24^{th} Feb. 2021 | Shift - 2
PHYSICS

1. Zener breakdown occurs in a p-n junction having p and n both :
(1) lightly doped and have wide depletion layer.
(2) heavily doped andhave narrow depletion layer.
(3) heavily doped and have wide depletion layer.
(4) lightly doped and have narrow depletion layer.

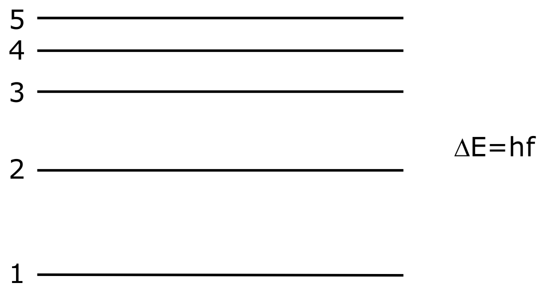
Ans. (2)

Sol. The zener breakdown occurs in the heavily doped p-n junction diode. Heavily doped p-n junction diodes have narrow depletion region.

2. According to Bohr atom model, in which of the following transitions will the frequency be maximum?
(1)n=2 to n=1 (2)n=4 to n =3 (3)n=5 to n=4 (4)n=3 to n=2

Ans. (1)

Sol.



f is more for transition from n = 2 to n = 1.

3. An X-ray tube is operated at 1.24 million volt. The shortest wavelength of the produced photon will be :
(1) 10^{-2} nm (2)10^{-3} nm (3)10^{-4} nm (4)10^{-1} nm

Ans. (2)

Sol. $\lambda_{min} = \frac{hc}{eV}$

$\lambda_{min} = \frac{1240nm - eV}{1.24 \times 10^6}$

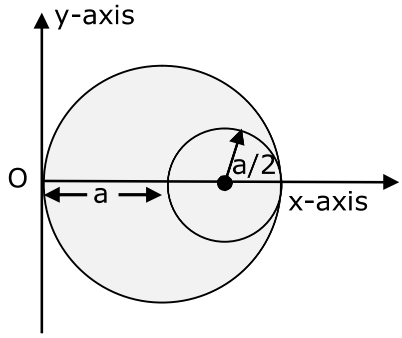
$\lambda_{min} = 10^{-3}nm$

4. On the basis of kinetic theory of gases, the gas exerts pressure because its molecules:
(1) suffer change in momentum when impinge on the walls of container.
(2) continuously stick to the walls of container.
(3) continuously lose their energy till it reaches wall.
(4) are attracted by the walls of container.

Ans. (1)

Sol. On the basis of kinetic theory of gases, the gas pressure is due to the molecules suffering change in momentum when impinge on the walls of container.

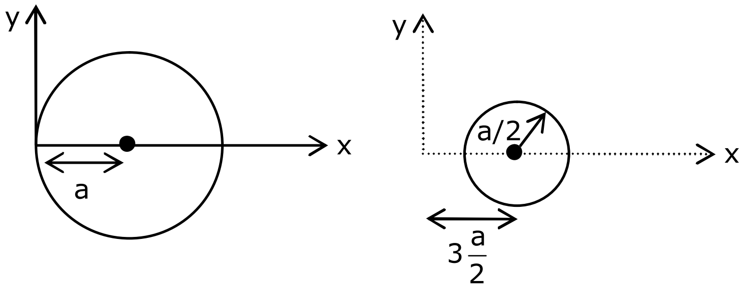
5. A circular hole of radius $\left(\frac{a}{2}\right)$ is cut out of a circular disc of radius 'a' shown in figure. The centroid of the remaining circular portion with respect to point 'O' will be :



(1) $\frac{10}{11}a$ (2) $\frac{2}{3}a$ (3) $\frac{1}{6}a$ (4) $\frac{5}{6}a$

Ans. **(4)**

Sol. Let σ is the surface mass density of disc.



$$X_{com} = \frac{(\sigma \times \pi a^2 \times a) - (\sigma \frac{\pi a^2}{4} \times \frac{3a}{2})}{\sigma\pi a^2 - \frac{\sigma\pi a^2}{4}}$$

$$X_{com} = \frac{a - 3\frac{a}{8}}{1 - \frac{1}{4}}$$

$$X_{com} = \frac{\frac{5a}{8}}{\frac{3}{4}}$$

$$X_{com} = \frac{5a}{6}$$

6. Given below are two statements :

Statement I : PN junction diodes can be used to function as transistor, simply by connecting two diodes, back to back, which acts as the base terminal.

Statement II : In the study of transistor, the amplification factor β indicates ratio of the collector current to the base current.

In the light of the above statements, choose the correct answer from the options given below.

(1) Statement I is false but Statement II is true.
(2) Both Statement I and Statement II are true
(3) Statement I is true but Statement II is false.
(4) Both Statement I and Statement II are false

Ans. **(1)**

Sol. **S-1**

Statement 1 is false because in case of two discrete back to back connected diodes, there are four doped regions instead of three and there is nothing that resembles a thin base region between an emitter and a collector.

S-2

Statement-2 is true, as

$$\beta = \frac{I_C}{I_B}$$

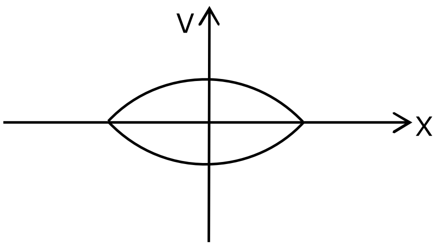
7. When a particle executes SHM, the nature of graphical representation of velocity as a function of displacement is :

(1)elliptical (2)parabolic (3)straight line (4)circular

Ans. **(1)**

Sol. We know that is SHM;

$$V = \omega\sqrt{A^2 - x^2}$$



elliptical

8. Match List – I with List – II.

	List – I	List - II
(a)	Source of microwave frequency	(i) Radioactive decay of nucleus
(b)	Source of infrared frequency	(ii) Magnetron
(c)	Source of Gamma Rays	(iii) Inner shell electrons
(d)	Source of X-rays	(iv) Vibration of atoms and molecules
		(v) LASER
		(vi) RC circuit

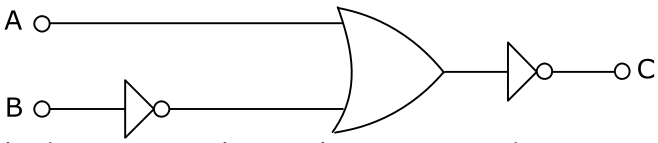
Choose the correct answer from the options given below :

(1) (a)-(ii),(b)-(iv),(c)-(i),(d)-(iii)
(2) (a)-(vi),(b)-(iv),(c)-(i),(d)-(v)
(3) (a)-(ii),(b)-(iv),(c)-(vi),(d)-(iii)
(4) (a)-(vi),(b)-(v),(c)-(i),(d)-(iv)

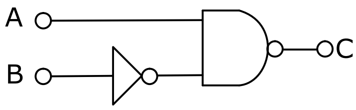
Ans. **(1)**

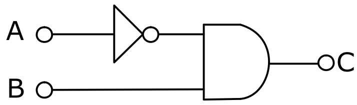
Sol. (a) Source of microwave frequency – (ii) Magnetron
(b) Source of infra red frequency – (iv) Vibration of atom and molecules
(c) Source of gamma ray – (i) Radio active decay of nucleus
(d) Source of X-ray – (iii) inner shell electron

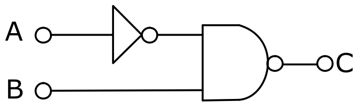
9.

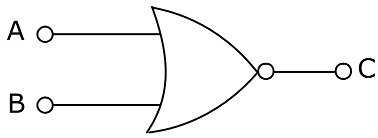


The logic circuit shown above is equivalent to :

(1)


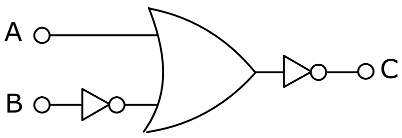
(2)


(3)


(4)


Ans. (2)

Sol.



$C = \overline{A + \overline{B}}$

$C = \overline{A}.B$

10. If the source of light used in a Young's double slit experiment is changed from red to violet:
(1)the fringes will become brighter.
(2)consecutive fringe lineswill come closer.
(3)the central bright fringe will become a dark fringe.
(4)the intensity of minima will increase.

Ans. (2)

Sol. $\beta = \frac{\lambda D}{d}$

As $\lambda_v < \lambda_R$

$\Rightarrow \beta_v < \beta_R$

$\therefore$ Consecutive fringe line will come closer.

$\therefore$ (2)

11. A body weighs 49 N on a spring balance at the north pole. What will be its weight recorded on the same weighing machine, if it is shifted to the equator?

[Use $g=\frac{GM}{R^2}$ = 9.8 ms^{-2} and radius of earth, R = 6400 km.]

(1)49 N (2) 49.83 N (3) 49.17 N (4) 48.83 N

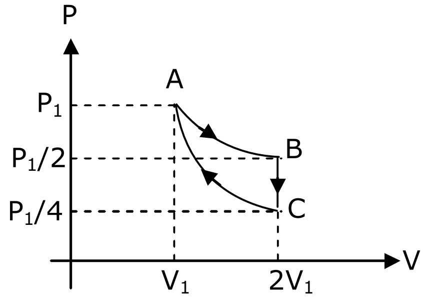
Ans. (4)

Sol. At north pole, weight
Mg = 49
Now, at equator
$g' = g-\underline{\omega}^2R$

$\Rightarrow Mg' = M(g - \omega^2 R)$

$\Rightarrow$ weight will be less than Mg at equator.

12. If one mole of an ideal gas at (P_1, V_1) is allowed to expand reversibly and isothermally (A to B) its pressure is reduced to one-half of the original pressure (see figure). This is followed by a constant volume cooling till its pressure is reduced to one-fourth of the initial value ($B \to C$). Then it is restored to its initial state by a reversible adiabatic compression (C to A). The net workdone by the gas is equal to :



(1)0 (2)$-\frac{RT}{2(\gamma-1)}$ (3)$RT\left[\ln 2-\frac{1}{2(\gamma-1)}\right]$ (4)RT ln2

Ans. (3)

Sol. $AB \to$ Isothermal process

$W_{AB} \to nRT \ln 2 = RT \ln 2$

$BC \to$ Isochoric process

$W_{BC} = 0$

$CA \to$ Adiabatic process

$$W_{CA} = \frac{P_1V_1 - \frac{P_1}{4} X 2V_1}{1-\gamma} = \frac{P_1V_1}{2(1-\gamma)} = \frac{RT}{2(1-\gamma)}$$

$$W_{ABCA} = RT\ell n2 + \frac{RT}{2(1-\gamma)}$$

$$= RT\left[\ell n2 - \frac{1}{2(\gamma-1)}\right]$$

13. The period of oscillation of a simple pendulum is $T = 2\pi\sqrt{\frac{L}{g}}$. Measured value of 'L' is 1.0 m from meter scale having a minimum division of 1 mm and time of one complete oscillation is 1.95 s measured from stopwatch of 0.01 s resolution. The percentage error in the determination of 'g' will be :

(1)1.33 % (2)1.30 % (3)1.13 % (4)1.03 %

Ans. (3)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g}}$

$$T^2 = 4\pi^2\left[\frac{\ell}{g}\right]$$

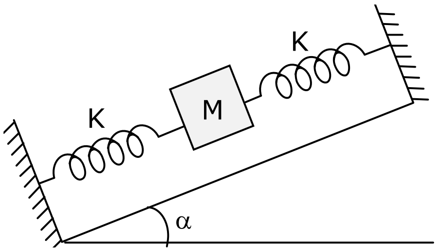
$$g = 4\pi^2\left[\frac{\ell}{T^2}\right]$$

$$\frac{\Delta g}{g} = \frac{\Delta\ell}{\ell} + \frac{2\Delta T}{T}$$

$$= \left[\frac{1mm}{1m} + \frac{2(10\times10^{-3})}{1.95}\right]\times 100$$

= 1.13 %

14. In the given figure, a body of mass M is held between two massless springs, on a smooth inclined plane. The free ends of the springs are attached to firm supports. If each spring has spring constant k, the frequency of oscillation of given body is :



(1) $\frac{1}{2\pi}\sqrt{\frac{2k}{Mg\sin\alpha}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{k}{Mg\sin\alpha}}$

(3) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{k}{2M}}$

Ans. **(1)**

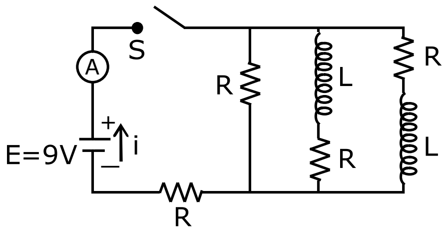
Sol. Equivalent K = K + K = 2K

Now, $T = 2\pi\sqrt{\frac{m}{K_{eq}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{m}{2k}}$

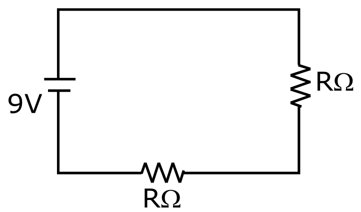
$\therefore f = \frac{1}{2\pi}\sqrt{\frac{2k}{m}}$

15. Figure shows a circuit that contains four identical resistors with resistance R = 2.0 Ω. Two identical inductors with inductance L = 2.0 mH and an ideal battery with emf E = 9.V. The current 'i' just after the switch 's' is closed will be :



(1) 9A (2) 3.0 A (3) 2.25 A (4) 3.37 A

Ans. **(3)**
Sol. When switch S is closed-



Given : v = 9v

From V = IR

$I = \frac{v}{R}$

$R_{eq.}$ = 2+2 = 4Ω

$I = \frac{9}{4}$ = 2.25 A

16. The de Broglie wavelength of a proton and α-particle are equal. The ratio of their velocities is :

(1) 4:2 (2) 4:1 (3) 1:4 (4) 4:3

Ans. **(2)**

Sol. From De-broglie's wavelength :-

$\lambda = \frac{h}{mv}$

Given $\lambda_P = \lambda_\alpha$

$v \alpha \frac{1}{m}$

$\frac{v_p}{v_\alpha} = \frac{m_\alpha}{m_p} = \frac{4m_p}{m_p} = \frac{4}{1}$

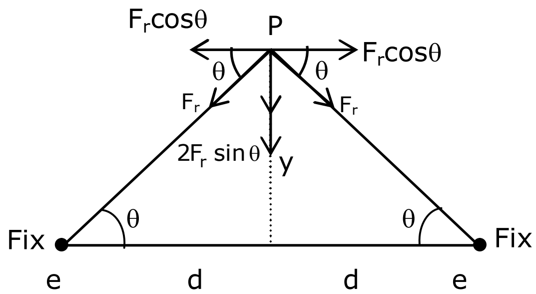
17. Two electrons each are fixed at a distance '2d'. A third charge proton placed at the midpoint is displaced slightly by a distance x (x<<d) perpendicular to the line joining the two fixed charges. Proton will execute simple harmonic motion having angular frequency:

(m = mass of charged particle)

(1) $\left(\frac{q^2}{2\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$ (2) $\left(\frac{\pi\varepsilon_0 md^3}{2q^2}\right)^{\frac{1}{2}}$ (3) $\left(\frac{2\pi\varepsilon_0 md^3}{q^2}\right)^{\frac{1}{2}}$ (4) $\left(\frac{2q^2}{\pi\varepsilon_0 md^3}\right)^{\frac{1}{2}}$

Ans. **(1)**

Sol.



Restoring force on proton :-

$$F_r = \frac{2Kq^2y}{\left[d^2 + y^2\right]^{\frac{3}{2}}}$$

Y <<< d

$$F_r = \frac{2kq^2y}{d^3} = \frac{q^2y}{2\pi\varepsilon_o d^3} = ky$$

$$K = \frac{q^2}{2\pi\varepsilon_0 d^3}$$

Angular Frequency :-

$$\omega = \sqrt{\frac{k}{m}}$$

$$\omega = \sqrt{\frac{q^2}{2\pi\varepsilon_0 m d^3}}$$

18. A soft ferromagnetic material is placed in an external magnetic field. The magnetic domains :

(1) decrease in size and changes orientation.

(2) may increase or decrease in size and change its orientation.

(3) increase in size but no change in orientation.

(4) have no relation with external magnetic field.

Ans. **(2)**

Sol. Atoms of ferromagnetic material in unmagnetised state form domains inside the ferromagnetic material. These domains have large magnetic moment of atoms. In the absence of magnetic field, these domains have magnetic moment in different directions. But when the magnetic field is applied, domains aligned in the direction of the field grow in size and those aligned in the direction opposite to the field reduce in size and also its orientation changes.

19. Which of the following equations represents a travelling wave?

(1) $y = Ae^{-x^2}(vt + \theta)$

(2) $y = A\sin(15x - 2t)$

(3) $y = Ae^{x}\cos(\omega t - \theta)$

(4) $y = A\sin x \cos \omega t$

Ans. **(2)**

Sol. Y = F(x,t)

For travelling wave y should be linear function of x and t and they must exist as (x±vt)

Y = A sin (15x-2t) $\rightarrow$ linear function in x and t.

20. A particle is projected with velocity v_0 along x-axis. A damping forceis acting on the particle which is proportional to the square of the distance from the origin i.e. ma = $-\alpha x^2$. The distance at which the particle stops :

(1) $\left(\frac{2v_0}{3\alpha}\right)^{\frac{1}{3}}$ (2) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{2}}$ (3) $\left(\frac{3v_0^2}{2\alpha}\right)^{\frac{1}{3}}$ (4) $\left(\frac{2v_0^2}{3\alpha}\right)^{\frac{1}{2}}$

Ans. **Bonus**

Sol. $a = \frac{vdv}{dx}$

$$\int_{v_i}^{v_f} Vdv = \int_{x_i}^{x_f} adx$$

Given :- $v_i = v_0$
$V_f = 0$
$X_i = 0$
$X_f = x$

From Damping Force : $a = -\frac{\alpha x^2}{m}$

$$\int_{V_0}^{O} VdV = -\int_{o}^{x} \frac{\alpha x^2}{m} dx$$

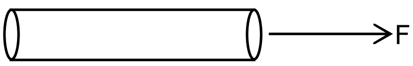
$$-\frac{v_0^2}{2} = \frac{-\alpha}{m}\left[\frac{x^3}{3}\right]$$

$$x = \left[\frac{3mv_0^2}{2\alpha}\right]^{\frac{1}{3}}$$

1. A uniform metallic wire is elongated by 0.04 m when subjected to a linear force F. The elongation, if its length and diameter is doubled andsubjected to the same force will be ________ cm.

Ans. **2**

Sol.



$$y = \frac{F/A}{\Delta\ell/\ell}$$

$$\Rightarrow \frac{F}{A} = y\frac{\Delta\ell}{\ell}$$

$$\Rightarrow \frac{F}{A} = y \times \frac{0.04}{\ell} \qquad ...(1)$$

When length & diameter is doubled.

$$\Rightarrow \frac{F}{4A} = y \times \frac{\Delta\ell}{2\ell} \qquad ...(2)$$

$(1) \div (2)$

$$\frac{F/A}{F/4A} = \frac{y \times \frac{0.04}{\ell}}{y \times \frac{\Delta\ell}{2\ell}}$$

$$4 = \frac{0.04 \times 2}{\Delta\ell}$$

$$\Delta\ell = 0.02$$

$$\Delta\ell = 2 \times 10^{-2}$$

$$\therefore x = 2$$

2. A cylindrical wire of radius 0.5 mm and conductivity 5×10^7 S/m is subjected to an electric field of 10 mV/m. The expected value of current in the wire will be $x^3\pi$ mA. The value of x is _____.

Ans. **5**

Sol. We know that

$J = \sigma E$

$\Rightarrow J = 5\times10^7\times10\times10^{-3}$

$\Rightarrow J = 50\times10^4\ A/m^2$

Currentflowing ;

$I = J \times \pi R^2$

$I = 50\times10^4 \times\pi(0.5\times10^{-3})^2$

$I = 5\times10^4\times\pi\times0.25\times10^{-6}$

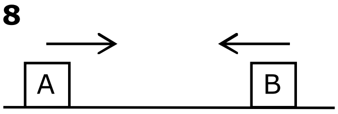
$I = 125\times10^{-3}\pi$

X = 5

3. Two cars are approaching each other at an equal speed of 7.2 km/hr. When they see each other, both blow horns having frequency of 676 Hz. The beat frequency heard by each driver will be ________ Hz. [Velocity of sound in air is 340 m/s.]

Ans. **8**

Sol.



Speed = 7.2 km/h = 2 m/s

Frequency as heard by A

$$f_A' = f_B\left(\frac{v+v_0}{v-v_s}\right)$$

$$f_A' = 676\left(\frac{340+2}{340-2}\right)$$

$$f_A' = 684\text{Hz}$$

$$\therefore f_{Beat} = f_A' - f_B$$

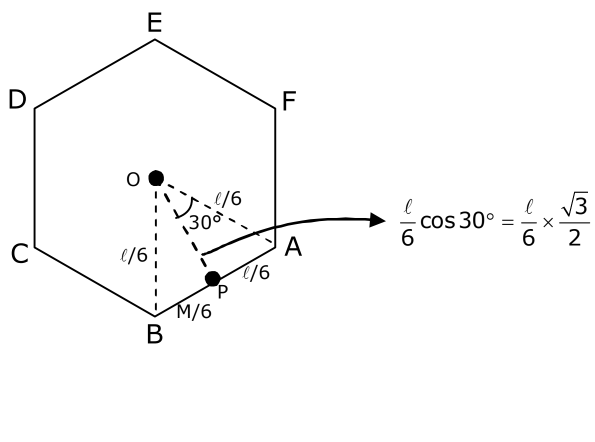
=684-676

= 8 Hz

4. A uniform thin bar of mass 6 kg and length 2.4 meter is bent to make an equilateral hexagon. The moment of inertia about an axis passing through the centre of mass and perpendicular to the plane of hexagon is ______$\times10^{-1}$ kg m^2.

Ans. **8**

Sol.



MOI of AB about P : $I_{AB_P} = \frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12}$

MOI of AB about O,

$$I_{AB_O} = \left[\frac{\frac{M}{6}\left(\frac{\ell}{6}\right)^2}{12} + \frac{M}{6}\left(\frac{\ell}{6}\frac{\sqrt{3}}{2}\right)^2\right]$$

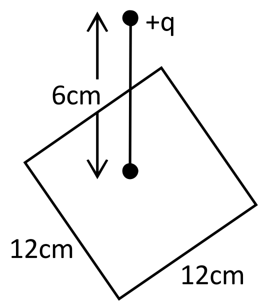
$$I_{Hexagon_0} = 6I_{AB_0} = M\left[\frac{\ell^2}{12\times 36} + \frac{\ell^2}{36}\times\frac{3}{4}\right]$$

$$= \frac{6}{100}\left[\frac{24\times 24}{12\times 36} + \frac{24\times 24}{36}\times\frac{3}{4}\right]$$

$= 0.8\ \text{kgm}^2$

$= 8\times 10^{-2}\ \text{kg/m}^2$

5. A point charge of +12 μC is at a distance 6 cm vertically above the centre of a square of side 12 cm as shown in figure. The magnitude of the electric flux through the square will be ______ $\times 10^3$ Nm^2/C.



Ans. **226**

Sol. Using Gauss law, it is a part of cube of side 12 cm and charge at centre so;

$$\phi = \frac{Q}{6\varepsilon_0} = \frac{12\mu c}{6\varepsilon_0} = 2\times 4\pi\times 9\times 10^9\times 10^{-6}$$

$= 226\times 10^3\ \text{Nm}^2/\text{C}$

6. Two solids A and B of mass 1 kg and 2 kg respectively are moving withequal linear momentum. The ratio of their kinetic energies $(K.E.)_A$: $(K.E.)_B$ will be $\frac{A}{1}$. So the value of A will be ________.

Ans. **2**

Sol. Given that, $\frac{M_1}{M_2} = \frac{1}{2}$

Also, $p_1 = p_2 = p$

$\Rightarrow M_1V_1 = M_2V_2 = p$

Also, we know that

$$K = \frac{p^2}{2M} \Rightarrow K_1 = \frac{p^2}{2M_1} \& \Rightarrow K_2 = \frac{p^2}{2M_2}$$

$$\Rightarrow \frac{K_1}{K_2} = \frac{p^2}{2M_1} \times \frac{2M_2}{p^2} \Rightarrow \frac{K_1}{K_2} = \frac{M_2}{M_1} = \frac{2}{1}$$

$$\Rightarrow \frac{A}{1} = \frac{2}{1} \Rightarrow \therefore A = 2$$

7. The root mean square speed of molecules of a given mass of a gas at 27°C and 1 atmosphere pressure is 200 ms^{-1}. The root mean square speed of molecules of the gas at 127°C and 2 atmosphere pressure is $\frac{x}{\sqrt{3}}$ ms^{-1}. The value of x will be ___________.

Ans. **400 m/s**

Sol. $V_{rms}\sqrt{\frac{3RT_1}{M_0}}$

$$200 = \sqrt{\frac{3R \times 300}{M_0}} \quad \text{...(1)}$$

Also, $$\frac{x}{\sqrt{3}} = \sqrt{\frac{3R \times 400}{M_0}} \quad \text{...(2)}$$

$(1) \div (2)$

$$\frac{200}{x/\sqrt{3}} = \sqrt{\frac{300}{400}} = \sqrt{\frac{3}{4}}$$

$$\Rightarrow x = 400 \text{ m/s}$$

8. A series LCR circuit is designed to resonate at an angular frequency $\omega_0 = 10^5 \text{rad/s}$. The circuit draws 16W power from 120 V source at resonance. The value of resistance 'R' in the circuit is _______ Ω.

Ans. **900**

Sol. $P = \frac{V^2}{R}$

$$16 = \frac{120^2}{R} \Rightarrow R = \frac{14400}{16}$$

$$\Rightarrow R = 900\Omega$$

9. An electromagnetic wave of frequency 3 GHz enters a dielectric medium of relative electric permittivity 2.25 from vacuum. The wavelength of this wave in that medium wil be _______ $\times 10^{-2}$ cm.

Ans. 667

Sol. $f = 3GHz$, $\varepsilon_r = 2.25$

$$v = \lambda f \Rightarrow \lambda = \frac{v}{f}$$

$$C = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$$

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} \Rightarrow \lambda = \frac{1}{f.\sqrt{\mu_0 \varepsilon_0}.\sqrt{\mu_r \varepsilon_r}.f}$$

$$\Rightarrow \lambda = \frac{C}{f.\sqrt{\mu_r}.\sqrt{\varepsilon_r}} \Rightarrow \lambda = \frac{3 \times 10^8}{3 \times 10^9 \times \sqrt{1} \times \sqrt{2.25}}$$

$$\Rightarrow \lambda = 667 \times 10^{-2} \text{ cm}$$

10. A signal of 0.1 kW is transmitted in a cable. The attenuation of cable is -5 dB per km and cable length is 20 km. the power received at receiver is 10^{-x}W. The value of x is ______.

[Gain in dB = 10 $\log_{10}\left(\frac{P_0}{P_i}\right)$]

Ans. **8**

Sol. Power of signal transmitted : P_i = 0.1 Kw = 100w

Rate of attenuation = -5 dB/Km

Total length of path = 20 km

Total loss suffered = -5×20=-100dB

Gain in dB = 10 $\log_{10}\frac{P_0}{P_i}$

$$-100 = 10\log_{10}\frac{P_0}{P_i}$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = 10$$

$$\Rightarrow \log_{10}\frac{P_i}{P_0} = \log_{10} 10^{10}$$

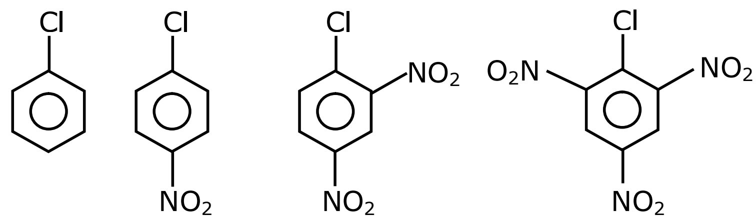
$$\Rightarrow \frac{100}{P_0} = 10^{10}$$

$$\Rightarrow P_0 = \frac{1}{10^8} = 10^{-8}$$

$\therefore$ x = 8

24^{th} Feb. 2021 | Shift - 2
CHEMISTRY

1. The correct order of the following compounds showing increasing tendency towards nucleophilic substitution reaction is :

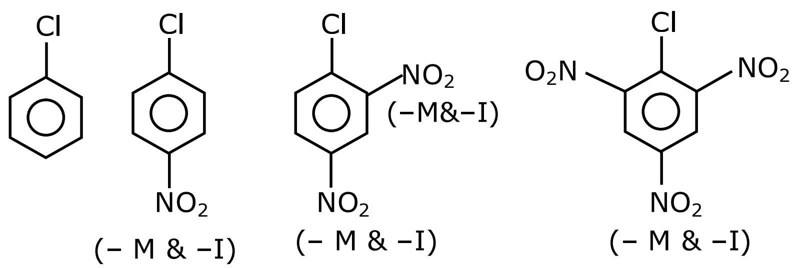


(i) (ii) (iii) (iv)

(1) (iv) < (i) < (iii) < (ii)
(2) (iv) < (i) < (ii) < (iii)
(3) (i) < (ii) < (iii) < (iv)
(4) (iv) < (iii) < (ii) < (i)

Ans. **(3)**

Sol.



(i) (ii) (iii) (iv)

Reactivity ∝ – m group present at O/P position.

2. Match List-I with List-II

List- I	List-II
(Metal)	(Ores)
(a) Aluminium	(i) Siderite
(b) Iron	(ii) Calamine
(c) Copper	(iii) Kaolinite
(d) Zinc	(iv) Malachite

Choose the correct answer from the options given below :

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(i), (b)-(ii), (c)-(iii), (d)-(iv)
(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. **(3)**

Sol.

Siderite	$FeCO_3$
Calamine	$ZnCO_3$
Kaolinite	$Si_2Al_2O_5(OH)_4$ or $Al_2O_3.2SiO_2.2H_2O$
Malachite	$CuCO_3.Cu(OH)_2$

3. Match List-I with List-II

List- I	List-II
(Salt)	(Flame colour wavelength)
(a) LiCl	(i) 455.5 nm
(b) NaCl	(ii) 970.8 nm
(c) RbCl	(iii) 780.0 nm
(d) CsCl	(iv) 589.2 nm

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(2) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
(3) (a)-(iv), (b)-(ii), (c)-(iii), (d)-(i)
(4) (a)-(i), (b)-(iv), (c)-(ii), (d)-(iii)

Ans. (2)

Sol. Range of visible region : - 390nm – 760nm
VIBGYOR
Violet Red

LiCl Crimson Red
NaCl Golden yellow
RbCl Violet
CsCl Blue

So Licl Which is crimson have wave length closed to red in the spectrum of visible region which is as per given data is.

4. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Hydrogen is the most abundant element in the Universe, but it is not the most abundant gas in the troposphere.

Reason R : Hydrogen is the lightest element.

In the light of the above statements, choose the correct answer from the given below

(1) A is false but R is true
(2) Both A and R are true and R is the correct explanation of A
(3) A is true but R is false
(4) Both A and R are true but R is NOT the correct explanation of A

Ans. (2)

Sol. Hydrogen is most abundant element in universe because all luminous body of universe i.e. stars & nebulae are made up of hydrogen which acts as nuclear fuel & fusion reaction is responsible for their light.

5. Given below are two statements :

Statement I : The value of the parameter ''Biochemical Oxygen Demand (BOD)'' is important for survival of aquatic life.

Statement II : The optimum value of BOD is 6.5 ppm.

In the light of the above statements, choose the most appropriate answer from the options given below.

(1) Both Statement I and Statement II are false

(2) Statement I is false but Statement II is true

(3) Statement I is true but Statement II is false

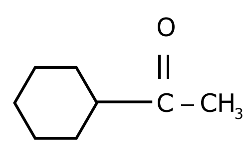
(4) Both Statement I and Statement II are true

Ans. **(3)**

Sol. For survival of aquatic life dissolved oxygen is responsible its optimum limit 6.5 ppm and optimum limit of BOD ranges from 10-20 ppm & BOD stands for biochemical oxygen demand.

6. Wich one of the following carbonyl compounds cannot be prepared by addition of wate on an alkyne in the presence of $HgSO_4$ and H_2SO_4 ?

(1) $CH_3-CH_2-\overset{\overset{\displaystyle O}{||}}{C}-H$

(2) 

(3) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-H$

(4) $CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_2CH_3$

Ans. **(1)**

Sol. Reaction of Alkyne with $HgSO_4$ & H_2SO_4 follow as

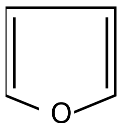
$CH\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3CHO$

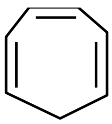
$CH_3-C\equiv CH \xrightarrow[H_2O]{HgSO_4, H_2SO_4} CH_3-\overset{\overset{\displaystyle O}{||}}{C}-CH_3$

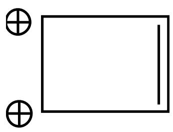
Hence, by this process preparation of CH_3CH_2CHO

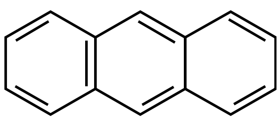
Cann't possible.

7. Which one of the following compounds is non-aromatic ?

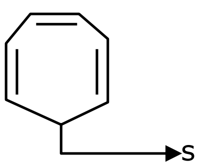
(1) 

(2) 

(3) 

(4) 

Ans. **(2)**

Sol. 

sp^3 carbon atom (Not planer)

Hence It is non-aromatic.

8. The incorrect statement among the following is :

(1) $VOSO_4$ is a reducing agent

(2) Red colour of ruby is due to the presence of CO^{3+}

(3) Cr_2O_3 is an amphoteric oxide

(4) RuO_4 is an oxidizing agent

Ans. **(2)**

Sol. Red colour of ruby is due to presence of CrO_3 or Cr^{+6} not CO^{3+}

9. According to Bohr's atomic theory :

(A) Kinetic energy of electron is $\propto \frac{Z^2}{n^2}$

(B) The product of velocity (v) of electron and principal quantum number (n). 'vn' $\propto Z^2$.

(C) Frequency of revolution of electron in an orbit is $\propto \frac{Z^3}{n^3}$.

(D) Coulombic force of attraction on the electron is $\propto \frac{Z^3}{n^4}$.

Choose the most appropriate answer from the options given below:

(1) (C) only
(2) (A) and (D) only
(3) (A) only
(4) (A), (C) and (D) only

Ans. (2) Correction on NTA

Sol. (A) KE = −TE = 13.6 × $\frac{Z^2}{n^2}$eV

$KE \propto \frac{Z^2}{n^2}$

(B) V = 2.188 × 10^6 × $\frac{Z}{n}$ m/sec.

So, Vn $\propto$ Z

(C) Frequency = $\frac{V}{2\pi r}$

So, $F \propto \frac{Z^2}{n^3}$ $\left[\therefore r \propto \frac{n^2}{z} \text{ and } v \propto \frac{Z}{n}\right]$

(D) Force $\propto \frac{z}{r^2}$

So, $F \propto \frac{Z^3}{n^4}$

So, only statement (A) is correct

10. Match List-I with List-II

List- I	List-II
(a) Valium	(i) Antifertility drug
(b) Morphine	(ii) Pernicious anaemia
(c) Norethindrone	(iii) Analgesic
(d) Vitamin B_{12}	(iv) Tranquilizer

(1) (a)-(iv), (b)-(iii), (c)-(ii), (d)-(i)
(2) (a)-(i), (b)-(iii), (c)-(iv), (d)-(ii)
(3) (a)-(ii), (b)-(iv), (c)-(iii), (d)-(i)
(4) (a)-(iv), (b)-(iii), (c)-(i), (d)-(ii)

Ans. (4)

Sol.

(a) Valium	(iv) Tranquilizer
(b) Morphine	(iii) Analgesic
(c) Norethindrone	(i) Antifertility drug
(d) Vitamin B_{12}	(ii) Pernicious anaemia

11. The Correct set from the following in which both pairs are in correct order of melting point is :

(1) LiF > LiCl ; NaCl > MgO
(2) LiF > LiCl ; MgO > NaCl
(3) LiCl > LiF ; NaCl > MgO
(4) LiCl > LiF ; MgO > NaCl

Ans. **(2)**

Sol. Generally

M.P. $\propto$ Lattice energy = $\frac{KQ_1Q_2}{r^+ + r^-}$

$\propto$ (packing efficiency)

12. The calculated magnetic moments (spin only value) for species $[FeCl_4]^{2-}$, $[Co(C_2O_4)_3]^{3-}$ and MnO_4^{2-} respectively are :

(1) 5.92, 4.90 and 0 BM
(2) 5.82, O and 0 BM
(3) 4.90, 0 and 1.73 BM
(4) 4.90, 0 and 2.83 BM

Ans. **(3)**

Sol. $[FeCl_4]^{2-}$ Fe^{2+} $3d^6 \rightarrow$ 4 unpaired electron. as Cl^- in a weak field liquid.

$\mu_{spin} = \sqrt{24}$ 8M

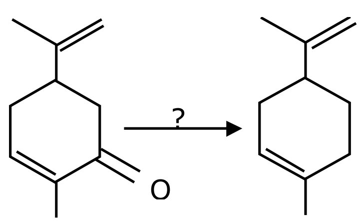
= 4.9 BM

$[Co(C_2O_4)_3]^{3-}$ Co^{3+} $3d^6 \rightarrow$ for Co^{3+} with coodination no. 6 $C_2O_4^{2-}$ is strong field ligend & causes pairing & hence no. unpaired electron

$\mu_{spin} = 0$

$[MnO_4]^{2-}$ Mn^{+6} it has one unpaired electron.

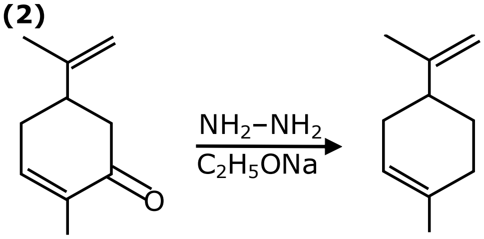
$\mu_{spin} = \sqrt{3}$ BM

13. 

Which of the following reagent is suitable for the preparation of the product in the above reaction.

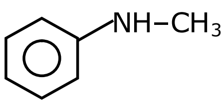
(1) Red P + Cl_2
(2) $NH_2-NH_2/C_2H_5\overset{\ominus\oplus}{ONa}$
(3) Ni/H_2
(4) $NaBH_4$

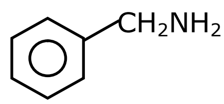
Ans. **(2)**

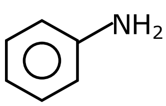
Sol. 

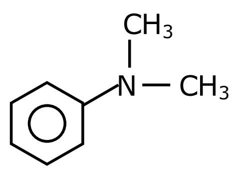
It is wolf-kishner reduction of carbonyl compounds.

14. The diazonium salt of which of the following compounds will form a coloured dye on reaction with β-Naphthol in NaOH ?

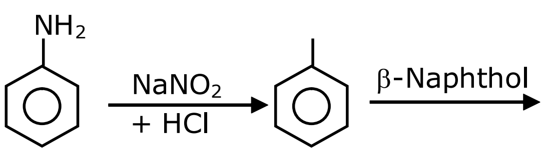
(1) 

(2) 

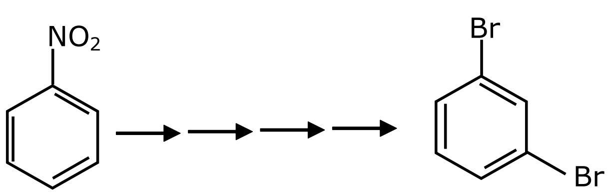
(2) 

(4) 

Ans. **(3)**

Sol.  Orange bright dye.

15. What is the correct sequence of reagents used for converting nitrobenzene into m-dibromobenzene ?



(1) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{NaNO_2}$ / $\xrightarrow{NaBr}$

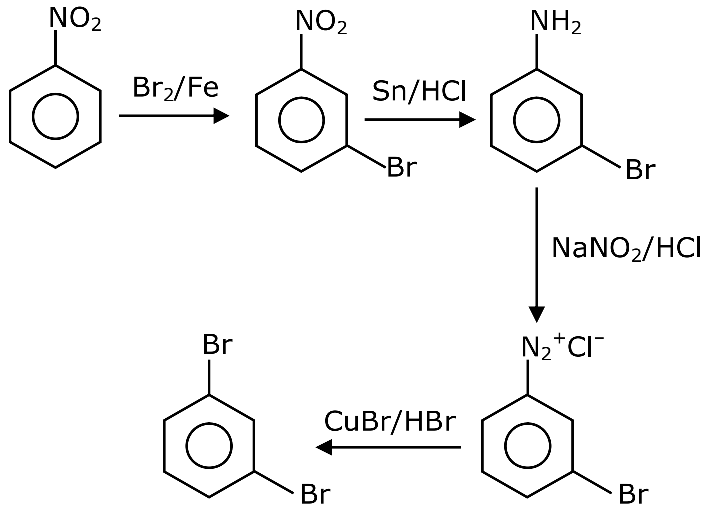
(2) $\xrightarrow{Sn/HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{Br_2}$ / $\xrightarrow{H^+}$

(3) $\xrightarrow{NaNO_2}$ / $\xrightarrow{HCl}$ / $\xrightarrow{KBr}$ / $\xrightarrow{H^+}$

(4) $\xrightarrow{Br_2/Fe}$ / $\xrightarrow{Sn/HCl}$ / $\xrightarrow{NaNO_2/HCl}$ / $\xrightarrow{CuBr/HBr}$

Ans. **(4)**

Sol.



16. The correct shape and I-I-I bond angles respectively in I_3^- ion are :

(1) Trigonal planar; 120º

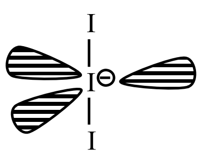
(2) Distorted trigonal planar; 135º and 90º

(3) Linear; 180º

(4) T-shaped; 180º and 90º

Ans. **(3)**

Sol. I_3^- sp^3d hybridisation (2BP + 3L.P.) Linear geometry



17. What is the correct order of the following elements with respect to their density ?

(1) Cr < Fe < Co < Cu < Zn

(2) Cr < Zn < Co < Cu < Fe

(3) Zn < Cu < Co < Fe < Cr

(4) Zn < Cr < Fe < Co < Cu

Ans. **(4)**

Sol. Fact Based

Density depend on many factor like atomic mass. atomic radius and packing efficiency.

18. **Match List-I and List-II.**

List – I | List-II

(a) $R-\overset{O}{\overset{||}{C}}-Cl \rightarrow R-CHO$ (i) $Br_2/NaOH$

(b) $R-CH_2-COOH \rightarrow R-\underset{Cl}{\underset{|}{CH}}-COOH$ (ii) $H_2/Pd\text{-}BaSO_4$

(c) $R-\overset{O}{\overset{||}{C}}-NH_2 \rightarrow R-NH_2$ (iii) $Zn(Hg)/Conc.\ HCl$

(d) $R-\overset{O}{\overset{||}{C}}-CH_3 \rightarrow R-CH_2-CH_3$ (iv) $Cl_2/Red\ P,\ H_2O$

Choose the correct answer from the options given below :

(1) (a)-(ii), (b)-(i), (c)-(iv), (d)-(iii)
(2) (a)-(iii), (b)-(iv), (c)-(i), (d)-(ii)
(3) (a)-(iii), (b)-(i), (c)-(iv), (d)-(ii)
(4) (a)-(ii), (b)-(iv), (c)-(i), (d)-(iii)

Ans. **(4)**

Sol. (a) $R-\overset{O}{\overset{||}{C}}-Cl \xrightarrow{H_2/Pd-BaSO_4} R-CHO$ (Rosenmunt reaction)

(b) $R-CH_2-COOH \xrightarrow{Cl_2/Red\ P,\ H_2O} R-\underset{Cl}{\underset{|}{CH}}-COOH$ (HVZ reaction)

(c) $R-\overset{O}{\overset{||}{C}}-NH_2 \xrightarrow{Br_2/NaOH} R-NH_2$ (Hoffmann Bromamide reaction)

(d) $R-\overset{O}{\overset{||}{C}}-CH_3 \xrightarrow{Zn(Hg)/conc.HCl} R-CH_2-CH_3$ (Clemmensen reaction)

19. In polymer Buna-S ; 'S' stands for :

(1) Styrene (2) Sulphur (3) Strength (4) Sulphonation

Ans. **(1)**

Sol. Buna-S is the co-polymer of buta- 1, 3 diene & styrene.

20. Most suitable salt which can be used for efficient clotting of blood will be :

(1) $Mg(HCO_3)_2$ (2) $FeSO_4$ (3) $NaHCO_3$ (4) $FeCl_3$

Ans. **(4)**

Sol. Blood is a negative sol. According to hardy-Schulz's rule, the cation with high charge has high coagulation power. Hence, $FeCl_3$ can be used for clotting blood.

Section -B

1. The magnitude of the change in oxidising power of the MnO_4^- / Mn^{2+} couple is x × 10^{-4} V, if the H^+ concentration is decreased from 1M to 10^{-4} M at 25ºC. (Assume concentration of MnO_4^- and Mn^{2+} to be same on change in H^+ concentration). The value of x is _____.
(Rounded off to the nearest integer)

$$\left[\text{Given : } \frac{2303RT}{F} = 0.059\right]$$

Ans. **3776**

Sol. $5e^- + MnO_4^- + 8H^+ \longrightarrow Mn^{+2} + 4H_2O$

$$Q = \frac{\left[Mn^{+2}\right]}{\left[H^+\right]^8\left[MnO_4^-\right]} \quad \Rightarrow \quad E_1 = E^\circ - \frac{0.059}{5}\log(Q_1)$$

$$E_2 = E^\circ - \frac{0.059}{5}\log(Q_2) \quad \Rightarrow \quad E_2 - E_1 = \frac{0.059}{5}\log\left(\frac{Q_1}{Q_2}\right)$$

$$= \frac{0.059}{5}\log\left\{\frac{\left[H^+\right]_{II}}{\left[H^+\right]_I}\right\}^8 \quad \Rightarrow \quad = \frac{0.059}{5}\log\left(\frac{10^{-4}}{1}\right)^8$$

$$(E_2 - E_1) = \frac{0.059}{5}\times(-32) \quad \Rightarrow \quad \left|(E_2 - E_1)\right| = 32\times\frac{0.059}{5} = x\times10^{-4}$$

$$= \frac{32\times590}{5}\times10^{-4} = x\times10^{-4} \quad \Rightarrow \quad = 3776 \times 10^{-4} \qquad x = 3776$$

2. Among the following allotropic forms of sulphur, the number of allotropic forms, which will show paramagnetism is _______.

(1) α-sulphur (2) β-sulphur (3) S_2-form

Ans. **(1)**

Sol. S_2 is like O_2 i;e paramagnetic as per molecular orbital theory.

3. C_6H_6 freezes at 5.5ºC. The temperature at which a solution of 10 g of C_4H_{10} in 200 g of C_6H_6 freeze is _________ºC. (The molal freezing point depression constant of C_6H_6 is) 5.12ºC/m)

Ans. 1

Sol. $\Delta T_f = i \times K_f \times m$

$= (1) \times 5.12 \times \frac{10/58}{200} \times 1000 \quad \Rightarrow \quad \Delta T_f = \frac{5.12 \times 50}{58} = 4.414$

$T_{f(solution)} = T_{K(solvent)} - \Delta T_f$

$= 5.5 - 4.414$

$= 1.086°C$

$\approx 1.09°C = 1$ (nearest integer)

4. The volume occupied by 4.75 g of acetylene gas at 50ºC and 740 mmHg pressure is ________L. (Rounded off to the nearest integer)
(Given R = 0.0826 L atm K^{-1} mol^{-1})

Ans. 5

Sol. T = 50ºC = 323.15 K

$P = 740 \text{ mm of Hg} = \frac{740}{760} \text{atm}$

V = ?

$\text{moles } (n) = \frac{4.75}{26}$

$$V = \frac{4.75}{26} \times \frac{0.0821 \times 323.15}{740} \times 760$$

$V = 4.97 \simeq 5$ Lit

5. The solubility product of PbI_2 is 8.0×10^{-9} . The solubility of lead iodide in 0.1 molar solution of lead nitrate is $x \times 10^{-6}$ mol/L. The value of x is ________(Rounded off to the nearest integer)
[Given $\sqrt{2} = 1.41$]

Ans. 141

Sol. $K_{SP}(PbI_2) = 8 \times 10^{-9}$

$$PbI_2(s) \rightleftharpoons Pb^{+2}(aq) + 2I^-(aq)$$

S + 0.1 2S

$K_{SP} = [Pb^{+2}][I^-]^2$

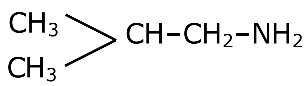
$8 \times 10^{-9} = (S + 0.1)(2S)^2 \Rightarrow 8 \times 10^{-9} \simeq 0.1 \times 4S^2$

$\Rightarrow S^2 = 2 \times 10^{-8}$

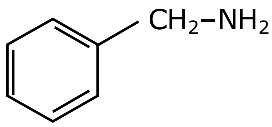
$S = 1.414 \times 10^{-4}$ mol/Lit

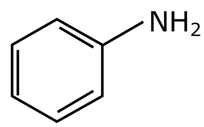
$= x \times 10^{-6}$ mol/Lit $\quad \therefore \quad x = 141.4 \simeq 141$

6. The total number of amines among the following which can be synthesized by Gabriel synthesis is ________.

(1) 

(2) $CH_3CH_2NH_2$

(3) 

(4) 

Ans. (3)

Sol. Only aliphatic amines can be prepared by Gabriel synthesis.

7. 1.86 g of aniline completely reacts to form acetanilide. 10% of the product is lost during purificatiion. Amount of acetanilide obtained after purification (in g) is _____ $\times 10^{-2}$.

Ans. 243

Sol.

$$\underset{\substack{(C_6H_7N) \\ \text{Molar mass} = 93}}{Ph-NH_2} \longrightarrow \underset{\substack{(\text{Acetanilide})(C_8H_9NO) \\ \text{Molar mass} = 135}}{Ph-NH-\overset{\overset{\displaystyle O}{||}}{C}-CH_3}$$

93 g Aniline produce 135 g acetanilide

1.86 g produce $\frac{135 \times 1.86}{93} = 2.70\,g$

At 10% loss, 90% product will be formed after purification.

$\therefore$ Amount of product obtained = $\frac{2.70 \times 90}{100} = 2.43\,g = 243 \times 10^{-2}$ g

8. The formula of a gaseous hydrocarbon which requires 6 times of its own volume of O_2 for complete oxidation and produces 4 times its own volume of CO_2 is C_xH_y. The value of y is …………

Ans. 8

Sol. $C_xH_y + 6O_2 \longrightarrow 4CO_2 + \frac{y}{2} H_2O$

Applying POAC on 'O' atoms

$6 \times 2 = 4 \times 2 + y/2 \times 1$

$y/2 = 4 \Rightarrow y = 8$

9. Sucrose hydrolyses in acid solution into glucose and fructose following first order rate law with a half-life of 3.33 h at 25ºC. After 9h, the fraction of sucrose remaining is f. The value of $\log_{10}\left(\frac{1}{f}\right)$ is _________$\times 10^{-2}$ (Rounded off to the nearest integer)
[Assume: ln10 = 2.303, ln2 = 0.693]

Ans. 81

Sol. Sucose $\xrightarrow{\text{Hydrolysis}}$ Glucose + Fructose

$t_{1/2} = 3.33h = \frac{10}{3}h \quad \Rightarrow \quad C_t = \frac{C_o}{2^{t/t_{1/2}}}$

Fraction of sucrose remaining = f = $\frac{C_t}{C_o} = \frac{1}{2^{t/t_{1/2}}}$

$\frac{1}{f} = 2^{t/t_{1/2}}$

$\log(1/f) = \log\left(2^{t/t_{1/2}}\right) = \frac{t}{t_{1/2}}\log(2)$

$= \frac{9}{10/3} \times 0.3 = \frac{8.1}{10} = 0.81 \quad = x \times 10^{-2} \quad x = 81$

10. Assuming ideal behaviour, the magnitude of log K for the following reaction at 25ºC is $x \times 10^{-1}$. The value of x is __________.(Integer answer)

$$3HC \equiv CH_{(g)} \rightleftharpoons C_6H_{6(\ell)}$$

[Given : $\Delta_f G°(HC \equiv CH) = -2.04 \times 10^5$] mol^{-1}; $\Delta_f G°(C_6H_6) = -1.24 \times 10^5$ J mol^{-1} ; $R = 8.314$ J K^{-1} mol^{-1}]

Ans. 855

Sol. $3HC \equiv CH(g) \rightleftharpoons C_6H_6(\ell)$

$\Delta G_r^o = \Delta G_f^o\left[C_6H_8(\ell)\right] - 3 \times \Delta G_f^o\left[HC \equiv CH\right]$

$= [-1.24 \times 10^5 - 3x(-2.04 \times 10^5)]$

$= 4.88 \times 10^5$ J/mol

$\Delta G_r^o = -RT\ln(K_{eq})$

$\log(K_{eq}) = \frac{-\Delta G^\circ}{2.303RT}$

$= \frac{-4.88 \times 10^5}{2.303 \times 8.314 \times 298}$

$= -8.55 \times 10^1 = 855 \times 10^{-1}$

24^{th} Feb. 2021 | Shift - 2
MATHEMATICS

1. Let $a, b \in R$. If the mirror image of the point $P(a, 6, 9)$ with respect to the line $\frac{x-3}{7} = \frac{y-2}{5} = \frac{z-1}{-9}$ is $(20, b, -a-9)$, then $|a+b|$ is equal to :

(1) 86 (2) 88 (3)84 (4)90

Ans. (2)

Sol. P(a, 6, 9), Q (20, b, –a–9)

mid point of $PQ = \left(\frac{a+20}{2}, \frac{b+6}{2}, -\frac{a}{2}\right)$

lie on line

$$\frac{\frac{a+20}{2}-3}{7} = \frac{\frac{b+6}{2}-2}{5} = \frac{-\frac{a}{2}-1}{-9}$$

$$\frac{a+20-6}{14} = \frac{b+6-4}{10} = \frac{-a-2}{-18}$$

$$\frac{a+14}{14} = \frac{a+2}{18}$$

$$18a + 252 = 14a + 28$$

$$4a = -224$$

$$\boxed{a = -56}$$

$$\frac{b+2}{10} = \frac{a+2}{18}$$

$$\frac{b+2}{10} = \frac{-54}{18}$$

$$\frac{b+2}{10} = -3 \Rightarrow b = -32$$

$$|a+b| = |-56-32| = 88$$

2. Let f be a twice differentiable function defined on R such that $f(0) = 1, f'(0) = 2$ and $f'(x) \neq 0$ for all x∈R. If $\begin{vmatrix} f(x) & f'(x) \\ f'(x) & f''(x) \end{vmatrix} = 0,$ for all x∈R then the value of $f(1)$ lies in the interval:

(1) (9, 12) (2) (6, 9) (3) (3, 6) (4) (0, 3)

Ans. **(2)**

Sol. Given $f(x)\, f''(X) - (f'(x))^2 = 0$

Let $h(x) = \dfrac{f(x)}{f'(x)}$

$\Rightarrow h'(x) = 0$ $\qquad\Rightarrow h(x) = k$

$\Rightarrow \dfrac{f(x)}{f'(x)} = k$ $\qquad\Rightarrow f(x) = k\, f'(x)$

$\Rightarrow f(0) = k\, f'(0)$ $\qquad\Rightarrow 1 = k(2) \Rightarrow k = \dfrac{1}{2}$

Now $f(x) = \dfrac{1}{2} f'(x) \Rightarrow \int 2dx = \int \dfrac{f'(x)}{f(x)} dx$

$\Rightarrow 2x = ln|f(x)| + C$

As $f(0) = 1 \Rightarrow C = 0$

$\Rightarrow 2x = ln|f(X)| \Rightarrow f(x) = \pm e^{2x}$

As $f(0) = 1 \Rightarrow f(x) = e^{2x} \Rightarrow f(1) = e^2$

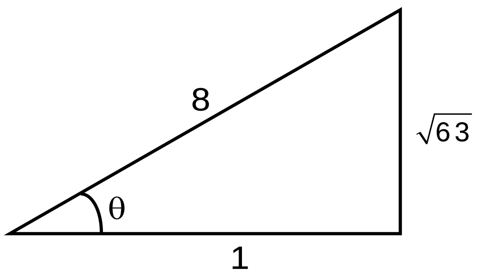
3. A possible value of $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$ is:

(1) $\dfrac{1}{2\sqrt{2}}$ (2) $\dfrac{1}{\sqrt{7}}$ (3) $\sqrt{7}-1$ (4) $2\sqrt{2}-1$

Ans. **(2)**

Sol. $\tan\left(\dfrac{1}{4}\sin^{-1}\dfrac{\sqrt{63}}{8}\right)$

Let $\sin^{-1}\left(\dfrac{\sqrt{63}}{8}\right) = \theta$ $\qquad \sin\theta = \dfrac{\sqrt{63}}{8}$



$\cos\theta = \dfrac{1}{8}$

$2\cos^2\frac{\theta}{2}-1=\frac{1}{8}$

$\cos^2\frac{\theta}{2}=\frac{9}{16}$

$\cos\frac{\theta}{2}=\frac{3}{4}$

$\frac{1-\tan^2\frac{\theta}{4}}{1+\tan^2\frac{\theta}{4}}=\frac{3}{4}$

$\tan\frac{\theta}{4}=\frac{1}{\sqrt{7}}$

4. The probability that two randomly selected subsets of the set {1,2,3,4,5} have exactly two elements in their intersection, is:

(1) $\frac{65}{2^7}$ (2) $\frac{135}{2^9}$ (3) $\frac{65}{2^8}$ (4) $\frac{35}{2^7}$

Ans. (2)

Sol. Required probability

$= \frac{{}^5C_2 \times 3^3}{4^5}$

$= \frac{10\times 27}{2^{10}}=\frac{135}{2^9}$

5. The vector equation of the plane passing through the intersection of the planes $\vec{r}\cdot(\hat{i}+\hat{j}+\hat{k})=1$ and $\vec{r}\cdot(\hat{i}-2\hat{j})=-2$, and the point (1,0,2) is :

(1) $\vec{r}.\left(\hat{i}-7\hat{j}+3\hat{k}\right)=\frac{7}{3}$ (2) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

(3) $\vec{r}.\left(3\hat{i}+7\hat{j}+3\hat{k}\right)=7$ (4) $\vec{r}.\left(\hat{i}+7\hat{j}+3\hat{k}\right)=\frac{7}{3}$

Ans. **(2)**

Sol. Plane passing through intersection of plane is

$\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}+\lambda\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

Passes through $\hat{i}+2\hat{k}$, we get

$\left(3-1\right)+\lambda\left(1+2\right)=0\Rightarrow\lambda=-\frac{2}{3}$

Hence, equation of plane is $3\left\{\vec{r}\cdot\left(\hat{i}+\hat{j}+\hat{k}\right)-1\right\}-2\left\{\vec{r}\cdot\left(\hat{i}-2\hat{j}\right)+2\right\}=0$

$\Rightarrow\quad \vec{r}\cdot\left(\hat{i}+7\hat{j}+3\hat{k}\right)=7$

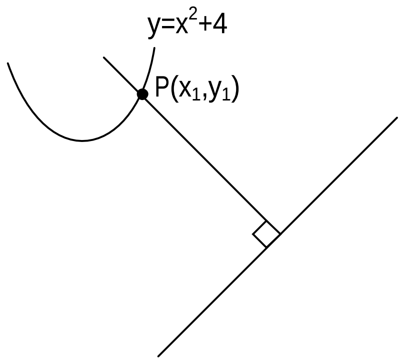
6. If P is a point on the parabola $y=x^2+4$ which is closest to the straight line $y=4x-1$, then the co-ordinates of P are :

(1) (–2, 8) (2) (1, 5) (3) (3, 13) (4) (2, 8)

Ans. **(4)**

Sol. $\frac{dy}{dx}\Big|_{p}=4$

$\therefore\ 2x_1=4$



$\Rightarrow\ x_1=2$

$\therefore$ Point will be (2, 8)

7. Let a, b, c be in arithmetic progression. Let the centroid of the triangle with vertices $(a,c),(2,b)$ and (a,b) be $\left(\frac{10}{3},\frac{7}{3}\right)$. If α,β are the roots of the equation $ax^2+bx+1=0$, then the value of $\alpha^2+\beta^2-\alpha\beta$ is:

(1) $\frac{71}{256}$ (2) $-\frac{69}{256}$ (3) $\frac{69}{256}$ (4) $-\frac{71}{256}$

Ans. (4)

Sol. 2b = a + c

$\frac{2a+2}{3}=\frac{10}{3}$ and $\frac{2b+c}{3}=\frac{7}{3}$

a = 4, $\left.\begin{matrix}2b+c=7\\2b-c=4\end{matrix}\right\}$, solving

$b=\frac{11}{4}$

$c=\frac{3}{2}$

$\therefore$ Quadratic Equation is $4x^2+\frac{11}{4}x+1=0$

$\therefore$ The value of $(\alpha+\beta)^2-3\alpha\beta=\frac{121}{256}-\frac{3}{4}=-\frac{71}{256}$

8. The value of the integral, $\int_1^3\left[x^2-2x-2\right]dx$, where [x] denotes the greatest integer less than or equal to x, is:

(1) –4 (2) –5 (3) $-\sqrt{2}-\sqrt{3}-1$ (4) $-\sqrt{2}-\sqrt{3}+1$

Ans. (3)

Sol. $I=\int_1^3 -3dx+\int_1^3\left[(x-1)^2\right]dx$

Put x –1 = t ; dx = dt

$I=(-6)+\int_0^2\left[t^2\right]dt$

$I=-6+\int_0^1 0\,dt+\int_1^{\sqrt{2}}1\,dt+\int_{\sqrt{2}}^{\sqrt{3}}2\,dt+\int_{\sqrt{3}}^{2}3\,dt$

$I=-6+\left(\sqrt{2}-1\right)+2\sqrt{3}-2\sqrt{2}+6-3\sqrt{3}$

$I=-1-\sqrt{2}-\sqrt{3}$

9. Let $f:\mathbf{R}\to\mathbf{R}$ be defined as

$$f(x)=\begin{cases}-55x, & \text{if } x<-5\\ 2x^3-3x^2-120x, & \text{if } -5\le x\le 4\\ 2x^3-3x^2-36x-336, & \text{if } x>4\end{cases}$$

Let $A=\{x\in R: f$ is increasing$\}$. Then A is equal to :

(1) $(-5,-4)\cup(4,\infty)$ (2) $(-5,\infty)$

(3) $(-\infty,-5)\cup(4,\infty)$ (4) $(-\infty,-5)\cup(-4,\infty)$

Ans. (1)

Sol. $f(x) = \begin{cases}-55 & ; \quad x<-5\\ 6\left(x^2-x-20\right) & ; \quad -5<x<4\\ 6\left(x^2-x-6\right) & ; \quad x>4\end{cases}$

$f(x) = \begin{cases}-55 & ; \quad x<-5\\ 6\left(x-5\right)\left(x+4\right) & ; \quad -5<x<4\\ 6\left(x-3\right)\left(x+2\right) & ; \quad x>4\end{cases}$

Hence, f(x) is monotonically increasing in interval $(-5, -4)\cup(4, \infty)$

10. If the curve $y=ax^2+bx+c, x\in R,$ passes through the point (1,2) and the tangent line to this curve at origin is $y=x$, then the possible values of a, b, c are :

(1). a =1, b=1, c=0 (2) a = –1, b=1, c =1

(3) a =1, b=0, c =1 (4) $a=\frac{1}{2}, b=\frac{1}{2}, c=1$

Ans. (1)

Sol. $2 = a + b + c$ ......(i)

$\frac{dy}{dx} = 2ax + b \Rightarrow \left.\frac{dy}{dx}\right|_{(0,0)} = 1$

$\Rightarrow b = 1 \Rightarrow a + c = 1$

(0,0) lie on curve

$\therefore c=0, a=1$

11. The negation of the statement

$\sim p \wedge (p \vee q)$ is :

(1) $\sim p \wedge q$ (2) $p \wedge \sim q$ (3) $\sim p \vee q$ (4) $p \vee \sim q$

Ans. (4)

Sol.

p	q	~ p	p ∨ q	(~ p) ∧ (p∨ q)	~ q	p∨ ~ q
T	T	F	T	F	F	T
T	F	F	T	F	T	T
F	T	T	T	T	F	F
F	F	T	F	F	T	T

$\therefore \sim p \wedge (p \vee q) \equiv p \vee \sim q$

12. For the system of linear equations:

$x-2y=1, x-y+kz=-2, ky+4z=6, k \in \mathbf{R}$

consider the following statements:

(A) The system has unique solution if $k \neq 2, k \neq -2$.

(B) The system has unique solution if $k=-2$.

(C) The system has unique solution if $k=2$.

(D) The system has no-solution if $k=2$.

(E) The system has infinite number of solutions if $k \neq -2$.

Which of the following statements are correct?

(1) (B) and (E) only (2)(C) and (D) only

(3) (A) and (D) only (4) (A) and (E) only

Ans. (3)

Sol. $x-2y+0.z=1$

$x-y+kz=-2$

$0.x+ky+4z=6$

$$\Delta = \begin{vmatrix} 1 & -2 & 0 \\ 1 & -1 & k \\ 0 & k & 4 \end{vmatrix} = 4 - k^2$$

For unique solution $4 - k^2 \neq 0$

$\boxed{k \neq \pm 2}$

For k=2

$x - 2y + 0.z = 1$

$x - y + 2z = -2$

$0.x + 2y + 4z = 6$

$$\Delta x = \begin{vmatrix} 1 & -2 & 0 \\ -2 & -1 & 2 \\ 6 & 2 & 4 \end{vmatrix} = (-8) + 2[-20]$$

$\Delta x = -48 \neq 0$

For k=2 $\Delta x \neq 0$

For K=2; The system has no solution

13. For which of the following curves, the line $x + \sqrt{3}y = 2\sqrt{3}$ is the tangent at the point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$?

(1) $x^2 + 9y^2 = 9$

(2) $2x^2 - 18y^2 = 9$

(3) $y^2 = \frac{1}{6\sqrt{3}}x$

(4) $x^2 + y^2 = 7$

Ans. (1)

Sol. Tangent to $x^2 + 9y^2 = 9$ at point $\left(\frac{3\sqrt{3}}{2}, \frac{1}{2}\right)$ is $x \left|\frac{3\sqrt{3}}{2}\right| + 9y\left(\frac{1}{2}\right) = 9$

$3\sqrt{3}x + 9y = 18 \Rightarrow x + \sqrt{3}y = 2\sqrt{3}$

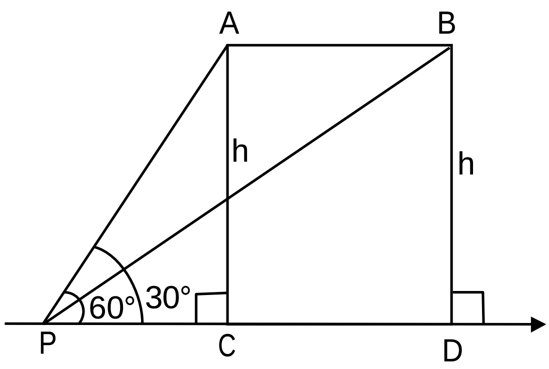
$\Rightarrow$ option (1) is true

14. The angle of elevation of a jet plane from a point A on the ground is 60°. After a flight of 20 seconds at the speed of 432 km/ hour, the angle of elevation changes to 30°. If the jet plane is flying at a constant height, then its height is:

(1) $1200\sqrt{3}$m (2) $1800\sqrt{3}$m (3) $3600\sqrt{3}$m (4) $2400\sqrt{3}$m

Ans. (1)

Sol.



$v = 432 \times \frac{1000}{60 \times 60}$ m/sec = 120 m/sec

Distance AB = v × 20 = 2400 meter

In ΔPAC

$\tan 60° = \frac{h}{PC} \Rightarrow PC = \frac{h}{\sqrt{3}}$

In ΔPBD

$\tan 30° = \frac{h}{PD} \Rightarrow PD = \sqrt{3}h$

PD = PC + CD

$\sqrt{3}h = \frac{h}{\sqrt{3}} + 2400 \Rightarrow \frac{2h}{\sqrt{3}} = 2400$

h = $1200\sqrt{3}$ meter

15. For the statements p and q, consider the following compound statements:

(a) $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$

(b) $((p \vee q)) \wedge \sim P) \rightarrow P$

Then which of the following statements is correct?

(1) (a) is a tautology but not (b)
(2) (a) and (b) both are not tautologies.
(3) (a) and (b) both are tautologies.
(4) (b) is a tautology but not (a).

Ans. (3)

Sol. (a)

p	q	$\sim q$	$p \rightarrow q$	$\sim q \wedge (p \rightarrow q)$	$\sim p$	$(\sim q) \wedge (p \rightarrow q) \rightarrow \sim p$
T	T	F	T	F	F	T
T	F	T	F	F	F	T
F	T	F	T	F	T	T
F	F	T	T	T	T	T

(a) is tautologies

(b)

p	q	$p \vee q$	$\sim p$	$(p \vee q) \wedge \sim p$	$((p \vee q) \wedge \sim p) \rightarrow q$
T	T	T	F	F	T
T	F	T	F	F	T
F	T	T	T	T	T
F	F	F	T	F	T

(b) is tautologies

$\therefore$ a & b are both tautologies.

16. Let A and B be 3×3 real matrices such that A is symmetric matrix and B is skew-symmetric matrix. Then the system of linear equations $\left(A^2 B^2 - B^2 A^2\right)X = O$, where X is a 3×1 column matrix of unknown variables and O is a 3×1 null matrix, has :

(1) a unique solution
(2) exactly two solutions
(3) infinitely many solutions
(4) no solution

Ans. **(3)**

Sol. $A^T = A, B^T = -B$

Let $A^2B^2 - B^2A^2 = P$

$P^T = (A^2B^2 - B^2A^2)^T = (A^2B^2)^T - (B^2A^2)^T$

$= (B^2)^T (A^2)^T - (A^2)^T (B^2)^T$

$= B^2A^2 - A^2B^2$

$\Rightarrow$ P is skew-symmetric matrix

$$\begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$\therefore$ $ay + bz = 0$ ...(1)

$-ax + cz = 0$ ...(2)

$-bx - cy = 0$ ...(3)

From equation 1,2,3

$\Delta = 0$ & $\Delta_1 = \Delta_2 = \Delta_3 = 0$

$\therefore$ equation have infinite number of solution

17. If $n \geq 2$ is a positive integer, then the sum of the series

${}^{n+1}C_2 + 2\left({}^2C_2 + {}^3C_2 + {}^4C_2 + \ldots + {}^nC_2\right)$ is :

(1) $\dfrac{n(n+1)^2(n+2)}{12}$ (2) $\dfrac{n(n-1)(2n+1)}{6}$

(3) $\dfrac{n(n+1)(2n+1)}{6}$ (4) $\dfrac{n(2n+1)(3n+1)}{6}$

Ans. **(3)**

Sol. ${}^2C_2 = {}^3C_3$

$S = {}^3C_3 + {}^3C_2 + \ldots\ldots + {}^nC_2 = {}^{n+1}C_3$

$\because$ ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

$\therefore {}^{n+1}C_2 + {}^{n+1}C_3 + {}^{n+1}C_3 = {}^{n+2}C_3 + {}^{n+1}C_3$

$= \dfrac{(n+1)!}{3!(n-1)!} + \dfrac{(n+1)!}{3!(n-2)!}$

$= \dfrac{(n+2)(n+1)n}{6} + \dfrac{(n+1)(n)(n-1)}{6} = \dfrac{n(n+1)(2n+1)}{6}$

18. If a curve $y = f(x)$ passes through the point (1,2) and satisfies $x\frac{dy}{dx} + y = bx^4$, then for what value of $b, \int_1^2 f(x)dx = \frac{62}{5}$?

(1)5 (2) $\frac{62}{5}$ (3) $\frac{31}{5}$ (4) 10

Ans. (4)

Sol. $\frac{dy}{dx} + \frac{y}{x} = bx^3$, I.F. $= e^{\int \frac{dx}{x}} = x$

$\therefore yx = \int bx^4 dx = \frac{bx^5}{5} + C$

Passes through (1,2), we get

$2 = \frac{b}{5} + C$ .....(i)

Also, $\int_1^2 \left(\frac{bx^4}{5} + \frac{c}{x}\right) dx = \frac{62}{5}$

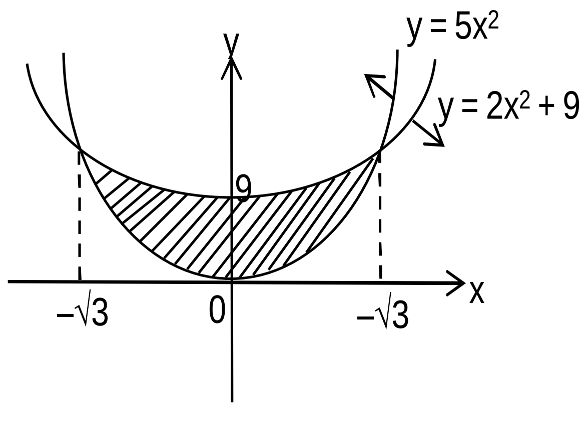
$\Rightarrow \frac{b}{25} \times 32 + C\ln 2 - \frac{b}{25} = \frac{62}{5} \Rightarrow C = 0 \ \& \ b = 10$

19. The area of the region : $R = \{(x, y) : 5x^2 \le y \le 2x^2 + 9\}$ is:

(1) $9\sqrt{3}$ square units (2) $12\sqrt{3}$ square units (3) $11\sqrt{3}$ square units (4) $6\sqrt{3}$ square units

Ans. (2)

Sol.



Required area

$$= 2\int_0^{\sqrt{3}} \left(2x^2 + 9 - 5x^2\right) dx$$

$$= 2\int_0^{\sqrt{3}} \left(9 - 3x^2\right) dx$$

$$= 2\,|\,9x - x^3\,|_0^{\sqrt{3}} = 12\sqrt{3}$$

20. Let $f(x)$ be a differentiable function defined on [0,2] such that $f'(x) = f'(2-x)$ for all $x \in (0,2)$, $f(0) = 1$ and $f(2) = e^2$. Then the value of $\int_0^2 f(x)dx$ is:

(1) $1 + e^2$ (2) $1 - e^2$ (3) $2(1 - e^2)$ (4) $2(1 + e^2)$

Ans. (1)

Sol. f '(x) = f ' (2–x)

On integrating both side f(x) = –f(2–x) +c

put x = 0

f(0) + f(2) = c $\Rightarrow$ c = 1 + e^2

$\Rightarrow$ f(x) + f(2–x) = $1 + e^2$ ......(i)

$$I = \int_0^2 f(x)\,dx = \int_0^1 \left\{f(x) + f(2-x)\right\} dx = \left(1 + e^2\right)$$

Section B

1. The number of the real roots of the equation $(x+1)^2 + |x-5| = \frac{27}{4}$ is_______.

Ans. 2

Sol. $x \geq 5$

$$(x+1)^2 + (x-5) = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - 4 = \frac{27}{4}$$

$$\Rightarrow x^2 + 3x - \frac{43}{4} = 0$$

$$\Rightarrow 4x^2 + 12x - 43 = 0$$

$$x = \frac{-12 \pm \sqrt{144+688}}{8}$$

$$x = \frac{-12 \pm \sqrt{832}}{8} = \frac{-12 \pm 28.8}{8}$$

$$= \frac{-3 \pm 7.2}{2}$$

$$= \frac{-3+7.2}{2}, \frac{-3-7.2}{2}$$ (Therefore no solution)

For $x \le 5$

$$(x+1)^2 - (x-5) = \frac{27}{4}$$

$$x^2 + x + 6 - \frac{27}{4} = 0$$

$$4x^2 + 4x - 3 = 0$$

$$x = \frac{-4 \pm \sqrt{16+48}}{8}$$

$$x = \frac{-4 \pm 8}{8} \Rightarrow x = -\frac{12}{8}, \frac{4}{8}$$

$\therefore$ 2 Real Root's

2. The students $S_1, S_2, \ldots, S_{10}$ are to be divided into 3 groups A, B and C such that each group has at least one student and the group C has at most 3 students. Then the total number of possibilities of forming such groups is_______.

Ans. 31650

Sol.

$$C \to 1 \qquad 9 \left[\begin{array}{l} A \\ \\ B \end{array} \right.$$

$$C \to 2 \qquad 8 \left[\begin{array}{l} A \\ \\ B \end{array} \right.$$

$$C \to 3 \qquad 7 \left[\begin{array}{l} A \\ \\ B \end{array} \right.$$

$= {}^{10}C_1\ [2^9 - 2] + {}^{10}C_2[2^8 - 2] + {}^{10}C_3\ [2^7 - 2]$

$= 2^7[{}^{10}C_1 \times 4 + {}^{10}C_2 \times 2 + {}^{10}C_3] - 20 - 90 - 240$

$= 128\ [40 + 90 + 120\ ] - 350$

$= (128 \times 250) - 350$

$= 10[3165] = 31650$

3. If $a+\alpha = 1, b+\beta = 2$ and $af(x)+\alpha f\left(\frac{1}{x}\right) = bx+\frac{\beta}{x}, x \neq 0,$ then the value of the expression $\dfrac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}}$ is________.

Ans. 2

Sol. $af(x) + \alpha f\left(\frac{1}{x}\right) = bx + \frac{\beta}{x}$ ....(i)

$x \to \frac{1}{x}$

$af\left(\frac{1}{x}\right) + \alpha f(x) = \frac{b}{x} + \beta x$ ....(ii)

(i) + (ii)

$(a+\alpha)\left[f(x)+f\left(\frac{1}{x}\right)\right] = \left(x+\frac{1}{x}\right)(b+\beta)$

$$\frac{f(x)+f\left(\frac{1}{x}\right)}{x+\frac{1}{x}} = \frac{2}{1} = 2$$

4. If the variance of 10 natural numbers $1,1,1,\ldots,1,k$ is less than 10, then the maximum possible value of k is________.

Ans. 11

Sol. $\sigma^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2$

$\sigma^2 = \frac{\left(9+k^2\right)}{10} - \left(\frac{9+k}{10}\right)^2 < 10$

(90 + k^2) 10–(81 + k^2+8k) < 1000

90 + 10k^2–k^2 – 18k –81<1000

9k^2–18k + 9 < 1000

$(k-1)^2 < \frac{1000}{9} \Rightarrow k-1 < \frac{10\sqrt{10}}{3}$

$k < \frac{10\sqrt{10}}{3} + 1$

Maximum integral value of k = 11

5. Let λ be an integer. If the shortest distance between the lines $x-\lambda = 2y-1 = -2z$ and $x = y+2\lambda = z-\lambda$ is $\frac{\sqrt{7}}{2\sqrt{2}}$, then the value of $|\lambda|$ is

Ans. 1

Sol. $\frac{x-\lambda}{1} = \frac{y-\frac{1}{2}}{\frac{1}{2}} = \frac{z}{-\frac{1}{2}}$

$\frac{x-\lambda}{2} = \frac{y-\frac{1}{2}}{1} = \frac{2}{-1}$ .....(1) Point on line = $\left(\lambda, \frac{1}{2}, 0\right)$

$\frac{x}{1} = \frac{y+2\lambda}{1} = \frac{z-\lambda}{1}$ .....(2) Point on line = $(0, -2\lambda, \lambda)$

Distance between skew lines = $\frac{\left[\vec{a}_2 - \vec{a}_1 \ \vec{b}_1 \ \vec{b}_2\right]}{\left|\vec{b}_1 \times \vec{b}_2\right|}$

$$\frac{\begin{vmatrix} \lambda & \frac{1}{2}+2\lambda & -\lambda \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 1 & 1 & 1 \end{vmatrix}}$$

= $\frac{\left|-5\lambda-\frac{3}{2}\right|}{\sqrt{14}}=\frac{\sqrt{7}}{2\sqrt{2}}$ (given)

= $|10\lambda+3|=7 \Rightarrow \lambda=-1$

$\Rightarrow |\lambda|=1$

6. Let $i=\sqrt{-1}$. If $\frac{(-1+i\sqrt{3})^{21}}{(1-i)^{24}}+\frac{(1+i\sqrt{3})^{21}}{(1+i)^{24}}=k,$ and $n=[|k|]$ be the greatest integral part of |k|. Then $\sum_{j=0}^{n+5}(j+5)^2-\sum_{j=0}^{n+5}(j+5)$ is equal to______.

Ans. 310

Sol. $\frac{\left(2e^{i\frac{2\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{-i\frac{\pi}{4}}\right)^{24}}+\frac{\left(2e^{i\frac{\pi}{3}}\right)^{21}}{\left(\sqrt{2}e^{i\frac{\pi}{4}}\right)^{24}}$

$\Rightarrow \frac{2^{21}.e^{i14\pi}}{2^{12}.e^{-i6\pi}}+\frac{2^{21}\left(e^{i7\pi}\right)}{2^{12}\left(e^{i6\pi}\right)}$

$\Rightarrow 2^9e^{i(20\pi)}+2^9e^{i\pi}$

$\Rightarrow 2^9+2^9(-1)=0$

n= 0

$\sum_{j=0}^{5}(j+5)^2-\sum_{j=0}^{5}(j+5)$

$\Rightarrow \left[5^2+6^2+7^2+8^2+9^2+10^2\right]-[5+6+7+8+9+10]$

$\Rightarrow \left[\left(1^2+2^2+....+10^2\right)-\left(1^2+2^2+3^2+4^2\right)\right]-\left[(1+2+3+.....+10)-(1+2+3+4)\right]$

$\Rightarrow (385-30)-[55-10]$

$\Rightarrow$ 355 – 45 $\Rightarrow$ 310 ans.

7. Let a point P be such that its distance from the point (5,0) is thrice the distance of P from the point $(-5,0)$. If the locus of the point P is a circle of radius r, then $4r^2$ is equal to

Ans. 56.25

Sol. Let P(h,k)

Given

PA = 3PB

$PA^2 = 9PB^2$

$\Rightarrow (h-5)^2 + k^2 = 9[(h+5)^2+k^2]$

$\Rightarrow 8h^2 + 8k^2 + 100h + 200 = 0$

$\therefore$ Locus

$x^2 + y^2 + \left(\frac{25}{2}\right)x + 25 = 0$

$\therefore c \equiv \left(\frac{-25}{4}, 0\right)$

$\therefore r^2 = \left(\frac{-25}{4}\right)^2 - 25$

$= \frac{625}{16} - 25$

$= \frac{225}{16}$

$\therefore 4r^2 = 4\times\frac{225}{16} = \frac{225}{4} = 56.25$

8. For integers n and r, let $\binom{n}{r} = \begin{cases} {}^nC_r, & \text{if } n \geq r \geq 0 \\ 0, & \text{otherwise} \end{cases}$

The maximum value of k for which the sum

$\sum_{i=0}^{k} \binom{10}{i}\binom{15}{k-i} + \sum_{i=0}^{k+1} \binom{12}{i}\binom{13}{k+1-i}$ exists, is equal to________.

Ans. 12

Sol. $(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1x + {}^{10}C_2x^2 + \ldots\ldots + {}^{10}C_{10}x^{10}$

$(1+x)^{15} = {}^{15}C_0 + {}^{15}C_1x + \ldots.{}^{15}C_{k-1}\,x^{k-1} + {}^{15}C_kx^k + {}^{15}C_{k+1}x^{k+1} + \ldots\ldots\ {}^{15}C_{15}x^{15}$

$\sum_{i=0}^{k}(10C_i)(15C_{k-i}) = {}^{10}C_0.\ {}^{15}C_k + {}^{10}C_1.{}^{15}C_{k-1} + \ldots.. + {}^{10}C_k.{}^{15}C_0$

Coefficient of x_k in $(1+x)^{25}$

$= {}^{25}C_k$

$\sum_{i=0}^{k+1}({}^{12}C_i)({}^{13}C_{k+1-i}) = {}^{12}C_0.{}^{13}C_{k+1} + {}^{12}C_1.\ {}^{13}C_k + \ldots\ldots + {}^{12}C_{k+1}.\ {}^{13}C_0$

Coefficient of x^{k+1} in $(1+x)^{25}$

$= {}^{25}C_{k+1}$

${}^{25}C_k + {}^{25}C_{k+1} = {}^{26}C_{k+1}$

For maximum value

k+1= 13

K= 12

9. The sum of first four terms of a geometric progression (G.P.) is $\frac{65}{12}$ and the sum of their respective reciprocals is $\frac{65}{18}$. If the product of first three terms of the G.P. is 1, and the third term is α, then 2α is__________.

Ans. 3

Sol. a, ar, ar^2, ar^3

$a + ar + ar^2 + ar^3 = \frac{65}{12}$ ....(1)

$\frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \frac{1}{ar^3} = \frac{65}{18}$

$$\frac{1}{a}\left(\frac{r^3+r^2+r+1}{r^3}\right)=\frac{65}{18} \qquad ....(2)$$

$$\frac{(i)}{(ii)}, a^2r^3=\frac{18}{12}=\frac{3}{2}$$

$$a^3r^3=1 \Rightarrow a\left(\frac{3}{2}\right)=1 \Rightarrow a=\frac{2}{3}$$

$$\frac{4}{9}r^3=\frac{3}{2} \Rightarrow r^3=\frac{3^3}{2^3} \Rightarrow r=\frac{3}{2}$$

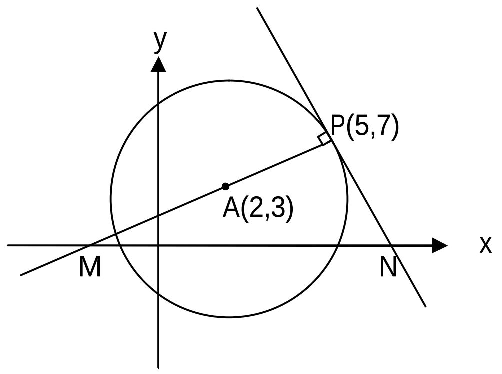
$$\alpha=ar^2=\frac{2}{3}.\left(\frac{3}{2}\right)^2=\frac{3}{2}$$

$2\alpha = 3$

10. If the area of the triangle formed by the positive x-axis, the normal and the tangent to the circle $(x-2)^2+(y-3)^2=25$ at the point (5,7) is A, then 24A is equal to______.

Ans. 1225

Sol.



Equation of normal at P

$$(y-7)=\left(\frac{7-3}{5-2}\right)(x-5)$$

3y –21 = 4x – 20

$\Rightarrow$ 4x – 3y + 1 = 0 ....(i)

$$\Rightarrow \quad M\left(-\frac{1}{4},0\right)$$

Equation of tangent at P

$$\left(y-7\right)=-\frac{3}{4}\left(x-5\right)$$

$4y - 28 = -3x + 15$

$\Rightarrow 3x + 4y = 43$ ....(ii)

$$\Rightarrow \quad N\left(\frac{43}{3},0\right)$$

Hence ar (ΔPMN) = $\frac{1}{2}\times MN\times 7$

$$\lambda=\frac{1}{2}\times\frac{175}{12}\times 7$$

$\Rightarrow 24\lambda = 1225$