FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

PHYSICS

TEST PAPER WITH ANSWER & SOLUTION

SECTION-A

1. An oil drop of radius 2 mm with a density 3g cm^{-3} is held stationary under a constant electric field 3.55×10^5 V m^{-1} in the Millikan's oil drop experiment. What is the number of excess electrons that the oil drop will possess ? (consider g = 9.81 m/s^2)

(1) 48.8×10^{11}

(2) 1.73×10^{10}

(3) 17.3×10^{10}

(4) 1.73×10^{12}

Official Ans. by NTA (2)

Sol. $qE = Mg$

$$neE = \rho\left(\frac{4}{3}\pi r^3\right) \times g$$

$$n \times 1.6 \times 10^{-19} \times 3.55 \times 10^5$$

$$= 3 \times 10^3 \times \frac{4}{3} \times \pi \times (2 \times 10^{-3})^3 \times 9.81$$

$$n = 173 \times 10^{(3 - 9 - 5 + 19)}$$

$$n = 1.73 \times 10^{10}$$

2. Match List–I with List–II.

List–I

(a) 10 km height over earth's surface

(b) 70 km height over earth's surface

(c) 180 km height over earth's surface

(d) 270 km height over earth's surface

List–II

(i) Thermosphere

(ii) Mesosphere

(iii) Stratosphere

(iv) Troposphere

(1) (a)–(iv), (b)–(iii), (c)–(ii), (d)–(i)

(2) (a)–(i), (b)–(iv), (c)–(iii), (d)–(ii)

(3) (a)–(iii), (b)–(ii), (c)–(i), (d)–(iv)

(4) (a)–(ii), (b)–(i), (c)–(iv), (d)–(iii)

Official Ans. by NTA (1)

Sol. Order of atmosphere stratification from bottom Troposphere, stralospherre, Mesosphre, Thermosphere

(a) → (iv)

(b) → (iii)

(c) → (ii)

(d) → (i)

3. Imagine that the electron in a hydrogen atom is replaced by a muon (μ). The mass of muon particle is 207 times that of an electron and charge is equal to the charge of an electron. The ionization potential of this hydrogen atom will be :-

(1) 13.6 eV (2) 2815.2 eV

(3) 331.2 eV (4) 27.2 eV

Official Ans. by NTA (2)

Sol. $E \propto \frac{1}{r}$ $\quad r \propto \frac{1}{m}$

$E \propto m$

$$\text{Ionization potential} = 13.6 \times \frac{(\text{Mass}_\mu)\text{eV}}{(\text{Mass}_e)}$$

$= 13.6 \times 207$ eV $= 2815.2$ eV

4. A plane electromagnetic wave of frequency 100 MHz is travelling in vacuum along the x-direction. At a particular point in space and time, $\vec{B} = 2.0 \times 10^{-8}\hat{k}T$. (where, $\hat{k}$ is unit vector along z-direction) What is $\vec{E}$ at this point ?

(1) $0.6\hat{j}$ V/m (2) $6.0\hat{k}$ V/m

(3) $6.0\hat{j}$ V/m (4) $0.6\hat{k}$ V/m

Official Ans. by NTA (3)

Sol. E = BC = 6

(Dir. of wave) $\parallel \left(\vec{E}\times\vec{B}\right)$

$\hat{i}=\hat{j}\times\hat{k}$

$\vec{E}=6\hat{j}\ V/m$

5. A thin circular ring of mass M and radius r is rotating about its axis with an angular speed ω. Two particles having mass m each are now attached at diametrically opposite points. The angular speed of the ring will become :

(1) $\omega\frac{M}{M+m}$ (2) $\omega\frac{M+2m}{M}$

(3) $\omega\frac{M}{M+2m}$ (4) $\omega\frac{M-2m}{M+2m}$

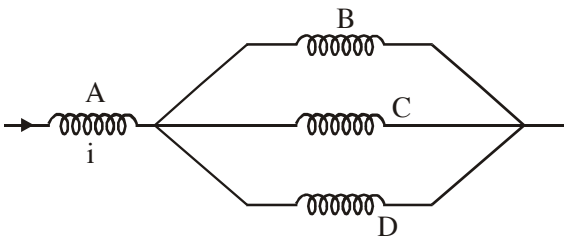
Official Ans. by NTA (3)

Sol. Using conservation of angular momentum

$(Mr^2)\omega = (Mr^2 + 2mr^2)\omega'$

$\omega'=\frac{M\omega}{M+2m}$

6. Four identical long solenoids A, B, C and D are connected to each other as shown in the figure. If the magnetic field at the center of A is 3T, the field at the center of C would be : (Assume that the magnetic field is confined with in the volume of respective solenoid).

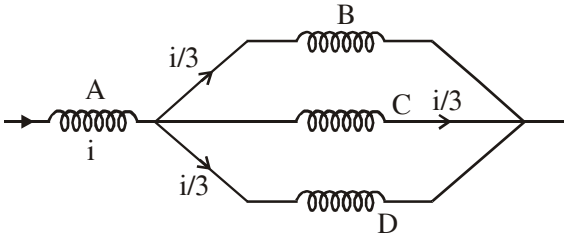


(1) 12T (2) 6T

(3) 9T (4) 1T

Official Ans. by NTA (4)

Sol.



$\phi \propto i$

$\Rightarrow B \propto i$

so, field at centre of C = $\frac{3}{3}=1T$

7. The time period of a simple pendulum is given by $T=2\pi\sqrt{\frac{\ell}{g}}$. The measured value of the length of pendulum is 10 cm known to a 1mm accuracy. The time for 200 oscillations of the pendulum is found to be 100 second using a clock of 1s resolution. The percentage accuracy in the determination of 'g' using this pendulum is 'x'. The value of 'x' to the nearest integer is:-

(1) 2% (2) 3%

(3) 5% (4) 4%

Official Ans. by NTA (2)

Sol. $g=\frac{4\pi^2\ell}{T^2}$

$$\frac{\Delta g}{g}=\frac{\Delta\ell}{\ell}+2\frac{\Delta T}{T}=\frac{0.1}{10}+2\left(\frac{\frac{1}{200}}{0.5}\right)$$

$$\frac{\Delta g}{g}=\frac{1}{100}+\frac{1}{50}$$

$$\frac{\Delta g}{g}\times 100=3\%$$

8. A constant power delivering machine has towed a box, which was initially at rest, along a horizontal straight line. The distance moved by the box in time 't' is proportional to :-

(1) $t^{2/3}$ (2) $t^{3/2}$

(3) t (4) $t^{1/2}$

Official Ans. by NTA (2)

Sol. $P = C$

$FV = C$

$$M\frac{dV}{dt}V = C$$

$$\frac{V^2}{2} \propto t$$

$$V \propto t^{1/2}$$

$$\frac{dx}{dt} \propto t^{1/2}$$

$$x \propto t^{3/2}$$

9. What will be the average value of energy along one degree of freedom for an ideal gas in thermal equilibrium at a temperature T ? (k_B is Boltzmann constant)

(1) $\frac{1}{2}k_BT$ (2) $\frac{2}{3}k_BT$

(3) $\frac{3}{2}k_BT$ (4) k_BT

Official Ans. by NTA (1)

Sol. Energy associated with each degree of freedom per molecule = $\frac{1}{2}k_BT$.

10. A radioactive sample disintegrates via two independent decay processes having half lives $T_{1/2}^{(1)}$ and $T_{1/2}^{(2)}$ respectively. The effective half-life $T_{1/2}$ of the nuclei is :

(1) None of the above (2) $T_{1/2} = T_{1/2}^{(1)} + T_{1/2}^{(2)}$

(3) $T_{1/2} = \frac{T_{1/2}^{(1)}T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$ (4) $T_{1/2} = \frac{T_{1/2}^{(1)} + T_{1/2}^{(2)}}{T_{1/2}^{(1)} - T_{1/2}^{(2)}}$

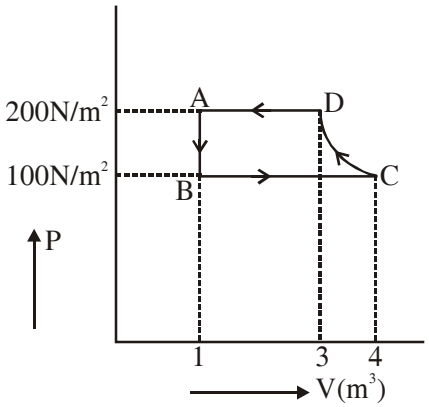
Official Ans. by NTA (3)

Sol. $\lambda_{eq} = \lambda_1 + \lambda_2$

$$\frac{1}{T_{1/2}} = \frac{1}{T_{1/2}^{(1)}} + \frac{1}{T_{1/2}^{(2)}}$$

$$T_{1/2} = \frac{T_{1/2}^{(1)}T_{1/2}^{(2)}}{T_{1/2}^{(1)} + T_{1/2}^{(2)}}$$

11. The P-V diagram of a diatomic ideal gas system going under cyclic process as shown in figure. The work done during an adiabatic process CD is (use $\gamma = 1.4$) :



(1) –500 J (2) –400 J

(3) 400 J (4) 200 J

Official Ans. by NTA (1)

Sol. Adiabatic process is from C to D

$$WD = \frac{P_2V_2 - P_1V_1}{1-\gamma}$$

$$= \frac{P_DV_D - P_CV_C}{1-\gamma}$$

$$= \frac{200(3) - (100)(4)}{1-1.4}$$

= –500 J Ans. (1)

12. In Young's double slit arrangement, slits are separated by a gap of 0.5 mm, and the screen is placed at a distance of 0.5 m from them. The distance between the first and the third bright fringe formed when the slits are illuminated by a monochromatic light of 5890 Å is :-

(1) 1178×10^{-9} m (2) 1178×10^{-6} m

(3) 1178×10^{-12} m (4) 5890×10^{-7} m

Official Ans. by NTA (2)

Sol. $\beta = \frac{\lambda D}{d} = \frac{5890 \times 10^{-10} \times 0.5}{0.5 \times 10^{-3}}$

$= 589 \times 10^{-6}$ m

Distance between first and third bright fringe is $2\beta = 2 \times 589 \times 10^{-6}$ m

$= 1178 \times 10^{-6}$ m Ans. (2)

13. A particle is travelling 4 times as fast as an electron. Assuming the ratio of de-Broglie wavelength of a particle to that of electron is 2 : 1, the mass of the particle is :-

(1) $\frac{1}{16}$ times the mass of e^-

(2) 8 times the mass of e^-

(3) 16 times the mass of e^-

(4) $\frac{1}{8}$ times the mass of e^-

Official Ans. by NTA (4)

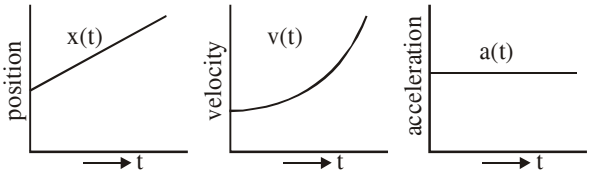
Sol. $\lambda = \frac{h}{p}$

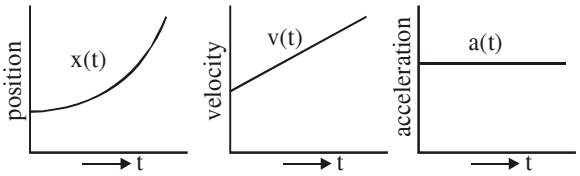
$$\frac{\lambda_p}{\lambda_e} = \frac{p_e}{p_p} = \frac{m_e v_e}{m_p v_p}$$

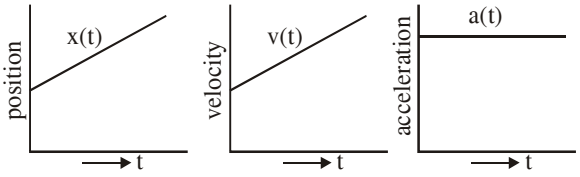
$$2 = \frac{m_e}{m_p}\left(\frac{v_e}{4v_e}\right)$$

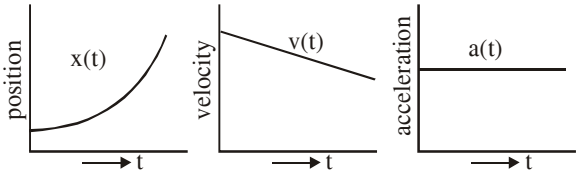
$\therefore \; m_p = \frac{m_e}{8}$ Ans. (4)

14. The position, velocity and acceleration of a particle moving with a constant acceleration can be represented by :

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol. Option (2) represent correct graph for particle moving with constant acceleration, as for constant acceleration velocity time graph is straight line with positive slope and x-t graph should be an opening upward parabola.

15. In the experiment of Ohm's law, a potential difference of 5.0 V is applied across the end of a conductor of length 10.0 cm and diameter of 5.00 mm. The measured current in the conductor is 2.00 A. The maximum permissible percentage error in the resistivity of the conductor is :-

(1) 3.9 (2) 8.4

(3) 7.5 (4) 3.0

Official Ans. by NTA (1)

Sol. $R = \frac{\rho\ell}{A} = \frac{V}{I}$

$$\rho = \frac{AV}{I\ell} = \frac{\pi d^2 V}{4I\ell} \qquad \left(A = \frac{\pi d^2}{4}\right)$$

$$\therefore \quad \frac{\Delta\rho}{\rho} = \frac{2\Delta d}{d} + \frac{\Delta V}{V} + \frac{\Delta I}{I} + \frac{\Delta\ell}{\ell}$$

$$\frac{\Delta\rho}{\rho} = 2\left(\frac{0.01}{5.00}\right) + \frac{0.1}{5.0} + \frac{0.01}{2.00} + \frac{0.1}{10.0}$$

$$\frac{\Delta\rho}{\rho} = 0.004 + 0.02 + 0.005 + 0.01$$

$$\frac{\Delta\rho}{\rho} = 0.039$$

$$\% \text{ error} = \frac{\Delta\rho}{\rho} \times 100 = 0.039 \times 100 = 3.90\%$$

Ans. (1)

16. In a scries LCR resonance circuit, if we change the resistance only, from a lower to higher value :

(1) The bandwidth of resonance circuit will increase.

(2) The resonance frequency will increase.

(3) The quality factor will increase.

(4) The quality factor and the resonance frequency will remain constant.

Official Ans. by NTA (1)

Sol. Bandwidth = R/L

Bandwidth $\propto$ R

So bandwidth will increase

17. An AC source rated 220 V, 50 Hz is connected to a resistor. The time taken by the current to change from its maximum to the rms value is :

(1) 2.5 ms (2) 25 ms

(3) 2.5 s (4) 0.25 ms

Official Ans. by NTA (1)

Sol. $i = i_0 \cos(\omega t)$

$i = i_0$ at $t = 0$

$$i = \frac{i_0}{\sqrt{2}} \text{ at } \omega t = \frac{\pi}{4}$$

$$t = \frac{\pi}{4\omega} = \frac{\pi}{4(2\pi f)} = \frac{1}{8f}$$

$$t = \frac{1}{400} = 2.5\,\text{ms}$$

18. Your friend is having eye sight problem. She is not able lo see clearly a distant uniform window mesh and it appears to her as non-uniform and distorted. The doctor diagnosed the problem as :

(1) Astigmatism

(2) Myopia with Astigmatism

(3) Presbyopia with Astigmatism

(4) Myopia and hypermetropia

Official Ans. by NTA (2)

Sol. If distant objects are blurry then problem is Myopia.

If objects are distorted then problem is Astigmatism

19. A loop of flexible wire of irregular shape carrying current is placed in an external magnetic field. Identify the effect of the field on the wire.

(1) Loop assumes circular shape with its plane normal to the field.

(2) Loop assumes circular shape with its plane parallel to the field.

(3) Wire gets stretched to become straight.

(4) Shape of the loop remains unchanged.

Official Ans. by NTA (1)

Sol. Every part ($d\ell$) of the wire is pulled by force $i(d\ell)B$ acting perpendicular to current & magnetic field giving it a shape of circle.

20. The time period of a satellite in a circular orbit of radius R is T. The period of another satellite in a circular orbit of radius 9R is :

(1) 9 T (2) 27 T

(3) 12 T (4) 3 T

Official Ans. by NTA (2)

Sol. $T^2 \propto R^3$

$$\left(\frac{T'}{T}\right)^2 = \left(\frac{9R}{R}\right)^3$$

$T'^2 = T^2 \times 9^3$

$T' = T \times 3^3$

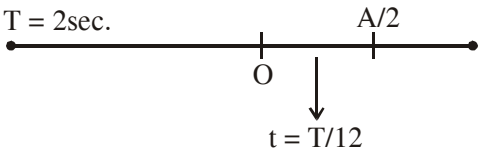
$T' = 27\ T$

SECTION-B

1. A particle performs simple harmonic motion with a period of 2 second. The time taken by the particle to cover a displacement equal to half of its amplitude from the mean position is $\frac{1}{a}$s. The value of 'a' to the nearest integer is _____ .

Official Ans. by NTA (6)

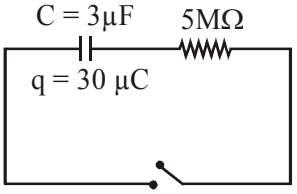
Sol.



$$t = \frac{2}{12} = \frac{1}{6}$$

$\therefore$ Correct answer = 6.00

2. The circuit shown in the figure consists of a charged capacitor of capacity 3 μF and a charge of 30 μC. At time t = 0, when the key is closed, the value of current flowing through the 5 MΩ resistor is 'x' μ-A. The value of 'x to the nearest integer is _____.

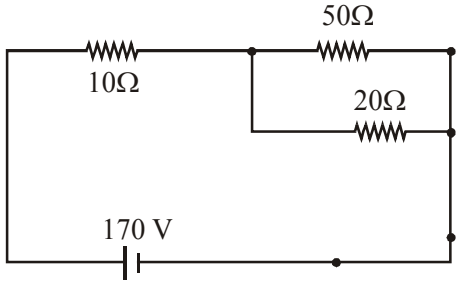


Official Ans. by NTA (2)

Sol. $i_0 = \frac{V}{R} = \frac{30/3}{5 \times 10^6} = 2 \times 10^{-6}$

$\therefore$ Ans. = 2.00

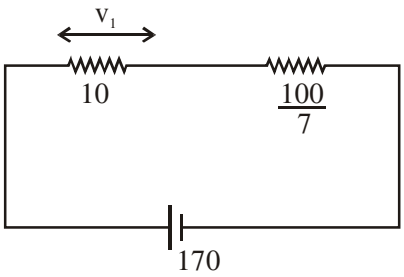
3. The voltage across the 10Ω resistor in the given circuit is x volt.



The value of 'x' to the nearest integer is _____.

Official Ans. by NTA (70)

Sol. $R_{eq_1} = \frac{50 \times 20}{70} = \frac{100}{7}$



$R_{eq} = \frac{170}{7}$

$v_1 = \left[\frac{\frac{170}{7}}{\frac{170}{7}}\right] \times 10 = 70v$

Ans. = 70.00

4. Two separate wires A and B are stretched by 2 mm and 4 mm respectively, when they are subjected to a force of 2 N. Assume that both the wires are made up of same material and the radius of wire B is 4 times that of the radius of wire A. The length of the wires A and B are in the ratio of a : b. Then a/b can be expressed as 1/x where x is _____ .

Official Ans. by NTA (32)

Sol. For A $\frac{E}{\pi r^2} = y\frac{2mm}{a}$ ....(1)

For B $\frac{E}{\pi.16r^2} = y\frac{4mm}{b}$ ....(2)

$\therefore$ (1)/(2)

$16 = \frac{2b}{4a}$

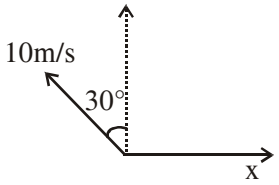
$\frac{a}{b} = \frac{1}{32}$

$\therefore$ Answer = 32

5. A person is swimming with a speed of 10 m/s at an angle of 120° with the flow and reaches to a point directly opposite on the other side of the river. The speed of the flow is 'x' m/s. The value of 'x' to the nearest integer is _____.

Official Ans. by NTA (5)

Sol.



10 sin 30° = x

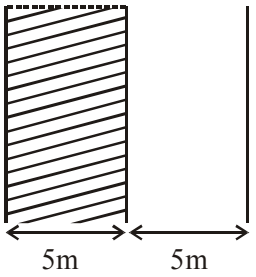
x = 5 m/s

6. A parallel plate capacitor has plate area 100 m^2 and plate separation of 10 m. The space between the plates is filled up to a thickness 5 m with a material of dielectric constant of 10. The resultant capacitance of the system is 'x' pF. The value of $\varepsilon_0 = 8.85 \times 10^{-12}$ $F.m^{-1}$.

The value of 'x' to the nearest integer is_____.

Official Ans. by NTA (161)

Sol.



A = 100 m^2

Using $C = \frac{k \in_0 A}{d}$

$C_1 = \frac{10 \in_0 (100)}{5}$

$= 200 \in_0$

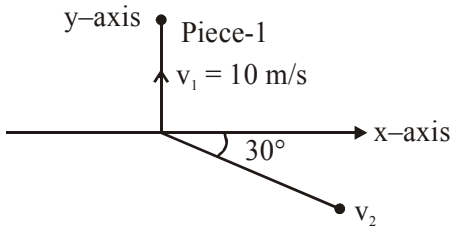
$C_2 = \frac{\in_0 (100)}{5} = 20 \in_0$

C_1 & C_2 are in series so $C_{eqv.} = \frac{C_1 C_2}{C_1 + C_2}$

$= \frac{4000 \in_0}{220}$

$= 160.9 \times 10^{-12} \simeq 161$ pF

7. A ball of mass 10 kg moving with a velocity $10\sqrt{3}$ m/s along the x-axis, hits another ball of mass 20 kg which is at rest. After the collision, first ball comes to rest while the second ball disintegrates into two equal pieces. One piece starts moving along y-axis with a speed of 10 m/s. The second piece starts moving at an angle of 30° with respect to the x-axis. The velocity of the ball moving at 30° with x-axis is x m/s. The configuration of pieces after collision is shown in the figure below. The value of x to the nearest integer is _____ .



Official Ans. by NTA (20)

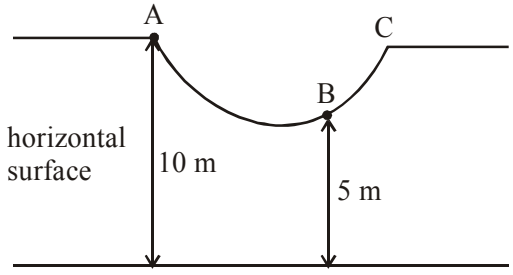
Sol. Let velocity of 2nd fragment is $\vec{v}$ then by conservation of linear momentum

$$10\left(10\sqrt{3}\right)\hat{i} = (10)\left(10\hat{j}\right) + 10\vec{v}$$

$$\Rightarrow \vec{v} = 10\sqrt{3}\hat{i} - 10\hat{j}$$

$$|\vec{v}| = \sqrt{300+100} = \sqrt{400} = 20\,m/s$$

8. As shown in the figure, a particle of mass 10 kg is placed at a point A. When the particle is slightly displaced to its right, it starts moving and reaches the point B. The speed of the particle at B is x m/s. (Take g = 10 m/s^2)
The value of 'x' to the nearest integer is_____.



Official Ans. by NTA (10)

Sol. Using work energy theorem,
$W_g = \Delta K.E.$

$$(10)\ (g)\ (5) = \frac{1}{2}(10)v^2 - 0$$

$v = 10$ m/s

9. An npn transistor operates as a common emitter amplifier with a power gain of 10^6. The input circuit resistance is 100Ω and the output load resistance is 10 KΩ. The common emitter current gain 'β' will be _____. (Round off to the Nearest Integer)

Official Ans. by NTA (100)

Sol. $$10^6 = \beta^2 \times \frac{R_0}{R_i}$$

$$10^6 = \beta^2 \times \frac{10^4}{10^2}$$

$$\beta^2 = 10^4 \Rightarrow \beta = 100$$

10. A bullet of mass 0.1 kg is fired on a wooden block to pierce through it, but it stops after moving a distance of 50 cm into it. If the velocity of bullet before hitting the wood is 10 m/s and it slows down with uniform deceleration, then the magnitude of effective retarding force on the bullet is 'x' N. The value of 'x' to the nearest integer is _____ .

Official Ans. by NTA (10)

Sol. $v^2 = u^2 + 2as$

$$0 = (10)^2 + 2\ (-a)\left(\frac{1}{2}\right)$$

$a = 100$ m/s^2
$F = ma = (0.1)\ (100) = 10$ N

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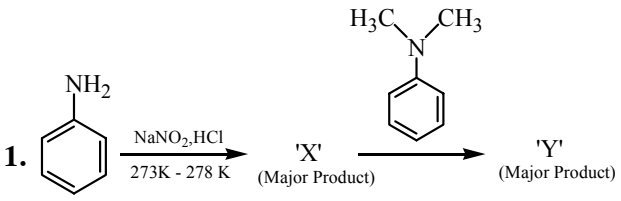
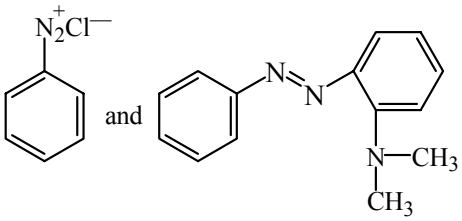
CHEMISTRY | TEST PAPER WITH ANSWER & SOLUTION

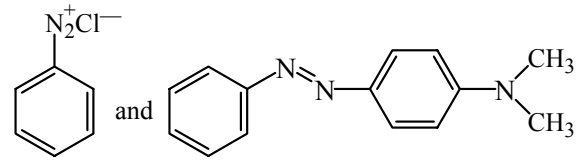
SECTION-A

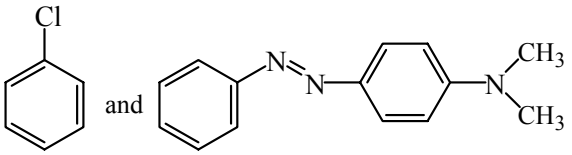
1.

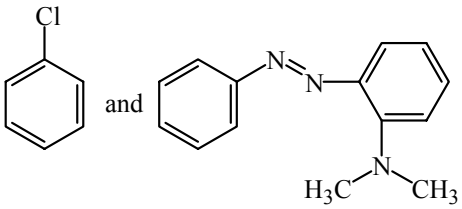


Considering the above reaction, X and Y respectively are :

(1) 

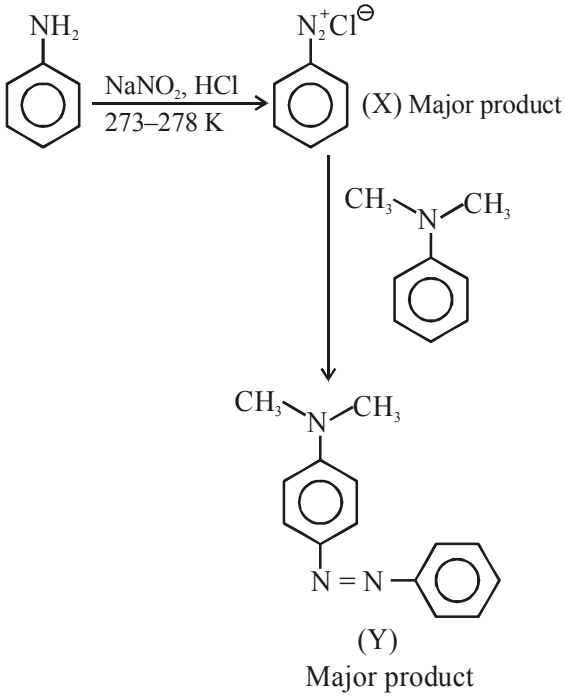
(2) 

(3) 

(4) 

Official Ans. by NTA (2)

Sol.



2. The ionic radius of Na^+ ions is 1.02 Å. The ionic radii (in Å) of Mg^{2+} and Al^{3+}, respectively, are-

(1) 1.05 and 0.99 (2) 0.72 and 0.54

(3) 0.85 and 0.99 (4) 0.68 and 0.72

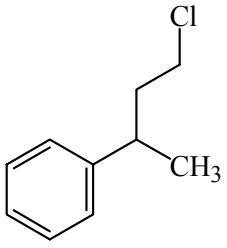
Official Ans. by NTA (2)

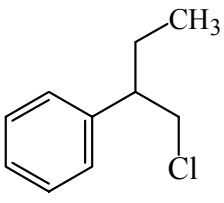
Sol. The ionic radii order is

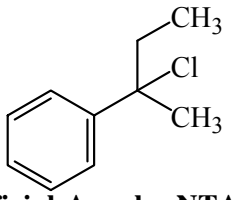
$Na^+ > Mg^{2+} > Al^{3+}$

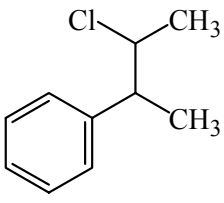
3. Reaction of Grignard reagent, C_2H_5MgBr with C_8H_8O followed by hydrolysis gives compound "A" which reacts instantly with Lucas reagent to give compound B, $C_{10}H_{13}Cl$.

The Compound B is :

(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (3)

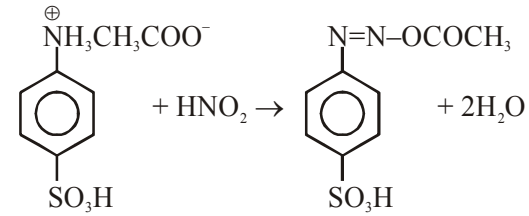
4. Reagent, 1-naphthylamine and sulphanilic acid in acetic acid is used for the detection of

(1) N_2O (2) NO_3^- (3) NO (4) NO_2^-

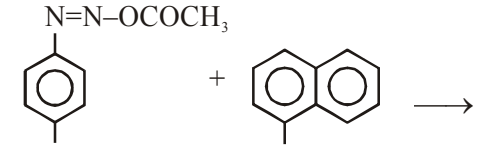
Official Ans. by NTA (4)

Sol. For detection of NO_2^-, the following test is used.

$NO_2^- + CH_3COOH \rightarrow HNO_2 + CH_3COO^-$

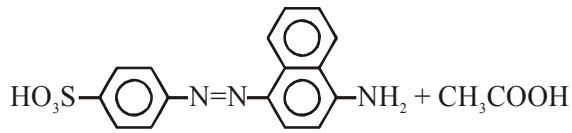


(Sulphanilic acid solution)



Diazotized acid

1-napthyl amine



(Red azo dye)

5. A non-reducing sugar "A" hydrolyses to give two reducing mono saccharides. Sugar A is-

(1) Fructose (2) Galactose

(3) Glucose (4) Sucrose

Official Ans. by NTA (4)

Sol. Sucrose $\xrightarrow{H_2O}$ glucose + Fructose

(Non reducing sugar) (Reducing sugar) (Reducing sugar)

6. Match the list -I with list - II

List-I (Class of Drug)	**List-II (Example)**
(a) Antacid	(i) Novestrol
(b) Artificial sweetener	(ii) Cimetidine
(c) Antifertility	(iii) Valium
(d) Tranquilizers	(iv) Alitame

(1) (a) – (ii), (b) – (iv),(c) – (i), (d) – (iii)

(2) (a) – (iv), (b) – (i),(c) – (ii), (d) – (iii)

(3) (a) – (iv), (b) – (iii),(c) – (i), (d) – (ii)

(4) (a) – (ii), (b) – (iv),(c) – (iii), (d) – (i)

Official Ans. by NTA (1)

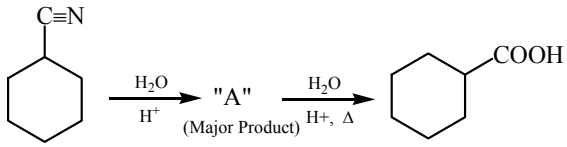
Sol. (a) Antacid : Cimetidine

(b) Artifical Sweetener : Alitame

(c) Antifertility : Novestrol

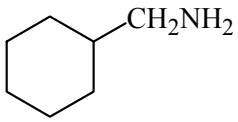
(d) Tranquilizers : Valium

7.

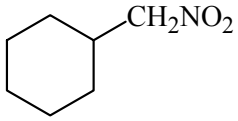


Consider the above chemical reaction and identify product "A"

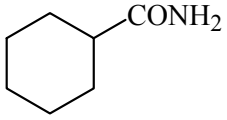
(1)



(2)



(3)



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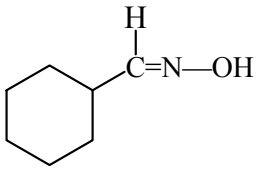
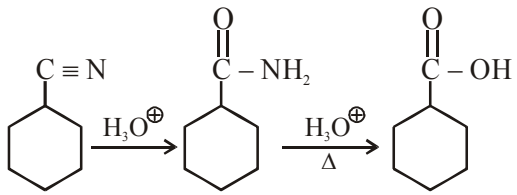


Official Ans. by NTA (3)

Sol.



'A'

Major product (partial hydrolysis)

Complete hydrolysis

8. Match List-I with List-II

List-I	List-II
(a) Chlorophyll	(i) Ruthenium
(b) Vitamin-B_{12}	(ii) Platinum
(c) Anticancer drug	(iii) Cobalt
(d) Grubbs catalyst	(iv) Magnesium

Choose the most appropriate answer from the options given below :

(a) a-iii, b-ii, c-iv, d-i

(b) a-iv, b-iii), c-ii, d-i

(c) a-iv, b-iii, c-i, d-ii

(d) a-iv, b-ii, c-iii, d-i

Official Ans. by NTA (2)

Sol. Chlorophyll is a coordination compound of magnesium.

Vitamin B-12, cyanocobalamine is a coordination compound of cobalt.

Cisplatin is used as an anti-cancer drug and is a coordination compound of platinum.

Grubbs catalyst is a compound of Ruthenium.

9. Match List-I with List-II :

List-I

(Chemicals)

(a) Alcoholic potassium hydroxide

(b) Pd/ $BaSO_4$

(c) BHC (Benzene hexachloride)

(d) Polyacetylene

List-II

(Use / Preparation / Constituent)

(i) Electrodes in batteries

(ii) Obtained by addition reaction

(iii) Used for β - elimination reaction

(iv) Lindlar's catalyst

Choose the most appropriate match :

(1) a-ii, b-i, c-iv, d-iii

(2) a-iii, b-iv, c-ii, d-i

(3) a-iii, b-i, c-iv, d-ii

(4) a-ii, b-iv, c-i, d-iii

Official Ans. by NTA (2)

Sol. (a) Alcoholic potassium hydroxide $\rightarrow$ used for β-elimination

(b) Pd/ $BaSO_4$ $\rightarrow$ Lindlar's catalyst

(c) BHC (Benzene hexachloride) $\rightarrow$ Obtained by addition reactions

(d) Polyacetylene $\rightarrow$ Electrodes in batteries

10. The satements that are TRUE :

(A) Methane leads to both global warming and photochemical smog

(B) Methane is generated from paddy fields

(C) Methane is a stronger global warming gas than CO_2

(D) Methane is a part of reducing smog

Choose the most appropriate answer from the options given below :

(1) (A), (B), (C) only

(2) (A) and (B) only

(3) (B), (C), (D) only

(4) (A), (B), (D) only

Official Ans. by NTA (1)

Sol. Methane leads to both global warming & photochemical smog.

Methane is generated in large amounts from paddy fields.

CO_2 can be absorbed by photosynthesis, or by formation of acid rain etc., while no such activities are there for methane.

Hence methane is stronger global warming gas than CH_4.

Methane is not a part of reducing smog.

11. Match List-I with List-II

List-I	**List-II**
(a) $Ca(OCl)_2$	(i) Antacid
(b) $CaSO_4 \cdot \frac{1}{2}H_2O$	(ii) Cement
(c) CaO	(iii) Bleach
(d) $CaCO_3$	(iv) Plaster of paris

Choose the most appropriate answer from the options given below :

(1) a-i, b-iv, c-iii, d-ii

(2) a-iii, b-ii, c-iv, d-i

(3) a-iii, b-iv, c-ii, d-i

(4) a-iii, b-ii, c-i, d-iv

Official Ans. by NTA (3)

Sol. $Ca(OCl)_2$ is Bleach.

$CaSO_4 \cdot \frac{1}{2}H_2O$ is plaster of paris.

$CaCO_3$ is used as an antacid.

CaO is major component of cement.

12. Compound with molecular formula C_3H_6O can show :

(1) Positional isomerism

(2) Both positional isomerism and metamerism

(3) Metamerism

(4) Functional group isomerism

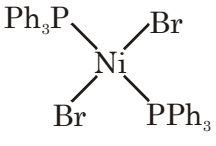
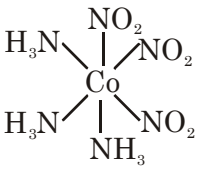
Official Ans. by NTA (4)

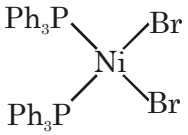
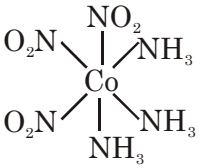
Sol. $C_3H_6O \Rightarrow CH_3–CH_2–CH=O$

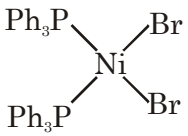
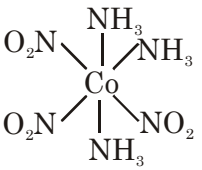
& $CH_3–\underset{\underset{O}{\|}}{C}–CH_3$

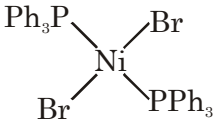
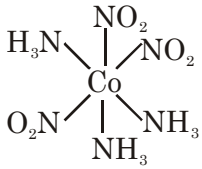
They are functional group isomerism.

13. The correct structures of trans-$[NiBr_2(PPh_3)_2]$ and meridonial-$[Co(NH_3)_3(NO_2)_3]$, respectively, are

(1)  and 

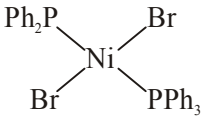
(2)  and 

(3)  and 

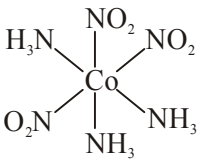
(4)  and 

Official Ans. by NTA (4)

Sol. trans-[Ni $Br_2(PPh_3)_2$] is



meridional - $[Co(NH_3)_3(NO_2)_3]$ is



14. A certain orbital has no angular nodes and two radial nodes. The orbital is :

(1) 2s (2) 3s (3) 3p (4) 2p

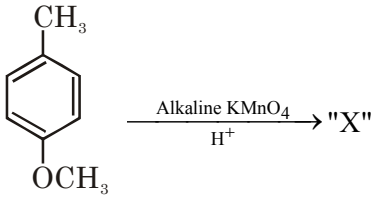
Official Ans. by NTA (2)

Sol. $l = 0 \Rightarrow$'s' orbital

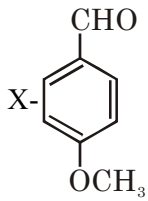
$n - l - 1 = 2$

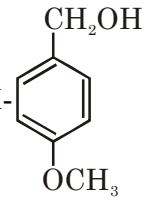
$n - 1 = 2$

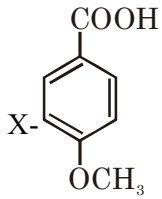
$n = 3$

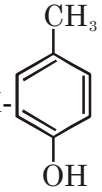
15. 

Considering the above chemical reaction, identify the product "X" :

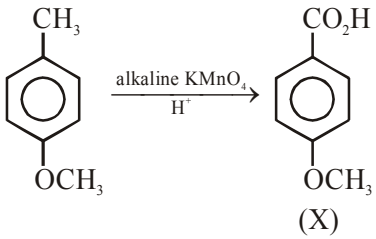
(1) X- 

(2) X- 

(3) X- 

(4) X- 

Official Ans. by NTA (3)

Sol. 

(X)

16. Match **List-I** with **List-II**

List-I(process)	**List-II (catalyst)**
(a) Deacron's process	(i) ZSM-5
(b) Contact process	(ii) $CuCl_2$
(c) Cracking of hydrocarbons	(iii) Particles 'Ni'
(d) Hydrogenation of vegetable oils	(iv) V_2O_5

Choose the most appropriate answer from the options given below -

(1) a-ii, b-iv, c-i, d-iii (2) a-i, b-iii, c-ii, d-iv

(3) a-iii, b-i, c-iv, d-ii (4) a-iv, b-ii, c-i, d-iii

Official Ans. by NTA (1)

Sol. In manufacture of H_2SO_4 (contact process), V_2O_5 is used as a catalyst.

Ni catalysts enables the hydrogenation of fats.

$CuCl_2$ is used as catalyst in Deacon's process.

ZSM-5 used as catalyst in cracking of hydrocarbons.

17. Given below are two statements : One is labelled as Assertion A and the other labelled as reason R

Assertion A : During the boiling of water having temporary hardness, $Mg(HCO_3)_2$ is converted to $MgCO_3$.

Reason R : The solubility product of $Mg(OH)_2$ is greater than that of $MgCO_3$.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Both A and R are true but R is not the correct explanation of A

(2) A is true but R is false

(3) Both A and R are true and R is the correct explanation of A

(4) A is false but R is true

Official Ans. by NTA (4)

Sol. For temporary hardness,

$Mg(HCO_3)_2 \xrightarrow{\text{heating}} Mg(OH)_2\downarrow + 2CO_2\uparrow$

Assertion is false.

$MgCO_3$ has high solubility product than $Mg(OH)_2$.

According to data of NCERT table 7.9 (Equilibrium chapter), the solubility product of magnesium carbonate is 3.5×10^{-8} and solubility product of $Mg(OH)_2$ is 1.8×10^{-11}.

Hence Reason is incorrect.

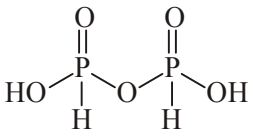
The question should be Bonus.

18. The number of ionisable hydrogens present in the product obtained from a reaction of phosphorus trichloride and phosphonic acid is:

(1) 3 (2) 0 (3) 2 (4) 1

Official Ans. by NTA (3)

Sol. $PCl_3 + H_3PO_3 \rightarrow H_4P_2O_5$



(Two ionisable H)

19. In a binary compound, atoms of element A form a hcp structure and those of element M occupy 2/3 of the tetrahedral voids of the hcp structure. The formula of the binary compound is :

(1) M_2A_3 (2) M_4A_3 (3) M_4A (4) MA_3

Official Ans. by NTA (2)

Sol. $M_{12\times\frac{2}{3}}A_6$

M_8A_6

M_4A_3

20. The chemical that is added to reduce the melting point of the reaction mixture during the extraction of aluminium is :

(1) Cryolite (2) Bauxite

(3) Calamine (4) Kaolite

Official Ans. by NTA (1)

Sol. To reduce the melting point of reaction mixture, cryolite is added.

SECTION-B

1. AX is a covalent diatomic molecule where A and X are second row elements of periodic table. Based on Molecular orbital theory, the bond order of AX is 25. The total number of electrons in AX is _____. (Round off to the Nearest Integer).

Official Ans. by NTA (15)

Sol. AX is a covalent diatomic molecule.

The molecule is NO.

Total no. of electrons is 15.

2. In order to prepare a buffer solution of pH 5.74, sodium acetate is added to acetic acid. If the concentration of acetic acid in the buffer is 1.0 M, the concentration of sodium acetate in the buffer is _____ M. (Round off to the Nearest Integer).

[Given : pKa (acetic acid) = 4.74]

Official Ans. by NTA (10)

Sol. $pH = pKa + \log \frac{[CB]}{[WA]}$

$5.74 = 4.74 + \log \frac{[CB]}{1}$

$\Rightarrow [CB] = 10\ M$

3. $2\ NO(g) + Cl_2(g) \rightleftharpoons 2\ NOCl(s)$

This reaction was studied at –10°C and the following data was obtained

run	$[NO]_0$	$[Cl_2]_0$	r_0
1	0.10	0.10	0.18
2	0.10	0.20	0.35
3	0.20	0.20	1.40

$[NO]_0$ and $[Cl_2]_0$ are the initial concentrations and r_0 is the initial reaction rate.

The overall order of the reaction is _____. (Round off to the Nearest Integer).

Official Ans. by NTA (3)

Sol. $r = k[NO]^m [Cl_2]^n$

$= k(0.1)^m (0.1)^n$ .....(1)

$= k(0.1)^m (0.2)^n$ .....(2)

$= k(0.2)^m (0.2)^n$ .....(3)

$n = 1$

$m = 2$

$m + n = 3$

4. For the reaction

$C_2H_6 \rightarrow C_2H_4 + H_2$

the reaction enthalpy $\Delta_r H$ = _____ kJ mol^{-1}. (Round off to the Nearest Integer).

[Given : Bond enthalpies in kJ mol^{-1} : C–C : 347, C=C : 611; C–H : 414, H–H : 436]

Official Ans. by NTA (128)

Sol. $\Delta_r H = [\in_{C-C} + 2\in_{C-H}] - [\in_{C=C} + \in_{H-H}]$

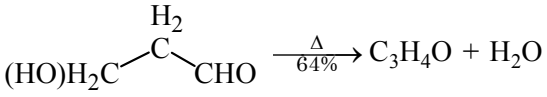
$= [347 + 2 \times 414] - [611 + 436]$

$= 128$

5. _____ grams of 3-Hydroxy propanal (MW=74) must be dehydrated to produce 7.8 g of acrolein (MW = 56) (C_3H_4O) if the percentage yield is 64. (Round off to the Nearest Integer).

[Given : Atomic masses : C : 12.0 u, H : 1.0 u, O : 16.0 u]

Official Ans. by NTA (16)

Sol. 

$\frac{x}{74}$ mol $\qquad\qquad \frac{x}{74} \times 0.64 = \frac{7.8}{56}$

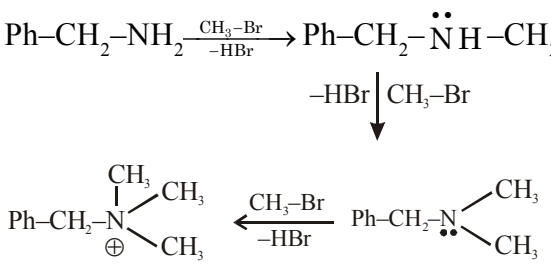
$x = 16.10$

$\simeq 16.00$

6. A reaction of 0.1 mole of Benzylamine with bromomethane gave 23 g of Benzyl trimethyl ammonium bromide. The number of moles of bromomethane consumed in this reaction are $n \times 10^{-1}$, when n = _____. (Round off to the Nearest Integer).

(Given : Atomic masses : C : 12.0 u, H : 1.0 u, N : 14.0 u, Br : 80.0 u]

Official Ans. by NTA (3)

Sol. 

no of moles = 3

7. The total number of unpaired electrons present in the complex $K_3[Cr(oxalate)_3]$ is _____.

Official Ans. by NTA (3)

Sol. $K_3[Cr(oxalate)_3]$

Chromium is in +3 oxidation state.

Number of unpaired electrons in Cr^{+3} will be 3.

8. 2 molal solution of a weak acid HA has a freezing point of 3.885°C. The degree of dissociation of this acid is _____ $\times 10^{-3}$. (Round off to the Nearest Integer).

[Given : Molal depression constant of water = 1.85 K kg mol^{-1} Freezing point of pure water = 0°C]

Official Ans. by NTA (50)

Sol. $\Delta T_f = (1 + \alpha)\, K_f.m$

$\alpha = 0.05 = 50 \times 10^{-3}$

9. For the reaction

$2Fe^{3+}(aq) + 2I^-(aq) \rightarrow 2Fe^{2+}(aq) + I_2(s)$

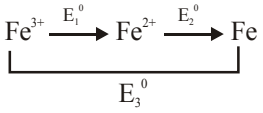
the magnitude of the standard molar free energy change, $\Delta_r G_m^\circ = -$ _____ kJ (Round off to the Nearest Integer).

$$\left[\begin{array}{l} E^o_{Fe^{2+}/Fe(s)} = -0.440 \text{ V}; \; E^o_{Fe^{3+}/Fe(s)} = -0.036 \text{ V} \\ E^o_{I_2/2I^-} = 0.539 \text{ V}; \qquad F = 96500 \text{ C} \end{array}\right]$$

Official Ans. by NTA (46)

Official Ans. by ALLEN (45)

Sol.



$E_1^0 + 2E_2^0 = 3E_3^0$

$E_1^0 = 3E_3^0 - 2E_2^0$

$= 3\ (-0.036) - 2(-0.44)$

$= +\ 0.772$ V

$E^0_{cell} = E^0_{Fe^{3+}/Fe^{2+}} + E^0_{I^-/I_2} = 0.233$

$\Delta_r G^0 = -\ 2 \times 96.5 \times 0.233 = -45$ kJ

10. Complete combustion of 3 g of ethane gives $x \times 10^{22}$ molecules of water. The value of x is _____. (Round off to the Nearest Integer).

[Use : $N_A = 6.023 \times 10^{23}$; Atomic masses in u : C : 12.0 ; O : 16.0 ; H : 1.0]

Official Ans. by NTA (18)

Sol.

C_2H_6	$\rightarrow$	$3H_2O$
0.1 mol		$0.3 = 0.3 \times 6 \times 10^{23} = 18 \times 10^{22}$ mol

No. of molecules $= 0.3 \times 6.023 \times 10^{23}$

$= 18.069 \times 10^{22}$

FINAL JEE–MAIN EXAMINATION – MARCH, 2021

(Held On Thursday 18th March, 2021) TIME : 9 : 00 AM to 12 : 00 NOON

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The differential equation satisfied by the system of parabolas $y^2 = 4a(x + a)$ is :

(1) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) - y = 0$

(2) $y\left(\frac{dy}{dx}\right)^2 - 2x\left(\frac{dy}{dx}\right) + y = 0$

(3) $y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$

(4) $y\left(\frac{dy}{dx}\right) + 2x\left(\frac{dy}{dx}\right) - y = 0$

Official Ans. by NTA (3)

Sol. $y^2 = 4ax + 4a^2$

differentiate with respect to x

$$\Rightarrow 2y\frac{dy}{dx} = 4a$$

$$\Rightarrow a = \left(\frac{y}{2}\frac{dy}{dx}\right)$$

so, required differential equation is

$$y^2 = \left(4 \times \frac{y}{2}\frac{dy}{dx}\right)x + 4\left(\frac{y}{2}\frac{dy}{dx}\right)^2$$

$$\Rightarrow y^2\left(\frac{dy}{dx}\right)^2 + 2xy\left(\frac{dy}{dx}\right) - y^2 = 0$$

$$\Rightarrow y\left(\frac{dy}{dx}\right)^2 + 2x\left(\frac{dy}{dx}\right) - y = 0$$

2. The number of integral values of m so that the abscissa of point of intersection of lines $3x + 4y = 9$ and $y = mx + 1$ is also an integer, is :

(1) 1 (2) 2 (3) 3 (4) 0

Official Ans. by NTA (2)

Sol. $3x + 4y = 9$

$y = mx + 1$

$\Rightarrow 3x + 4mx + 4 = 9$

$\Rightarrow (3 + 4m)x = 5$

$\Rightarrow$ x will be an integer when

$3 + 4m = 5, -5, 1, -1$

$$\Rightarrow m = \frac{1}{2}, -2, -\frac{1}{2}, -1$$

so, number of integral values of m is 2

3. Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to

(1) $2^{20}(2^{20} - 21)$

(2) $2^{19}(2^{20} - 21)$

(3) $2^{19}(2^{20} + 21)$

(4) $2^{20}(2^{20} + 21)$

Official Ans. by NTA (2)

Sol. $(1 + x + 2x^2)^{20} = a_0 + a_1x + ... + a_{40}x^{40}$ put x = 1, – 1

$\Rightarrow a_0 + a_1 + a_2 + ... + a_{40} = 2^{20}$

$a_0 - a_1 + a_2 + ... + a_{40} = 2^{20}$

$$\Rightarrow a_1 + a_3 + ... + a_{39} = \frac{4^{20} - 2^{20}}{2}$$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$

here $a_{39} = \frac{20!(2)^{19} \times 1}{19!} = 20 \times 2^{19}$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{19}(2^{20} - 1 - 20)$

$= 2^{19}(2^{20} - 21)$

4. The solutions of the equation

$$\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0, (0 < x < \pi),$$ are

(1) $\frac{\pi}{12}, \frac{\pi}{6}$ (2) $\frac{\pi}{6}, \frac{5\pi}{6}$

(3) $\frac{5\pi}{12}, \frac{7\pi}{12}$ (4) $\frac{7\pi}{12}, \frac{11\pi}{12}$

Official Ans. by NTA (4)

Sol. $\begin{vmatrix} 1+\sin^2 x & \sin^2 x & \sin^2 x \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$

use $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow (2+4\sin 2x)\begin{vmatrix} 1 & 1 & 1 \\ \cos^2 x & 1+\cos^2 x & \cos^2 x \\ 4\sin 2x & 4\sin 2x & 1+4\sin 2x \end{vmatrix} = 0$$

$$\Rightarrow \sin 2x = -\frac{1}{2}$$

$$\Rightarrow 2x = \pi + \frac{\pi}{6}, 2\pi - \frac{\pi}{6}$$

$$x = \frac{\pi}{2} + \frac{\pi}{12}, \pi - \frac{\pi}{12}$$

5. Choose the correct statement about two circles whose equations are given below :

$x^2 + y^2 - 10x - 10y + 41 = 0$

$x^2 + y^2 - 22x - 10y + 137 = 0$

(1) circles have same centre

(2) circles have no meeting point

(3) circles have only one meeting point

(4) circles have two meeting points

Official Ans. by NTA (3)

Sol. $x^2 + y^2 - 10x - 10y + 41 = 0$

A(5,5), $R_1 = 3$

$x^2 + y^2 - 22x - 10y + 137 = 0$

B(11,5), $R_2 = 3$

$AB = 6 = R_1 + R_2$

Touch each other externally

$\Rightarrow$ circles have only one meeting point.

6. Let α,β,γ be the real roots of the equation, $x^3 + ax^2 + bx + c = 0$, $(a,b,c \in R$ and $a,b \neq 0)$. If the system of equations (in, u,v,w) given by $\alpha u + \beta v + \gamma w = 0$, $\beta u + \gamma v + \alpha w = 0$; $\gamma u + \alpha v + \beta w = 0$ has non-trivial solution, then the value of $\frac{a^2}{b}$ is

(1) 5 (2) 3 (3) 1 (4) 0

Official Ans. by NTA (2)

Sol. $\begin{vmatrix} \alpha & \beta & \gamma \\ \beta & \gamma & \alpha \\ \gamma & \alpha & \beta \end{vmatrix} = 0$

$\Rightarrow -(\alpha + \beta + \gamma)(\alpha^2 + \beta^2 + \gamma^2 - \sum\alpha\beta) = 0$

$\Rightarrow -(-a)(a^2 - 2b - b) = 0$

$\Rightarrow a(a^2 - 3b) = 0$

$\Rightarrow a^2 = 3b \Rightarrow \frac{a^2}{b} = 3$

7. The integral $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{4x^2-4x+6}}dx$ is equal to

(where c is a constant of integration)

(1) $\frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

(2) $\frac{1}{2}\cos\sqrt{(2x+1)^2+5}+c$

(3) $\frac{1}{2}\cos\sqrt{(2x-1)^2+5}+c$

(4) $\frac{1}{2}\sin\sqrt{(2x+1)^2+5}+c$

Official Ans. by NTA (1)

Sol. $\int \frac{(2x-1)\cos\sqrt{(2x-1)^2+5}}{\sqrt{(2x-1)^2+5}}dx$

$(2x-1)^2 + 5 = t^2$

$2(2x-1)\,2dx = 2t\,dt$

$2\sqrt{t^2-5}dx = t\,dt$

So $\int \frac{\sqrt{t^2-5}\cos t}{2\sqrt{t^2-5}}dt = \frac{1}{2}\sin t + c$

$= \frac{1}{2}\sin\sqrt{(2x-1)^2+5}+c$

8. The equation of one of the straight lines which passes through the point (1,3) and makes an angles $\tan^{-1}\left(\sqrt{2}\right)$ with the straight line, $y+1=3\sqrt{2}\,x$ is

(1) $4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$

(2) $5\sqrt{2}x+4y-\left(15+4\sqrt{2}\right)=0$

(3) $4\sqrt{2}x+5y-4\sqrt{2}=0$

(4) $4\sqrt{2}x-5y-\left(5+4\sqrt{2}\right)=0$

Official Ans. by NTA (1)

Sol. $y = mx + c$

$3 = m + c$

$$\sqrt{2}=\left|\frac{m-3\sqrt{2}}{1+3\sqrt{2}m}\right|$$

$$=6m+\sqrt{2}=m-3\sqrt{2}$$

$$=\sin=-4\sqrt{2}\ \rightarrow\ m=\frac{-4\sqrt{2}}{5}$$

$$=\ =6m-\sqrt{2}=m-3\sqrt{2}$$

$$=7m-2\sqrt{2}\ \rightarrow\ m=\frac{2\sqrt{2}}{7}$$

According to options take $m=\frac{-4\sqrt{2}}{5}$

So $y=\frac{-4\sqrt{2}x}{5}+\frac{3+4\sqrt{2}}{5}$

$$4\sqrt{2}x+5y-\left(15+4\sqrt{2}\right)=0$$

9. If $\lim\limits_{x\to0}\frac{\sin^{-1}x-\tan^{-1}x}{3x^3}$ is equal to L, then the value of (6L + 1) is

(1) $\frac{1}{6}$ (2) $\frac{1}{2}$

(3) 6 (4) 2

Official Ans. by NTA (4)

Sol. $$\lim_{x\to0}\frac{\left(x+\frac{x^3}{3!}...\right)-\left(x-\frac{x^3}{3}...\right)}{3x^3}=\frac{1}{6}$$

So 6L + 1 = 2

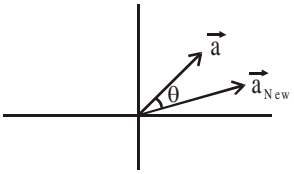
10. A vector $\vec{a}$ has components 3p and 1 with respect to a rectangular cartesian system. This system is rotated through a certain angle about the origin in the counter clockwise sense. If, with respect to new system, $\vec{a}$ has components p + 1 and $\sqrt{10}$, then a value of p is equal to:

(1) 1 (2) $-\frac{5}{4}$ (3) $\frac{4}{5}$ (4) –1

Official Ans. by NTA (4)

Sol. $\vec{a}_{Old}=3p\hat{i}+\hat{j}$

$\vec{a}_{New}=(p+1)\hat{i}+\sqrt{10}\hat{j}$



$\Rightarrow|\vec{a}_{Old}|=|\vec{a}_{New}|$

$\Rightarrow ap^2 + 1 = p^2 + 2p + 1 + 10$

$8p^2 - 2p - 10 = 0$

$4p^2 - p - 5 = 0$

$(4p - 5)(p + 1) = 0 \rightarrow p = \frac{5}{4}, -1$

11. If the equation $a|z|^2+\overline{\bar{\alpha}z+\alpha\bar{z}}+d=0$ represents a circle where a,d are real constants then which of the following condition is correct ?

(1) $|\alpha|^2-ad\neq0$

(2) $|\alpha|^2 - ad > 0$ and $a \in R - \{0\}$

(3) $|\alpha|^2 - ad \geq 0$ and $a \in R$

(4) $\alpha = 0$, $a,d \in R^+$

Official Ans. by NTA (2)

Sol. $az\bar{z}+\alpha\bar{z}+\bar{\alpha}z+d=0\ \rightarrow$ Circle

centre = $\frac{-\alpha}{a}$ $\qquad 2=\sqrt{\frac{\alpha\bar{\alpha}}{a^2}-\frac{d}{a}}=\sqrt{\frac{\alpha\bar{\alpha}-ad}{a^2}}$

So $|\alpha|^2 - ad > 0$ & $a \in R - \{0\}$

12. For the four circles M, N, O and P, following four equations are given :

Circle M : $x^2 + y^2 = 1$

Circle N : $x^2 + y^2 - 2x = 0$

Circle O : $x^2 + y^2 - 2x - 2y + 1 = 0$

Circle P : $x^2 + y^2 - 2y = 0$

If the centre of circle M is joined with centre of the circle N, further centre of circle N is joined with centre of the circle O, centre of circle O is joined with the centre of circle P and lastly, centre of circle P is joined with centre of circle M, then these lines form the sides of a :

(1) Rhombus

(2) Square

(3) Rectangle

(4) Parallelogram

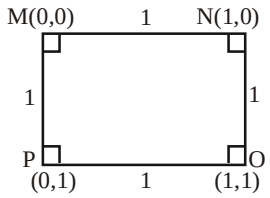
Official Ans. by NTA (2)

Sol. M : $x^2 + y^2 = 1$ (0,0)

N : $x^2 + y^2 - 2x = 0$ (1,0)

O : $x^2 + y^2 - 2x - 2y + 1 = 0$ (1,1)

P : $x^2 + y^2 - 2y = 0$ (0,1)



13. If α, β are natural numbers such that $100^{\alpha} - 199\beta = (100)(100) + (99)(101) + (98)(102) + \ldots + (1)(199)$, then the slope of the line passing through (α,β) and origin is :

(1) 540

(2) 550

(3) 530

(4) 510

Official Ans. by NTA (2)

Sol. $S = (100)(100) + (99)(101) + (98)(102) \ldots$

$$\ldots (2)(198) + (1)(199)$$

$$S = \sum_{x=0}^{99} (100 - x)(100 + x) = \sum 100^2 - x^2$$

$$= 100^3 - \frac{99 \times 100 \times 199}{6}$$

$\alpha = 3$ $\beta = 1650$

$$\text{slope} = \frac{1650}{3} = 550$$

14. The real valued function $f(x) = \frac{\text{cosec}^{-1} x}{\sqrt{x - [x]}}$, where [x] denotes the greatest integer less than or equal to x, is defined for all x belonging to :

(1) all reals except integers

(2) all non-integers except the interval [–1,1]

(3) all integers except 0,–1,1

(4) all reals except the Interval [–1,1]

Official Ans. by NTA (2)

Sol. $f(x) = \frac{\text{cosec}^{-1} x}{\sqrt{\{x\}}}$

Domain $\in (-\infty, -1] \cup [1, \infty)$

$\{x\} \neq 0$ so $x \neq$ integers

15. $\frac{1}{3^2 - 1} + \frac{1}{5^2 - 1} + \frac{1}{7^2 - 1} + \ldots + \frac{1}{(201)^2 - 1}$ is equal to

(1) $\frac{101}{404}$ (2) $\frac{25}{101}$ (3) $\frac{101}{408}$ (4) $\frac{99}{400}$

Official Ans. by NTA (2)

Sol. $T_n = \frac{1}{(2n+1)^2 - 1} \frac{1}{(2n+2)2n} = \frac{1}{4(n)(n+1)}$

$$= \frac{(n+1) - n}{4n(n+1)} = \frac{1}{4}\left(\frac{1}{n} - \frac{1}{n+1}\right)$$

$$S = \frac{1}{4}\left(1 - \frac{1}{101}\right) = \frac{1}{4}\left(\frac{100}{101}\right) = \frac{25}{101}$$

16. If the functions are defined as $f(x)=\sqrt{x}$ and $g(x)=\sqrt{1-x}$, then what is the common domain of the following functions :

$f + g$, $f - g$, f/g, g/f, $g - f$ where $(f \pm g)(x) = f(x) \pm g(x)$, $(f/g)(x) = \dfrac{f(x)}{g(x)}$

(1) $0 \le x \le 1$
(2) $0 \le x < 1$
(3) $0 < x < 1$
(4) $0 < x \le 1$

Official Ans. by NTA (3)

Sol. $f(x) + g(x) = \sqrt{x} + \sqrt{1-x}$, domain [0, 1]

$f(x) - g(x) = \sqrt{x} - \sqrt{1-x}$, domain [0, 1]

$g(x) - f(x) = \sqrt{1-x} - \sqrt{x}$, domain [0, 1]

$\dfrac{f(x)}{g(x)} = \dfrac{\sqrt{x}}{\sqrt{1-x}}$, domain [0, 1)

$\dfrac{g(x)}{f(x)} = \dfrac{\sqrt{1-x}}{\sqrt{x}}$, domain (0, 1]

So, common domain is (0, 1)

17. If $f(x)=\begin{cases} \dfrac{1}{|x|} & ; \ |x| \ge 1 \\ ax^2+b & ; \ |x|<1 \end{cases}$ is differentiable at every point of the domain, then the values of a and b are respectively :

(1) $\dfrac{1}{2}, \dfrac{1}{2}$

(2) $\dfrac{1}{2}, -\dfrac{3}{2}$

(3) $\dfrac{5}{2}, -\dfrac{3}{2}$

(4) $-\dfrac{1}{2}, \dfrac{3}{2}$

Official Ans. by NTA (4)

Sol. $f(x)=\begin{cases} \dfrac{1}{|x|}, & |x| \ge 1 \\ ax^2+b, & |x|<1 \end{cases}$

at x = 1 function must be continuous

So, $1 = a + b$ ...(1)

differentiability at x = 1

$$\left(-\frac{1}{x^2}\right)_{x=1} = (2ax)_{x=1}$$

$$\Rightarrow -1 = 2a \Rightarrow a = -\frac{1}{2}$$

$$(1) \Rightarrow b = 1 + \frac{1}{2} = \frac{3}{2}$$

18. Let $A+2B=\begin{bmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{bmatrix}$

and $2A-B=\begin{bmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{bmatrix}$. If Tr(A) denotes the sum of all diagonal elements of the matrix A, then Tr(A) – Tr(B) has value equal to

(1) 1 (2) 2 (3) 0 (4) 3

Official Ans. by NTA (2)

Sol. $A+2B=\begin{pmatrix} 1 & 2 & 0 \\ 6 & -3 & 3 \\ -5 & 3 & 1 \end{pmatrix}$ ...(1)

$$2A-B=\begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

$$\Rightarrow 4A - 2B = \begin{pmatrix} 4 & -2 & 10 \\ 4 & -2 & 12 \\ 0 & 2 & 4 \end{pmatrix} \quad ...(2)$$

$$(1) + (2) \Rightarrow 5A = \begin{pmatrix} 5 & 0 & 10 \\ 10 & -5 & 15 \\ -5 & 5 & 5 \end{pmatrix}$$

$$A = \begin{pmatrix} 1 & 0 & 2 \\ 2 & -1 & 3 \\ -1 & 1 & 1 \end{pmatrix} \text{ and } 2A = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix}$$

$$\therefore B = \begin{pmatrix} 2 & 0 & 4 \\ 4 & -2 & 6 \\ -2 & 2 & 2 \end{pmatrix} - \begin{pmatrix} 2 & -1 & 5 \\ 2 & -1 & 6 \\ 0 & 1 & 2 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 & 1 & -1 \\ 2 & -1 & 0 \\ -2 & 1 & 0 \end{pmatrix}$$

$$\text{tr}(A) = 1 - 1 + 1 = 1$$
$$\text{tr}(B) = -1$$

tr(A) = 1 and tr(B) = –1

$\therefore$ tr(A) – tr(B) = 2

19. The sum of all the 4-digit distinct numbers that can be formed with the digits 1, 2, 2 and 3 is:

(1) 26664 (2) 122664

(3) 122234 (4) 22264

Official Ans. by NTA (1)

Sol. Digits are 1, 2, 2, 3

total distinct numbers $\frac{4!}{2!} = 12$.

total numbers when 1 at unit place is 3.
2 at unit place is 6
3 at unit place is 3.

So, sum $= (3 + 12 + 9)(10^3 + 10^2 + 10 + 1)$
$= (1111) \times 24$
$= 26664$

20. The value of $3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + ...\infty}}}}$ is equal to

(1) $1.5 + \sqrt{3}$ (2) $2 + \sqrt{3}$

(3) $3 + 2\sqrt{3}$ (4) $4 + \sqrt{3}$

Official Ans. by NTA (1)

Sol. Let $x = 3 + \cfrac{1}{4 + \cfrac{1}{3 + \cfrac{1}{4 + \cfrac{1}{3 + ...\infty}}}}$

So, $x = 3 + \cfrac{1}{4 + \cfrac{1}{x}} = 3 + \cfrac{1}{\cfrac{4x+1}{x}}$

$$\Rightarrow (x - 3) = \frac{x}{(4x+1)}$$

$\Rightarrow (4x + 1)(x - 3) = x$

$\Rightarrow 4x^2 - 12x + x - 3 = x$

$\Rightarrow 4x^2 - 12x - 3 = 0$

$$x = \frac{12 \pm \sqrt{(12)^2 + 12 \times 4}}{2 \times 4} = \frac{12 \pm \sqrt{12(16)}}{8}$$

$$= \frac{12 \pm 4 \times 2\sqrt{3}}{8} = \frac{3 \pm 2\sqrt{3}}{2}$$

$$x = \frac{3}{2} \pm \sqrt{3} = 1.5 \pm \sqrt{3}.$$

But only positive value is accepted

So, $x = 1.5 + \sqrt{3}$

SECTION-B

1. The number of times the digit 3 will be written when listing the integers from 1 to 1000 is

Official Ans. by NTA (300)

Sol. 3_ _ = 10 × 10 = 100

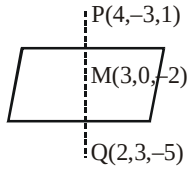
_3 _ = 10 × 10 = 100

_ _ 3 = 10 × 10 = 100

2. Let the plane ax + by + cz + d = 0 bisect the line joining the points (4,–3,1) and (2, 3, –5) at the right angles. If a, b, c, d are integers, then the minimum value of $(a^2 + b^2 + c^2 + d^2)$ is

Official Ans. by NTA (28)

Sol.



Plane is $1(x - 3) - 3(y - 0) + 3(z + 2) = 0$

$x - 3y + 3z + 3 = 0$

$(a^2 + b^2 + c^2 + d^2)_{min} = 28$

3. Let $f(x)$ and $g(x)$ be two functions satisfying $f(x^2) + g(4 - x) = 4x^3$ and $g(4 - x) + g(x) = 0$, then the value of $\int_{-4}^{4} f(x)^2 \, dx$ is

Official Ans. by NTA (512)

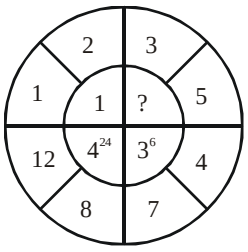
Sol. $I = 2\int_0^4 f(x^2)dx$ {Even funtion}

$$= 2\int_0^4 \left(4x^3 - g(4-x)\right)dx$$

$$= 2\left(\frac{4x^4}{4}\Big|_0^4 - \int_0^4 g(4-x)dx\right)$$

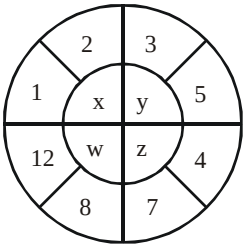
$= 2(256 - 0) = 512$

4. The missing value in the following figure is



Official Ans. by NTA (4)

Sol. $x = (2 - 1)^{1!} = 1$



$w = (12 - 8)^{4!} = 4^{24}$

$z = (7 - 4)^{3!} = 3^6$

hence $y = (5 - 3)^{2!} = 2^2$

5. Let z_1, z_2 be the roots of the equation $z^2 + az + 12 = 0$ and z_1, z_2 form an equilateral triangle with origin. Then, the value of |a| is

Official Ans. by NTA (6)

Sol. If $0, z, z_2$ are vertices of equilateral triangles

$\Rightarrow a^2 + z_1^2 + z_2^2 = 0\ (z_1 + z_2) + z_1 z_2$

$\Rightarrow (z_1 + z_2)^2 = 3z_1 z_2$

$\Rightarrow a^2 = 3 \times 12$

$\Rightarrow |a| = 6$

6. The equation of the planes parallel to the plane x – 2y + 2z – 3 = 0 which are at unit distance from the point (1, 2, 3) is ax + by + cz + d =0. If (b – d) = K(c – a), then the positive value of K is

Official Ans. by NTA (4)

Sol. Let plane is $x - 2y + 2z + \lambda = 0$

distance from (1,2,3) = 1

$$\Rightarrow \frac{|\lambda + 3|}{5} = 1 \Rightarrow \lambda = 0, -6$$

$\Rightarrow a = 1, b = -2, c = 2, d = -6$ or 0

$b - d = 4$ or -2, $c - a = 1$

$\Rightarrow k = 4$ or -2

7. The mean age of 25 teachers in a school is 40 years. A teacher retires at the age of 60 years and a new teacher is appointed in his place. If the mean age of the teachers in this school now is 39 years, then the age (in years) of the newly appointed teacher is_.

Official Ans. by NTA (35)

Sol. $\frac{\sum x_i}{25} = 40 \;\&\; \frac{\sum x_i - 60 + N}{25} = 39$

Let age of newly appointed teacher is N

$\Rightarrow 1000 - 60 + N = 975$

$\Rightarrow N = 35$ years

8. If $f(x) = \int \frac{5x^8 + 7x^6}{\left(x^2 + 1 + 2x^7\right)^2} dx, (x \geq 0), f(0) = 0$

and $f(1) = \frac{1}{K}$, then the value of K is

Official Ans. by NTA (4)

Sol. $f(x) = \int \frac{\left(5x^8 + 7x^6\right)dx}{x^{14}\left(x^{-5} + x^{-7} + 2\right)^2}$

Let $x^{-5} + x^{-7} + 2 = t$

$(-5x^{-6} - 7x^{-8})dx = dt$

$\Rightarrow f(x) = \int -\frac{dt}{t^2} = \frac{1}{t} + c$

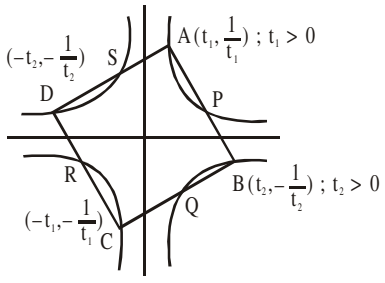
$f(x) = \frac{x^7}{x^2 + 1 + 2x^7}$

$f(1) = \frac{1}{4}$

9. A square ABCD has all its vertices on the curve $x^2y^2 = 1$. The midpoints of its sides also lie on the same curve. Then, the square of area of ABCD is

Official Ans. by NTA (80)

Sol. $xy = 1, -1$



$\frac{t_1 + t_2}{2} \cdot \frac{\frac{1}{t_1} - \frac{1}{t_2}}{2} = 1$

$\Rightarrow t_1^2 - t_2^2 = 4t_1t_2$

$\frac{1}{t_1^2} \times \left(-\frac{1}{t_2^2}\right) = -1 \Rightarrow t_1t_2 = 1$

$\Rightarrow (t_1t_2)^2 = 1 \Rightarrow t_1t_2 = 1$

$t_1^2 - t_2^2 = 4$

$\Rightarrow t_1^2 + t_2^2 = \sqrt{4^2 + 4} = 2\sqrt{5}$

$\Rightarrow t_1^2 = 2 + \sqrt{5} \Rightarrow \frac{1}{t_1^2} = \sqrt{5} - 2$

$AB^2 = (t_1 - t_2)^2 + \left(\frac{1}{t_1} + \frac{1}{t_2}\right)^2$

$= 2\left(t_1^2 + \frac{1}{t_1^2}\right) = 4\sqrt{5} \Rightarrow \text{Area}^2 = 80$

10. The number of solutions of the equation $|\cot x| = \cot x + \frac{1}{\sin x}$ in the interval $[0, 2\pi]$ is

Official Ans. by NTA (1)

Sol. If $\cot x > 0 \Rightarrow \frac{1}{\sin x} = 0$ (Not possible)

If $\cot x < 0 \Rightarrow 2\cot x + \frac{1}{\sin x} = 0$

$\Rightarrow 2\cos x = -1$

$\Rightarrow x = \frac{2\pi}{3} \text{or} \frac{4\pi}{3}$ (reject)