} + \frac{1}{C_2}$

$\frac{1}{C'} = \frac{(3d/4)}{\epsilon_0 KA} + \frac{d/4}{\epsilon_0 A}$

$\frac{1}{C'} = \frac{d}{4\epsilon_0 A}\left(\frac{3+K}{K}\right)$

$C' = \frac{4KC_0}{(3+K)}$

3. A block of mass m slides along a floor while a force of magnitude F is applied to it at an angle θ as shown in figure. The coefficient of kinetic friction is μ_K. Then, the block's acceleration 'a' is given by : (g is acceleration due to gravity)



(1) $-\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

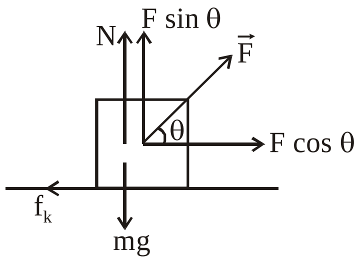
(2) $\frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

(3) $\frac{F}{m}\cos\theta - \mu_K\left(g + \frac{F}{m}\sin\theta\right)$

(4) $\frac{F}{m}\cos\theta + \mu_K\left(g - \frac{F}{m}\sin\theta\right)$

Official Ans. by NTA (2)

Sol.



$N = mg - f \sin\theta$

$F\cos\theta - \mu_k N = ma$

$F\cos\theta - \mu_k(mg - F\sin\theta) = ma$

$$a = \frac{F}{m}\cos\theta - \mu_K\left(g - \frac{F}{m}\sin\theta\right)$$

4. The pressure acting on a submarine is 3×10^5 Pa at a certain depth. If the depth is doubled, the percentage increase in the pressure acting on the submarine would be : (Assume that atmospheric pressure is 1×10^5 Pa density of water is 10^3 kg m^{-3}, g = 10 ms^{-2})

(1) $\frac{200}{3}\%$ (2) $\frac{200}{5}\%$

(3) $\frac{5}{200}\%$ (4) $\frac{3}{200}\%$

Official Ans. by NTA (1)

Sol. $P_1 = \rho g d + P_0 = 3 \times 10^5$ Pa

$\therefore \rho g d = 2 \times 10^5$ Pa

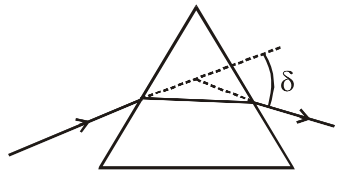
$P_2 = 2\rho g d + P_0$

$= 4 \times 10^5 + 10^5 = 5 \times 10^5$ Pa

$$\%\text{increase} = \frac{P_2 - P_1}{P_1} \times 100$$

$$= \frac{5\times10^5 - 3\times10^5}{3\times10^5}\times100 = \frac{200}{3}\%$$

5. The angle of deviation through a prism is minimum when



(A) Incident ray and emergent ray are symmetric to the prism

(B) The refracted ray inside the prism becomes parallel to its base

(C) Angle of incidence is equal to that of the angle of emergence

(D) When angle of emergence is double the angle of incidence

Choose the correct answer from the options given below :

(1) Statements (A), (B) and (C) are true

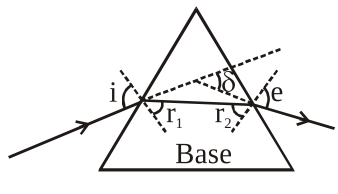
(2) Only statement (D) is true

(3) Only statements (A) and (B) are true

(4) Statements (B) and (C) are true

Official Ans. by NTA (1)

Sol. Deviation is minimum in a prism when :
$i = e$, $r_1 = r_2$ and ray (2) is parallel to base of prism.



6. A plane electromagnetic wave of frequency 500 MHz is travelling in vacuum along y-direction. At a particular point in space and time, $\vec{B} = 8.0 \times 10^{-8}\,\hat{z}T$. The value of electric field at this point is :
(speed of light = 3×10^8 ms^{-1})
$\hat{x}, \hat{y}, \hat{z}$ are unit vectors along x, y and z direction.

(1) $-24\hat{x}$ V / m (2) $2.6\hat{x}$ V / m

(3) $24\hat{x}$ V / m (4) $-2.6\hat{y}$ V / m

Official Ans. by NTA (1)

Sol. $f = 5 \times 10^8$ Hz

EM wave is travelling towards $+\hat{j}$

$\vec{B} = 8.0 \times 10^{-8} \hat{z} T$

$\vec{E} = \vec{B} \times \vec{C}$ = $(8 \times 10^{-8} \hat{z}) \times (3 \times 10^8 \hat{y})$

$= -24\hat{x}$ V / m

7. The maximum and minimum distances of a comet from the Sun are 1.6×10^{12} m and 8.0×10^{10} m respectively. If the speed of the comet at the nearest point is 6×10^4 ms^{-1}, the speed at the farthest point is :

(1) 1.5×10^3 m/s (2) 6.0×10^3 m/s

(3) 3.0×10^3 m/s (4) 4.5×10^3 m/s

Official Ans. by NTA (3)

Sol. By angular momentum conservation :

$mv_1r_1 = mv_2r_2$

$v_1 = \frac{48 \times 10^{14}}{1.6 \times 10^{12}}$ = 3000 m/sec

$= 3 \times 10^3$ m/sec.

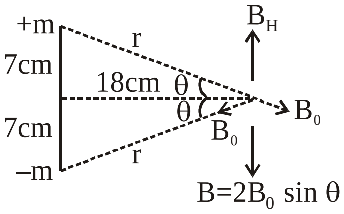
8. A bar magnet of length 14 cm is placed in the magnetic meridian with its north pole pointing towards the geographic north pole. A neutral point is obtained at a distance of 18 cm from the center of the magnet. If $B_H = 0.4$ G, the magnetic moment of the magnet is ($1 \text{ G} = 10^{-4}$T)

(1) 2.880×10^3 J T^{-1} (2) 2.880×10^2 J T^{-1}

(3) 2.880 J T^{-1} (4) 28.80 J T^{-1}

Official Ans. by NTA (3)

Sol.



i.e. $\frac{2\mu_0}{4\pi} \frac{m}{r^2} \times \frac{7}{r}$ = 0.4×10^{-4}

$\Rightarrow 2 \times 10^{-7} \times \frac{m \times 7}{(7^2 + 18^2)^{3/2}} \times 10^4$

$= 0.4 \times 10^{-4}$

$m = \frac{4 \times 10^{-2} \times (373)^{3/2}}{14}$

$M = m \times 14 \text{ cm} = m \times \frac{14}{100}$

$= \frac{0.04 \times (373)^{3/2}}{14} \times \frac{14}{100}$

$= 4 \times 10^{-4} \times 7203.82 = 2.88$ J/T

9. The volume V of an enclosure contains a mixture of three gases, 16 g of oxygen, 28 g of nitrogen and 44 g of carbon dioxide at absolute temperature T. Consider R as universal gas constant. The pressure of the mixture of gases is :

(1) $\frac{88RT}{V}$ (2) $\frac{3RT}{V}$

(3) $\frac{5}{2}\frac{RT}{V}$ (4) $\frac{4RT}{V}$

Official Ans. by NTA (3)

Sol. $PV = (n_1 + n_2 + n_3)RT$

$P \times V = \left[\frac{16}{32} + \frac{28}{28} + \frac{44}{44}\right]RT$

$PV = \left[\frac{1}{2} + 1 + 1\right]RT$

$P = \frac{5}{2}\frac{RT}{V}$

10. In thermodynamics, heat and work are :

(1) Path functions

(2) Intensive thermodynamic state variables

(3) Extensive thermodynamic state variables

(4) Point functions

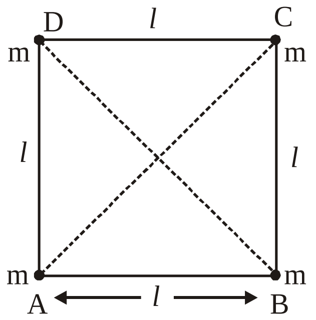
Official Ans. by NTA (1)

Sol. Heat and work are treated as path functions in thermodynamics.

$\Delta Q = \Delta U + \Delta W$

Since work done by gas depends on type of process i.e. path and ΔU depends just on initial and final states, so ΔQ i.e. heat, also has to depend on process is path.

11. Four equal masses, m each are placed at the corners of a square of length (l) as shown in the figure. The moment of inertia of the system about an axis passing through A and parallel to DB would be :



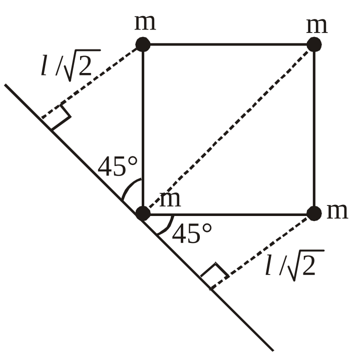
(1) ml^2 (2) $2\ ml^2$

(3) $3\ ml^2$ (4) $\sqrt{3}\,ml^2$

Official Ans. by NTA (3)

Sol. Moment of inertia of point mass

= mass × (Perpendicular distance from axis)^2



Moment of Inertia

$= m(0)^2 + m(l\sqrt{2})^2 + m\left(\frac{l}{\sqrt{2}}\right)^2 + m\left(\frac{l}{\sqrt{2}}\right)^2$

$= 3\ ml^2$

12. A conducting wire of length 'l', area of cross-section A and electric resistivity ρ is connected between the terminals of a battery. A potential difference V is developed between its ends, causing an electric current.

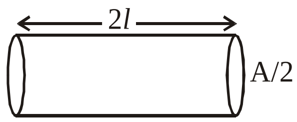
If the length of the wire of the same material is doubled and the area of cross-section is halved, the resultant current would be :

(1) $\frac{1}{4}\frac{VA}{\rho l}$ (2) $\frac{3}{4}\frac{VA}{\rho l}$

(3) $\frac{1}{4}\frac{\rho l}{VA}$ (4) $4\frac{VA}{\rho l}$

Official Ans. by NTA (1)

Sol. As per the question



Resistance $= \frac{\rho(2l)}{(A/2)} = \frac{4\rho l}{A}$

$\Rightarrow$ Current $= \frac{V}{R} = \frac{VA}{4\rho l}$

13. Time period of a simple pendulum is T inside a lift when the lift is stationary. If the lift moves upwards with an acceleration g/2, the time period of pendulum will be :

(1) $\sqrt{3}\,T$

(2) $\frac{T}{\sqrt{3}}$

(3) $\sqrt{\frac{3}{2}}\,T$

(4) $\sqrt{\frac{2}{3}}\,T$

Official Ans. by NTA (4)

Sol. When lift is stationary

$$T = 2\pi\sqrt{\frac{L}{g}}$$

When lift is moving upwards

$\Rightarrow$ Pseudo force acts downwards

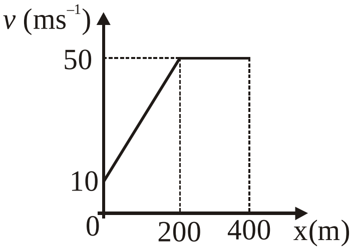
$$\Rightarrow g_{eff} = g + \frac{g}{2} = \frac{3g}{2}$$

$\Rightarrow$ New time period

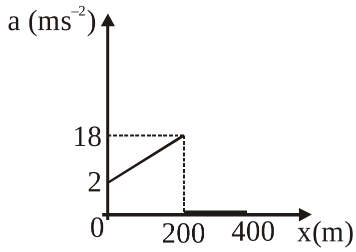
$$T' = 2\pi\sqrt{\frac{L}{g_{eff}}} = 2\pi\sqrt{\frac{2L}{3g}}$$

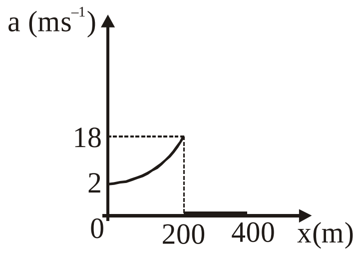
$$\boxed{T' = \sqrt{\frac{2}{3}}\,T}$$

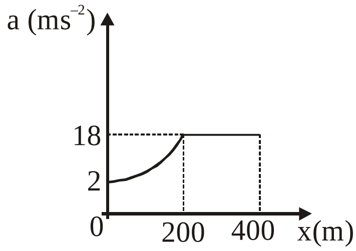
14. The velocity-displacement graph describing the motion of a bicycle is shown in the figure.

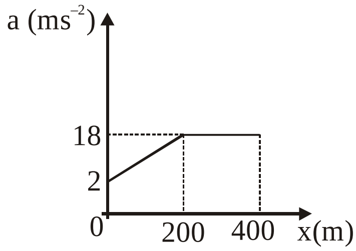


The acceleration-displacement graph of the bicycle's motion is best described by :

(1)


(2)


(3)


(4)


Official Ans. by NTA (1)

Official Ans. by ALLEN (1 or Bonus)

Sol. For $0 \le x \le 200$

$v = mx + C$

$$v = \frac{1}{5}x + 10$$

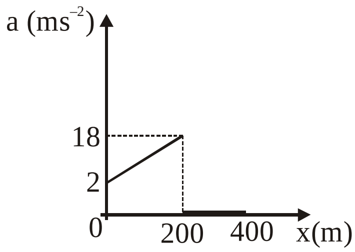
$$a = \frac{vdv}{dx} = \left(\frac{x}{5}+10\right)\left(\frac{1}{5}\right)$$

$$a = \frac{x}{25}+2 \Rightarrow \text{Straight line till } x = 200$$

for $x > 200$

v = constant

$\Rightarrow a = 0$

$\Rightarrow$


Hence most approriate option will be (1), otherwise it would be BONUS.

15. A 25 m long antenna is mounted on an antenna tower. The height of the antenna tower is 75 m. The wavelength (in meter) of the signal transmitted by this antenna would be :

(1) 300 (2) 400

(3) 200 (4) 100

Official Ans. by NTA (4)

Sol. Length of Antena = 25m = $\frac{\lambda}{4}$

$\Rightarrow \boxed{\lambda = 100\,\text{m}}$

16. For an electromagnetic wave travelling in free space, the relation between average energy densities due to electric (U_e) and magnetic (U_m) fields is :

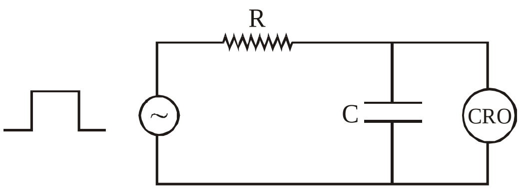
(1) $U_e = U_m$ (2) $U_e > U_m$

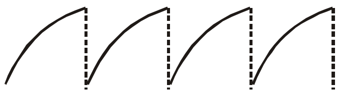
(3) $U_e < U_m$ (4) $U_e \neq U_m$

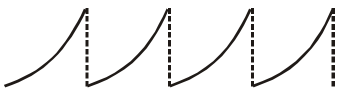
Official Ans. by NTA (1)

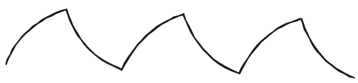
Sol. In EMW, Average energy density due to electric (U_e) and magnetic (U_m) fields is same.

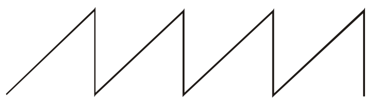
17. An RC circuit as shown in the figure is driven by a AC source generating a square wave. The output wave pattern monitored by CRO would look close to :



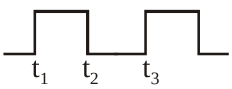
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol. 

For $t_1 - t_2$ Charging graph

$t_2 - t_3$ Discharging graph

18. The stopping potential in the context of photoelectric effect depends on the following property of incident electromagnetic radiation :

(1) Phase (2) Intensity

(3) Amplitude (4) Frequency

Official Ans. by NTA (4)

Sol. Stopping potential changes linearly with frequency of incident radiation.

19. A block of 200 g mass moves with a uniform speed in a horizontal circular groove, with vertical side walls of radius 20 cm. If the block takes 40 s to complete one round, the normal force by the side walls of the groove is :

(1) 0.0314 N

(2) 9.859×10^{-2} N

(3) 6.28×10^{-3} N

(4) 9.859×10^{-4} N

Official Ans. by NTA (4)

Sol. $N = m\omega^2 R$

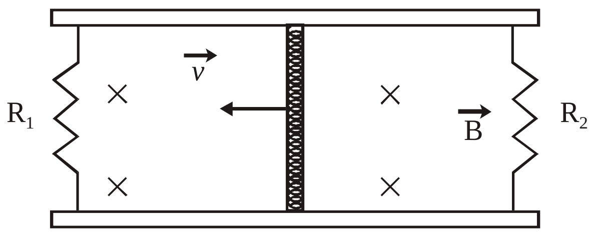
$$N = m\left[\frac{4\pi^2}{T^2}\right]R \qquad .....(1)$$

Given m = 0.2 kg, T = 40 S, R = 0.2 m

Put values in equation (1)

$N = 9.859 \times 10^{-4}$ N

20. A conducting bar of length L is free to slide on two parallel conducting rails as shown in the figure



Two resistors R_1 and R_2 are connected across the ends of the rails. There is a uniform magnetic field $\vec{B}$ pointing into the page. An external agent pulls the bar to the left at a constant speed v.

The correct statement about the directions of induced currents I_1 and I_2 flowing through R_1 and R_2 respectively is :

(1) Both I_1 and I_2 are in anticlockwise direction

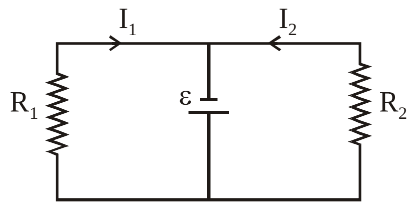
(2) Both I_1 and I_2 are in clockwise direction

(3) I_1 is in clockwise direction and I_2 is in anticlockwise direction

(4) I_1 is in anticlockwise direction and I_2 is in clockwise direction

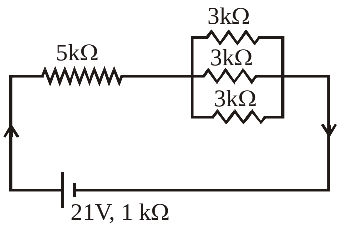
Official Ans. by NTA (3)

Sol.



SECTION-B

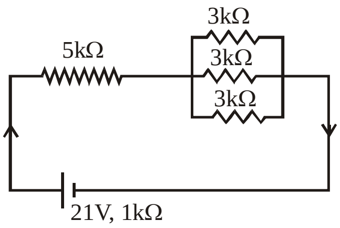
1. In the figure given, the electric current flowing through the 5 kΩ resistor is 'x' mA.



The value of x to the nearest integer is _______.

Official Ans. by NTA (3)

Sol.



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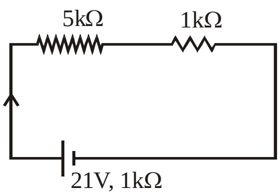


$$I = \frac{21}{5+1+1} = 3 \text{ mA}$$

2. A fringe width of 6 mm was produced for two slits separated by 1 mm apart. The screen is placed 10 m away. The wavelength of light used is 'x' nm. The value of 'x' to the nearest integer is ______.

Official Ans. by NTA (600)

Sol. $\beta = \frac{\lambda D}{d}$

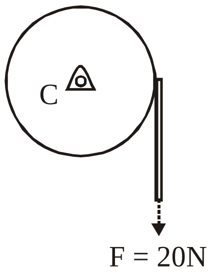
$$\lambda = \frac{\beta d}{D}$$

$$\lambda = \frac{6\times10^{-3}\times10^{-3}}{10}$$

$\lambda = 6 \times 10^{-7}$ m $= 600 \times 10^{-9}$ m

$\lambda = 600$ nm

3. Consider a 20 kg uniform circular disk of radius 0.2 m. It is pin supported at its center and is at rest initially. The disk is acted upon by a constant force F = 20 N through a massless string wrapped around its periphery as shown in the figure.



Suppose the disk makes n number of revolutions to attain an angular speed of 50 rad s^{-1}. The value of n, to the nearest integer, is _______. [Given : In one complete revolution, the disk rotates by 6.28 rad]

Official Ans. by NTA (20)

Sol. $\alpha = \frac{\tau}{I} = \frac{F.R.}{mR^2/2} = \frac{2F}{mR}$

$$\alpha = \frac{2\times200}{20\times(0.2)} = 10 \text{ rad/s}^2$$

$$\omega^2 = \omega_0^2 + 2\alpha\Delta\theta$$

$$(50)^2 = 0^2 + 2(10)\,\Delta\theta \Rightarrow \Delta\theta = \frac{2500}{20}$$

$\Delta\theta = 125$ rad

No. of revolution $= \frac{125}{2\pi} \approx 20$ revolution

4. The first three spectral lines of H-atom in the Balmer series are given λ_1, λ_2, λ_3 considering the Bohr atomic model, the wave lengths of first and third spectral lines $\left(\frac{\lambda_1}{\lambda_3}\right)$ are related by a factor of approximately 'x' × 10^{-1}. The value of x, to the nearest integer, is ________.

Official Ans. by NTA (15)

Sol. For 1st line

$$\frac{1}{\lambda_1} = Rz^2\left(\frac{1}{2^2}-\frac{1}{3^2}\right)$$

$$\frac{1}{\lambda_1} = Rz^2\ \frac{5}{36} \qquad .....(i)$$

For 3rd line

$$\frac{1}{\lambda_3} = Rz^2\left(\frac{1}{2^2}-\frac{1}{5^2}\right)$$

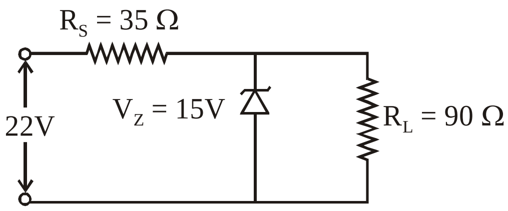
$$\frac{1}{\lambda_3} = Rz^2\ \frac{21}{100} \qquad .....(ii)$$

(ii) + (i)

$$\frac{\lambda_1}{\lambda_3} = \frac{21}{100}\times\frac{36}{5} = 1.512 = 15.12\times 10^{-1}$$

x ≈ 15

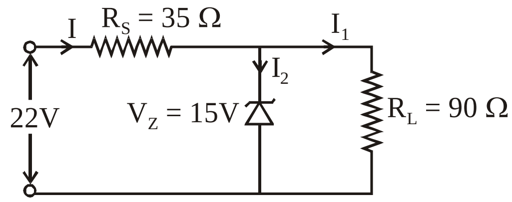
5. The value of power dissipated across the zener diode (V_z = 15 V) connected in the circuit as shown in the figure is x × 10^{-1} watt.



The value of x, to the nearest integer, is ______.

Official Ans. by NTA (5)

Sol.



Voltage across R_S = 22 – 15 = 7V

Current through R_S = I = $\frac{7}{35} = \frac{1}{5}$ A

Current through 90Ω = I_2 = $\frac{15}{90} = \frac{1}{6}$ A

Current through zener = $\frac{1}{5} - \frac{1}{6} = \frac{1}{30}$ A

Power through zener diode

P = VI

$$P = 15\times\frac{1}{30} = 0.5 \text{ watt}$$

P = 5 × 10^{-1} watt

6. A sinusoidal voltage of peak value 250 V is applied to a series LCR circuit, in which R = 8Ω, L = 24 mH and C = 60μF. The value of power dissipated at resonant condition is 'x' kW. The value of x to the nearest integer is ________.

Official Ans. by NTA (4)

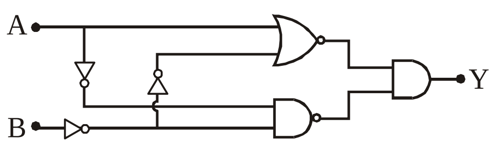
Sol. At resonance power (P)

$$P = \frac{(V_{rms})^2}{R}$$

$$P = \frac{(250/\sqrt{2})^2}{8} = 3906.25 \text{ W}$$

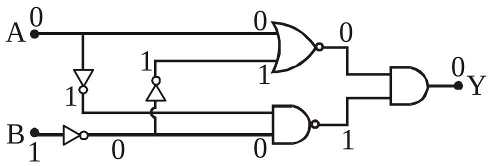
≈ 4 kW

7. In the logic circuit shown in the figure, if input A and B are 0 to 1 respectively, the output at Y would be 'x'. The value of x is _______.



Official Ans. by NTA (0)

Sol.



8. The resistance $R = \frac{V}{I}$, where V = (50 ± 2)V and I = (20 ± 0.2)A. The percentage error in R is 'x' %. The value of 'x' to the nearest integer is _________.

Official Ans. by NTA (5)

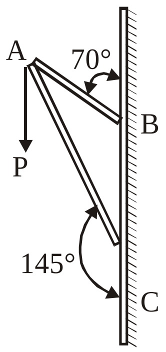
Sol. $\frac{\Delta R}{R} \times 100 = \frac{\Delta V}{V} \times 100 + \frac{\Delta I}{I} \times 100$

% error in R = $\frac{2}{50} \times 100 + \frac{0.2}{20} \times 100$

% error in R = 4 + 1

% error in R = 5%

9. Consider a frame that is made up of two thin massless rods AB and AC as shown in the figure. A vertical force $\vec{P}$ of magnitude 100 N is applied at point A of the frame.



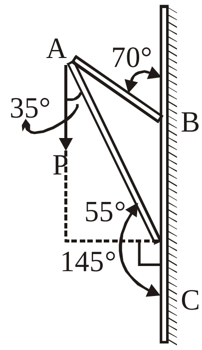
Suppose the force is $\vec{P}$ resolved parallel to the arms AB and AC of the frame. The magnitude of the resolved component along the arm AC is xN. The value of x, to the nearest integer, is ________.

[Given : sin(35°) = 0.573, cos(35°) = 0.819

sin(110°) = 0.939, cos(110°) = –0.342]

Official Ans. by NTA (82)

Sol.



Component along AC

= 100 cos 35°N

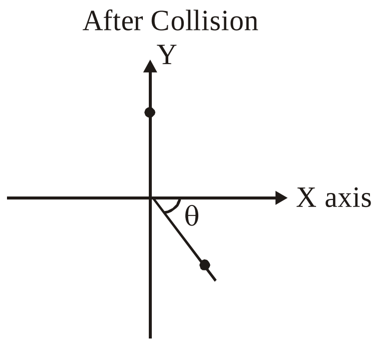
= 100 × 0.819 N

= 81.9 N

≈ 82 N

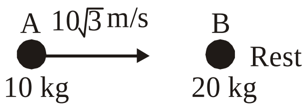
10. A ball of mass 10 kg moving with a velocity $10\sqrt{3}\,ms^{-1}$ along X-axis, hits another ball of mass 20 kg which is at rest. After collision, the first ball comes to rest and the second one disintegrates into two equal pieces. One of the pieces starts moving along Y-axis at a speed of 10 m/s. The second piece starts moving at a speed of 20 m/s at an angle θ (degree) with respect to the X-axis.

The configuration of pieces after collision is shown in the figure. The value of θ to the nearest integer is _______.

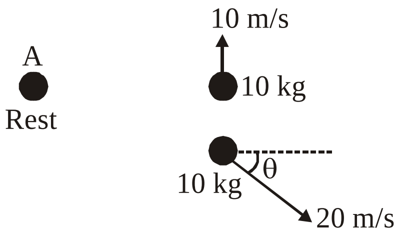


Official Ans. by NTA (30)

Sol. **Before Collision**



After Collision



From conservation of momentum along x axis;

$\vec{P}_i = \vec{P}_f$

$10 \times 10\sqrt{3} = 200 \cos \theta$

$\cos \theta = \dfrac{\sqrt{3}}{2}$

$\theta = 30°$

CHEMISTRY — TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. Given below are two statement : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : Size of Bk^{3+} ion is less than Np^{3+} ion.

Reason R : The above is a consequence of the lanthanoid contraction.

In the light of the above statements, choose the correct answer from the options given below :

(1) A is false but R is true

(2) Both A and R are true but R is not the correct explanation of A

(3) Both A and R are true and R is the correct explanation of A

(4) A is true but R is false

Official Ans. by NTA (3)

Official Ans. by ALLEN (4)

Sol. Size of ${}_{97}Bk^{3+}$ ion is less than that of ${}_{93}Np^{3+}$ due to actinoid contraction.

As we know that in a period from left to right ionic radius decreases and in actinide series it is due to actinoid contraction.

2. Which among the following pairs of Vitamins is stored in our body relatively for longer duration ?

(1) Thiamine and Vitamin A

(2) Vitamin A and Vitamin D

(3) Thiamine and Ascorbic acid

(4) Ascorbic acid and Vitamin D

Official Ans. by NTA (2)

Sol. Vitamin-A & Vitamin-D

3. Given below are two statements :

Statement I : Both $CaCl_2.6H_2O$ and $MgCl_2.8H_2O$ undergo dehydration on heating.

Statement II : BeO is amphoteric whereas the oxides of other elements in the same group are acidic.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is false but statement II is true

(2) Both statement I and statement II are false

(3) Both statement I and statement II are true

(4) Statement I is true but statement II is false

Official Ans. by NTA (2)

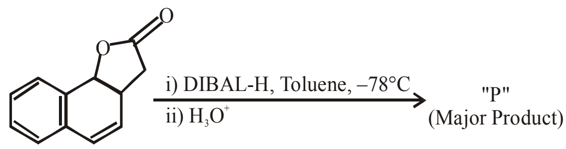
Sol. (a) $CaCl_2.6H_2O \xrightarrow{\Delta} \underset{\text{(Anhydrous)}}{CaCl_2} + 6H_2O$

(b) $MgCl_2.8H_2O \xrightarrow{\Delta} MgO + 2HCl + 6H_2O$

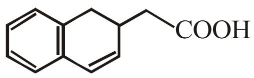
The dehydration of hydrated chloride of calcium can be achieved. The corresponding hydrated chloride of magnesium on heating suffer hydrolysis.

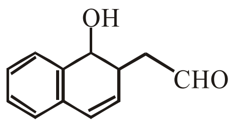
(c) BeO $\rightarrow$ Amphoteric

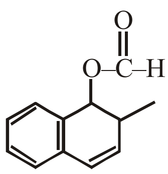
$\left.\begin{matrix} MgO \\ CaO \\ SrO \\ BaO \end{matrix}\right] \Rightarrow$ All are basic oxide

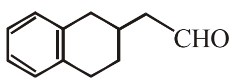
4. 

The product "P" in the above reaction is :

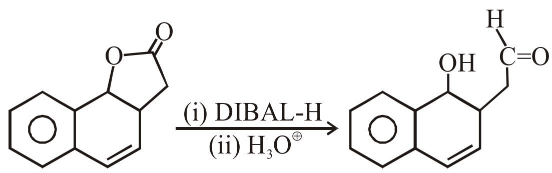
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. 

DIBAL can not reduce double bond

It can reduce cyclic ester.

5. Match List-I with List-II :

List-I Industrial process	List-II Application
(a) Haber's process	(i) HNO_3 synthesis
(b) Ostwald's process	(ii) Aluminium extraction
(c) Contact process	(iii) NH_3 synthesis
(d) Hall-Heroult process	(iv) H_2SO_4 synthesis

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(iii), (c)-(iv), (d)-(i)

(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)

(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)

(4) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

Official Ans. by NTA (3)

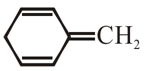
Sol. (a) Haber's process is used for NH_3 synthesis.

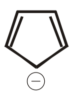
(b) Ostwald's process is used for HNO_3 synthesis.

(c) Contact process is used for H_2SO_4 synthesis.

(d) In Hall-Heroult process, electrolytic reduction of impure alumina can be done. (Aluminium extraction)

6. Among the following, the aromatic compounds are :

(A) 
(B) 

(C) 
(D) 

Choose the correct answer from the following options :

(1) (A) and (B) only

(2) (B) and (C) only

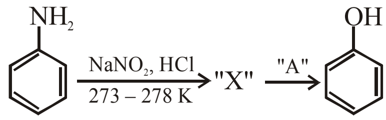
(3) (B), (C) and (D) only

(4) (A), (B) and (C) only

Official Ans. by NTA (2)

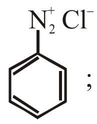
Sol. (A) Non-Aromatic (B) Aromatic

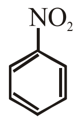
(C) Aromatic (D) Anti-Aromatic

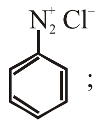
7. 

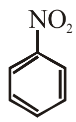
Major Product

In the above chemical reaction, intermediate "X" and reagent/condition "A" are :

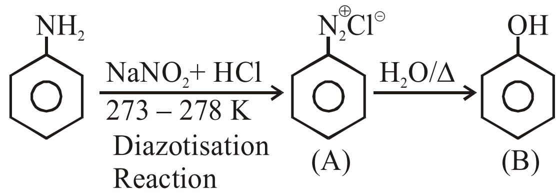
(1) X-  ; A- $H_2O/NaOH$

(2) X-  ; A- H_2O/Δ

(3) X-  ; A- H_2O/Δ

(4) X-  ; A- $H_2O/NaOH$

Official Ans. by NTA (3)

Sol. 

8. Given below are two statements :

Statement I : The E° value of Ce^{4+} / Ce^{3+} is + 1.74 V.

Statement II : Ce is more stable in Ce^{4+} state than Ce^{3+} state.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both statement I and statement II are correct

(2) Statement I is incorrect but statement II is correct

(3) Both statement I and statement II are incorrect

(4) Statement I is correct but statement II is incorrect

Official Ans. by NTA (4)

Sol. The E° value for Ce^{4+}/Ce^{3+} is +1.74 V because the most stable oxidation state of lanthanide series elements is +3.
It means Ce^{3+} is more stable than Ce^{4+}.

9. The functions of antihistamine are :

(1) Antiallergic and Analgesic

(2) Antacid and antiallergic

(3) Analgesic and antacid

(4) Antiallergic and antidepressant

Official Ans. by NTA (2)

10. Which of the following is Lindlar catalyst ?

(1) Zinc chloride and HCl

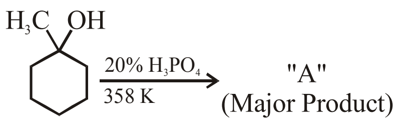
(2) Cold dilute solution of $KMnO_4$

(3) Sodium and Liquid NH_3

(4) Partially deactivated palladised charcoal

Official Ans. by NTA (4)

Sol. Partially deactivated palladised charcoal (H_2/pd/$CaCO_3$) is lindlar catalyst.

11. 

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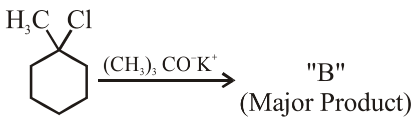
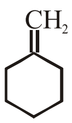
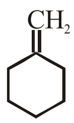
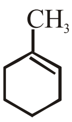
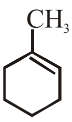
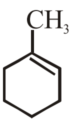
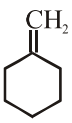


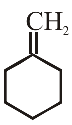
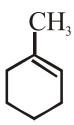
,

The product "A" and "B" formed in above reactions are :

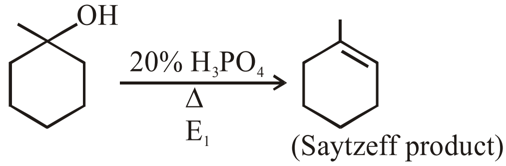
(1) A-  B- 

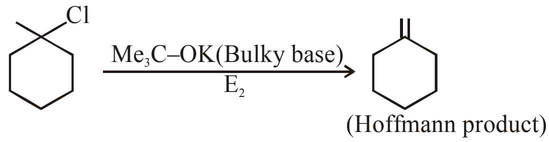
(2) A-  B- 

(3) A-  B- 

(4) A-  B- 

Official Ans. by NTA (3)

Sol. 



12. Given below are two statements :
Statement I : H_2O_2 can act as both oxidising and reducing agent in basic medium.
Statement II : In the hydrogen economy, the energy is transmitted in the form of dihydrogen.
In the light of the above statements, choose the correct answer from the options given below :
(1) Both statement I and statement II are false
(2) Both statement I and statement II are true
(3) Statement I is true but statement II is false
(4) Statement I is false but statement II is true
Official Ans. by NTA (2)

Sol. (a) H_2O_2 can acts as both oxidising and reducing agent in basic medium.
(i) $2Fe^{2+} + H_2O_2 \rightarrow 2Fe^{3+} + 2OH^-$
In this reaction, H_2O_2 acts as oxiding agent.
(ii) $2\overset{+7}{M}nO_4^- + 3H_2O_2 \rightarrow 2\overset{+4}{M}nO_2 + 3O_2 + 2H_2O + 2OH^-$
In this reaction, H_2O_2 acts as reducing agent.
(b) The basic principle of hydrogen economy is the transportation and storage of energy in the form of liquids or gaseous dihydrogen.
Advantage of hydrogen economy is that energy is transmitted in the form of dihydrogen and not as electric power

13. The type of pollution that gets increased during the day time and in the presence of O_3 is :
(1) Reducing smog (2) Oxidising smog
(3) Global warming (4) Acid rain
Official Ans. by NTA (2)

Sol. In presence of ozone(O_3), oxidising smog gets increased during the day time because automobiles and factories produce main components of the photochemcial smog (oxidising smog) results from the action of sunlight on unsaturated hydrocarbon and nitrogen oxide.
Ozone is strong oxidising agent and can react with the unburnt hydrocarbons in the polluted air to produce chemicals.

14. Assertion A : Enol form of acetone [CH_3COCH_3] exists in < 0.1% quantity. However, the enol form of acetyl acetone [$CH_3COCH_2OCCH_3$] exists in approximately 15% quantity.

Reason R : enol form of acetyl acetone is stabilized by intramolecular hydrogen bonding, which is not possible in enol form of acetone.

Choose the correct statement :

(1) A is false but R is true

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is not the correct explanation of A

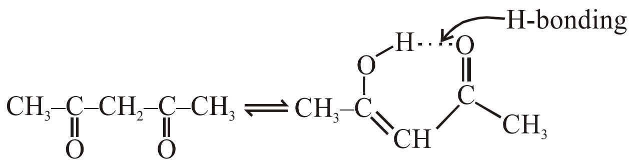
(4) A is true but R is false

Official Ans. by NTA (2)

Sol.

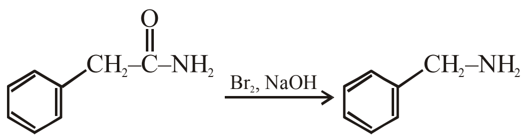
$$\underset{\text{(Keto form)}}{CH_3-\underset{\overset{\|}{O}}{C}-CH_3} \rightleftharpoons \underset{\text{(enol form)}}{CH_2=\underset{\overset{|}{OH}}{C}-CH_3}$$

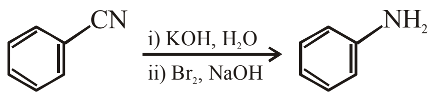
enol from of acetone is very less (< 0.1 %)

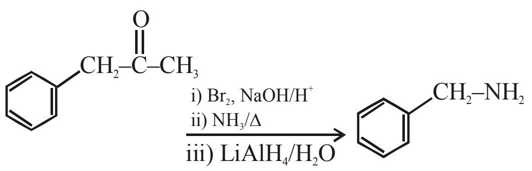


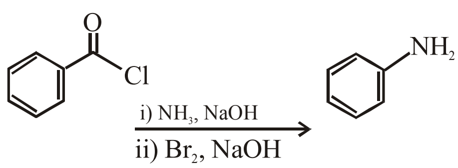
enol from (more than 50%)

15. Which of the following reaction DOES NOT involve Hoffmann Bromamide degradation ?

(1) 

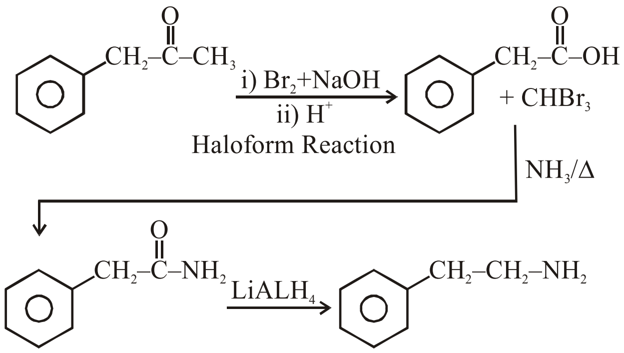
(2) 

(3) 

(4) 

Official Ans. by NTA (3)

Sol.



⇒ This reaction does not involve haffmann bromanide degradation.

⇒ Rest all options involve haffmann bromamide degradation during the reaction of Br_2+NaOH with amide.

16. The process that involves the removal of sulphur from the ores is :

(1) Smelting

(2) Roasting

(3) Leaching

(4) Refining

Official Ans. by NTA (2)

Sol. In roasting process, metal sulphide (MS) ore are converted into metal oxide and sulphur is remove in the form of SO_2 gas.

$$2MS + 3O_2 \xrightarrow{\Delta} 2MO + 2SO_2\uparrow$$

17. Match List-I with List-II :

	List-I	**List-II**
	Name of oxo acid	Oxidation state of 'P'
(a)	Hypophosphorous acid	(i) +5
(b)	Orthophosphoric acid	(ii) +4
(c)	Hypophosphoric acid	(iii) +3
(d)	Orthophosphorous acid	(iv) +2
		(v) +1

Choose the correct answer from the options given below :

(1) (a)-(v), (b)-(i), (c)-(ii), (d)-(iii)

(2) (a)-(iv), (b)-(i), (c)-(ii), (d)-(iii)

(3) (a)-(iv), (b)-(v), (c)-(ii), (d)-(iii)

(4) (a)-(v), (b)-(iv), (c)-(ii), (d)-(iii)

Official Ans. by NTA (1)

Sol. (a) Hypophosphorus acid : $H_3\underline{P}O_2$

$(+1)\ 3 + x + (-2)2 = 0$

$x = +1$

(b) Orthophosphoric acid : $H_3\underline{P}O_4$

$(+1)\ 3 + x + (-2)4 = 0$

$x = +5$

(c) Hypophosphoric acid : $H_4P_2O_6$

$(+1)\ 4 + 2x + (-2)6 = 0$

$x = +4$

(d) Orthophosphorous acid : $H_3\underline{P}O_3$

$(+1)3 + x + (-2)3 = 0$

$x = +3$

18. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R :

Assertion A : The H–O–H bond angle in water molecule is 104.5°.

Reason R : The lone pair – lone pair repulsion of electrons is higher than the bond pair - bond pair repulsion.

(1) A is false but R is true

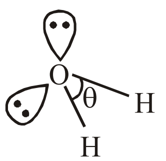
(2) Both A and R are true, but R is not the correct correct explanation of A

(3) A is true but R is false

(4) Both A and R are true, and R is the correct explanation of A

Official Ans. by NTA (4)

Sol. H_2O



$\theta = 104.5°$

the hybridisation of oxygen is water molecule is sp^3.

So electron geometry of water molecule is tetrahedral and the bond angle should be 109°28" but as we know that lone pair-lone pair repulsion of electrons is higher than the bond pair-bond pair repulsion because lone pair is occupied more space areound central atom than that of bond pair.

19. In chromotography technique, the purification of compound is independent of :

(1) Mobility or flow of solvent system

(2) Solubility of the compound

(3) Length of the column or TLC Plate

(4) Physical state of the pure compound

Official Ans. by NTA (4)

Sol. In chromotography technique, the purification of a compound is independent of the physical state of the pure compound.

20. A group 15 element, which is a metal and forms a hydride with strongest reducing power among group 15 hydrides. The element is :

(1) Sb (2) P (3) As (4) Bi

Official Ans. by NTA (4)

Sol. In group 15

$\left.\begin{matrix} N \\ P \end{matrix}\right] \rightarrow$ Non metal

$\left.\begin{matrix} As \\ Sb \end{matrix}\right] \rightarrow$ Metalloid

$\left.Bi\ \right] \rightarrow$ Metal

Hydrides of group 15 elements are

NH_3

PH_3

AsH_3

SbH_3

BiH_3

In NH_3, hydrogen atom gets partial positive charge due to less electronegativity.

But in BiH_3, hydrogen atom gets partial negative charge because hydrogen is more electronegative than bismuth.

i.e. BiH_3 is a strong reducing agent than others because we know that H^- is a strong reducing agent.

SECTION-B

1. For the reaction $A(g) \rightleftharpoons B(g)$ at 495 K, $\Delta_rG^o = -9.478\ kJ\ mol^{-1}$.

If we start the reaction in a closed container at 495 K with 22 millimoles of A, the amount of B is the equilibrium mixture is _______ millimoles. (Round off to the Nearest Integer).

[$R = 8.314\ J\ mol^{-1}\ K^{-1}$; $\ell n\ 10 = 2.303$]

Official Ans. by NTA (20)

Sol. $\Delta G^\circ = -RT\ \ell n\ K_{eq}$

Given $\Delta G^\circ = -9.478$ KJ/mole

$T = 495K \quad R = 8.314\ J\ mol^{-1}$

So $-9.478 \times 10^3 = -495 \times 8.314 \times \ell n\ K_{eq}$

$\ell n\ K_{eq} = 2.303$

$= \ell n\ 10$

So $K_{eq} = 10$

Now $A(g) \rightleftharpoons B(g)$

$t = 0 \quad 22 \quad 0$

$t = t \quad 22-x \quad x$

$$K_{eq} = \frac{[B]}{[C]} = \frac{x}{22-x} = 10$$

or $x = 20$

So millmoles of B = 20

2. Complete combustion of 750 g of an organic compound provides 420 g of CO_2 and 210 g of H_2O. The percentage composition of carbon and hydrogen in organic compound is 15.3 and ________ respectively. (Round off to the Nearest Integer)

Official Ans. by NTA (3)

Sol. 44 gm CO_2 have 12 gm carbon

So, 420 gm $CO_2 \Rightarrow \frac{12}{44} \times 420$

$\Rightarrow \frac{1260}{11}$ gm carbon

$\Rightarrow$ 114.545 gram carbon

So, % of carbon $= \frac{114.545}{750} \times 100$

$\simeq 15.3\%$

18 gm $H_2O \Rightarrow$ 2 gm H_2

210 gm $\Rightarrow \frac{2}{18} \times 210$

$= 23.33$ gm H_2

So, % $H_2 \Rightarrow \frac{23.33}{750} \times 100 = 3.11\%$

$\approx 3\%$

3. $2\,MnO_4^- + b\ C_2O_4^{2-} + c\ H^+ \rightarrow x\ Mn^{2+} + y\ CO_2 + z\ H_2O$

If the above equation is balanced with integer coefficients, the value of c is _______. (Round off to the Nearest Integer).

Official Ans. by NTA (16)

Sol. Writting the half reaction

oxidation half reaction

$MnO_4^- \rightarrow Mn^{2+}$

balancing oxygen

$MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing Hydrogen

$8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

balancing charge

$5e^- + 8H^+ + MnO_4^- \rightarrow Mn^{2+} + 4H_2O$

Reduction half

$C_2O_4^{2-} \rightarrow CO_2$

Balancing carbon

$C_2O_4^{2-} \rightarrow 2CO_2$

Balancing charge

$C_2O_4^{2-} \rightarrow 2CO_2 + 2e^-$

Net equation

$$16H^+ + 2MnO_4^- + 5C_2O_4^{2-} \rightarrow 10CO_2 + 2Mn^{2+} + 8H_2O$$

So c = 16

4. AB_2 is 10% dissociated in water to A^{2+} and B^-. The boiling point of a 10.0 molal aqueous solution of AB_2 is _______ ^oC. (Round off to the Nearest Integer).

[Given : Molal elevation constant of water $K_b = 0.5$ K kg mol^{-1} boiling point of pure water $= 100^oC$]

Official Ans. by NTA (106)

Sol. $AB_2 \rightarrow A^{2+} + 2B^-$

$t = 0 \quad a \quad 0 \quad 0$

$t = t \quad a-a\alpha \quad a\alpha \quad 2a\alpha$

$n_T = a - a\alpha + a\alpha + 2a\alpha$

$= a(1 + 2\alpha)$

so $i = 1 + 2\alpha$

Now $\Delta T_b = i \times m \times K_b$

$\Delta T_b = (1 + 2\alpha) \times m \times K_b$

$\alpha = 0.1 \quad m = 10 \quad K_b = 0.5$

$\Delta T_b = 1.2 \times 10 \times 0.5$

$= 6$

So boiling point = 106

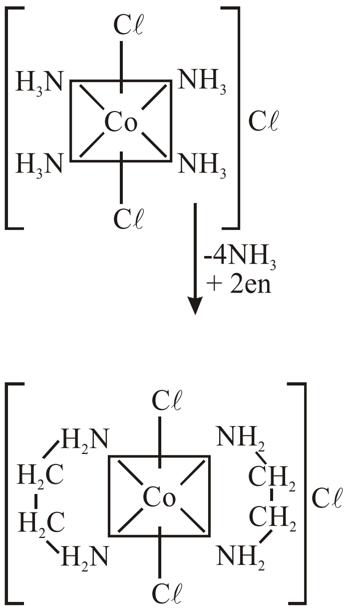
5. The equivalents of ethylene diamine required to replace the neutral ligands from the coordination sphere of the trans-complex of $CoCl_3.4NH_3$ is _______. (Round off to the Nearest Integer).

Official Ans. by NTA (2)

Sol. trans - $CoCl_3.4NH_3$

or

trans-$[Co(NH_3)_4Cl_2]C\ell$



As we know that ethylene diamine is a bidentate ligand and ammonia is a mono dentate ligand.

It means overall two ethylene diamine is required to replace the all neutral ligands (four ammonia) from the coordination sphere of this complex.

6. A 6.50 molal solution of KOH (aq.) has a density of 1.89 g cm^{-3}. The molarity of the solution is ________ mol dm^{-3}. (Round off to the Nearest Integer).

[Atomic masses: K :39.0 u; O :16.0 u; H :1.0 u]

Official Ans. by NTA (9)

Sol. 6.5 molal KOH = 1000gm solvent has 6.5 moles KOH

so wt of solute = 6.5 × 56

= 364 gm

wt of solution = 1000 + 364 = 1364

Volume of solution = $\frac{1364}{1.89}m\ell$

$$\text{Molarity} = \frac{\text{mole of solute}}{V_{solution}\text{ in Litre}}$$

$$= \frac{6.5 \times 1.89 \times 1000}{1364}$$

= 9.00

7. When light of wavelength 248 nm falls on a metal of threshold energy 3.0 eV, the de-Broglie wavelength of emitted electrons is _______ Å. (Round off to the Nearest Integer).

[Use : $\sqrt{3}$ = 1.73, h = 6.63 × 10^{-34} Js

m_e = 9.1 × 10^{-31} kg ; c = 3.0 × 10^8 ms^{-1} ; 1eV = 1.6 × 10^{-19}J]

Official Ans. by NTA (9)

Sol. Energy incident = $\frac{hc}{\lambda}$

$$= \frac{6.63 \times 10^{-34} \times 3.0 \times 10^8}{248 \times 10^{-9} \times 1.6 \times 10^{-19}} \text{ eV}$$

$$= \frac{6.63 \times 3 \times 100}{248 \times 1.6}$$

= 0.05 eV × 100 = 5 eV

Now using

E = ϕ + K.E.

5 = 3 + K.E.

K.E. = 2eV = 3.2 × 10^{-19} J

for debroglie wavelength $\lambda = \frac{h}{mv}$

$$K.E = \frac{1}{2}mv^2$$

so $v = \sqrt{\frac{2KE}{m}}$

hence $\lambda = \frac{h}{\sqrt{2KE \times m}}$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 3.2 \times 10^{-19} \times 9.1 \times 10^{-31}}}$$

$$= \frac{6.63}{7.6} \times \frac{10^{-34}}{10^{-25}} = \frac{66.3 \times 10^{-10}\text{ m}}{7.6}$$

= 8.72 × 10^{-10} m

$\approx 9 \times 10^{-10}$ m

$= 9$Å

8. Two salts A_2X and MX have the same value of solubility product of 4.0×10^{-12}. The ratio of their molar solubilities i.e. $\frac{S(A_2X)}{S(MX)} =$ _______. (Round off to the Nearest Integer).

Official Ans. by NTA (50)

Sol. For A_2X

$A_2X \rightarrow 2A^+ + X^{2-}$

$\quad 2S_1 \quad S_1$

$K_{sp} = 4S_1^3 = 4 \times 10^{-12}$

$S_1 = 10^{-4}$

for MX

$MX \rightarrow M^+ + X^-$

$\quad S_2 \quad S_2$

$K_{sp} = S_2^2 = 4 \times 10^{-12}$

$S_2 = 2 \times 10^{-6}$

so $\frac{S_{A_2X}}{S_{MX}} = \frac{10^{-4}}{2 \times 10^{-6}} = 50$

9. A certain element crystallises in a bcc lattice of unit cell edge length 27 Å. If the same element under the same conditions crystallises in the fcc lattice, the edge length of the unit cell in Å will be ________. (Round off to the Nearest Integer).

[Assume each lattice point has a single atom]

[Assume $\sqrt{3} = 1.73$, $\sqrt{2} = 1.41$]

Official Ans. by NTA (33)

Sol. For BCC $\sqrt{3}\,a = 4r$

so $\quad r = \frac{\sqrt{3}}{4} \times 27$

for FCC $\quad a = 2\sqrt{2}\,r$

$= 2 \times \sqrt{2} \times \frac{\sqrt{3}}{4} \times 27$

$= \frac{\sqrt{3}}{\sqrt{2}} \times 27$

$= 33$

10. The decomposition of formic acid on gold surface follows first order kinetics. If the rate constant at 300 K is $1.0 \times 10^{-3}\ s^{-1}$ and the activation energy $E_a = 11.488$ kJ mol^{-1}, the rate constant at 200 K is _______ $\times 10^{-5}\ s^{-1}$. (Round of to the Nearest Integer).

(Given : R = 8.314 J mol^{-1} K^{-1})

Official Ans. by NTA (10)

Sol. $K_{300} = 10^{-4} \qquad K_{200} = ?$

$E_a = 11.488$ KJ/mole $\quad$ R = 8.314 J/mole-K

so $\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{E_a}{R}\left(\frac{1}{200} - \frac{1}{300}\right)$

$\ell n\left(\frac{K_{300}}{K_{200}}\right) = \frac{11.488 \times 1000 \times 100}{8.314 \times 200 \times 300}$

$= 2.303$

$= \ell n 10$

so $\frac{K_{300}}{K_{200}} = 10$

$K_{200} = \frac{1}{10} \times K_{300} = 10^{-4}$

$= 10 \times 10^{-5}\ sec^{-1}$

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The number of elements in the set $\{x \in \mathbb{R} : (|x| - 3)\,|x + 4| = 6\}$ is equal to

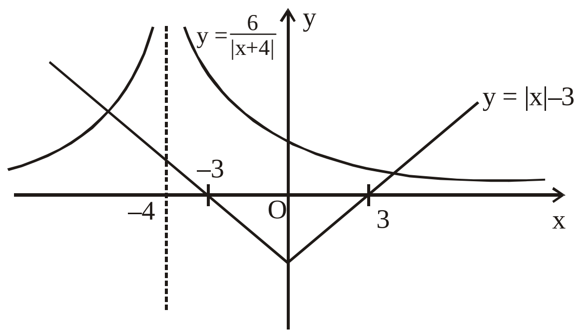
(1) 3 (2) 2 (3) 4 (4) 1

Official Ans. by NTA (2)

Sol. $x \neq -4$

$(|x| - 3)(|x + 4|) = 6$

$\Rightarrow |x| - 3 = \dfrac{6}{|x+4|}$



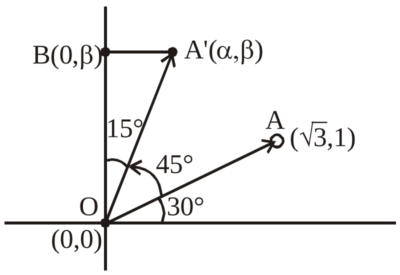
No. of solutions = 2

2. Let a vector $\alpha\hat{i} + \beta\hat{j}$ be obtained by rotating the vector $\sqrt{3}\hat{i} + \hat{j}$ by an angle 45° about the origin in counterclockwise direction in the first quadrant. Then the area of triangle having vertices (α, β), $(0, \beta)$ and $(0, 0)$ is equal to

(1) $\frac{1}{2}$ (2) 1 (3) $\frac{1}{\sqrt{2}}$ (4) $2\sqrt{2}$

Official Ans. by NTA (1)

Sol.



Area of $\Delta(OA'B) = \frac{1}{2} OA'\cos 15° \times OA'\sin 15°$

$= \frac{1}{2}(OA')^2 \frac{\sin 30°}{2}$

$= (3+1) \times \frac{1}{8} = \frac{1}{2}$

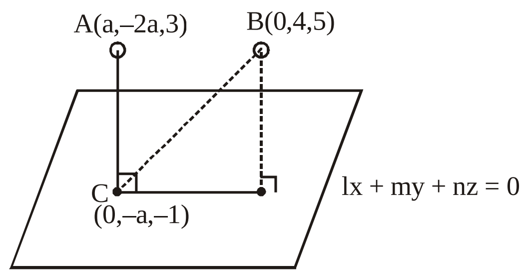
3. If for $a > 0$, the feet of perpendiculars from the points $A(a, -2a, 3)$ and $B(0, 4, 5)$ on the plane $lx + my + nz = 0$ are points $C(0, -a, -1)$ and D respectively, then the length of line segment CD is equal to :

(1) $\sqrt{31}$ (2) $\sqrt{41}$

(3) $\sqrt{55}$ (4) $\sqrt{66}$

Official Ans. by NTA (4)

Sol.



C lies on plane $\Rightarrow -ma - n = 0 \Rightarrow \frac{m}{n} = -\frac{1}{a}$ ....(1)

$\overrightarrow{CA} \parallel l\hat{i} + m\hat{j} + n\hat{k}$

$\frac{a-0}{l} = \frac{-a}{m} = \frac{4}{n} \Rightarrow \frac{m}{n} = -\frac{a}{4}$ ....(2)

From (1) & (2)

$-\frac{1}{a} = \frac{-a}{4} \Rightarrow a^2 = 4 \Rightarrow a = 2$ (since $a > 0$)

From (2) $\frac{m}{n} = \frac{-1}{2}$

Let $m = -t \Rightarrow n = 2t$

$\frac{2}{l} = \frac{-2}{-t} \Rightarrow l = t$

So plane : $t(x - y + 2z) = 0$

$BD = \frac{6}{\sqrt{6}} = \sqrt{6}$ $C \cong (0, -2, -1)$

$CD = \sqrt{BC^2 - BD^2}$

$= \sqrt{(0^2 + 6^2 + 6^2) - (\sqrt{6})^2}$

$= \sqrt{66}$

4. Consider three observations a, b and c such that b = a + c. If the standard deviation of a + 2, b + 2, c + 2 is d, then which of the following is true ?

(1) $b^2 = 3(a^2 + c^2) + 9d^2$

(2) $b^2 = a^2 + c^2 + 3d^2$

(3) $b^2 = 3(a^2 + c^2 + d^2)$

(4) $b^2 = 3(a^2 + c^2) - 9d^2$

Official Ans. by NTA (4)

Sol. For a, b, c

mean $= \dfrac{a+b+c}{3}\ (=\bar{x})$

b = a + c

$\Rightarrow \quad \bar{x} = \dfrac{2b}{3}$ ......(1)

S.D. (a + 2, b + 2, c + 2) = S.D. (a, b, c) = d

$\Rightarrow \quad d^2 = \dfrac{a^2+b^2+c^2}{3} - (\bar{x})^2$

$\Rightarrow \quad d^2 = \dfrac{a^2+b^2+c^2}{3} - \dfrac{4b^2}{9}$

$\Rightarrow \quad 9d^2 = 3(a^2 + b^2 + c^2) - 4b^2$

$\Rightarrow \quad b^2 = 3(a^2 + c^2) - 9d^2$

5. If for $x \in \left(0, \dfrac{\pi}{2}\right)$, $\log_{10}\sin x + \log_{10}\cos x = -1$ and $\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10}n - 1)$, n > 0, then the value of n is equal to :

(1) 20 (2) 12 (3) 9 (4) 16

Official Ans. by NTA (2)

Sol. $x \in \left(0, \dfrac{\pi}{2}\right)$

$\log_{10}\sin x + \log_{10}\cos x = -1$

$\Rightarrow \quad \log_{10}\sin x.\cos x = -1$

$\Rightarrow \quad \sin x.\cos x = \dfrac{1}{10}$ ......(1)

$\log_{10}(\sin x + \cos x) = \dfrac{1}{2}(\log_{10} n - 1)$

$\Rightarrow \quad \sin x + \cos x = 10^{\left(\log_{10}\sqrt{n} - \frac{1}{2}\right)} = \sqrt{\dfrac{n}{10}}$

by squaring

$1 + 2\sin x.\cos x = \dfrac{n}{10}$

$\Rightarrow \quad 1 + \dfrac{1}{5} = \dfrac{n}{10} \quad \Rightarrow \quad n = 12$

6. Let $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$, $i = \sqrt{-1}$. Then, the system of linear equations $A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$ has :

(1) A unique solution

(2) Infinitely many solutions

(3) No solution

(4) Exactly two solutions

Official Ans. by NTA (3)

Sol. $A = \begin{bmatrix} i & -i \\ -i & i \end{bmatrix}$

$A^2 = \begin{bmatrix} -2 & 2 \\ 2 & -2 \end{bmatrix} = 2\begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix}$

$A^4 = 2^2\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 8\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8 = 64\begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} = 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$

$A^8\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad 128\begin{bmatrix} x-y \\ -x+y \end{bmatrix} = \begin{bmatrix} 8 \\ 64 \end{bmatrix}$

$\Rightarrow \quad x - y = \dfrac{1}{16}$ .....(1)

& $\quad -x + y = \dfrac{1}{2}$ .....(2)

$\Rightarrow$ From (1) & (2) : No solution.

7. If the three normals drawn to the parabola, $y^2 = 2x$ pass through the point (a, 0) a ≠ 0, then 'a' must be greater than :

(1) $\dfrac{1}{2}$ (2) $-\dfrac{1}{2}$ (3) –1 (4) 1

Official Ans. by NTA (4)

Sol. For standard parabola

For more than 3 normals (on axis)

$x > \dfrac{L}{2}$ (where L is length of L.R.)

For $y^2 = 2x$

L.R. = 2

for (a, 0)

$$a > \frac{\text{L.R.}}{2} \Rightarrow a > 1$$

8. Let the position vectors of two points P and Q be $3\hat{i} - \hat{j} + 2\hat{k}$ and $\hat{i} + 2\hat{j} - 4\hat{k}$, respectively. Let R and S be two points such that the direction ratios of lines PR and QS are (4, –1, 2) and (–2, 1, –2), respectively. Let lines PR and QS intersect at T. If the vector $\overrightarrow{TA}$ is perpendicular to both $\overrightarrow{PR}$ and $\overrightarrow{QS}$ and the length of vector $\overrightarrow{TA}$ is $\sqrt{5}$ units, then the modulus of a position vector of A is :

(1) $\sqrt{482}$ (2) $\sqrt{171}$

(3) $\sqrt{5}$ (4) $\sqrt{227}$

Official Ans. by NTA (2)

Sol. P(3, –1, 2)

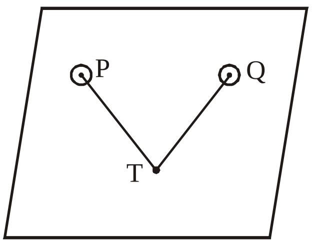
Q(1, 2, –4)

$\overrightarrow{PR} \parallel 4\hat{i} - \hat{j} + 2\hat{k}$

$\overrightarrow{QS} \parallel -2\hat{i} + \hat{j} - 2\hat{k}$

dr's of normal to the plane containing P, T & Q will be proportional to :

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -1 & 2 \\ -2 & 1 & -2 \end{vmatrix}$$



$$\therefore \quad \frac{\ell}{0} = \frac{m}{4} = \frac{n}{2}$$

For point, T : $\overrightarrow{PT} = \dfrac{x-3}{4} = \dfrac{y+1}{-1} = \dfrac{z-2}{2} = \lambda$

$$\overrightarrow{QT} = \frac{x-1}{-2} = \frac{y-1}{1} = \frac{z+4}{-2} = \mu$$

T : $(4\lambda + 3, -\lambda - 1, 2\lambda + 2)$

$$\cong (2\mu + 1, \mu + 2, -2\mu - 4)$$

$4\lambda + 3 = -2\mu + 1 \quad \Rightarrow \quad 2\lambda + \mu = -1$

$\lambda + \mu = -3 \quad \Rightarrow \quad \lambda = 2$

& $\mu = -5 \qquad \lambda + \mu = -3 \quad \Rightarrow \quad \lambda = 2$

So point T : (11, –3, 6)

$$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(\frac{2\hat{j} + \hat{k}}{\sqrt{5}}\right)\sqrt{5}$$

$$\overrightarrow{OA} = \left(11\hat{i} - 3\hat{j} + 6\hat{k}\right) \pm \left(2\hat{j} + \hat{k}\right)$$

$$\overrightarrow{OA} = 11\hat{i} - \hat{j} + 7\hat{k}$$

or

$$9\hat{i} - 5\hat{j} + 5\hat{k}$$

$$|\overrightarrow{OA}| = \sqrt{121 + 1 + 49} = \sqrt{171}$$

or

$$\sqrt{81 + 25 + 25} = \sqrt{131}$$

9. Let the functions $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} x + 2, & x < 0 \\ x^2, & x \geq 0 \end{cases} \text{ and } g(x) = \begin{cases} x^3, & x < 1 \\ 3x - 2, & x \geq 1 \end{cases}$$

Then, the number of points in $\mathbb{R}$ where $(f\text{og})(x)$ is NOT differentiable is equal to :

(1) 3 (2) 1

(3) 0 (4) 2

Official Ans. by NTA (2)

Sol. $$f(g(x)) = \begin{cases} g(x) + 2, & g(x) < 0 \\ (g(x))^2, & g(x) \geq 0 \end{cases}$$

$$= \begin{cases} x^3 + 2, & x < 0 \\ x^6, & x \in [0, 1) \\ (3x - 2)^2, & x \in [1, \infty) \end{cases}$$

$$(f\text{og}(x))' = \begin{cases} 3x^2, & x < 0 \\ 6x^5, & x \in (0, 1) \\ 2(3x - 2) \times 3, & x \in (1, \infty) \end{cases}$$

At 'O'

L.H.L. $\neq$ R.H.L. (Discontinuous)

At '1'

L.H.D. = 6 = R.H.D.

$\Rightarrow \quad f\text{og}(x)$ is differentiable for $x \in \mathbb{R} - \{0\}$

10. Which of the following Boolean expression is a tautology ?

(1) $(p \wedge q) \vee (p \vee q)$

(2) $(p \wedge q) \vee (p \to q)$

(3) $(p \wedge q) \wedge (p \to q)$

(4) $(p \wedge q) \to (p \to q)$

Official Ans. by NTA (4)

Sol.

p	q	$p\wedge q$	$p\to q$	$(p\wedge q)\to(p\to q)$
T	T	T	T	T
T	F	F	F	T
F	T	F	T	T
F	F	F	T	T

$(p \wedge q) \to (p \to q)$ is tautology

11. Let a complex number z, $|z| \neq 1$,

satisfy $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$. Then, the largest value of $|z|$ is equal to _____ .

(1) 8 (2) 7 (3) 6 (4) 5

Official Ans. by NTA (2)

Sol. $\log_{\frac{1}{\sqrt{2}}}\left(\frac{|z|+11}{(|z|-1)^2}\right) \leq 2$

$\frac{|z|+11}{(|z|-1)^2} \geq \frac{1}{2}$

$2|z| + 22 \geq (|z| - 1)^2$

$2|z| + 22 \geq |z|^2 + 1 - 2|z|$

$|z|^2 - 4|z| - 21 \leq 0$

$\Rightarrow$ $|z| \leq 7$

$\therefore$ Largest value of $|z|$ is 7

12. If n is the number of irrational terms in the expansion of $(3^{1/4} + 5^{1/8})^{60}$, then$(n - 1)$ is divisible by :

(1) 26 (2) 30 (3) 8 (4) 7

Official Ans. by NTA (1)

Sol. $(3^{1/4} + 5^{1/8})^{60}$

${}^{60}C_r(3^{1/4})^{60-r}.(5^{1/8})^r$

${}^{60}C_r(3)^{\frac{60-r}{4}}.5^{\frac{r}{8}}$

For rational terms.

$\frac{r}{8} = k$; $0 \leq r \leq 60$

$0 \leq 8k \leq 60$

$0 \leq k \leq \frac{60}{8}$

$0 \leq k \leq 7.5$

$k = 0, 1, 2, 3, 4, 5, 6, 7$

$\frac{60-8k}{4}$ is always divisible by 4 for all value of k.

Total rational terms = 8

Total terms = 61

irrational terms = 53

$n - 1 = 53 - 1 = 52$

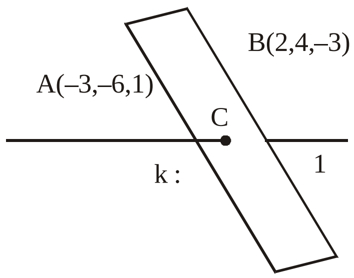
52 is divisible by 26.

13. Let P be a plane $lx + my + nz = 0$ containing the line, $\frac{1-x}{1} = \frac{y+4}{2} = \frac{z+2}{3}$. If plane P divides the line segment AB joining points $A(-3, -6, 1)$ and $B(2, 4, -3)$ in ratio $k : 1$ then the value of k is equal to :

(1) 1.5 (2) 3 (3) 2 (4) 4

Official Ans. by NTA (3)

Sol.



Point C is

$\left(\frac{2k-3}{k+1}, \frac{4k-6}{k+1}, \frac{-3k+1}{k+1}\right)$

$\frac{x-1}{-1} = \frac{y+4}{2} = \frac{z+2}{3}$

Plane $lx + my + nz = 0$

$l(-1) + m(2) + n(3) = 0$

$-l + 2m + 3n = 0$ ....(1)

It also satisfy point $(1, -4, -2)$

$l - 4m - 2n = 0$ ....(2)

Solving (1) and (2)

$2m + 3n = 4m + 2n$

$n = 2m$

$l - 4m - 4m = 0$

$l = 8m$

$\frac{l}{8} = \frac{m}{1} = \frac{n}{2}$

$l : m : n = 8 : 1 : 2$

Plane is $8x + y + 2z = 0$

It will satisfy point C

$8\left(\frac{2k-3}{k+1}\right) + \left(\frac{4k-6}{k+1}\right) + 2\left(\frac{-3k+1}{k+1}\right) = 0$

$16k - 24 + 4k - 6 - 6k + 2 = 0$

$14k = 28$ $\therefore$ $k = 2$

14. The range of $a \in \mathbb{R}$ for which the function

$$f(x) = (4a-3)(x+\log_e 5) + 2(a-7)\cot\left(\frac{x}{2}\right)\sin^2\left(\frac{x}{2}\right),$$

$x \neq 2n\pi, n \in \mathbb{N}$, has critical points, is :

(1) $(-3, 1)$ (2) $\left[-\frac{4}{3}, 2\right]$

(3) $[1, \infty)$ (4) $(-\infty, -1]$

Official Ans. by NTA (2)

Sol. $f(x) = (4a - 3)(x + \log_e 5) + (a - 7)\sin x$

$f(x) = (4a - 3)(1) + (a - 7)\cos x = 0$

$$\Rightarrow \quad \cos x = \frac{3-4a}{a-7}$$

$$-1 \le \frac{3-4a}{a-7} < 1$$

$$\frac{3-4a}{a-7} + 1 \ge 0 \qquad \frac{3-4a}{a-7} < 1$$

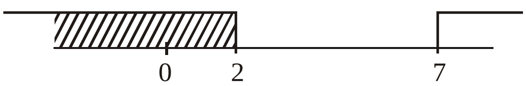
$$\frac{3-4a+a-7}{a-7} \ge 0 \qquad \frac{3-4a}{a-7} - 1 < 0$$

$$\frac{-3a-4}{a-7} \ge 0 \qquad \frac{3-4a-a+7}{a-7} < 0$$

$$\frac{3a+4}{a-7} \le 0 \qquad \frac{-5a+10}{a-7} < 0$$

$$\frac{5a-10}{a-7} > 0$$

$$\frac{5(a-2)}{a-7} > 0$$



$$\alpha \in \left[-\frac{4}{3}, 2\right)$$

Check end point $\left[-\frac{4}{3}, 2\right)$

15. A pack of cards has one card missing. Two cards are drawn randomly and are found to be spades. The probability that the missing card is not a spade, is :

(1) $\frac{3}{4}$ (2) $\frac{52}{867}$

(3) $\frac{39}{50}$ (4) $\frac{22}{425}$

Official Ans. by NTA (3)

Sol. E_1 : Event denotes spade is missing

$$P(E_1) = \frac{1}{4};\ P\left(\bar{E}_1\right) = \frac{3}{4}$$

A : Event drawn two cards are spade

$$P(A) = \frac{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}{\frac{1}{4}\times\left(\frac{{}^{12}C_2}{{}^{51}C_2}\right) + \frac{3}{4}\times\left(\frac{{}^{13}C_2}{{}^{51}C_2}\right)}$$

$$= \frac{39}{50}$$

16. Let $[x]$ denote greatest integer less than or equal to x. If for $n \in \mathbb{N}$, $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$,

then $\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ is equal to :

(1) 2 (2) 2^{n-1}

(3) 1 (4) n

Official Ans. by NTA (3)

Sol. $(1-x+x^3)^n = \sum_{j=0}^{3n} a_j x^j$

$(1 - x + x^3)^n = a_0 + a_1x + a_2x^2 \ldots\ldots. + a_{3n}x^{3n}$

$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j}$ = Sum of $a_0 + a_2 + a_4$ ....

$\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1}$ = Sum of $a_1 + a_3 + a_5$ ....

put x = 1

$1 = a_0 + a_1 + a_2 + a_3 \ldots\ldots\ldots + a_{3n}$ ....(A)

Put x = –1

$1 = a_0 - a_1 + a_2 - a_3 \ldots\ldots\ldots + (-1)^{3n}a_{3n}$ ....(B)

Solving (A) and (B)

$a_0 + a_2 + a_4 \ldots\ldots = 1$

$a_1 + a_3 + a_5 \ldots\ldots. = 0$

$$\sum_{j=0}^{\left[\frac{3n}{2}\right]} a_{2j} + 4\sum_{j=0}^{\left[\frac{3n-1}{2}\right]} a_{2j+1} = 1$$

17. If y = y(x) is the solution of the differential equation, $\frac{dy}{dx}+2y\tan x=\sin x,\ y\left(\frac{\pi}{3}\right)=0$, then the maximum value of the function y(x) over $\mathbb{R}$ is equal to :

(1) 8 (2) $\frac{1}{2}$ (3) $-\frac{15}{4}$ (4) $\frac{1}{8}$

Official Ans. by NTA (4)

Sol. $\frac{dy}{dx}+2y\tan x=\sin x$

I.F. = $e^{\int 2\tan x dx}=e^{2\ell n \sec x}$

I.F. = $\sec^2 x$

$y.(\sec^2 x)=\int \sin x.\sec^2 x dx$

$y.(\sec^2 x)=\int \sec x\tan x dx$

$y.(\sec^2 x)=\sec x + C$

$x=\frac{\pi}{3};\ y=0$

$\Rightarrow$ C = –2

$\Rightarrow \quad y=\frac{\sec x-2}{\sec^2 x}=\cos x-2\cos^2 x$

$y = t - 2t^2 \Rightarrow \frac{dy}{dt}=1-4t=0 \Rightarrow t=\frac{1}{4}$

$\therefore \quad \max = \frac{1}{4}-\frac{1}{8}=\frac{2-1}{8}=\frac{1}{8}$

18. The locus of the midpoints of the chord of the circle, $x^2 + y^2 = 25$ which is tangent to the hyperbola, $\frac{x^2}{9}-\frac{y^2}{16}=1$ is :

(1) $(x^2 + y^2)^2 - 16x^2 + 9y^2 = 0$

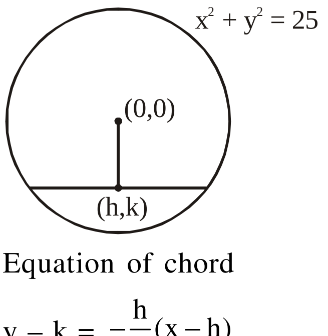
(2) $(x^2 + y^2)^2 - 9x^2 + 144y^2 = 0$

(3) $(x^2 + y^2)^2 - 9x^2 - 16y^2 = 0$

(4) $(x^2 + y^2)^2 - 9x^2 + 16y^2 = 0$

Official Ans. by NTA (4)

Sol.



Equation of chord

$y - k = -\frac{h}{k}(x-h)$

$ky - k^2 = -hx + h^2$

$hx + ky = h^2 + k^2$

$y=-\frac{hx}{k}+\frac{h^2+k^2}{k}$

tangent to $\frac{x^2}{9}-\frac{y^2}{16}=1$

$c^2 = a^2m^2 - b^2$

$\left(\frac{h^2+k^2}{k}\right)^2=9\left(-\frac{h}{k}\right)^2-16$

$(x^2 + y^2)^2 = 9x^2 - 16y^2$

19. The number of roots of the equation,

$(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$

in the interval [0, π] is equal to :

(1) 3 (2) 4 (3) 8 (4) 2

Official Ans. by NTA (2)

Sol. $(81)^{\sin^2 x}+(81)^{\cos^2 x}=30$

$(81)^{\sin^2 x}+\frac{(81)^1}{(18)^{\sin^2 x}}=30$

$(81)^{\sin^2 x}=t$

$t+\frac{81}{t}=30$

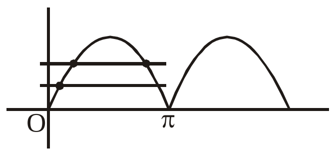
$t^2 - 30t + 81 = 0$

$(t - 3)(t - 27) = 0$

$(81)^{\sin^2 x}=3^1$ or $(81)^{\sin^2 x}=3^3$

$3^{4\sin^2 x}=3^1$ or $3^{4\sin^2 x}=3^3$

$\sin^2 x=\frac{1}{4}$ or $\sin^2 x=\frac{3}{4}$



Total sol. = 4

20. Let $S_k=\sum_{r=1}^{k}\tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$. Then $\lim_{k\to\infty} S_k$ is equal to :

(1) $\tan^{-1}\left(\frac{3}{2}\right)$ (2) $\frac{\pi}{2}$

(3) $\cot^{-1}\left(\frac{3}{2}\right)$ (4) $\tan^{-1}(3)$

Official Ans. by NTA (3)

Sol. $S_k = \sum_{r=1}^{k} \tan^{-1}\left(\frac{6^r}{2^{2r+1}+3^{2r+1}}\right)$

Divide by 3^{2r}

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{\left(\frac{2}{3}\right)^{2r}.2+3}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\left(\frac{2}{3}\right)^r}{3\left(\left(\frac{2}{3}\right)^{2r+1}+1\right)}\right)$$

Let $\left(\frac{2}{3}\right)^r = t$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{\frac{t}{3}}{1+\frac{2}{3}t^2}\right)$$

$$\sum_{r=1}^{k} \tan^{-1}\left(\frac{t-\frac{2t}{3}}{1+t.\frac{2t}{3}}\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}(t)-\tan^{-1}\left(\frac{2t}{3}\right)\right)$$

$$\sum_{r=1}^{k}\left(\tan^{-1}\left(\frac{2}{3}\right)^r-\tan^{-1}\left(\frac{2}{3}\right)^{r+1}\right)$$

$$S_k = \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}$$

$$S_\infty = \lim_{k\to\infty}\left(\tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}\left(\frac{2}{3}\right)^{k+1}\right)$$

$$= \tan^{-1}\left(\frac{2}{3}\right)-\tan^{-1}(0)$$

$$\therefore \quad S_\infty = \tan^{-1}\left(\frac{2}{3}\right)=\cot^{-1}\left(\frac{3}{2}\right)$$

SECTION-B

1. Consider an arithmetic series and a geometric series having four initial terms from the set {11, 8, 21, 16, 26, 32, 4}. If the last terms of these series are the maximum possible four digit numbers, then the number of common terms in these two series is equal to ______ .

Official Ans. by NTA (3)

Sol. **GP :** 4, 8, 16, 32, 64, 128, 256, 512, 1024, 2048, 4096, 8192

AP : 11, 16, 21, 26, 31, 36

Common terms : 16, 256, 4096 only

2. Let $f : (0, 2) \to \mathbb{R}$ be defined as

$$f(x) = \log_2\left(1+\tan\left(\frac{\pi x}{4}\right)\right).$$

Then, $\lim_{n\to\infty}\frac{2}{n}\left(f\left(\frac{1}{n}\right)+f\left(\frac{2}{n}\right)+....+f(1)\right)$ is equal to _______ .

Official Ans. by NTA (1)

Sol. $E = 2\lim_{n\to\infty}\sum_{r=1}^{n}\frac{1}{n}f\left(\frac{r}{n}\right)$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi x}{4}\right)dx \qquad .....(i)$$

replacing $x \to 1-x$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\frac{\pi}{4}(1-x)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\tan\left(\frac{\pi}{4}-\frac{\pi}{4}x\right)\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(1+\frac{1+\tan\frac{\pi}{4}x}{1+\tan\frac{\pi}{4}x}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1 \ell n\left(\frac{2}{1+\tan\frac{\pi x}{4}}\right)dx$$

$$E = \frac{2}{\ell n2}\int_0^1\left(\ell n2-\ell n\left(1+\tan\frac{\pi x}{4}\right)\right)dx \qquad .....(ii)$$

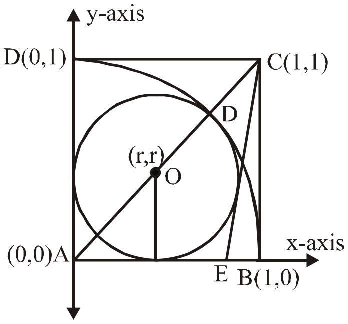
equation (i) + (ii)

E = 1

3. Let ABCD be a square of side of unit length. Let a circle C_1 centered at A with unit radius is drawn. Another circle C_2 which touches C_1 and the lines AD and AB are tangent to it, is also drawn. Let a tangent line from the point C to the circle C_2 meet the side AB at E. If the length of EB is $\alpha + \sqrt{3}\beta$, where α, β are integers, then $\alpha + \beta$ is equal to_______.

Official Ans. by NTA (1)

Sol.



Here AO + OD = 1 or $(\sqrt{2}+1)r = 1$

$\Rightarrow$ $r = \sqrt{2-1}$

equation of circle $(x - r)^2 + (y - r)^2 = r^2$

Equation of CE

$y - 1 = m(x - 1)$

$mx - y + 1 - M = 0$

It is tangent to circle

$$\therefore \quad \left|\frac{mr - r + 1 - m}{\sqrt{m^2+1}}\right| = r$$

$$\left|\frac{(m-1)r + 1 - m}{\sqrt{m^2+1}}\right| = r$$

$$\frac{(m-1)^2 (r-1)^2}{m^2+1} = r^2$$

Put $r = \sqrt{2} - 1$

On solving $m = 2 - \sqrt{3}$, $2 + \sqrt{3}$

Taking greater slope of CE as $2 + \sqrt{3}$

$y - 1 = (2 + \sqrt{3})(x - 1)$

Put $y = 0$

$-1 = (2 + \sqrt{3})(x - 1)$

$$\frac{-1}{2+\sqrt{3}} \times \left(\frac{2-\sqrt{3}}{2-\sqrt{3}}\right) = x - 1$$

$x - 1 = \sqrt{3} - 1$

$EB = 1 - x = 1 - (\sqrt{3} - 1)$

$EB = 2 - \sqrt{3}$

4. If $\lim\limits_{x\to 0} \dfrac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$, then a + b + c is equal to _______.

Official Ans. by NTA (4)

Sol. $$\lim_{x\to 0} \frac{ae^x - b\cos x + ce^{-x}}{x\sin x} = 2$$

$$\Rightarrow \lim_{x\to 0} \frac{a\left(1 + x + \frac{x^2}{2!}...\right) - b\left(1 - \frac{x^2}{2!} + ...\right) + c\left(1 - x + \frac{x^2}{2!}\right)}{\left(\frac{x\sin x}{x}\right)x} = 2$$

$a - b + c = 0$ .....(1)

$a - c = 0$ .....(2)

& $\dfrac{a+b+c}{2} = 2$

$\Rightarrow$ $\boxed{a+b+c=4}$

5. The total number of 3×3 matrices A having enteries from the set (0, 1, 2, 3) such that the sum of all the diagonal entries of AA^T is 9, is equal to _____.

Official Ans. by NTA (766)

Sol. Let $A = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$

diagonal elements of

AA^T, $a^2 + b^2 + c^2$, $d^2 + e^2 + f^2$, $g^2 + b^2 + c^2$

Sum $= a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 + i^2 = 9$

a, b, c, d, e, f, g, h, i $\in$ {0, 1, 2, 3}

	Case	No. of Matrices
(1)	All – 1s	$\frac{9!}{9!}=1$
(2)	One → 3 remaining-0	$\frac{9!}{1!\times 8!}=9$
(3)	One-2 five-1s three-0s	$\frac{9!}{1!\times 5!\times 3!}=8\times 63$
(4)	two – 2's one-1 six-0's	$\frac{9!}{2!\times 6!}=63\times 4$

Total no. of ways = 1 + 9 + 8 × 63 + 63 × 4

= $\boxed{766}$

6. Let

$$P=\begin{bmatrix}-30 & 20 & 56\\ 90 & 140 & 112\\ 120 & 60 & 14\end{bmatrix} \text{ and } A=\begin{bmatrix}2 & 7 & \omega^2\\ -1 & -\omega & 1\\ 0 & -\omega & -\omega+1\end{bmatrix}$$

where $\omega=\frac{-1+i\sqrt{3}}{2}$, and I_3 be the identity matrix of order 3. If the determinant of the matrix $(P^{-1}AP - I_3)^2$ is $\alpha\omega^2$, then the value of α is equal to ______ .

Official Ans. by NTA (36)

Sol. Let $M = (P^{-1}AP - I)^2$

$= (P^{-1}AP)^2 - 2P^{-1}AP + I$

$= P^{-1}A^2P - 2P^{-1}AP + I$

$PM = A^2P - 2AP + P$

$= (A^2 - 2A.I + I^2)P$

$\Rightarrow$ Det(PM) = Det$((A - I)^2 \times P)$

$\Rightarrow$ DetP.DetM = Det$(A - I)^2 \times$ Det(P)

$\Rightarrow$ Det M = $(\text{Det}(A - I))^2$

Now $A-I=\begin{bmatrix}1 & 7 & w^2\\ -1 & -w-1 & 1\\ 0 & -w & -w\end{bmatrix}$

$\text{Det}(A - I) = (w^2 + w + w) + 7(-w) + w^3 = -6w$

$\text{Det}((A - I))^2 = 36w^2$

$\Rightarrow \alpha = 36$

7. If the normal to the curve $y(x)=\int_0^x (2t^2-15t+10)\,dt$ at a point (a,b) is parallel to the line x + 3y = –5, a > 1, then the value of |a + 6b| is equal to ________ .

Official Ans. by NTA (406)

Sol. $y(x)=\int_0^x (2t^2-15t+10)\,dt$

$y'(x)\big]_{x=a}=\left[2x^2-15x+10\right]_a=2a^2-15a+10$

Slope of normal $=-\frac{1}{3}$

$\Rightarrow \quad 2a^2-15a+10=3 \Rightarrow a=7$

& $\quad a = \frac{1}{2}$ (rejected)

$b = y(7) = \int_0^7 (2t^2-15t+10)dt$

$=\left[\frac{2t^3}{3}-\frac{15t^2}{2}+10t\right]_0^7$

$\Rightarrow \quad 6b=4\times 7^3-45\times 49+60\times 7$

|a + 6b| = 406

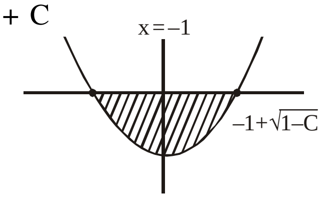
8. Let the curve y = y(x) be the solution of the differential equation, $\frac{dy}{dx}=2(x+1)$. If the numerical value of area bounded by the curve y = y(x) and x-axis is $\frac{4\sqrt{8}}{3}$, then the value of y(1) is equal to _______ .

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx}=2(x+1)$

$\Rightarrow \quad \int dy=\int 2(x+1)dx$

$\Rightarrow \quad y(x) = x^2 + 2x + C$



Area $=\frac{4\sqrt{8}}{3}$

$-1+\sqrt{1-C}$

$\Rightarrow \quad 2\int_{-1}^{-1+\sqrt{1-C}} (-(x+1)^2-C+1)dx=\frac{4\sqrt{8}}{3}$

$\Rightarrow \quad 2\left[-\frac{(x+1)^3}{3}-Cx+x\right]_{-1}^{-1+\sqrt{1-C}}=\frac{4\sqrt{8}}{3}$

$\Rightarrow \quad -(\sqrt{1-C})^3+3c-3C\sqrt{1-C}$

$-3+3\sqrt{1-C}-3C+3=2\sqrt{8}$

$\Rightarrow \quad C = -1$

$\Rightarrow \quad f(x) = x^2 + 2x - 1, \quad f(1) = 2$

9. Let $f : \mathbb{R} \to \mathbb{R}$ be a continuous function such that $f(x) + f(x + 1) = 2$, for all $x \in \mathbb{R}$. If $I_1 = \int_0^8 f(x)dx$ and $I_2 = \int_{-1}^{3} f(x)dx$, then the value of $I_1 + 2I_2$ is equal to ________ .

Official Ans. by NTA (16)

Sol. $f(x) + f(x + 1) = 2$

$\Rightarrow$ $f(x)$ is periodic with period = 2

$$I_1 = \int_0^8 f(x)dx = 4\int_0^2 f(x)dx$$

$$= 4\int_0^1 \left(f(x) + f(1+x)\right)dx = 8$$

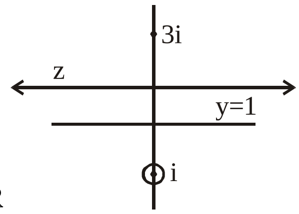
Similarly $I_2 = 2 \times 2 = 4$

$I_1 + 2I_2 = 16$

10. Let z and w be two complex numbers such that $w = z\bar{z} - 2z + 2$, $\left|\frac{z+i}{z-3i}\right| = 1$ and $Re(w)$ has minimum value. Then, the minimum value of $n \in \mathbb{N}$ for which w^n is real, is equal to _____.

Official Ans. by NTA (4)

Sol. $\omega = z\bar{z} - 2z + 2$



$\left|\frac{z+i}{z-3i}\right| = 1$

$\Rightarrow$ $|z+i| = |z-3i|$

$\Rightarrow$ $z = x + i,\ x \in \mathbb{R}$

$\omega = (x + i)(x - i) - 2(x + i) + 2$

$= x^2 + 1 - 2x - 2i + 2$

$Re(\omega) = x^2 - 2x + 3$

For min $(Re(\omega))$, $x = 1$

$\Rightarrow$ $\omega = 2 - 2i = 2(1 - i) = 2\sqrt{2}\, e^{-i\frac{\pi}{4}}$

$\omega^n = (2\sqrt{2})^n e^{-i\frac{n\pi}{4}}$

For real & minimum value of n,

n = 4