## JEE-Main-20-07-2021-Shift-1 (Memory Based)

## PHYSICS

Question: A butterfly in North East direction with a velocity of $4 \sqrt{2} \mathrm{~m} / \mathrm{s}$. Wind is blowing from North to South with a velocity of $1 \mathrm{~m} / \mathrm{s}$. Find the displacement of the bird in three seconds.

## Options:

(a) $15 \mathrm{~m}, 37^{\circ}$ North to East
(b) $15 \mathrm{~m}, 37^{\circ}$ East to North
(c) $15 \mathrm{~m}, 37^{\circ}$ North to West
(d) None of these

Answer: (a)

## Solution:


$\vec{V}_{\text {butt }}=4 m / s \hat{i}+4 m / s \hat{j}$
$\vec{V}_{\text {wind }}=-1 m / s \hat{j}$
$\vec{V}_{\text {butt wind }}=4 m / s \hat{i}+3 m / s \hat{j}$
Displacement of bird in three second.
$D=\sqrt{(4 \times 3)^{2}+(3 \times 3)^{2}}$
D $=15 \mathrm{~m}$
Direction $\theta=\tan ^{-1}\left(\frac{3}{4}\right)$
$\theta=37^{\circ}$
$\therefore 15 \mathrm{~m}, 37^{\circ}$ North to East.

Question: A travelling wave moving with a velocity v. The equation of wave at two different instants $\mathrm{t}=0$ and $\mathrm{t}=3$ is given by $y=\frac{1}{1+x^{2}}$ and $y=\frac{1}{1+(x+1)^{2}}$ respectively. Find the speed v of the wave. ( y is in $\mathrm{mm}, \mathrm{x}$ is in cm )
Options:
(a) $\frac{1}{3} \mathrm{~mm} / \mathrm{s}$
(b) $3 \mathrm{~mm} / \mathrm{s}$
(c) $\frac{1}{3} \mathrm{~cm} / \mathrm{s}$
(d) $3 \mathrm{~cm} / \mathrm{s}$

Answer: (c)

## Solution:

$y(x, 0)=\frac{1}{1+x^{2}}$
$y(x, 3)=\frac{1}{1+(x+1)^{2}}$
If v is the velocity of the wave, then,
$y(x, t)=y(x+v t, 0)$
Therefore,
$y(x, 3)=(x+3 v, 0)$
Replacing values we get
$\frac{1}{1+(x+1)^{2}}=\frac{1}{1+(x+3 v)^{2}}$
$x+1=x+3 v$
$x=+\frac{1}{3} \mathrm{~cm} / \mathrm{s}$

Question: A block of mass 20 kg is placed on a horizontal platform and three blocks each with mass 10 kg are arranged as given in figure below. If platform accelerated downward with acceleration $2 \mathrm{~m} / \mathrm{s}^{2}$, then the normal force between 10 kg and 20 kg block is-


## Options:

(a) 100 N
(b) 150 N
(c) 120 N
(d) 140 N

Answer: (c)

## Solution:



20 kg is rest w.r.t platform.
$2 N=200+20 \times 2$
$2 N=240 N$
$N=120 N$

Question: A ball having charge to mass $8 \mu c / \mathrm{g}$ is placed at a distance of 10 cm from a wall. Suddenly an Electric field $100 \mathrm{Nm}^{-1}$ is switched on. Assuming collisions to be Elastic. Find Time period of oscillations.


Options:
(a) 0.5 sec
(b) 1 sec
(c) $\sqrt{2} \mathrm{sec}$
(d) $\sqrt{3} \mathrm{sec}$

Answer: ()

## Solution:

$\frac{q}{m}=8 \mu c / g$

$F=q E$
$a=\frac{q E}{m}$
$a=\frac{8 \times 10^{-6}}{1 \times 10^{-3}} \times 100$
$a=0.8 \mathrm{~m} / \mathrm{s}^{2}$
Time period (T) $=2 \sqrt{\frac{2 S}{a}}$
$=2 \times \sqrt{\frac{2 \times 0.1}{8}}$
$\mathrm{T}=1 \mathrm{sec}$

Question: If an ideal gas is taken through process AB then find work done on gas by external agent. Given curve AB is an ellipse.


## Options:

(a) $200 \pi \mathrm{~J}$
(b) $-200 \pi J$
(c) $100 \pi J$
(d) $-100 \pi J$

Answer: (a)
Solution:

$W_{\text {external }}+W_{\text {gas }}=0$
$W_{\text {external }}=-W_{\text {gas }}$
$=-\left\{-\frac{\pi \times 20 \times 20}{2}\right\} J$
$W_{\text {external }}=200 \pi J$

Question: If $\vec{A} \cdot \vec{B}=|\vec{A} \times \vec{B}|$, find $|\vec{A}-\vec{B}|$

## Options:

(a) $\sqrt{A^{2}+B^{2}+2 A \cdot B}$
(b) $\sqrt{A^{2}+B^{2}+\sqrt{2} A B}$
(c) $\sqrt{A^{2}+B^{2}-\sqrt{2} A B}$
(d) $\sqrt{A^{2}+B^{2}}$

Answer: (c)

## Solution:

$\vec{A} \cdot \vec{B}=|\vec{A} \times \vec{B}|$
$A B \cos \theta=A B \sin \theta$
$\cos \theta=\sin \theta$
$\Rightarrow \theta=\frac{\pi}{4}$
then
$|\vec{A}-\vec{B}|=\sqrt{A^{2}+B^{2}-2 A B \cos 45^{\circ}}$
$|\vec{A}-\vec{B}|=\sqrt{A^{2}+B^{2}-\sqrt{2} A B}$

Question: Two charges are kept at a fixed distance from each other. The sum of both charges is Q . What should be charge on each of them in order to maximize force between them.

## Options:

(a) $\frac{Q}{2}, \frac{Q}{2}$
(b) $\frac{Q}{3}, \frac{2 Q}{3}$
(c) $\frac{2 Q}{3}, \frac{Q}{3}$
(d) $\frac{Q}{4}, \frac{3 Q}{4}$

## Answer: (a)

## Solution:

Let one charge be $q_{1}$ and other be $\theta-q_{1}$

$$
F=K \frac{q_{1}\left(Q-q_{1}\right)}{r_{2}}
$$

For maximum force $\frac{d F}{d q_{1}}=0$
and $\frac{d^{2} F}{d q_{1}^{2}}<0$
$\frac{d F}{d q_{1}}=\frac{K\left(Q-q_{1}\right)}{r^{2}}-\frac{K q_{1}}{r^{2}}$
$\frac{d F}{d q_{1}}=0$
$Q-q_{1}-q_{1}=0 \Rightarrow q_{1}=\frac{Q}{2}$
$\frac{d^{2} F}{d q_{1}^{2}}=-\frac{K}{r^{2}}-\frac{K}{r^{2}}=\frac{-2 K}{r^{2}}$
$-\frac{2 K}{r^{2}}<0$ for all values of $q_{1}$
So the value of $q_{1}=\frac{Q}{2}$ and $Q-q_{1}=\frac{Q}{2}$

Question: A car is moving towards a stationary wall making Horn of frequency 400 Hz . The Reflected frequency heard by driver of car is 500 Hz . Find speed of car [ $\mathrm{v}=$ speed of sound]


Options:
(a) $\frac{V}{3}$
(b) $\frac{V}{4}$
(c) $\frac{V}{9}$
(d) $\frac{V}{12}$

Answer: (c)

## Solution:


$f^{\prime}$ be the frequency heard at wall.
$f^{\prime}=\left(\frac{V_{S}}{V_{S}-V_{C}}\right) f$

Then this $f^{\prime}$ reflected back and the frequency heard by the driver is $f^{\prime \prime}$
$f^{\prime \prime}=\left(\frac{V_{S}+V_{C}}{V_{S}}\right) f^{\prime}$
From equation (1) and (2)
$f^{\prime \prime}=\left(\frac{V_{S}+V_{C}}{V_{S}-V_{C}}\right) f$
$f^{\prime \prime}=\left(\frac{V+V_{C}}{V-V_{C}}\right) f$
$500=\left(\frac{V+V_{C}}{V-V_{C}}\right) \times 400$
$\frac{5}{4}=\frac{V+V_{C}}{V-V_{C}}$
$5 V-5 V_{C}=4 V+4 V_{C}$
$9 V_{C}=V$
$V_{C}=\frac{V}{9}$

Question: Light emitted by hydrogen gas corresponding to transition from $\mathrm{n}=3$ to $\mathrm{n}=2$ incident on a metal plate. The electron emitted from metal plate with maximum kinetic energy enters a magnetic field $5 \times 10^{-4} T$ perpendicularly. If the radius of path of electron is 7 mm then find the work function of metal.

## Options:

(a) 0.91 eV
(b) 0.81 eV
(c) 0.01 eV
(d) 0.5 eV

Answer: (b)

## Solution:

Energy of photon emitted by Hydrogen atom is
$E=-13.6\left[\frac{1}{3^{2}}-\frac{1}{2^{2}}\right]$
$E=-13.6\left[\frac{1}{9}-\frac{1}{4}\right]$
$E=-13.6 \frac{(-5)}{36}=\frac{13.6 \times 5}{36} \mathrm{eV}$
$E=1.889 \mathrm{eV}$
Now we have equation for kinetic energy
$\left(\frac{1}{2} m v^{2}\right)_{\max }=h v-\phi$

If radius of electron R
$R=\frac{m v}{q B}$
Squaring both side
$R^{2}=\frac{m^{2} v^{2}}{q^{2} B^{2}}$
$R^{2}=\frac{2 m(K E)}{q^{2} B^{2}}$
$K E=\frac{q^{2} B^{2} R^{2}}{2 m}$
$K E=\frac{\left[1.6 \times 10^{19} \times 5 \times 10^{-4} \times 7 \times 10^{-3}\right]^{2}}{2 \times 9.1 \times 10^{-31}}$
$K E=\frac{3136 \times 10^{-52}}{2 \times 9.1 \times 10^{-31}}$
$K E=172.30 \times 10^{-21} J$
$K E=107.68 \times 10^{-2} \mathrm{eV}$
$K E=1.077 \mathrm{eV}$
From eq (1),(2) and (3)
$1.077=1.889-\phi$
$\phi=0.811 \mathrm{eV} \sim 0.81 \mathrm{eV}$

Question: Consider on equation $S=\alpha^{2} \beta \ln \left[\frac{n k R}{J \beta^{2}}+1\right]$
Where S = Enkopy
$\mathrm{n}=$ No. of moles
$\mathrm{k}=$ Bolzmann constant
R = Universal Gas constant
$\mathrm{J}=$ Mechanical equivalent of Heat
Then Dimensions of $\propto$ and $\beta$ respectively.

## Options:

(a) $\left[M^{0} L^{0} T^{0}\right],\left[M^{1} L^{2} T^{-2} K^{-1}\right]$
(b) $\left[M^{1} L^{2} T^{-2}\right],\left[M^{1} L^{2} T^{-2} K^{-1}\right]$
(c) $\left[M^{1} L^{2} T^{-2} K^{-1}\right],\left[M^{0} L^{0} T^{0}\right]$
(d) None of these

Answer: (a)

## Solution:

$[S]=\left[M L^{2} T^{-2} K^{-1}\right]$
$n=[\mathrm{mol}]$
$[k]=\left[M L^{2} T^{-2} K^{-1}\right]$
$[J]=\left[M^{0} L^{0} T^{0}\right]$
$[R]=\left[M^{1} L^{2} T^{-2} K^{-1} \mathrm{~mol}^{-1}\right]$
$S=\alpha^{2} \beta \ln \left[\frac{n k R}{J \beta^{2}}+1\right]$
The quantity inside $\log$ must be dimensionless.
$[n k R]=\left[J \beta^{2}\right]$
$[\mathrm{mol}]\left[M L^{2} \mathrm{~T}^{-2} \mathrm{~K}^{-1}\right]\left[M L^{2} \mathrm{~T}^{-2} \mathrm{~K}^{-1}\right]\left[\mathrm{mol}^{-1}\right]=[\beta]^{2}$
$\left[M L^{2} T^{-2} K^{-1}\right]^{2}=[\beta]^{2}$
$\beta=\left[M L^{2} T^{-2} K^{-1}\right]$
Now
$[S]=\left[\alpha^{2} \beta\right]$
$\left[M L^{2} T^{-2} K^{-1}\right]=\alpha^{2}\left[M L^{2} T^{-2} K^{-1}\right]$
$\alpha=\left[M^{0} L^{0} T^{0} K^{0}\right]$

Question: An alpha particle and a deuteron enter a region of magnetic field which is perpendicular to their velocity. If their kinetic energies are equal, then find the ratio of their radii.

## Options:

(a) $1: 1$
(b) $1: \sqrt{2}$
(c) $\sqrt{2}: 1$
(d) $4: 2$

Answer: (b)

## Solution:

$\frac{m v^{2}}{R}=q v B$
$R=\frac{m \nu}{q B}=\frac{\sqrt{2 m E}}{q B}$
$R_{d}=\frac{\sqrt{2 m_{d}} E}{q_{d} B}, R_{\alpha}=\frac{\sqrt{2 m_{\alpha} E}}{q_{\alpha} B}$

$$
R_{\alpha}: R_{d}=\frac{\sqrt{m_{\alpha}}}{q_{\alpha}}: \frac{\sqrt{m_{d}}}{q_{d}}=\frac{\sqrt{4}}{2}: \frac{\sqrt{2}}{1}=1: \sqrt{2}
$$

Question: A zener diode is used in a circuit as shown. Find the current 'I' passing through the zener diode.


## Options:

(a) 25 mA
(b) 50 mA
(c) 100 mA
(d) zero

Answer: (a)

## Solution:



For zener breakdown, potential difference across $2 k \Omega$ resistor will be 5.0 V
$V_{z}=50 \mathrm{~V}$
$i_{2}=\frac{V_{z}}{2 k}=\frac{50}{2 k}=25 \mathrm{~mA}$
$\Delta V$ across $1 \mathrm{k} \Omega=100-50=50 \mathrm{~V}$
$i_{1}=\frac{50}{1 k}=50 \mathrm{~mA}$
$i_{1}=i_{2}+i_{z}$
$i_{z}=50 \mathrm{~mA}-25 \mathrm{~mA}=25 \mathrm{~mA}$

Question: A current of 5 A is flowing through magnesium wire. The current density is making an angle of $60^{\circ}$ with Area vector. Find the electric field. (Given : Area $=2 \mathrm{~m}^{2}, \rho=$ Resistivity of Magnesium $=11 \times 10^{-4}$ SI units)

## Options:

(a) $55 \times 10^{-4} \mathrm{~V} / \mathrm{m}$
(b) $\frac{5}{11} \times 10^{-4} \mathrm{~V} / \mathrm{m}$
(c) $\frac{11}{5} \times 10^{-4} \mathrm{~V} / \mathrm{m}$
(d) $\frac{55}{2} \times 10^{-4} \mathrm{~V} / \mathrm{m}$

Answer: (a)

## Solution:

$E=J \rho$
$=\frac{I}{A \cos \theta} \rho$
$=\frac{5}{2 \cos 60} \times 11 \times 10^{-4}$
$=55 \times 10^{-4} \mathrm{~V} / \mathrm{m}$

Question: A body of mass emits a photon of frequency ' $v$ ', then loss in its internal energy is

## Options:

(a) $h v$
(b) $h \nu\left(1-\frac{h \nu}{2 m c^{2}}\right)$
(c) $h v\left(1+\frac{h v}{2 m c^{2}}\right)$
(d) zero

Answer: (c)

## Solution:

K.E. of body $=\left(\frac{1}{2}\right) m v^{2}=\left(\frac{1}{2}\right) m\left(\frac{E}{m} c\right)^{2}=\frac{E^{2}}{2 m c^{2}}$

Energy emitted by photon $=E$
Total decrease in internal energy $=E+\frac{E^{2}}{2 m c^{2}}$
$=E\left(1+\frac{E}{2 m c^{2}}\right)$
$=h v\left(1+\frac{h v}{2 m c^{2}}\right) \quad(a s E=h v)$
Question: Tension in a spring is $T_{1}$ when length of the spring is $L_{1}$ and tension is $T_{2}$ when its
length is $L_{2}$. The natural length of the spring is

## Options:

(a) $\frac{T_{2} l_{2}+T_{1} l_{1}}{T_{2}+T_{1}}$
(b) $\frac{T_{2} l_{2}-T_{1} l_{1}}{T_{2}-T_{1}}$
(c) $\frac{T_{2} l_{1}+T_{1} l_{2}}{T_{2}+T_{1}}$
(d) $\frac{T_{2} l_{1}-T_{1} l_{2}}{T_{2}-T_{1}}$

Answer: (a)

## Solution:

Let natural length of spring be L then on extending it by $x_{1}$ new length becomes $L_{1}$
$\stackrel{\text { nn }}{\rightleftarrows}$
-9000
$\leftarrow \mathrm{L} \rightarrow \leftarrow x_{2} \rightarrow$
$\longleftarrow \mathrm{L}_{2} \longrightarrow$
$T_{1}=K x_{1}$
$K=\frac{T_{1}}{x_{1}}$.
$L_{1}=L+x_{1}$
$x_{1}=L_{1}-L$
On extending it by $x_{2}$ new length becomes $L_{2}$
mon
$\longleftarrow L \longrightarrow$
mmo
$\stackrel{L}{\leftarrow} \stackrel{x_{1} \rightarrow}{\longrightarrow}$
$T_{2}=K x_{2}$
$K=\frac{T_{2}}{x_{2}}$.
$L_{2}=L+x_{2} \Rightarrow x_{2}=L_{2}-L$
Now spring constant remains same so from (1) and (2)
$\frac{T_{1}}{x_{1}}=\frac{T_{2}}{x_{2}} \Rightarrow \frac{T_{1}}{L_{1}-L}=\frac{T_{2}}{L_{2}-L}$
$T_{1} L_{2}-T_{1} L=T_{2} L_{1}-T_{2} L \Rightarrow L=\frac{T_{2} L_{2}}{T_{2}-T_{1}}$

Question: A person is standing on weighing machine and is slowly taken from the surface of Earth to the surface of mars. Given that the value of $\$ \mathrm{~g} \$$ on Earth is $10 \mathrm{~ms}^{-2}$ and that on Mars is $4 \mathrm{~ms}^{-2}$. Draw graph of weight vs distance from earth's surface

## Options:

(a)

(b)

(c)

(d)


Answer: (c)
Solution:

$$
g \propto \frac{1}{(\text { distance })^{2}} \cdots(1)
$$

Weight of the man goes from $W_{o}=m g$ on surface of earth to zero somewhere in the path (where gravitational
pull from both planets is equal to opposite) and finally to $\frac{W_{o}}{2.5}$ on surface of mass
g on earth $=10$
$g$ on mars $4=\frac{10}{2.5}=\frac{g}{2.5}$

From (1) the decline from $\mathrm{W}_{\mathrm{o}}$ to zero and rise from o to $\mathrm{W}_{\mathrm{o}} / 4$ is non-linear Hence (c) is the right answer.

Question: A chamber containing 4 moles of diatomic gas is heated such that the temperature of the gas increases from $0^{\circ} \mathrm{C}$ to $50^{\circ} \mathrm{C}$. Find the change in internal energy of the gas. Assume that molecules are rigid.

## Options:

(a) 500 R
(b) 400 R
(c) 300 R
(d) 50 R

Answer: (a)

## Solution:

Change in internal energy
$d u=n C_{v} d T$
$C_{v}$ for a diatomic gas is
$\frac{5}{2} R(\therefore$ temp given is not very high, so neglect vibrational degree of freedom)
Given $n=4 ; C_{v}=\frac{5}{2} R$ and $d T=50$
$\therefore d u=4(5 / 2 R)(50)$
$=500 \mathrm{R}$

Question: A particle of mass $\$ \mathrm{~m} \$$ and speed $\$ \mathrm{u} \$$ collides elastically with the end of a uniform rod of mass $\$ M \$$ and length $\$ L \$$ as shown. If the particle comes to rest after collision, find $\frac{m}{M}$


## Options:

(a) $\frac{1}{2}$
(b) 1
(c) $\frac{1}{3}$
(d) $\frac{1}{4}$

Answer: (a)

## Solution:

Conserving angular momentum about the hinge at the center

$\vec{L}$ before collision $=m u(L / 2)$
$\vec{L}$ after collision $=I \omega$
$\omega=\frac{u}{(L / 2)}$
$\therefore \mathrm{mu}\left(\frac{L}{2}\right)=\frac{M L^{2}}{12}\left(\frac{u}{L / 2}\right)$
$\frac{m u}{2}=\frac{M u}{6}$
$\therefore \frac{m}{m}=\frac{1}{3}$

Question: A radioactive material is undergoing simultaneous disintegration into two different products with half lives 1400 years and 700 years respectively. Find the time taken by the sample to decay to one third of its initial value

## Options:

(a) $\left(\frac{\ln 2}{\ln 3}, \frac{1400}{3}\right)$ years
(b) $\frac{\ln 3}{\ln 2}(1400)$ years
(c) $\frac{\ln 3}{\ln 2} \frac{1400}{3}$ years
(d) $\frac{\ln 3}{\ln 2}(700)$ years

Answer: (c)

## Solution:

We know that the number of atoms left after half-lives is given by

$$
N=N_{0}\left(\frac{1}{2}\right)^{n}
$$

where $\mathrm{n}=\mathrm{no}$ of half lives
Now decay constant
$(\lambda)=\left(\frac{\ln 2}{T_{1 / 2}}\right)$
For a material containing 2 radioactive substances effective disintegration constant ( $\lambda_{\text {eff }}$ )
$\lambda_{\text {eff }}=\lambda_{A}+\lambda_{B}$ [A and B are the substances]
$\Rightarrow \frac{\ln 2}{T_{e f f}}-\frac{\ln 2}{T_{1 / 2 A}}+\frac{\ln 2}{T_{1 / 2 B}}$
$\Rightarrow \frac{1}{T_{e f f}}=\frac{1}{T_{1 / 2 A}}+\frac{1}{T_{1 / 2 B}}$
Here $T_{e f f}=$ Effective or equivalent half-life of the material.
Given $T_{1 / 2 A}=1400 \mathrm{yrs}$
$\& T_{1 / 2 B}=700 \mathrm{yrs}$
We get
$\mathrm{T}_{\text {eff }}=\frac{1400 \times 700}{1400+700}$
$=\frac{1400}{3} y r s$.
$\therefore$ no. of half-lives $(n)=\frac{t}{(1400 / 3)}$
where $t$ is time required for material to become one-third.
Now, $N=N_{o}\left(\frac{1}{2}\right)^{n}$
$\frac{N}{N_{o}}=\left(\frac{1}{2}\right)^{t /(1400 / 3)}$
$\Rightarrow\left(\frac{1}{3}\right)=\left(\frac{1}{2}\right)^{t /(1400 / 3)}$
$\Rightarrow 3=2^{3 t / 1400}$
$\Rightarrow \ln 3=(3 t / 1400) \ln 2 \Rightarrow t=\frac{\ln 3}{\ln 2}\left(\frac{1400}{3}\right) y r s$

## JEE-Main-20-07-2021-Shift-1 (Memory Based)

## CHEMISTRY

Question: 10000 kJ energy is needed per day and heat of combustion $2700 \mathrm{~kJ} / \mathrm{mol}$, then find the grams of glucose needed?

## Options:

(a) 666.67
(b) 650.33
(c) 459.50
(d) 576.62

Answer: (a)

## Solution:

2700 kJ of energy $\rightarrow 180 \mathrm{~g}$ of glucose
1 kJ of energy $\rightarrow \frac{180}{2700} \mathrm{~g}$ of glucose
10000 kJ of energy $\rightarrow \frac{180}{2700} \times 10000=666.67 \mathrm{~g}$

Question: The difference in energy between the 2 nd and 3 rd orbit of $\mathrm{He}^{+}$ion will be?
Options:
(a) 5.34 eV
(b) 2.01 eV
(c) 7.54 eV
(d) 9.24 eV

Answer: (c)
Solution: Energy in $2^{\text {nd }}$ orbit $=-13.6 \frac{\mathrm{z}^{2}}{\mathrm{n}^{2}}=-13.6 \frac{2^{2}}{2^{2}}=-13.6 \mathrm{eV}$
Energy in $3^{\text {nd }}$ orbit $=-13.6 \frac{\mathrm{z}^{2}}{\mathrm{n}^{2}}=-13.6 \frac{2^{2}}{3^{2}}=-13.6 \mathrm{eV} \times \frac{4}{9}=-6.04 \mathrm{eV}$
$\therefore$ Energy difference $=\mathrm{E}_{3}-\mathrm{E}_{2}=7.54 \mathrm{eV}$

Question: Tyndall effect conditions

## Options:

(a) The diameter of the dispersed particles is much smaller than the wavelength of the light used
(b) The refractive indices of the dispersed phase and the dispersion medium are same in magnitude
(c) The radius of the dispersed particles is much smaller than the wavelength of the light used
(d) The refractive indices of the dispersed phase and the dispersion medium is not differ greatly in magnitude
Answer: (a)
Solution: Tyndall effect is observed only when the following two conditions are satisfied. (i) The diameter of the dispersed particles is not much smaller than the wavelength of the light used; and (ii) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.


Question: Intensity of colour for $\mathrm{NiCl}_{4}, \mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right) 4, \mathrm{Ni}(\mathrm{CN})_{4}$
Options:
(a) $\mathrm{Ni}(\mathrm{CN})_{4}>\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}>\mathrm{NiCl}_{4}$
(b) $\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}>\mathrm{NiCl}_{4}>\mathrm{Ni}(\mathrm{CN})_{4}$
(c) $\mathrm{Ni}(\mathrm{CN})_{4}>\mathrm{NiCl}_{4}>\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$
(d) $\mathrm{NiCl}_{4}>\mathrm{Ni}(\mathrm{CN})_{4}>\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}$

Answer: (a)
Solution: Order of intensity: $\mathrm{Ni}(\mathrm{CN})_{4}>\mathrm{Ni}\left(\mathrm{H}_{2} \mathrm{O}\right)_{4}>\mathrm{NiCl}_{4}$
The intensity of colour depends on the strength of the ligand attached with the central metal atom. And strength of ligand is in the order:
$\mathrm{CN}^{-}>\mathrm{H}_{2} \mathrm{O}>\mathrm{Cl}^{-}$

Question: Azimuthal quantum number of valence electrons of $\mathrm{Ga}^{+1}$ is
(Atomic number of $\mathrm{Ga}=31$ )
Options:
(a) $l=0$
(b) $l=1$
(c) $l=2$
(d) $l=3$

Answer: (a)

## Solution:

Ga: $1 s^{2} 2 s^{2} 2 p^{6} 3 s^{2} 3 p^{6} 3 d^{10} 4 s^{2} 4 p^{1}$
Ga: $1 s^{2} 2 s^{2} 2 p^{6} 3 s^{2} 3 p^{6} 3 d^{10} 4 s^{2}$

$$
\begin{gathered}
\downarrow \\
l=0
\end{gathered}
$$

Question: Structure of ruhemann's purple from ninhydrin test Options:
(a)

(b)

(c)

(d) Both (a) and (c)

Answer: (d)

## Solution:

Ninhydrin test


Question: How many equivalents of $\mathrm{CH}_{3} \mathrm{MgBr}$ to make 2-methylpropan-2-ol from ethyl ethanoate?
Options:
(a) 4
(b) 3
(c) 5
(d) 2

Answer: (d)
Solution:


Question: s-block element having formula of oxide $\mathrm{MO}_{2}$, which is yellow and paramagnetic is

Options:
(a) Na
(b) K
(c) Ca
(d) Mg

Answer: (b)
Solution: $\mathrm{KO}_{2}$ (yellowish colour) $\rightarrow \mathrm{K}^{+}+\mathrm{O}_{2}^{-}$(paramagnetic)

Question: Which of the following can be purified using fractional distillation?

## Options:

(a) Fe
(b) Cu
(c) Zn
(d) Ni

Answer: (c)
Solution: Extraction of zinc oxide
The reduction of zinc oxide is done using coke. The temperature in this case is higher than that in case of copper. For the purpose of heating, the oxide is made into briquettes with coke and clay.
$\mathrm{ZnO}+\mathrm{C} \xrightarrow{\text { coke, } 1673 \mathrm{~K}} \mathrm{Zn}+\mathrm{CO}$
The metal is distilled off and collected by rapid chilling

Question: The hybridisation of Xenon in $\mathrm{XeOF}_{4}$ is

## Options:

(a) $\mathrm{sp}^{3}$
(b) $\mathrm{sp}^{3} \mathrm{~d}$
(c) $\mathrm{sp}^{3} \mathrm{~d}^{2}$
(d) $\operatorname{sp}^{3} \mathrm{~d}^{3}$

Answer: (c)

## Solution:



Hybridisation of Xe in $\mathrm{XeOF}_{4}$ is $\mathrm{sp}^{3} \mathrm{~d}^{2}$

Question: Number of lone pairs on central atom in $\mathrm{I}_{3}^{-}$

## Options:

(a) 2
(b) 1
(c) 0
(d) 3

Answer: (d)

## Solution:



Question: Which of these have different nature?

## Options:

(a) $\mathrm{Be}(\mathrm{OH})_{2}$ and $\mathrm{Al}(\mathrm{OH})_{3}$
(b) $\mathrm{B}(\mathrm{OH})_{3}$ and $\mathrm{Al}(\mathrm{OH})_{3}$
(c) $\mathrm{B}(\mathrm{OH})_{3}$ and $\mathrm{H}_{3} \mathrm{PO}_{3}$
(d) NaOH and $\mathrm{Ca}(\mathrm{OH})_{2}$

Answer: (b)

## Solution:

$\mathrm{Be}(\mathrm{OH})_{2} \& \mathrm{Al}(\mathrm{OH})_{3}$ are amphoteric.
$\mathrm{B}(\mathrm{OH})_{3} \& \mathrm{H}_{2} \mathrm{PO}_{3}$ are acidic.
$\mathrm{Ca}(\mathrm{OH})_{2} \& \mathrm{NaOH}$ are basic.

Question: Vapour pressure of benzene and methylbenzene are 70 and 20 respectively. Both are equimolar mixture. Find the mole fraction of benzene in vapour phase?

## Options:

(a) 0.33
(b) 0.77
(c) 0.50
(d) 0.66

## Answer: (b)

Solution: Gaseous phase
X 'Benzene $=$ ?
Liquid phase
$\mathrm{P}_{\mathrm{T}}=\mathrm{P}_{\text {Benzene }} \mathrm{X}_{\text {benzene }}+\mathrm{P}_{\text {methylbenzene }} \mathrm{X}_{\text {methylbenzene }}$
$=70 \times 1 / 2+20 \times 1 / 2$
$=35+10=45$
$X{ }^{\prime}{ }_{\text {Benzene }}=\frac{35}{45}=0.77$

Question: Orlon is
Options:
(a) polyamide
(b) polyester
(c) polyacrylonitrile
(d) Amino acid

Answer: (c)

## Solution:



Orlon is the name that is used in trade purposes for Polyacrylonitrile (PAN). It is a polymer of acrylonitrile which is commonly known as vinyl cyanide (VCN)

Question: An example of green chemistry in daily life:
Options:
(a) $\mathrm{CO}_{2}$ in cleaning clothes
(b) $\mathrm{Cl}_{2}$ for bleaching
(c) Tetrafluoroethane in laundry
(d) All of these

Answer: (a)

Solution: While $\mathrm{CO}_{2}$ is a main greenhouse gas, no new $\mathrm{CO}_{2}$ is generated with this technology, so it does not contribute to global warming. Liquid $\mathrm{CO}_{2}$ companies recapture the $\mathrm{CO}_{2}$ that's already a by-product of several manufacturing processes, and they then recycle it into the liquid solvent for cleaning clothes. The main drawback is that, while the $\mathrm{CO}_{2}$ itself is both cheap and abundant, the cost of a $\mathrm{CO}_{2}$ dry cleaning machine is very high. Few dry cleaners are adopting this technique for this reason.

Question: Which of the following carbocation can show resonance?
Options:
(a)

(b)

(c)

(d)


Answer: (b)
Solution:


Question: Assertion: In gas phase, the angle of $\mathrm{H}_{2} \mathrm{O}_{2}$ is $90.2^{\circ}$ and in solid phase, it is $112^{\circ}$
Reason: It is due to intermolecular forces
Options:
(a) If both assertion and reason are true and the reason is the correct explanation of the assertion.
(b) If both assertion and reason are true but reason is not the correct explanation of the assertion.
(c) If assertion is true but reason is false.
(d) If assertion is false but reason is true.

Answer: (a)
Solution: Hydrogen peroxide has a non-planar structure. The molecular dimension in the gas phase and solid are shown in figure

In solid-state, there are stronger intermolecular forces. So, the angle is less.

(a) $\mathrm{H}_{2} \mathrm{O}_{2}$ structures in gas phase, dihedral angle is $111.5^{\circ}$
(b) $\mathrm{H}_{2} \mathrm{O}_{2}$ structure in solid phase at 110 K , dihedral angle is $90.2^{\circ}$

Question: Organic compound A with $\mathrm{CHCl}_{3} / \mathrm{KOH}$ gives product B which can be decomposed by $\mathrm{H}^{+} / \mathrm{H}_{2} \mathrm{O}$ ?

## Options:

(a) $\mathrm{A}=$ Primary amine, $\mathrm{B}=$ isonitrile
(b) $\mathrm{A}=$ Secondary amine, $\mathrm{B}=$ Primary amine
(c) $\mathrm{A}=$ isonitrile, $\mathrm{B}=$ Primary amine
(d) A = Carboxylic acid, B = Primary amine

Answer: (a)
Solution: Primary amine on reaction with $\mathrm{CHCl}_{3} \& \mathrm{KOH}$ gives isocyanide or isonitrile.


Question: Which of the following does not disproportionate?
Options:
(a) $\mathrm{BrO}^{-}$
(b) $\mathrm{BrO}_{2}^{-}$
(c) $\mathrm{BrO}_{3}^{-}$
(d) $\mathrm{BrO}_{4}^{-}$

Answer: (d)

Solution: $\mathrm{In} \mathrm{BrO}_{4}^{-}$the oxidation state of Br is +7 is the maximum possible oxidation state of Br.

So, it can only undergo reduction.

Question: IF $\mathrm{P}_{4} \mathrm{P}_{10}+\mathrm{HNO}_{3}$ are mixed in $1: 4$ ratio, then nature of nitrogen oxide obtained is Options:
(a) Acidic
(b) Basic
(c) Amphoteric
(d) Neutral

Answer: (a)
Solution: The reaction as follows
$4 \mathrm{HNO}_{3}+\mathrm{P}_{4} \mathrm{O}_{10} \rightarrow 2 \mathrm{~N}_{2} \mathrm{O}_{5}+4 \mathrm{HPO}_{3}$

Question: Product A and B for the following reaction is



## Options:

(a) $\mathrm{A}=$ Cyclohexane-1,6-diol, $\mathrm{B}=$ Hexan-1,6-dioic acid
(b) $\mathrm{A}=$ Cyclopentane-1,5-diol $\mathrm{B}=$ Pentan-1,5-dioic acid
(c) $\mathrm{A}=$ Benzene, $\mathrm{B}=$ Benzoic acid
(d) $\mathrm{A}=$ Phenol, $\mathrm{B}=$ Hexane

Answer: (a)
Solution:

(A)

(B)

Question: 250 ml 0.5 M NaOH , and 500 ml of 1 M HCl . Find the number of molecules of HCl remaining after reaction in the form of $10^{21}$.
Answer: 225.82

## Solution:

Base ( NaOH )
$\mathrm{n}=0.5 \mathrm{M} \times 250 \mathrm{ml}$
$=125$ millimoles
$=0.125$ moles
Acid ( HCl )
$\mathrm{N}=1 \mathrm{M} \times 500 \mathrm{ml}$
$=500$ millimoles
$=0.5$ moles
Remaining moles of $\mathrm{HCl}=0.5-0.125=0.375$ moles
1 mole $\equiv \mathrm{N}_{\mathrm{A}}$ molecules
0.375 moles $\equiv \mathrm{N}_{\mathrm{A}} \times 0.375$ molecules $=225.825 \times 10^{21}$ molecules

## JEE-Main-20-07-2021-Shift-1 (Memory Based)

## MATHEMATICS

Question: Tangent and normal are drawn to $y^{2}=2 x$ at $A(2,2)$. Tangent cuts x-axis at T and normal cuts parabola again at p . Find area of $\triangle A T P$.
Options:
(a) $\frac{25}{2}$
(b) 25
(c)
(d)

Answer: (a)

## Solution:

Equation of tangent is $2 y=x+2$
And equation of normal is $2 x+y=6$
$\therefore T(-2,0)$ and $P\left(\frac{9}{2},-3\right)$
$\therefore$ Area $=\frac{1}{2}\left|\begin{array}{ccc}-2 & 0 & 1 \\ \frac{9}{2} & -3 & 1 \\ 2 & 2 & 1\end{array}\right|$
$=\frac{1}{2}[10+15]=\frac{25}{2}$

Question: Find the coefficient of $x^{256}$ in $[1-x]^{101} \times\left(x^{2}+x+1\right)^{100}$

## Options:

(a) ${ }^{100} C_{15}$
(b) $-{ }^{100} C_{15}$
(c) ${ }^{100} C_{18}$
(d) $-{ }^{100} C_{16}$

Answer: (a)
Solution:
$(1-x)^{101}\left(x^{2}+x+1\right)^{100}$
$=(1-x)\left[(1-x)\left(x^{2}+x+1\right)\right]^{100}$
$=(1-x)\left(1-x^{3}\right)^{100}$
For $\left(1-x^{3}\right)^{100}$
$T_{r+1}={ }^{100} C_{r}\left(-x^{3}\right)^{r}$
For $x^{256}, 1$ will multiply with $x^{256}$ and $x$ will multiply with $x^{255}$.
$x^{256}$ in $\left(1-x^{3}\right)^{100}$ is not possible
For $x^{255}, r=85$
$\therefore$ coefficient $=1 \times 0+(-1) \times\left(-{ }^{100} C_{85}\right)$
$={ }^{100} C_{85}={ }^{100} C_{15}$

Question: If the shortest distance between the lines
$\bar{r}_{1}=\alpha \hat{i}+2 \hat{j}+2 \hat{k}+\lambda(\hat{i}-2 \hat{j}+2 \hat{k}), \lambda \in R, \alpha>0$ and $\bar{r}_{2}=-4 \hat{i}-\hat{k}+\mu(3 \hat{i}-2 \hat{j}-2 \hat{k}), \mu \in R$ is 9 , then $\alpha$ is equal to

## Options:

(a)
(b)
(c)
(d)

Answer: 6

## Solution:

S.D. $=\frac{\left|\begin{array}{ccc}\alpha+4 & 2 & 3 \\ 1 & -2 & 2 \\ 3 & -2 & -2\end{array}\right|}{\left|\begin{array}{ccc}i & j & k \\ 1 & -2 & 2 \\ 3 & -2 & -2\end{array}\right|}$
$=\frac{8(\alpha+4)+16+12}{\sqrt{64+64+16}}=9$
$=8 \alpha+60=108$
$\Rightarrow \alpha=6$

Question: $x^{2}+3^{\frac{1}{4}} x+\sqrt{3}=0$ roots are $\alpha, \beta$. Find $\alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)$

## Options:

(a) $3^{24} .52$
(b) $3^{24} .56$
(c) $3^{25} .52$
(d) $3^{25} .56$

Answer: (a)

## Solution:

Let $a=3^{\frac{1}{4}}$
$\therefore x^{2}+a x+a^{2}=0$
$x=\frac{-a \pm \sqrt{3} a i}{2}$
$\therefore \alpha=\frac{-a-i \sqrt{3} a}{2}=a \omega$
$\beta=\frac{-a+i \sqrt{3} a}{2}=a \omega^{2}$
$\therefore \alpha^{96}\left(\alpha^{12}-1\right)+\beta^{96}\left(\beta^{12}-1\right)=2 a^{96}\left(a^{12}-1\right)=3^{24} \cdot 52$

Question: The word EXAMINATION is given : then the Probability that the $M$ is at the $4^{\text {th }}$ place is-

## Options:

(a) $\frac{1}{11}$
(b) $\frac{9}{11}$
(c)
(d)

Answer: (a)

## Solution:

Total case $=\frac{11!}{2!\cdot 2!\cdot 2!}$
Favourable case $=\frac{10!}{2!\cdot 2!\cdot 2!}$
$\therefore$ Required probability $=\frac{\frac{10!}{2!\cdot 2!\cdot 2!}}{\frac{11!}{2!\cdot 2!\cdot 2!}}$
$=\frac{1}{11}$

Question: $a_{i j}=\left\{\begin{array}{cc}1, & i=j \\ -x, & |i-j|=1 \quad A=\left[a_{i j}\right]_{3 \times 3} . f(x)=\operatorname{det}(A) \text {. Sum of maximum and } \\ 2 x+1, & \text { otherwise }\end{array}\right.$ minimum values of $f(x)$.
Options:
(a) $\frac{20}{27}$
(b) $\frac{-20}{27}$
(c) $\frac{88}{27}$
(d) $\frac{-88}{27}$

Answer: (a)
Solution:
$A=\left[\begin{array}{ccc}1 & -x & 2 x+1 \\ -x & 1 & -x \\ 2 x+1 & -x & 1\end{array}\right]$
$|A|=4 x^{3}-4 x^{2}-4 x$
$f(x)=4 x^{3}-4 x^{2}-4 x$
$f^{\prime}(x)=12 x^{2}-8 x-4$
$=4\left(3 x^{2}-2 x-1\right)$
$=4(x-1)(3 x+1)$
$f^{\prime}(x)=0$ gives $x=1, \frac{-1}{3}$
$f(1)=-4$
$f\left(\frac{-1}{3}\right)=\frac{20}{27}$
Sum $=-4+\frac{20}{27}=\frac{-88}{27}$

Question: $\int_{0}^{a} e^{x-[x]} d x=10 e-9$, find ' $a$ '.

## Options:

(a) $10+\ln (1+e)$
(b) $10-\ln (1+e)$
(c) $10+\ln 2$
(d) $10+\ln 3$

Answer: (c)

## Solution:

$\int_{0}^{a} e^{x-[x]} d x=\int_{0}^{[a]} e^{\{x\}} d x+\int_{[a]}^{a} e^{\{x\}} d x$
$=[a] \int_{0}^{1} e^{x} d x+\int_{[a]}^{a} e^{x-[a]} d x$
$=[a](e-1)+e^{-[a]}\left(a^{a}-2^{[a]}\right)$
$=[a] e+\left(-1-[a]-e^{a-[a]}\right)$
$=10 e-9$
On equality, $a=10+\ln 2$

Question: $|z . \omega|=1 \arg (z)-\arg (\omega)=\frac{3 \pi}{2}$ find $\arg \left[\frac{1-2 \bar{z} \omega}{1+3 \bar{z} \omega}\right]$
Options:
(a) $\frac{\pi}{4}$
(b) $\frac{-\pi}{4}$
(c) $\frac{3 \pi}{4}$
(d) $\frac{-3 \pi}{4}$

Answer: (d)

## Solution:

Let $\omega=r e^{i \theta}$
$\therefore z=\frac{1}{r} e^{i\left(\theta+\frac{3 \pi}{2}\right)}$
$\therefore \bar{z} \omega=\left[\frac{1}{r} e^{-i\left(\theta+\frac{3 \pi}{2}\right)}\right][r \cdot e i \theta]=e-i \frac{3 \pi}{2}=i$
$\Rightarrow\left(\frac{1-2 i}{1+3 i}\right)\left(\frac{1-3 i}{1-3 i}\right)=\frac{-5-5 i}{10}$
$\Rightarrow \arg =\frac{-3 \pi}{4}$

Question: $f(x)=\left\{\begin{array}{cc}a+[-x], & (0,1) \\ 2 x-b, & {[1, \infty)} \\ x^{2}-1, & (-\infty, 0]\end{array}\right.$ Find $(a+b)$ if $f(x)$ is continuous on R.

## Options:

(a) 2
(b) 3
(c) 4
(d) 5

Answer: (b)
Solution:
$f(x)=\left\{\begin{array}{cc}a+[-x], & (0,1) \\ 2 x-b, & {[1, \infty)} \\ x^{2}-1, & (-\infty, 0]\end{array}\right.$
$f\left(1^{-}\right)=f\left(1^{+}\right)$
$\Rightarrow \lim _{h \rightarrow 0} a+[-(1-h)]=\lim _{h \rightarrow 0} 2(1+h)-b$
$\Rightarrow a-1=2-b$
$\Rightarrow a+b=3$

Question: $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$ total number of integral terms is $\qquad$ -

## Options:

(a)
(b) 11
(c)
(d)

Answer: (b)

## Solution:

For $\left(4^{\frac{1}{4}}+5^{\frac{1}{6}}\right)^{120}$
$T_{r}={ }^{120} C_{r} \cdot 4^{\frac{120-r}{4}} \cdot 5^{\frac{r}{6}}$
$r$ is a multiple of 6 and 4
Hence multiple of 12
$r=0,12,24, \ldots \ldots .120$
Hence 11 integral terms

Question: From a team of 15 players, 6 are bowlers, 7 are batsman, 2 are wicket keepers. Find the number of ways to form a team of II players having atleast 4 bowlers 5 batsman, 1 wicket keeper.
Answer: 777

## Solution:

Ways to form team of 11
5 Bowlers, 5 Batsman, 1 wicket keeper
${ }^{6} C_{5} \times{ }^{7} C_{5} \times{ }^{2} C_{1}=252$
$6 \times 7 \times 6$
4 Bowlers, 6 Batsman, 1 wicket keeper
${ }^{6} C_{4} \times{ }^{7} C_{6} \times{ }^{2} C_{1}=210$
4 Bowler, 5 Batsman, 2 wicket keeper
${ }^{6} C_{4} \times{ }^{7} C_{5} \times{ }^{2} C_{2}=315$
Total number of ways $=252+210+315=777$

Question: $\bar{a}, \bar{b}, \bar{c}$ are manually $\perp$ unit vector equally inclined to $\bar{a}+\bar{b}+\bar{c}$ at angle $\theta$. Find $36 \cos ^{2} 2 \theta \ldots$
Answer: 4
Solution:
$\cos \theta=\frac{\bar{a} \cdot(\bar{a}+\bar{b}+\bar{c})}{|\bar{a}||\bar{a}+\bar{b}+\bar{c}|}$
Also, $(a+\bar{b}+\bar{c})^{2}=3+2 \times 0$
$\therefore|\bar{a}+\bar{b}+\bar{c}|=\sqrt{3}$
From (1) and (2)
$\cos \theta=\frac{1}{\sqrt{3}} \Rightarrow \cos 2 \theta=-\frac{1}{3}$
$36 \cos ^{2} 2 \theta=\frac{36}{9}=4$

Question: Let $a, b, c, d$ be in A.P with common difference $\lambda$. If $\left|\begin{array}{ccc}x+a-c & x+b & x+a \\ x-1 & x+c & x+b \\ x-b+d & x+d & x+c\end{array}\right|=2$ then $\lambda^{2}=?$
Answer: -1

## Solution:

Put $x=0$

$$
\left|\begin{array}{ccc}
-2 \lambda & a+\lambda & a \\
-1 & a+2 \lambda & a+\lambda \\
2 \lambda & a+3 \lambda & a+2 \lambda
\end{array}\right|=2
$$

$\Rightarrow\left|\begin{array}{ccc}-2 \lambda & a+\lambda & a \\ 2 \lambda-1 & \lambda & \lambda \\ 2 \lambda+1 & \lambda & \lambda\end{array}\right|=2$
$\Rightarrow\left|\begin{array}{ccc}-2 \lambda & a+\lambda & -\lambda \\ 2 \lambda-1 & \lambda & 0 \\ 2 \lambda+1 & \lambda & 0\end{array}\right|=2$
$\Rightarrow-\lambda\left[\left(2 \lambda^{2}-\lambda\right)-\left(2 \lambda^{2}+\lambda\right)\right]=2$
$\Rightarrow \lambda^{2}=1$

Question: The probability of selecting integers $a \in(-5,30]$, such that $x^{2}+2(a+4) x-5 a+64>0$, for all $x \in R$, is:

Answer: $\frac{2}{9}$

## Solution:

$\Delta<0$
$4(a+4)^{2}-4(-5 a+64)<0$
$a^{2}+16+8 a+5 a-64<0$
$a^{2}+13 a-48<0$
$(a+16)(a-3)<0$
$a \in(-16,3)$
$\therefore$ Probability $=\frac{8}{36}=\frac{2}{9}$

Question: Let $y=m x+c, m>0$ be the focal chord of $y^{2}=-64 x$ which is tangent to $(x+10)^{2}+y^{2}=4$. Then the value of $4 \sqrt{2}(m+c)$ is equal to $\qquad$ .
Answer: 34

## Solution:

Focus of $y^{2}=-64 x$ is $(-16,0)$
Hence $y=m x+c$ passes through $(-16,0)$
$\Rightarrow 0=m(-16)+c$
$\Rightarrow c-16 m=0$
$\Rightarrow c=16 m$
Centre of $(x+10)^{2}+y^{2}=4$ is $(-10,0)$ and radius is 2 Since $y=m x+c$ is tangent hence $\left|\frac{0-m(-10)-c}{\sqrt{1+m^{2}}}\right|=2$
$\left|\frac{10 m-c}{\sqrt{1+m^{2}}}\right|=2$
$\left|\frac{10 m-16 m}{\sqrt{1+m^{2}}}\right|=2$
$6 m=2 \sqrt{1+m^{2}}$
$3 m=\sqrt{1+m^{2}}$
$9 m^{2}=1+m^{2}$
$m^{2}=\frac{1}{8}$
$m=\frac{1}{2 \sqrt{2}}$
$c=16 m=\frac{16}{2 \sqrt{2}}$
$m+c=\frac{16}{2 \sqrt{2}}+\frac{1}{2 \sqrt{2}}=\frac{17}{2 \sqrt{2}}$
$4 \sqrt{2}(m+c)=34$

Question: The mean of 6 numbers is 6.5 and there variance is 10.25 . If 4 numbers are $2,4,5$, 7. Find the other two numbers.

Answer: 10, 11

## Solution:

$\frac{a+b+c+d+e+f}{6}=6.5$
$10.25=\frac{\sum x_{i}^{2}}{6}-(6.5)^{2}$
$\Rightarrow \sum x_{i}^{2}=315$
or $a^{2}+b^{2}=221$
and $a+b=21$
$\therefore a, b=10,11$

