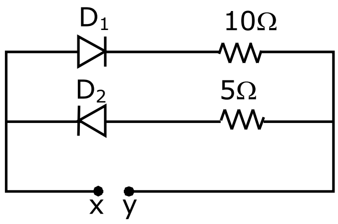
25^{th} Feb. 2021 | Shift - 1
PHYSICS

Section - A

1. A 5V battery is connected across the points X and Y. Assume D_1 and D_2 to be normal silicon diodes. Find the current supplied by the battery if the +ve terminal of the battery is connected to point X.



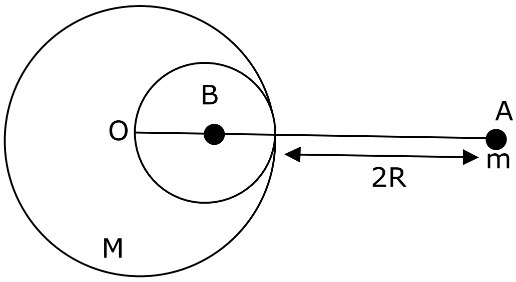
(1) ~0.86 A (2) ~0.5 A (3) ~ 0.43 A (4)~ 1.5 A

Sol. **3**

Since silicon diode is used so 0.7 Volt is drop across it, only D_1 will conduct so current through cell

$I = \frac{5-0.7}{10} = 0.43$ A

2. A solid sphere of radius R gravitationally attracts a particle placed at 3R from its centre with a force F_1. Now a spherical cavity of radius $\left(\frac{R}{2}\right)$ is made in the sphere (as shown in figure) and the force becomes F_2. The value of $F_1 : F_2$ is :



(1)41 : 50 (2) 36 : 25 (3) 50 : 41 (4) 25 : 36

Sol. **1**

$$g_1 = \frac{GM}{(3R)^2} = \frac{GM}{9R^2}$$

$$g_2 = \frac{GM}{9R^2} - \frac{G\left(\frac{M}{8}\right)}{\left(3R - \frac{R}{2}\right)^2}$$

$$= \frac{GM}{9R^2} - \frac{GM}{R^2 50} = \frac{41}{9 \times 50}\frac{GM}{R^2}$$

$$\frac{g_1}{g_2} = \frac{41}{50}$$

Force $\Rightarrow \frac{F_1}{F_2} = \frac{mg_1}{mg_2} = \frac{41}{50}$

3. A student is performing the experiment of resonance column. The diameter of the column tube is 6 cm. The frequency of the tuning fork is 504 Hz. Speed of the sound at the given temperature is 336 m/s. The zero of the metre scale coincides with the top end of the resonance column tube. The reading of the water level in the column when the first resonance occurs is :

(1) 13 cm (2) 14.8 cm (3) 16.6 cm (4) 18.4 cm

Sol. **2**

$\lambda = \frac{v}{f} = \frac{336}{504} = 66.66\text{cm}$

$\frac{\lambda}{4} = l + e = l + 0.3d$

$= l + 1.8$

$16.66 = l + 1.8$ cm

$l = 14.86$ cm

4. A diatomic gas, having $C_p = \frac{7}{2}R$ and $C_v = \frac{5}{2}R$, is heated at constant pressure.

The ratio dU : dQ : dW

(1) 3 : 7 : 2 (2) 5 : 7 : 2 (3) 5 : 7 : 3 (4) 3 : 5 : 2

Sol. **2**

$C_p = \frac{7}{2}R$

$C_v = \frac{5}{2}R$

$dU = nC_vdT$

$dQ = nC_pdT$

$dW = nRdT$

$dU : dQ : dW$

$C_v : C_p : R$

$\frac{5}{2}R : \frac{7}{2}R : R$

$5 : 7 : 2$

5. Given below are two statements :

Statement I : A speech signal of 2 kHz is used to modulate a carrier signal of 1 MHz. The bandwidth requirement for the signal is 4 kHz.

Statement II : The side band frequencies are 1002 kHz and 998 kHz.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both statement I and statement II are false

(2) Statement I is false but statement II is true

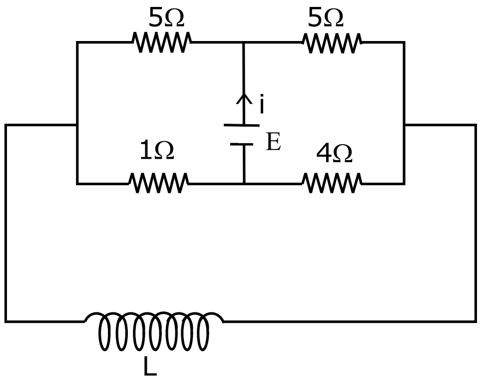
(3) Statement I is true but statement II is false

(4) Both statement I and statement II are true

Sol. **4**

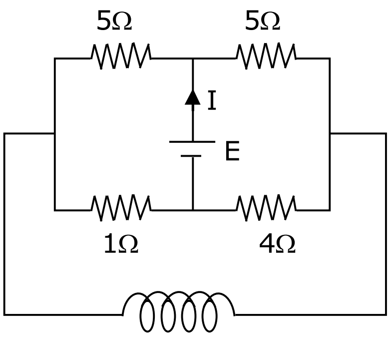
Side band = (f_c - f_m) to (f_c + f_m)
= (1000 – 2) KHz to (1000 + 2) KHz
= 998 KHz to 1002 kHz
Band width = $2f_m$
= 2 ×2 KHz
= 4 KHz
Both statements are true

6. The current (i) at time t=0 and t = ∞ respectively for the given circuit is :

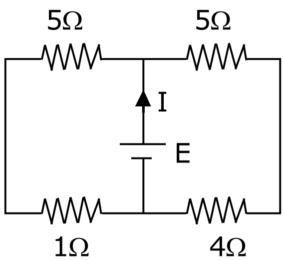


(1) $\frac{18E}{55}, \frac{5E}{18}$ (2) $\frac{5E}{18}, \frac{18E}{55}$ (3) $\frac{5E}{18}, \frac{10E}{33}$ (4) $\frac{10E}{33}, \frac{5E}{18}$

Sol. **3**



at t = 0, inductor is removed, so circuit will look like this
at t = 0

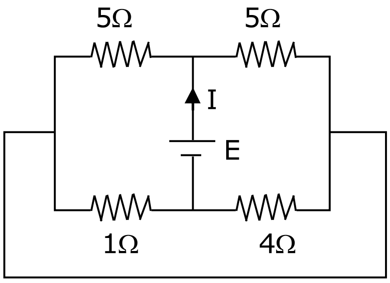


$R_{eq} = \frac{6 \times 9}{6+9} = \frac{54}{15}$

$I\ (t = 0) = \frac{E \times 15}{54} = \frac{5E}{18}$

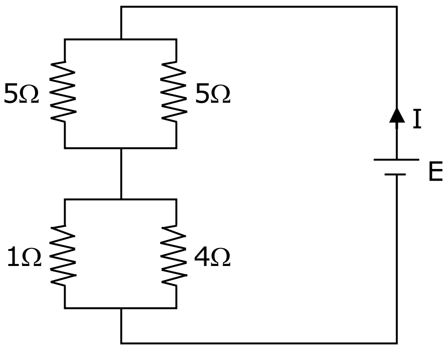
at $t = \infty$, inductor is replaced by plane wire, so circuit will look like this

at $t = \infty$



$$I\ (t = \infty) = \frac{E}{\frac{5}{2} + \frac{4}{5}} = \frac{10E}{33}$$

Now,



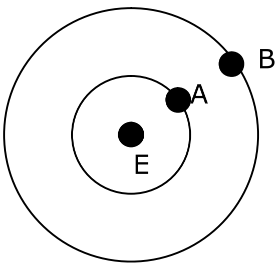
$$R_{eq} = \frac{1 \times 4}{1 + 4} + \frac{5 \times 5}{5 + 5}$$

$$= \frac{4}{5} + \frac{5}{2} = \frac{8 + 25}{10} = \frac{33}{10}$$

$$I = \frac{E}{R_{eq}} = \frac{10E}{33}$$

7. Two satellites A and B of masses 200 kg and 400 kg are revolving round the earth at height of 600 km and 1600 km respectively.

If T_A and T_B are the time periods of A and B respectively then the value of $T_B - T_A$:

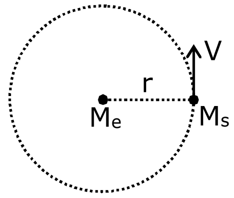


[Given : radius of earth = 6400 km, mass of earth = 6×10^{24} kg]

(1) 4.24×10^2 s (2) 3.33×10^2 s (3) 1.33×10^3 s (4) 4.24×10^3 s

Sol. **3**

$V = \sqrt{\dfrac{GM_e}{r}}$



$T = \dfrac{2\pi r}{\sqrt{\dfrac{GM_e}{r}}} = 2\pi r\sqrt{\dfrac{r}{GM_e}}$

$T = \sqrt{\dfrac{4\pi^2 r^3}{GM_e}} = \sqrt{\dfrac{4\pi^2 r^3}{GM_e}}$

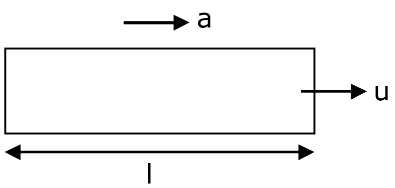
$T_2 - T_1 = \sqrt{\dfrac{4\pi^2(8000\times10^3)^3}{G\times6\times10^{24}}} - \sqrt{\dfrac{4\pi^2(7000\times10^3)^3}{G\times6\times10^{24}}}$

$\cong 1.33 \times 10^3 s$

8. An engine of a train, moving with uniform acceleration, passes the signal post with velocity u and the last compartment with velocity v. The velocity with which middle point of the train passes the signal post is :

(1) $\sqrt{\dfrac{v^2 - u^2}{2}}$ (2) $\dfrac{v-u}{2}$ (3) $\sqrt{\dfrac{v^2 + u^2}{2}}$ (4) $\dfrac{u+v}{2}$

Sol. **3**



a = uniform acceleration
u = velocity of first compartment
v = velocity of last compartment
l = length of train
$v^2 = u^2 + 2as$ (3rd equation of motion)
$v^2 = u^2 + 2al$ .....(1)

$v^2_{middle} = u^2 + 2a\,\dfrac{l}{2}$

$\therefore v^2_{middle} = u^2 + al$ ....(2)

from equation (1) and (2)

$v^2_{middle} = u^2 + \left(\dfrac{v^2 - u^2}{2}\right)$

$= \dfrac{v^2 + u^2}{2}$

$\therefore v_{middle} = \sqrt{\dfrac{v^2 + u^2}{2}}$

9. A proton, a deuteron and an α particle are moving with same momentum in a uniform magnetic field. The ratio of magnetic forces action on them is ________ and their speed is ______, in the ratio.

(1) 2 : 1 : 1 and 4 : 2 : 1 (2) 1 : 2 : 4 and 2 : 1 :1

(3) 1 : 2 : 4 and 1 : 1 : 2 (4) 4 : 2 : 1 and 2 : 1 : 1

Sol. **1**

As $v = \frac{p}{m}$ & $F = qvB$

$\therefore F = \frac{qp}{m}B$

$F_1 = \frac{qpB}{m}, v_1 = \frac{p}{m}$

$F_2 = \frac{qpB}{2m}, v_2 = \frac{p}{2m}$

$F_3 = \frac{2qpB}{4m}, v_3 = \frac{p}{4m}$

$F_1 : F_2 : F_3$ & $V_1 : V_2 : V_3$

$1 : \frac{1}{2} : \frac{1}{2}$ & $1 : \frac{1}{2} : \frac{1}{4}$

2 : 1 : 1 & 4 : 2 : 1

10. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : When a rod lying freely is heated, no thermal stress is developed in it.

Reason R : On heating, the length of the rod increases.

In the light of the above statements, choose the corect answer from the options given below :

(1) A is true but R is false

(2) Both A and R are true and R is the correct explanation of A

(3) Both A and R are true but R is NOT the correct explanation of A

(4) A is false but R is true

Sol. **3**

When a rod is free and it is heated then there is no thermal stress produced in it.

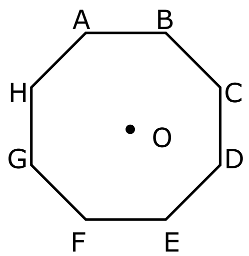
The rod will expand due to increase in temperature.

so both a & R are true.

11. In an octagon ABCDEFGH of equal side, what is the sum of

$\overrightarrow{AB} + \overrightarrow{AC} + \overrightarrow{AD} + \overrightarrow{AE} + \overrightarrow{AF} + \overrightarrow{AG} + \overrightarrow{AH}$

If, $\overrightarrow{AO} = 2\hat{i} + 3\hat{j} - 4\hat{k}$



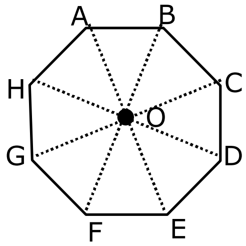
(1) $16\hat{i} + 24\hat{j} - 32\hat{k}$ (2) $-16\hat{i} - 24\hat{j} - 32\hat{k}$ (3) $-16\hat{i} - 24\hat{j} + 32\hat{k}$ (4) $-16\hat{i} + 24\hat{j} + 32\hat{k}$

Sol. **1**

$\overrightarrow{AO}+\overrightarrow{OB}=\overrightarrow{AB}$

$\overrightarrow{AO}+\overrightarrow{OC}=\overrightarrow{AC}$

$\overrightarrow{AO}+\overrightarrow{OD}=\overrightarrow{AD}$



$\overrightarrow{AO}+\overrightarrow{OE}=\overrightarrow{AE}$

$\overrightarrow{AO}+\overrightarrow{OF}=\overrightarrow{AF}$

$\overrightarrow{AO}+\overrightarrow{OG}=\overrightarrow{AG}$

$\overrightarrow{AO}+\overrightarrow{OH}=\overrightarrow{AH}$

$8\ \overrightarrow{AO} = (\overrightarrow{AB}+\overrightarrow{AC}+\overrightarrow{AD}+\overrightarrow{AE}+\overrightarrow{AF}+\overrightarrow{AG}+\overrightarrow{AH})$

$=8(2\hat{i}+3\hat{j}-4\hat{k})$.

$=16\hat{i}+24\hat{j}-32\hat{k}$

12. Two radioactive substances X and Y originally have N_1 and N_2nuclei respectively. Half life of X is half of the half life of Y. After there half lives of Y, number of nuclei of both are equal. The ratio $\frac{N_1}{N_2}$ will be equal to :

(1) $\frac{8}{1}$ (2) $\frac{1}{8}$ (3) $\frac{3}{1}$ (4) $\frac{1}{3}$

Sol. **1**

After n half life no of nuclei undecayed = $\frac{N_o}{2^n}$

given $T_{\frac{1}{2}x} = \frac{T_{\frac{1}{2}y}}{2}$

So 3 half life of y = 6 half life of x

Given, $N_x = N_y\left(\text{after } 3T_{\frac{1}{2}y}\right)$

$\frac{N_1}{2^6}=\frac{N_2}{2^3}$

$\frac{N_1}{N_2}=\frac{2^6}{2^3}=2^3=\frac{8}{1}$

13. Match List –I with List- II :

List-I	List-II
(a)h (Planck's constant)	(i) $[M\ L\ T^{-1}]$
(b)E (kinetic energy)	(ii) $[M\ L^2\ T^{-1}]$
(c)V (electric potential)	(iii) $[M\ L^2\ T^{-2}]$
(d)P (linear momentum)	(iv) $[ML^2I^{-1}\ T^{-3}]$

Choose the correct answer from the options given below :

(1) (a) $\rightarrow$ (ii), (b) $\rightarrow$ (iii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)

(2) (a) $\rightarrow$ (i), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (iii)

(3) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (ii), (c) $\rightarrow$ (iv), (d) $\rightarrow$ (i)

(4) (a) $\rightarrow$ (iii), (b) $\rightarrow$ (iv), (c) $\rightarrow$ (ii), (d) $\rightarrow$ (i)

Sol. **1**

K.E. = $[ML^2T^{-2}]$

P (linear momentum) = $[MLT^{-1}]$

h (planck's constant) = $[ML^2T^{-1}]$

v (electric potential) = $[ML^2T^{-3}I^{-1}]$

14. The pitch of the screw guage is 1 mm and there are 100 divisions on the circular scale. When nothing is put in between the jaws, the zero of the circular scale lines 8 divisions below the reference line. When a wire is placed between the jaws, the first linear scale division is clearly visible while 72^{nd} division on circular scale coincides with the reference line. The radius of the wire is :

(1) 1.64 mm (2) 1.80 mm (3) 0.82 mm (4) 0.90 mm

Sol. **3**

Least count. = $\frac{\text{pitch}}{\text{no. of div.}} = \frac{1\text{mm}}{100}$ = 0.01 m

+ve error = 8 × L.C. = +0.08 mm

measured reading = 1mm + 72 × L.C.

= 1mm + 0.72 mm

= 1.72 mm

True reading = 1.72 – 0.08

= 1.64 mm

Radius = $\frac{1.64}{2}$ = 0.82 mm

15. If the time period of a two meter long simple pendulum is 2 s, the acceleration due to gravity at the place where pendulum is executing S.H.M. is :

(1) $2\pi^2 ms^{-2}$ (2) $16 m/s^2$ (3) $9.8 ms^{-2}$ (4) $\pi^2 ms^{-2}$

Sol. **1**

$$T = 2\pi\sqrt{\frac{l}{g}}$$

$$T^2 = \frac{4\pi^2 l}{g}$$

$$g = \frac{4\pi^2 l}{T^2}$$

$$= \frac{4\pi^2 \times 2}{(2)^2} = 2\pi^2 ms^{-2}$$

16. An α particle and a proton are accelerated from rest by a potential difference of 200 V. After this, their de Broglie wavelengths are λ_α and λ_p respectively. The ratio $\frac{\lambda_p}{\lambda_\alpha}$ is :

(1) 8 (2) 2.8 (3) 3.8 (4) 7.8

Sol. **2**

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mqv}}$$

$$\frac{\lambda_p}{\lambda_\alpha} = \sqrt{\frac{m_\alpha q_\alpha}{m_p q_p}} = \sqrt{\frac{4 \times 2}{1 \times 1}}$$

$$= 2\sqrt{2} = 2.8$$

17. Given below are two statements : one is labelled as Assertion A and the other is labelled as reason R.

Assertion A : The escape velocities of planet A and B are same. But A and B are of unequal mass.

Reason R : The product of their mass and radius must be same. $M_1R_1 = M_2R_2$

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Both A and R are correct but R is NOT the correct explanation of A
(2) A is correct but R is not correct
(3) Both A and R are correct and R is the correct explanation of A
(4) A is not correct but R is correct

Sol. **2**

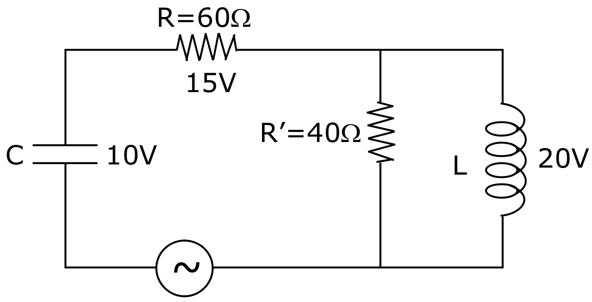
V_e = escape velocity

$$v_e = \sqrt{\frac{2GM}{R}}$$

so for same v_e, $\frac{M_1}{R_1} = \frac{M_2}{R_2}$

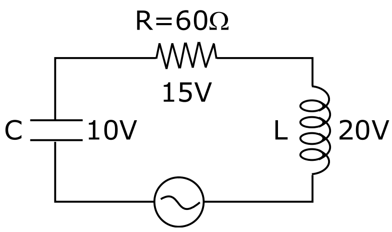
A is true but R is false

18. The angular frequency of alterlating current in a L-C-R circuit is 100 rad/s. The components connected are shown in the figure. Find the value of inductance of the coil and capacity of condenser.



(1) 0.8 H and 250 μF
(2) 0.8 H and 150 μF
(3) 1.33 H and 250 μF
(4) 1.33 H and 150 μF

Sol. **1**



Since key is open, circuit is series

$15 = i_{rms}\ (60)$

$\therefore i_{rms} = \frac{1}{4}$ A

Now, $20 = \frac{1}{4} X_L = \frac{1}{4} (\omega L)$

$\therefore L = \frac{4}{5} = 0.8$ H

& $10 = \frac{1}{4} \frac{1}{(100C)}$

$C = \frac{1}{4000}$ F = 250 μF

19. Two coherent light sources having intensity in the ratio 2x produce an interference pattern. The ratio $\frac{I_{max} - I_{min}}{I_{max} + I_{min}}$ will be :

(1) $\frac{2\sqrt{2x}}{x+1}$
(2) $\frac{\sqrt{2x}}{2x+1}$
(3) $\frac{2\sqrt{2x}}{2x+1}$
(4) $\frac{\sqrt{2x}}{x+1}$

Sol. **3**

Let $I_1 = 2x$

$I_2 = 1$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2$$

$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2$$

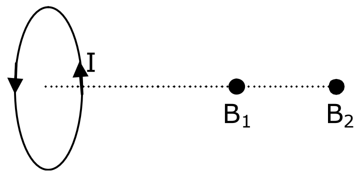
$$\frac{I_{max} - I_{min}}{I_{max} + I_{min}} = \frac{\left(\sqrt{2x}+1\right)^2 - \left(\sqrt{2x}-1\right)^2}{\left(\sqrt{2x}+1\right)^2 + \left(\sqrt{2x}-1\right)^2}$$

$$= \frac{4\sqrt{2x}}{2+4x} = \frac{2\sqrt{2x}}{1+2x}$$

20. Magnetic fields at two points on the axis of a circular coil at a distance of 0.05 m and 02 m from the centre are in the rato 8 : 1. The radius of coil is _______

(1) 0.15 m (2) 0.2 m (3) 0.1 m (4) 1.0 m

Sol. **3**



$$B = \frac{\mu_0 NiR^2}{2(R^2 + x^2)^{3/2}}$$

at $x_1 = 0.05$m, $B_1 = \dfrac{\mu_0 NiR^2}{2(R^2 + (0.05)^2)^{3/2}}$

at $x_2 = 0.2$m, $B_2 = \dfrac{\mu_0 NiR^2}{2(R^2 + (0.2)^2)^{3/2}}$

$$\frac{B_1}{B_2} = \frac{(R^2 + 0.04)^{3/2}}{(R^2 + 0.0025)^{3/2}}$$

$$\left(\frac{8}{1}\right)^{2/3} = \frac{R^2 + 0.04}{R^2 + 0.0025}$$

$4\ (R^2 + 0.0025) = R^2 + 0.04$

$3R^2 = 0.04 - 0.0100$

$$R^2 = \frac{0.03}{3} = 0.01$$

$$R = \sqrt{0.01} = 0.1 \text{ m}$$

Section - B

1. The same size images are formed by a convex lens when the object is placed at 20 cm or at 10 cm from the lens. The focal length of convex lens is ______ cm.

Sol. **15**

$\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$ ...(1)

$m = \frac{v}{u}$ ....(2)

from (1) and (2) we get

$m = \frac{f}{f+u}$

given conditions

$m_1 = -m_2$

$\frac{f}{f-10} = \frac{-f}{f-20}$

$f - 20 = -f + 10$

$2f = 30$

$f = 15$ cm

2. The electric field in a region is given by $\vec{E} = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right)\frac{N}{C}$. The ratio of flux of reported field through the rectangular surface of area 0.2 m^2 (parallel to y-z plane) to that of the surface of area 0.3 m^2(parallel to x-z plane) is a : b, where a = ________

[Here $\hat{i}, \hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axes respectively]

Sol. **0.5**

$\phi = \vec{E}.\vec{A}$

$\vec{A}_a = 0.2\,\hat{i}$

$\vec{A}_b = 0.3\,\hat{j}$

$\phi_a = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).0.2\hat{i}$

$\phi_a = \frac{3}{5}E_0 \times 0.2$

$\phi_a = \left(\frac{3}{5}E_0\hat{i} + \frac{4}{5}E_0\hat{j}\right).0.3\hat{j}$

$\phi_b = \frac{4}{5}E_0 \times 0.3$

$\frac{a}{b} = \frac{\phi_a}{\phi_b} = \frac{\frac{3}{5}E_0 \times 0.2}{\frac{4}{5}E_0 \times 0.3} = \frac{6}{12} = 0.5$

3. 512 identical drops of mercury are charged to a potential of 2 V each. The drops are joined to form a single drop. The potential of this drop is _____ V.

Sol. **128**

Let charge on each drop = q

radius = r

$v = \frac{kq}{r}$

$2 = \frac{kq}{r}$

radius of bigger

$\frac{4}{3}\pi R^3 = 512 \times \frac{4}{3}\pi r^3$

R = 8r

$v = \frac{k(512)q}{R} = \frac{512}{8}\frac{kq}{r} = \frac{512}{8} \times 2$

= 128 V

4. The potential energy (U) of a diatomic molecule is a function dependent on r (interatomic distance) as $U = \frac{\alpha}{r^{10}} - \frac{\beta}{r^5} - 3$ Where, a and b are positive constants. The equilibrium distance between two atoms will $\left(\frac{2\alpha}{\beta}\right)^{\frac{a}{b}}$. Where a =________

Sol. **1**

$F = -\frac{dU}{dr}$

$F = -\left[-\frac{10\alpha}{r^{11}} + \frac{5\beta}{r^6}\right]$

for equilibrium, F = 0

$\frac{10\alpha}{r^{11}} = \frac{5\beta}{r^6}$

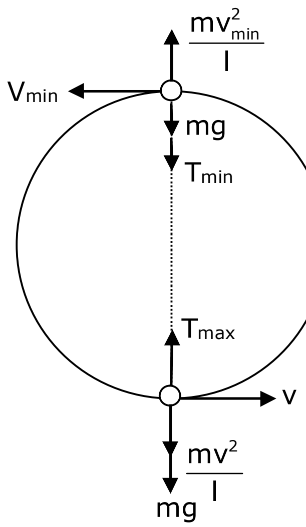
$\frac{2\alpha}{\beta} = r^5$

$r = \left(\frac{2\alpha}{\beta}\right)^{1/5}$

a = 1

5. A small bob tied at one end of a thin string of length 1m is describing a vertical circle so that the maximum and minimum tension in the string are in the rato 5 : 1. The velocity of the bob at the highest position is _______ m/s. (take g=10 m/s^2)

Sol. **5**



by conservation of energy,

$v^2_{min} = V^2 - 4gl$ ....(1)

$T_{max} = mg + \frac{mv^2}{l}$ ....(2)

$T_{min} = \frac{mv^2_{min}}{l} - mg$ ....(3)

from equation (1) and (3)

$T_{min} = \frac{m}{l}(v^2 - 4gl) - mg$

$$\frac{T_{max}}{T_{min}} = \frac{\frac{v^2}{l} + g}{\frac{v^2}{l} - 5g}$$

$$\frac{5}{1} = \frac{\frac{v^2}{1} + 10}{\frac{v^2}{1} - 50}$$

$5v^2 - 250 = v^2 + 10$

$v^2 = 65$ ....(4)

from equation (4) and (1)

$v^2_{min} = 65 - 40 = 25$

$v_{min} = 5$

6. In a certain themodynamical process, the pressure ofa gas depends on its volume as kV^3 . The work done when the temperature changes from 100ºC to 300ºC will be ____ nR, where n denotes number of moles of a gas.

Sol. 50

$P = kv^3$

$pv^{-3} = k$

$x = -3$

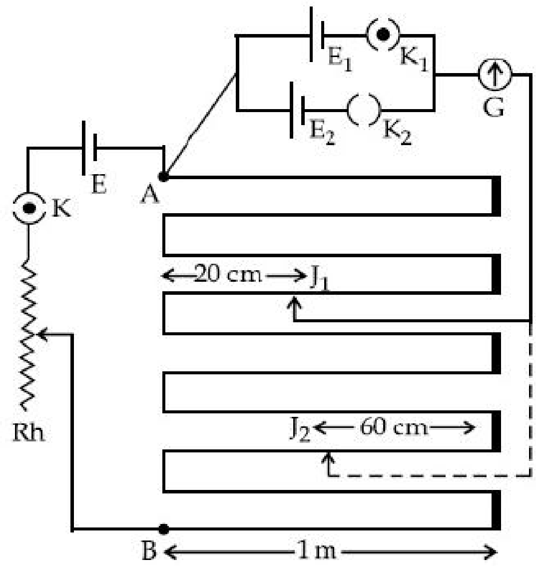
$w = \dfrac{nR(T_1 - T_2)}{x - 1}$

$= \dfrac{nR(100 - 300)}{-3 - 1}$

$= \dfrac{nR(-200)}{-4}$

$= 50nR$

7. In the given circuit of potentiometer, the potentital difference E across AB (10 m length) is larger than E_1 and E_2 as well. For key K_1 (closed), the jockey is adjusted to touch the wire at point J_1 so that there is no deflection in the galvanometer. Now the first battery (E_1) is replaced by second battery (E_2) for working by making K_1 open and E_2 closed. The galvanometer gives then null deflection at J_2. The value of $\frac{E_1}{E_2}$ is $\frac{a}{b}$, where a = ____.



Sol. 1

$\dfrac{E_1}{E_2} = \dfrac{l_1}{l_2}$

$= \dfrac{3 \times 100cm + (100 - 20)cm}{7 \times 100cm + 60cm}$

$= \dfrac{380}{760} = \dfrac{1}{2} = \dfrac{a}{b}$

$a = 1$

8. A monoatomic gas of mass 4.0 u is kept in an insulated container. Container is moving with velocity 30 m/s. If container is suddenly stopped then change in temperature of the gas (R=gas constant) is $\frac{x}{3R}$. Value of x is _______ .

Sol. **3600**

$\Delta K_E = \Delta U$

$\Delta U = nC_V\Delta T$

$\frac{1}{2}mv^2 = \frac{3}{2}nR\Delta T$

$\frac{mv^2}{3nR} = \Delta T$

$\frac{4 \times (30)^2}{3 \times 1 \times R} = \Delta T$

$\Delta T = \frac{1200}{R}$

$\frac{x}{3R} = \frac{1200}{R}$

$x = 3600$

9. A coil of inductance 2 H having negligible resistance is connected to a source of supply whose voltage is given by V =3t volt. (where t is in second). If the voltage is applied when t = 0, then the energy stored in the coil after 4 s is ________ J.

Sol. **144**

$L\frac{di}{dt} = \varepsilon$

$= 3t$

$L\int di = 3\int tdt$

$Li = \frac{3t^2}{2}$

$i = \frac{3t^2}{2L}$

energy, $E = \frac{1}{2}Li^2$

$= \frac{1}{2}L\left(\frac{3t^2}{2L}\right)^2$

$= \frac{1}{2} \times \frac{9t^4}{4L}$

$= \frac{9}{8} \times \frac{(4)^4}{4 \times 2} = 144$ J

10. A transmitting station releases waves of wavelength 960 m. A capacitor of 256 μF is used in the resonant circuit. The self inductance of coil necessary for resonance is _____ $\times 10^{-8}$H.

Sol. **10**

Since resonance

$$\omega_r = \frac{1}{\sqrt{LC}}$$

$$\therefore\ 2\pi f = \frac{1}{\sqrt{LC}}$$

$$\therefore\ 4\pi^2\frac{C^2}{\lambda^2} = \frac{1}{LC}$$

$$\therefore \frac{4\pi^2 \times 9 \times 10^8 \times 9 \times 10^8}{960 \times 960} = \frac{1}{L \times 2.56 \times 10^{-6}}$$

$$L = \frac{375 \times 960}{10^{-6} \times 4 \times \pi^2 \times 9 \times 10^{16}} = \frac{10^3}{10^{10}}$$

$= 10^{-7}$ H

$= 10 \times 10^{-8}$

25^{th} Feb. 2021 | Shift - 1
CHEMISTRY

SECTION - A

1. Ellingham diagram is a graphical representation of:

(1) ΔG vs T (2) $(\Delta G - T\Delta S)$ vs T (3) ΔH vs T (4) ΔG vs P

Sol. **(1)**

Ellingham diagram tells us about the spontanity of a reaction with temperature.

2. Which of the following equation depicts the oxidizing nature of H_2O_2?

(1) $Cl_2 + H_2O_2 \rightarrow 2HCl + O_2$ (2) $KIO_4 + H_2O_2 \rightarrow KIO_3 + H_2O + O_2$

(3) $2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$ (4) $I_2 + H_2O_2 + 2OH^- \rightarrow 2I^- + 2H2O + O_2$

Sol. **(3)**

$2I^- + H_2O_2 + 2H^+ \rightarrow I_2 + 2H_2O$

Oxygen reduces from -1 to -2

So, its reduction will takes place. Hence it will behave as oxidising agent or it shows oxidising nature.

While in other option it change from (-1) to 0.

3. In Freundlich adsorption isotherm at moderate pressure, the extent of adsorption $\left(\frac{x}{m}\right)$ is directly proportional to P^x. The value of x is:

(1) ∞ (2) 1 (3) zero (4) $\frac{1}{n}$

Sol. **(4)**

$\frac{x}{m} = p^x$

the formula is $\frac{x}{m} = p^{1/n}$

Hence $x = \frac{1}{n}$

The value of 'n' is any natural number.

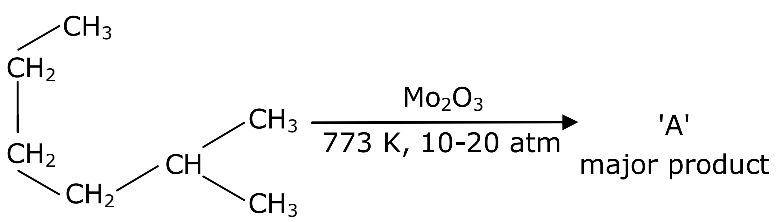
4. According to molecular orbital theory, the species among the following that does not exist is:

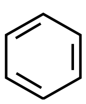
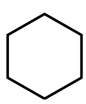
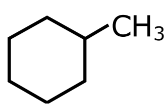
(1) He_2^- (2) He_2^+ (3) O_2^{2-} (4) Be_2

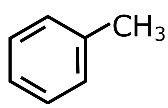
Sol. **(4)**

B.O. of Be_2 is zero, So it does not exist.

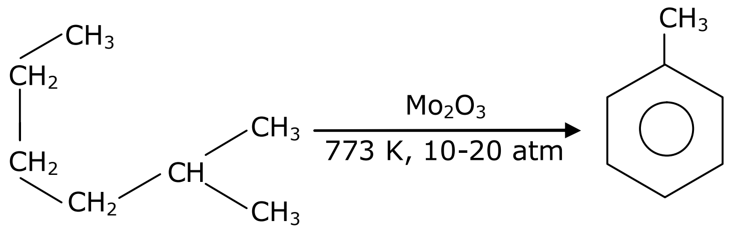
5. Identify A in the given chemical reaction.



(1)  (2)  (3) 

(4) 

Sol. **(4)**



Aromatization reaction or hydroforming reaction.

6. Given below are two statements:
Statement-I : CeO_2 can be used for oxidation of aldehydes and ketones.
Statement-II : Aqueous solution of $EuSO_4$ is a strong reducing agent.
(1) Statement I is true, statement II is false
(2) Statement I is false, statement II is true
(3) Both Statement I and Statement II are false
(4) Both Statement I and Statement II are true

Sol. **(4)**
CeO_2 can be used as oxidising agent like seO_2.
Similarly $EuSO_4$ used as a reducing agent.

7. The major product of the following chemical reaction is:

$$CH_3CH_2CN \xrightarrow[\text{3) Pd/BaSO}_4\text{, H}_2]{\substack{\text{1) } H_3O^+,\ \Delta \\ \text{2) } SOCl_2}} ?$$

(1) $(CH_3CH_2CO)_2O$ (2) CH_3CH_2CHO (3) $CH_3CH_2CH_3$ (4) $CH_3CH_2CH_2OH$

Sol. **(2)**

$$CH_3CH_2CN \xrightarrow{H_3O^+} CH_3-CH_2-\overset{\overset{O}{||}}{C}-OH \xrightarrow{SOCl_2} CH_3-CH_2-\overset{\overset{O}{||}}{C}-Cl$$

$$\xrightarrow{Pd/Ba,SO_4,} CH_3-CH_2-CH=O$$

8. Complete combustion of 1.80 g of an oxygen containing compound ($C_xH_yO_z$) gave 2.64 g of CO_2 and 1.08 g of H_2O. The percentage of oxygen in the organic compound is:
(1) 63.53 (2) 53.33 (3) 51.63 (4) 50.33

Sol. **(2)**

$$n_{CO_2} = \frac{2.64}{44} = 0.06$$

$n_c = 0.06$

weight of carbon = $0.06 \times 12 = 0.72$ gm

$$n_{H_2O} = \frac{1.08}{18} = 0.06$$

$n_H = 0.06 \times 2 = 0.12$

weight of H = 0.12 gm

$\therefore$ Weight of oxygen in $C_xH_yO_z$

$= 1.8 - (0.72 + 0.12)$

$= 0.96$ gram

$$\% \text{ weight of oxygen} = \frac{0.96}{1.8} \times 100$$

$$= 53.3\%$$

9. The correct statement about B_2H_6 is:

(1) All B–H–B angles are of 120°.

(2) Its fragment, BH_3, behaves as a Lewis base.

(3) Terminal B–H bonds have less p-character when compared to bridging bonds.

(4) The two B–H–B bonds are not of same length.

Sol. **(3)**

Terminal bond angle is greater than that of bridge bond angle

Bond angle $\propto$ S-character

$$\propto \frac{1}{p - \text{character}}$$

10. In which of the following pairs, the outer most electronic configuration will be the same?

(1) Fe^{2+} and Co^{+}

(2) Cr^{+} and Mn^{2+}

(3) Ni^{2+} and Cu^{+}

(4) V^{2+} and Cr^{+}

Sol. **(2)**

$Cr^{+} \rightarrow [Ar]3d^5$

$Mn^{2+} \Rightarrow [Ar]3d^5$

11. Which statement is correct?

(1) Buna-S is a synthetic and linear thermosetting polymer

(2) Neoprene is addition copolymer used in plastic bucket manufacturing

(3) Synthesis of Buna-S needs nascent oxygen

(4) Buna-N is a natural polymer

Sol. **(3)**

Synthesis of Buna-S needs nascent oxygen.

12. Given below are two statements:

Statement-I : An allotrope of oxygen is an important intermediate in the formation of reducing smog.

Statement-II : Gases such as oxides of nitrogen and sulphur present in troposphere contribute to the formation of photochemical smog.

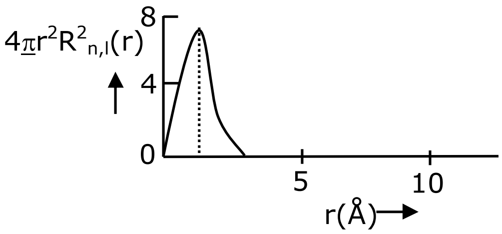
In the light of the above statements, choose the correct answer from the options given below:

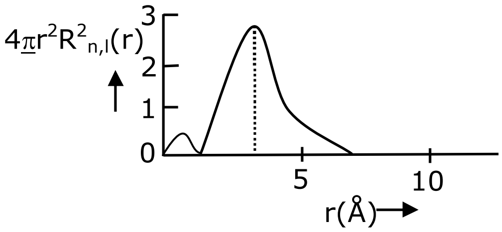
(1) Statement I and Statement II are true
(2) Statement I is true about Statement II is false
(3) Both Statement I and Statement II are false
(4) Statement I is false but Statement II is true

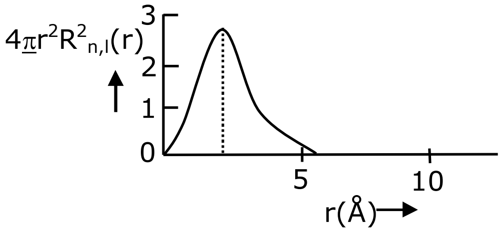
Sol. (3)

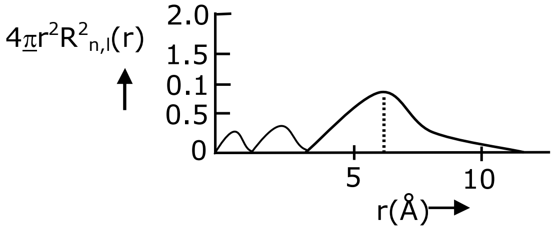
Reducing smog as is acts as reducing agent, the reducing character is due to presence of sulphur dioxide and carbon particles.

13. The plots of radial distribution functions for various orbitals of hydrogen atom against 'r' are given below:

(A)


(B)


(C)


(D)


The correct plot for 3s orbital is:

(1) D (2) B (3) A (4) C

Sol. **(1)**

3s orbital

Number of radial nodes = n − ℓ − 1

For 3s orbital n = 3 ℓ = 0

Number of radial nodes = 3 − 0 − 1 = 2

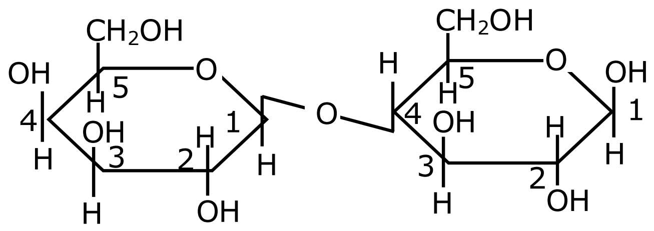
It is correctly represented in graph of option D

14. Which of the glycosidic linkage galactose and glucose is present in lactose?

(1) C-1 of glucose and C-6 of galactose (2) C-1 of galactose and C-4 of glucose

(3) C-1 of glucose and C-4 of galactose (4) C-1 of galactose and C-6 of glucose

Sol. **(2)**



β-D-Galactose β-D-Glucose

15. Which one of the following reactions will not form acetaldehyde?

(1) $CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4}$

(2) $CH_3CN \xrightarrow[\text{ii) } H_2O]{\text{i) DIBAL-H}}$

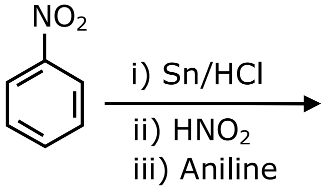
(3) $CH_2 = CH_2 + O_2 \xrightarrow[H_2O]{Pd(II)/Cu(II)}$

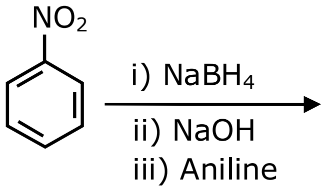
(4) $CH_3CH_2OH \xrightarrow[573\ K]{Cu}$

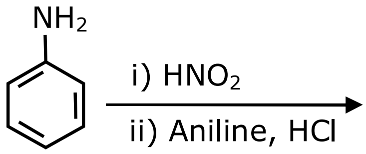
Sol. **(1)**

$$CH_3CH_2OH \xrightarrow{CrO_3 - H_2SO_4} CH_3 - COOH$$

16. Which of the following reaction/s will not give p-aminoazobenzene?

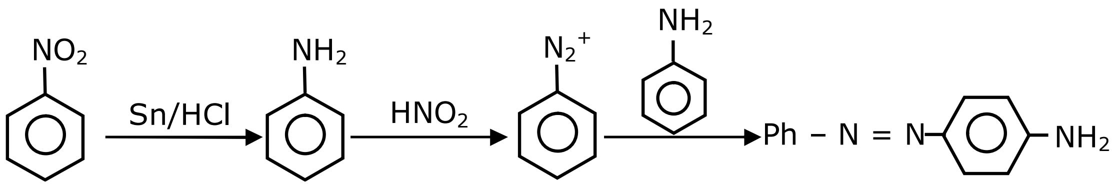
(A) 

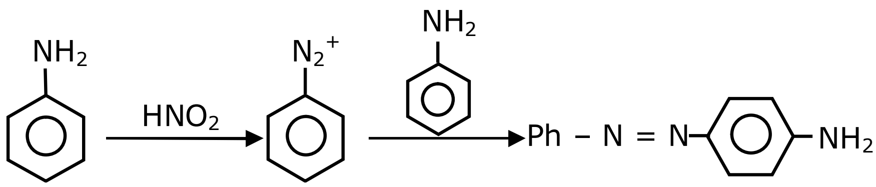
(B) 

(C) 

(1) B only (2) A and B (3) C only (4) A only

Sol. **(1)**

(A) 

(C) 

17. The hybridization and magnetic nature of $[Mn(CN)_6]^{4-}$ and $[Fe(CN)_6]^{3-}$, respectively are:

(1) d^2sp^3 and paramagnetic (2) sp^3d^2 and paramagnetic

(3) d^2sp^3 and diamagnetic (4) sp^3d^2 and diamagnetic

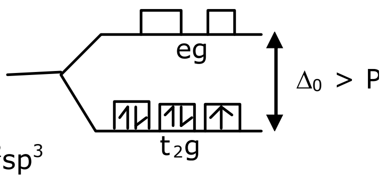
Sol. **(1)**

1. $(Mn(CN)_6)^{4-}$

$Mn^{++} = 3d^5$

$\mu = \sqrt{3}$

hybridization = d^2sp^3

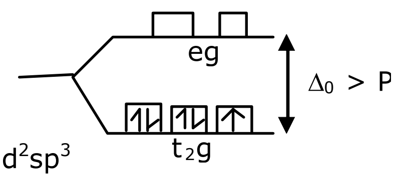


2. $[Fe(CN)_6]^{3-}$

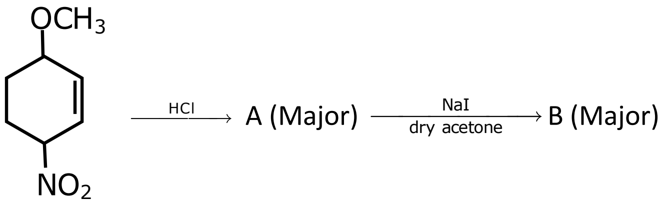
$Fe^{3+} = 3d^5$

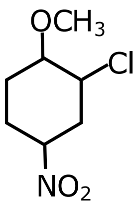
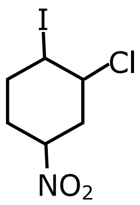
$\mu = \sqrt{3}$

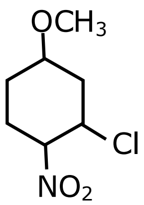
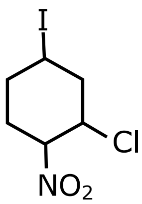
Hybridization – d^2sp^3

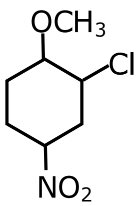
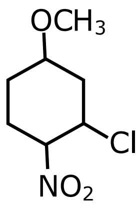


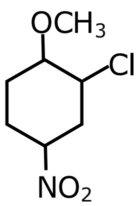
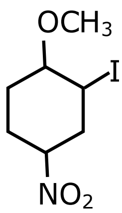
18. Identify A and B in the chemical reaction.



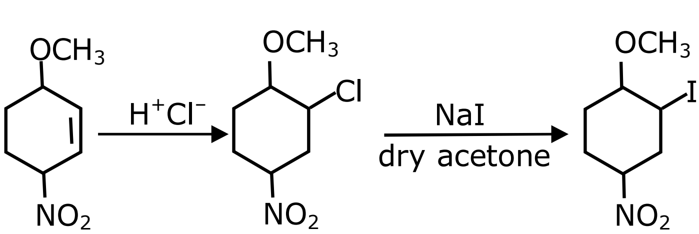
(1) A=  B= 

(2) A=  B= 

(3) A=  B= 

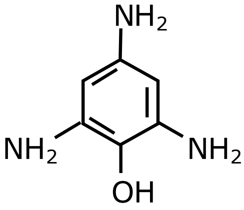
(4) A=  B= 

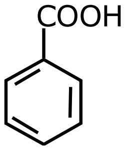
Sol. **(4)**

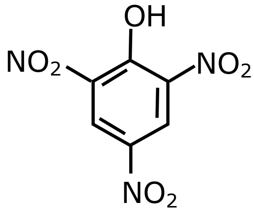


(A) (B)

19. Compound(s) which will liberate carbon dioxide with sodium bicarbonate solution is/are:

A = 

B = 

C = 

(1) B and C only (2) B only (3) A and B only (4) C only

Sol. **(1)**

Compounds which are more acidic then H_2CO_3, gives CO_2 gas on reaction with $NaHCO_3$. Compound B i.e. Benzoic acid and compound C i.e. picric acid both are more acidic than H_2CO_3.

20. The solubility of AgCN in a buffer solution of pH = 3 is x. The value of x is:
[Assume: No cyano complex is formed; $K_{sp}(AgCN) = 2.2 \times 10^{-16}$ and K_a (HCN) = 6.2×10^{-10}]
(1) 0.625×10^{-6} (2) 1.6×10^{-6} (3) 2.2×10^{-16} (4) 1.9×10^{-5}

Sol. **(4)**

Let solubility is x

$AgCN \rightleftharpoons \underset{x}{Ag^+} + \underset{x}{CN^-}$ $\qquad K_{sp} = 2.2 \times 10^{-16}$

$H^+ + CN^- \rightleftharpoons HCN$ $\qquad K = \frac{1}{K_a} = \frac{1}{6.2 \times 10^{-10}}$

$$K_{sp} \times \frac{1}{K_a} = [Ag^+]\,[CN^-] \times \frac{[HCN]}{[H^+][CN^-]}$$

$$2.2 \times 10^{-16} \times \frac{1}{6.2 \times 10^{-10}} = \frac{[S][S]}{10^{-3}}$$

$$S^2 = \frac{2.2}{6.2} \times 10^{-9}$$

$S^2 = 3.55 \times 10^{-10}$

$S = \sqrt{3.55 \times 10^{-10}}$

$S = 1.88 \times 10^{-5} \Rightarrow 1.9 \times 10^{-5}$

SECTION - B

1. The reaction of cyanamide, $NH_2CN_{(s)}$ with oxygen was run in a bomb calorimeter and ΔU was found to be -742.24 kJ mol^{-1}. The magnitude of ΔH_{298} for the reaction

$NH_2CN_{(s)} + \frac{3}{2}O_2(g) \rightarrow N_{2(g)} + O_{2(g)} + H_2O_{(l)}$ is ______ kJ. (Rounded off to the nearest integer)

[Assume ideal gases and R = 8.314 J mol^{-1} K^{-1}]

Sol. **741 kJ/mol**

$$NH_2CN(s) + \frac{3}{2}O_2(g) \longrightarrow N_2(g) + CO_2(g) + H_2O(\ell)$$

$$\Delta ng = (1 + 1) - \frac{3}{2} = \frac{1}{2}$$

$\Delta H = \Delta U + \Delta ng\ RT$

$= -742.24 + \frac{1}{2} \times \frac{8.314 \times 298}{1000}$

$= -742.24 + 1.24$

$= 741$ kJ/mol

2. In basic medium CrO_4^{2-} oxidizes $S_2O_3^{2-}$ to form SO_4^{2-} and itself changes into $Cr(OH)_4^-$. The volume of 0.154 M CrO_4^{2-} required to react with 40 mL of 0.25 M $S_2O_3^{2-}$ is _______ mL. (Rounded-off to the nearest integer)

Sol. **173 mL**

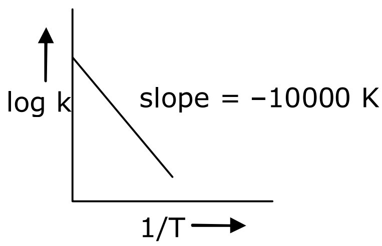
$17H_2O + 8CrO_4 + 3S_2O_3 \longrightarrow 6SO_4 + 8Cr(OH)_4^- + 2OH^-$

Applying mole-mole analysis

$\frac{0.154 \times v}{8} = \frac{40 \times 0.25}{3}$

V = 173 mL

3. For the reaction, aA + bB $\rightarrow$ cC + dD, the plot of log k vs $\frac{1}{T}$ is given below:



The temperature at which the rate constant of the reaction is $10^{-4} s^{-1}$ is _________ K.
[Rounded off to the nearest integer)
[Given: The rate constant of the reaction is 10^{-5} s^{-1} at 500 K]

Sol. **526 K**

$\log_{10}K = \log_{10}A - \frac{E_a}{2.303RT}$

Slope $= \frac{E_a}{2.303R} = -10000$

$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R} \times \left[\frac{1}{T_1} - \frac{1}{T_2}\right]$

$\log_{10}\frac{10^{-4}}{10^{-5}} = 10000 \times \left[\frac{1}{500} - \frac{1}{T}\right]$

$1 = 10000 \times \left[\frac{1}{500} - \frac{1}{T}\right]$

$\frac{1}{10000} = \frac{1}{500} - \frac{1}{T}$

$$\frac{1}{T} = \frac{1}{500} - \frac{1}{10000}$$

$$\frac{1}{T} = \frac{20-1}{10000} = \frac{19}{10000}$$

$$T = \frac{10,000}{19} \Rightarrow 526\text{ K}$$

4. 0.4g mixture of NaOH, Na_2CO_3 and some inert impurities was first titrated with $\frac{N}{10}$ HCl using phenolphthalein as an indicator, 17.5 mL of HCl was required at the end point. After this methyl orange was added and titrated. 1.5 mL of same HCl was required for the next end point. The weight percentage of Na_2CO_3 in the mixture is ______. (Rounded-off to the nearest integer)

Sol. **3%**

1st end point reaction

$NaOH + HCl \longrightarrow NaCl + H_2O$

nf = 1

$NaCO_3 + HCl \longrightarrow NaHCO_3$

nf = 1

Eq of HCl used $= n_{NaOH} \times 1 + n_{Na_2CO_3} \times 1$

$$17.5 \times \frac{1}{10} \times 10^{-3} = n_{NaOH} + n_{Na_2CO_3}$$

2nd end point

$NaHCO_3 + HCl \longrightarrow H_2CO_3$

$$1.5 \times \frac{1}{10} \times 10^{-3} = n_{NaHCO_3} \times 1 = n_{NaHCO_3}$$

$0.15\text{ mmol} = n_{Na_2CO_3}$

$0.15 = n_{Na_2CO_3}$

$$w_{Na_2CO_3} = \frac{0.15 \times 106 \times 10^{-3}}{0.5} \times 100 \times 10$$

$= 3 \times 106 \times 10^{-2}$

$= 3 \times 1.06 = 3.18\%$

5. The ionization enthalpy of Na^+ formation from $Na_{(g)}$ is 495.8 kJ mol^{-1}, while the electron gain enthalpy of Br is −325.0 kJ mol^{-1}. Given the lattice enthalpy of NaBr is −728.4 kJ mol^{-1}. The energy for the formation of NaBr ionic solid is (−)_____ $\times 10^{-1}$ kJ mol^{-1}.

Sol. **5576 kJ**

$Na(s) \longrightarrow Na^+(g)$ $\quad \Delta H = 495.8$

$\frac{1}{2}Br_2(\ell) + e^- \longrightarrow Br^-(g)$ $\quad \Delta H = 325$

$Na^+(g) + Br^-(g) \longrightarrow NaBr(s)$ $\quad \Delta H = -728.4$

$Na(s) + \frac{1}{2}Br_2(\ell) \longrightarrow NaBr(s).$ $\quad \Delta H = ?$

$\Delta H = 495.8 - 325 - 728.4$

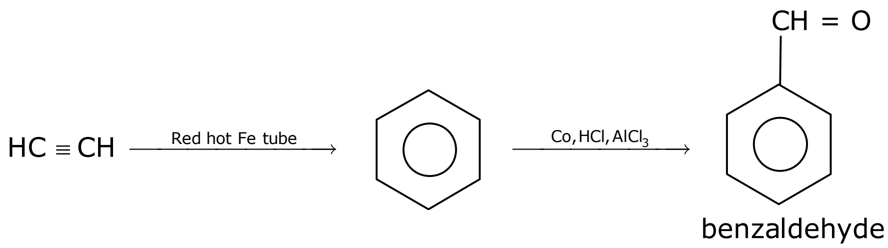
$-557.6\text{ kJ} = -5576 \times 10^{-1}\text{ kJ}$

6. Consider the following chemical reaction.

$$HC \equiv CH \xrightarrow[\text{(2)CO+HCl/AlCl}_3]{\text{(1)Red hot Fe tube, 873K}} \text{Product}$$

The number of sp^2 hybridized carbon atom(s) present in the product is _______.

Sol. **7**



All carbon atoms in benzaldehyde are sp^2 hybridised

7. A car tyre is filled with nitrogen gas at 35 psi at 27°C. It will burst if pressure exceeds 40 psi. The temperature in °C at which the car tyre will burst is ______. (Rounded-off to the nearest integer)

Sol. **69.85°C ≃ 70°C**

$$\frac{P_1}{T_1} = \frac{P_2}{T_2}$$

$$\frac{35}{300} = \frac{40}{T_2}$$

$$T_2 = \frac{40 \times 300}{35}$$

= 342.86 K

= 69.85°C ≃ 70°C

8. Among the following, the number of halide(s) which is/are inert to hydrolysis is ______.

(A) BF_3 (B) $SiCl_4$ (C) PCl_5 (D) SF_6

Sol. **1**

Due to crowding SF_6 is not hydrolysed.

9. 1 molal aqueous solution of an electrolyte A_2B_3 is 60% ionised. The boiling point of the solution at 1 atm is _____ K. (Rounded-off to the nearest integer)

[Given K_b for (H_2O) = 0.52 K kg mol^{-1}]

Sol. **375 K**

$A_2B_3 \longrightarrow 2A^{+3} + 3B^{-2}$

No. of ions = 2 + 3 = 5

$i = 1 + (n - 1) \propto$

$= 1 + (5 - 1) \times 0.6$

$= 1 + 4 \times 0.6 = 1 + 2.4 = 3.4$

$\Delta T_b = K_b \times m \times i$

$= 0.52 \times 1 \times 3.4 = 1.768°C$

$\Delta T_b = (T_b)_{solution} - [(T_b)_{H_2O}]_{Solution}$

$1.768 = (T_b)_{solution} - 100$

$(T_b)_{solution} = 101.768$ °C

$= 375$ K

10. Using the provided information in the following paper chromatogram:

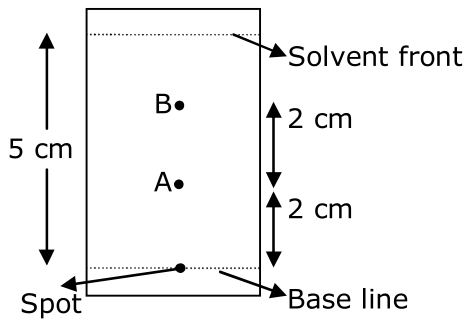


Fig: Paper chromatography for compounds A and B

the calculated R_f value of A _______ $\times 10^{-1}$.

Sol. **4**

$$R_f = \frac{\text{Distance travelled by compound}}{\text{Distance travelled by solvent}}$$

On chromatogram distance travelled by compound is $\rightarrow$ 2 cm

Distance travelled by solvent = 5 cm

So $R_f = \frac{2}{5} = 4 \times 10^{-1} = 0.4$

25^{th} Feb. 2021 | Shift - 1
MATHEMATICS

Section : Mathematics Section A

1. The coefficients a, b and c of the quadratic equation, $ax^2 + bx + c = 0$ are obtained by throwing a dice three times. The probability that this equation has equal roots is :

(1) $\frac{1}{54}$

(2) $\frac{1}{72}$

(3) $\frac{1}{36}$

(4) $\frac{5}{216}$

Ans. (4)

Sol. $ax^2 + bx + c = 0$

$a, b, c \in \{1,2,3,4,5,6\}$

$n(s) = 6 \times 6 \times 6 = 216$

$D = 0 \Rightarrow b^2 = 4ac$

$ac = \frac{b^2}{4}$ If $b = 2, ac = 1 \Rightarrow a = 1, c = 1$

If $b = 4, ac = 4 \Rightarrow a = 1, c = 4$

$a = 4, c = 1$

$a = 2, c = 2$

If $b = 6, ac = 9 \Rightarrow a = 3, c = 3$

$\therefore$ probability $= \frac{5}{216}$

2. Let α be the angle between the lines whose direction cosines satisfy the equations $l + m - n = 0$ and $l^2 + m^2 - n^2 = 0$. Then the value of $\sin^4\alpha + \cos^4\alpha$ is :

(1) $\frac{3}{4}$

(2) $\frac{1}{2}$

(3) $\frac{5}{8}$

(4) $\frac{3}{8}$

Ans. (3)

Sol. $l^2 + m^2 + n^2 = 1$

$\therefore 2n^2 = 1 \Rightarrow n = \pm\frac{1}{\sqrt{2}}$

$\therefore l^2 + m^2 = \frac{1}{2}$ & $l + m = \frac{1}{\sqrt{2}}$

$\Rightarrow \frac{1}{2} - 2lm = \frac{1}{2}$

$\Rightarrow$l m = 0 or m = 0

$\therefore$ l = 0, m = $\frac{1}{\sqrt{2}}$ or l = $\frac{1}{\sqrt{2}}$

$< 0, \frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} >$ or $< \frac{1}{\sqrt{2}}, 0, \frac{1}{\sqrt{2}} >$

$\therefore \cos\alpha = 0 + 0 + \frac{1}{2} = \frac{1}{2}$

$\therefore \sin^4\alpha + \cos^4\alpha = 1 - \frac{1}{2}\sin^2(2\alpha) = 1 - \frac{1}{2}, \frac{3}{4} = \frac{5}{8}$

3. The value of the integral

$$\int \frac{\sin\theta.\sin 2\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{1-\cos 2\theta} d\theta \text{ is}$$

(where c is a constant of integration)

(1) $\frac{1}{18}\left[9 - 2\sin^6\theta - 3\sin^4\theta - 6\sin^2\theta\right]^{\frac{3}{2}} + c$

(2) $\frac{1}{18}\left[11 - 18\sin^2\theta + 9\sin^4\theta - 2\sin^6\theta\right]^{\frac{3}{2}} + c$

(3) $\frac{1}{18}\left[11 - 18\cos^2\theta + 9\cos^4\theta - 2\cos^6\theta\right]^{\frac{3}{2}} + c$

(4) $\frac{1}{18}\left[9 - 2\cos^6\theta - 3\cos^4\theta - 6\cos^2\theta\right]^{\frac{3}{2}} + c$

Ans. (3)

Sol. $$\int \frac{2\sin^2\theta\cos\theta(\sin^6\theta + \sin^4\theta + \sin^2\theta)\sqrt{2\sin^4\theta + 3\sin^2\theta + 6}}{2\sin^2\theta} d\theta$$

Let $\sin\theta = t$, $\cos\theta\, d\theta = dt$

$= \int (t^6 + t^4 + t^2)\sqrt{2t^4 + 3t^2 + 6}\, dt = \int (t^5 + t^3 + t)\sqrt{2t^6 + 3t^4 + 6t^2}\, dt$

Let $2t^6 + 3t^4 + 6t^2 = z$

$12(t^5 + t^3 + t)\, dt = dz$

$= \frac{1}{12}\int \sqrt{z}\, dz = \frac{1}{18} z^{3/2} + c$

$= \frac{1}{18}[(2\sin^6\theta + 3\sin^4\theta + 6\sin^2\theta)^{3/2} + C$

$= \frac{1}{18}\left[\left(1 - \cos^2\theta\right)\left(2(1-\cos^2\theta)^2 + 3 - 3\cos^2\theta + 6\right)\right]^{3/2} + C$

$= \frac{1}{18}[(1 - \cos^2\theta)(2\cos^4\theta - 7\cos^2\theta + 11)]^{3/2} + C$

$= \frac{1}{18}[-2\cos^6\theta + 9\cos^4\theta - 18\cos^2\theta + 11]^{3/2} + C$

4. A man is observing, from the top of a tower, a boat speeding towards the tower from a certain point A, with uniform speed. At that point, angle of depression of the boat with the man's eye is 30° (Ignore man's height). After sailing for 20 seconds towards the base of the tower (which is at the level of water), the boat has reached a point B, where the angle of depression is 45°. Then the time taken (in seconds) by the boat from B to reach the base of the tower is :

(1) $10(\sqrt{3}-1)$

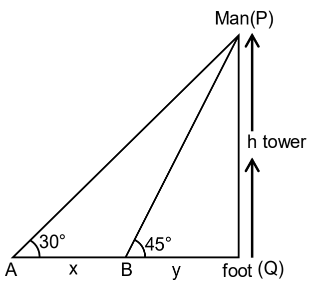
(2) $10\sqrt{3}$

(3) 10

(4) $10(\sqrt{3}+1)$

Ans. (4)

Sol.



$\frac{h}{x+y} = \tan 30°$

x + y = $\sqrt{3}h$ ....(1)

Also

$\frac{h}{y} = \tan 45°$

h = y ....(2)

put in (1)

x + y = $\sqrt{3}y$

x = $(\sqrt{3}-1)y$

$\frac{x}{20} = 'v'\text{speed}$

$\therefore$ time taken to reach

Foot from B

$\Rightarrow \frac{y}{V}$

$\Rightarrow \frac{x}{(\sqrt{3}-1).x} \times 20$

$\Rightarrow 10(\sqrt{3}+1)$

5. If $0 < \theta, \phi < \frac{\pi}{2}, x = \sum_{n=0}^{\infty} \cos^{2n}\theta, y = \sum_{n=0}^{\infty} \sin^{2n}\phi$ and

$z = \sum_{n=0}^{\infty} \cos^{2n}\theta . \sin^{2n}\phi$ then :

(1) xyz = 4
(2) xy − z = (x + y)z
(3) xy + yz + zx = z
(4) xy + z = (x +y)z

Ans. (4)

Sol. $x = 1 + \cos^2\theta + \ldots\ldots\infty$

$$x = \frac{1}{1-\cos^2\theta} = \frac{1}{\sin^2\theta} \quad \ldots\ldots(1)$$

$y = 1 + \sin^2\phi + \ldots\ldots\infty$

$$y = \frac{1}{1-\sin^2\phi} = \frac{1}{\cos^2\phi} \quad \ldots\ldots(2)$$

$$z = \frac{1}{1-\cos^2\theta.\sin^2\phi} = \frac{1}{1-\left(1-\frac{1}{x}\right)\left(1-\frac{1}{y}\right)} = \frac{xy}{xy-(x-1)(y-1)}$$

xz + yz − z = xy
xy + z = (x + y)z

6. The equation of the line through the point (0, 1, 2) and perpendicular to the line $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2}$ is :

(1) $\frac{x}{-3} = \frac{y-1}{4} = \frac{z-2}{3}$

(2) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{3}$

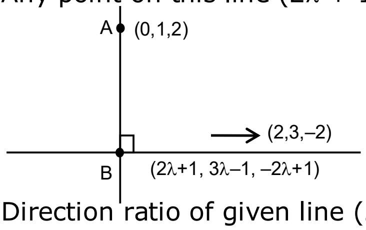
(3) $\frac{x}{3} = \frac{y-1}{-4} = \frac{z-2}{3}$

(4) $\frac{x}{3} = \frac{y-1}{4} = \frac{z-2}{-3}$

Ans. (1)

Sol. $\frac{x-1}{2} = \frac{y+1}{3} = \frac{z-1}{-2} = \lambda$

Any point on this line (2λ + 1, 3λ − 1, −2λ+1)



Direction ratio of given line (2, 3, −2)

Direction ratio of line to be found $(2\lambda + 1, 3\lambda - 2, -2\lambda - 1)$

$\therefore\ \vec{d}_1.\vec{d}_2 = 0$

$\lambda = 2/17$

Direction ratio of line $(21, -28, -21) \equiv (3, -4, -3) \equiv (-3, 4, 3)$

7. The statement $A \rightarrow (B \rightarrow A)$ is equivalent to:

(1) $A \rightarrow (A \wedge B)$

(2) $A \rightarrow (A \vee B)$

(3) $A \rightarrow (A \rightarrow B)$

(4) $A \rightarrow (A \leftrightarrow B)$

Ans. (2)

Sol. $A \rightarrow (B \rightarrow A)$

$\Rightarrow A \rightarrow (\sim B \vee A)$

$\Rightarrow \sim A \vee (\sim B \vee A)$

$\Rightarrow \sim B \vee (\sim A \vee A)$

$\Rightarrow \sim B \vee t$

$= t$ (tantology)

From options :

(2) $A \rightarrow (A \vee B)$

$\Rightarrow \sim A \vee (A \vee B)$

$\Rightarrow (\sim A \vee A) \vee B$

$\Rightarrow t \vee B$

$\Rightarrow t$

8. The integer 'k', for which the inequality $x^2 - 2(3k - 1)x + 8k^2 - 7 > 0$ is valid for every x in R is :

(1) 3

(2) 2

(3) 4

(4) 0

Ans. (1)

Sol. $D < 0$

$(2(3k-1))^2 - 4(8k^2 - 7) < 0$

$4(9k^2 - 6k + 1) - 4(8k^2 - 7) < 0$

$k^2 - 6k + 8 < 0$

$(k-4)(k-2) < 0$

$2 < k < 4$

then $k = 3$

9. A tangent is drawn to the parabola $y^2 = 6x$ which is perpendicular to the line $2x + y = 1$. Which of the following points does NOT lie on it ?

(1) (0, 3)
(2) (−6, 0)
(3) (4, 5)
(4) (5, 4)

Ans. (4)

Sol. Equation of tangent : $y = mx + \frac{3}{2m}$

$m_T = \frac{1}{2}$ ($\because$ perpendicular to line $2x + y = 1$)

$\therefore$ tangent is : $y = \frac{x}{2}+3$ $\Rightarrow x - 2y + 6 = 0$

10. Let f, g: $N \rightarrow N$ such that $f(n + 1) = f(n) + f(1)$ $\forall n \in N$ and g be any arbitrary function. Which of the following statements is NOT true ?

(1) f is one-one
(2) If fog is one-one, then g is one-one
(3) If g is onto, then fog is one-one
(4) If f is onto, then $f(n) = n \, \forall n \in N$

Ans. (3)

Sol. $f(n + 1) = f(n) + 1$
$f(2) = 2f(1)$
$f(3) = 3f(1)$
$f(4) = 4f(1)$
....
$f(n) = nf(1)$
f(x) is one-one

11. Let the lines $(2 - i)z = (2 + i)\bar{z}$ and $(2 + i)z + (i - 2)\bar{z} - 4i = 0$, (here $i^2 = -1$) be normal to a circle C. If the line $iz+\bar{z}+1+i=0$ is tangent to this circle C, then its radius is :

(1) $\frac{3}{\sqrt{2}}$
(2) $3\sqrt{2}$
(3) $\frac{3}{2\sqrt{2}}$
(4) $\frac{1}{2\sqrt{2}}$

Ans. (3)

Sol. $(2-i)z=(2+i)\bar{z}$

$\Rightarrow (2-i)(x+iy)=(2+i)(x-iy)$

$\Rightarrow 2x - ix + 2iy + y = 2x + ix - 2\text{-}iy + y$

$\Rightarrow 2ix - 4iy = 0$

$L_1 : x - 2y = 0$

$\Rightarrow (2+i)z + (i-2)\bar{z} - 4i = 0.$

$\Rightarrow (2+i)(x+iy) + (i-2)(x-iy) - 4i = 0.$

$\Rightarrow 2x + ix + 2iy - y + ix - 2x + y + 2iy - 4i = 0$

$\Rightarrow 2ix + 4iy - 4i = 0$

$L_2 : x + 2y - 2 = 0$

Solve L_1 and L_2 $4y=2$, $y=\frac{1}{2}$

$\therefore x=1$

Centre$\left(1,\frac{1}{2}\right)$

$L_3 : iz + \bar{z} + 1 + i = 0$

$\Rightarrow i(x+iy) + x - iy + 1 + i = 0$

$\Rightarrow ix - y + x - iy + 1 + i = 0$

$\Rightarrow (x - y + 1) + i(x - y + 1) = 0$

Radius = distance from$\left(1,\frac{1}{2}\right)$ to $x - y + 1 = 0$

$$r = \frac{1-\frac{1}{2}+1}{\sqrt{2}}$$

$$r = \frac{3}{2\sqrt{2}}$$

12. All possible values of $\theta \in [0, 2\pi]$ for which $\sin 2\theta + \tan 2\theta > 0$ lie in:

(1) $\left(0,\frac{\pi}{2}\right)\cup\left(\pi,\frac{3\pi}{2}\right)$

(2) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{5\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{7\pi}{4}\right)$

(3) $\left(0,\frac{\pi}{2}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\pi,\frac{7\pi}{6}\right)$

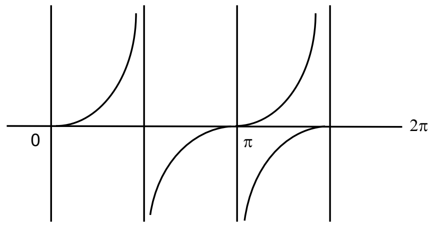
(4) $\left(0,\frac{\pi}{4}\right)\cup\left(\frac{\pi}{2},\frac{3\pi}{4}\right)\cup\left(\frac{3\pi}{2},\frac{11\pi}{6}\right)$

y

1

Ans. (2)

Sol.



$\tan 2\theta\,(1 + \cos 2\theta) > 0$

$2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\pi, \frac{3\pi}{2}\right) \cup \left(2\pi, \frac{5\pi}{2}\right) \cup \left(3\pi, \frac{7\pi}{2}\right)$

$\Rightarrow \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{2}, \frac{3\pi}{4}\right) \cup \left(\pi, \frac{5\pi}{4}\right) \cup \left(\frac{3\pi}{2}, \frac{7\pi}{4}\right)$

13. The image of the point (3,5) in the line $x - y + 1 = 0$, lies on :

(1) $(x - 2)^2 + (y - 4)^2 = 4$

(2) $(x - 4)^2 + (y + 2)^2 = 16$

(3) $(x - 4)^2 + (y - 4)^2 = 8$

(4) $(x - 2)^2 + (y - 2)^2 = 12$

Ans. (1)

Sol. Image of P(3, 5) on the line $x - y + 1 = 0$ is

$\frac{x-3}{1} = \frac{y-5}{-1} = \frac{-2(3-5+1)}{2} = 1$

$x = 4,\ y = 4$

$\therefore$ Image is (4, 4)

Which lies on

$(x - 2)^2 + (y - 4)^2 = 4$

14. If Rolle's theorem holds for the function $f(x) = x^3 - ax^2 + bx - 4,\ x \in [1, 2]$ with $f'\left(\frac{4}{3}\right) = 0$, then ordered pair (a, b) is equal to :

(1) (−5, 8)

(2) (5, 8)

(3) (5, −8)

(4) (−5, −8)

Ans. (2)

Sol. $f(1) = f(2)$

$\Rightarrow 1 - a + b - 4 = 8 - 4a + 2b - 4$

$3a - b = 7 \qquad \ldots(1)$

$f'(x) = 3x^2 - 2ax + b$

$\Rightarrow f'\left(\frac{4}{3}\right) = 0 \Rightarrow 3 \times \frac{16}{9} - \frac{8}{3}a + b = 0$

$\Rightarrow -8a + 3b = -16 \qquad \ldots(2)$

$a = 5,\ b = 8$

15. If the curves, $\frac{x^2}{a}+\frac{y^2}{b}=1$ and $\frac{x^2}{c}+\frac{y^2}{d}=1$ intersect each other at an angle of 90°, then which of the following relations is true ?

(1) a + b = c + d

(2) a − b = c − d

(3) ab = $\frac{c+d}{a+b}$

(4) a − c = b + d

Ans. (2)

Sol. $\frac{x^2}{a}+\frac{y^2}{b}=1$ ....(1)

diff : $\frac{2x}{a}+\frac{2y}{b}\frac{dy}{dx}=0 \Rightarrow \frac{y}{b}\frac{dy}{dx}=\frac{-x}{a}$

$\frac{dy}{dx}=\frac{-bx}{ay}$ ......(2)

$\frac{x^2}{c}+\frac{y^2}{d}=1$ ....(3)

Diff : $\frac{dy}{dx}=\frac{-dx}{cy}$ ....(4)

$m_1m_2 = -1 \Rightarrow \frac{-bx}{ay}\times\frac{-dx}{cy} = -1$

$\Rightarrow bdx^2 = -acy^2$ ....(5)

(1)−(3) $\Rightarrow \left(\frac{1}{a}-\frac{1}{c}\right)x^2 + \left(\frac{1}{b}-\frac{1}{d}\right)y^2 = 0$

$\Rightarrow \frac{c-a}{ac}x^2+\frac{d-b}{bd}\times\left(\frac{-bd}{ac}\right)x^2=0$ (using 5)

$\Rightarrow (c - a) - (d - b) = 0$

$\Rightarrow c - a = d - b$

$\Rightarrow c - d = a - b$

16. $\lim\limits_{n\to\infty}\left(1+\frac{1+\frac{1}{2}+\ldots\ldots+\frac{1}{n}}{n^2}\right)^n$ is equal to :

(1) $\frac{1}{2}$

(2) $\frac{1}{e}$

(3) 1

(4) 0

Ans. (3)

Sol. It is 1^{∞} form

$$L = e^{\lim_{n\to\infty}\left(\frac{1+\frac{1}{2}+\frac{1}{3}+.....+\frac{1}{n}}{n}\right)}$$

$$S = 1+\left(\frac{1}{2}+\frac{1}{3}\right)+\left(\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}\right)+\left(\frac{1}{8}+........+\frac{1}{15}\right)$$

$$S < 1+\left(\frac{1}{2}+\frac{1}{2}\right) +\left(\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}\right).........+\underbrace{\left(\frac{1}{2^P}+........+\frac{1}{2^P}\right)}_{2^P \text{ times}}$$

$$S < 1+ 1 + 1 + 1 +.........\ +1$$

$$S < P + 1$$

$$\therefore \quad L = e^{\lim_{n\to\infty}\frac{(P+1)}{2^P}}$$

$$\Rightarrow L = e^{\circ} = 1$$

17. The total number of positive integral solutions (x, y, z) such that xyz = 24 is

(1) 36

(2) 45

(3) 24

(4) 30

Ans. (4)

Sol. x.y.z = 24

$x.y.z=2^3.3^1$

Now using beggars method.

3 things to be distributed among 3 persons

Each may receive none, one or more

$\therefore\ {}^5C_2$ ways

Similarly for '1' $\quad\therefore\ {}^3C_2$ ways

Total ways = ${}^5C_2 \cdot {}^3C_2$ = 30 ways

18. If a curve passes through the origin and the slope of the tangent to it at any point (x, y) is $\frac{x^2-4x+y+8}{x-2}$,then this curve also passes through the point :

(1) (4, 5)

(2) (5, 4)

(3) (4, 4)

(4) (5, 5)

Ans. (4)

Sol. $\frac{dy}{dx}=\frac{(x-2)^2+y+4}{(x-2)}=(x-2)+\frac{y+4}{(x-2)}$

Let x – 2 = t $\Rightarrow$ dx = dt

and y + 4 = u $\Rightarrow$ dy = du

$$\frac{dy}{dx}=\frac{du}{dt}$$

$$\frac{du}{dt} = t + \frac{u}{t} \Rightarrow \frac{du}{dt}-\frac{u}{t}=t$$

I.F = $e^{\int \frac{-1}{t}dt} = e^{-\ln t} = \frac{1}{t}$

u. $\frac{1}{t}=\int t.\frac{1}{t}dt \Rightarrow \frac{u}{t}=t+c$

$\frac{y+4}{x-2}$ = (x – 2) + c

Passing through (0, 0)

c = 0

$\Rightarrow$ (y + 4) = $(x - 2)^2$

19. The value of $\int_{-1}^{1} x^2 e^{[x^3]}dx$, where [t] denotes the greatest integer $\leq$ t, is :

(1) $\frac{e+1}{3}$

(2) $\frac{e-1}{3e}$

(3) $\frac{e+1}{3e}$

(4) $\frac{1}{3e}$

Ans. (3)

Sol. I = $\int_{-1}^{0} x^2.e^{-1}dx+\int_{0}^{1} x^2 dx$

$\therefore$ I = $\left.\frac{x^3}{3e}\right|_{-1}^{0} + \left.\frac{x^3}{3}\right|_{0}^{1}$

$\Rightarrow$ I = $\frac{1}{3e}+\frac{1}{3}$

20. When a missile is fired from a ship, the probability that it is intercepted is $\frac{1}{3}$ and the probability that the missile hits the target, given that it is not intercepted, is $\frac{3}{4}$. If three missiles are fired independently from the ship, then the probability that all three hit the target, is:

(1) $\frac{1}{8}$

(2) $\frac{1}{27}$

(3) $\frac{3}{4}$

(4) $\frac{3}{8}$

Ans. (1)

Sol. Probability of not getting intercepted = $\frac{2}{3}$

Probability of missile hitting target = $\frac{3}{4}$

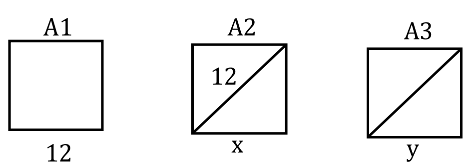
$\therefore$ Probability that all 3 hit the target = $\left(\frac{2}{3}\times\frac{3}{4}\right)^3 = \frac{1}{8}$

Section : Mathematics Section B

1. Let $A_1, A_2, A_3, \ldots\ldots$ be squares such that for each $n \geq 1$, the length of the side of A_n equals the length of diagonal of A_{n+1}. If the length of A_1 is 12 cm, then the smallest value of n for which area of A_n is less than one, is ____________.

Ans. (9)

Sol.



$x = \frac{12}{\sqrt{2}}$ $y = \frac{12}{(\sqrt{2})^2}$

$\therefore$ Side lengths are in G.P.

$T_n = \frac{12}{(\sqrt{2})^{n-1}}$

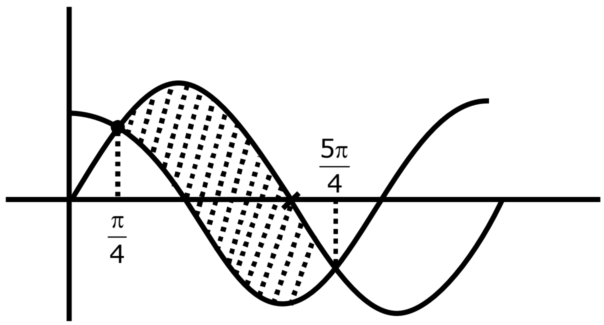
$\therefore$ Area = $\frac{144}{2^{n-1}} < 1$ $\Rightarrow 2^{n-1} > 144$

Smallest n = 9

2. The graphs of sine and cosine functions, intersect each other at a number of points and between two consecutive points of intersection, the two graphs enclose the same area A. Then A^4 is equal to _________

Ans. (64)

Sol.



$$A = \int_{\frac{\pi}{4}}^{\frac{5\pi}{4}} (\sin x - \cos x)\,dx = [-\cos x - \sin x]_{\pi/4}^{5\pi/4}$$

$$= -\left[\left(\cos\frac{5\pi}{4} + \sin\frac{\pi}{4}\right) - \left(\cos\frac{\pi}{4} + \sin\frac{\pi}{4}\right)\right]$$

$$= -\left[\left(-\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}\right) - \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right]$$

$$= \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

$$\Rightarrow A^4 = \left(2\sqrt{2}\right)^4 = 64$$

3. The locus of the point of intersection of the lines $(\sqrt{3})kx + ky - 4\sqrt{3} = 0$ and $\sqrt{3}\,x - y - 4(\sqrt{3})\,k = 0$ is a conic, whose eccentricity is _________.

Ans. (2)

Sol. $\sqrt{3}kx + ky = 4\sqrt{3}$ .... (1)

$\sqrt{3}kx - ky = 4\sqrt{3}\,k^2$ ....(2)

Adding equation (1) & (2)

$2\sqrt{3}kx = 4\sqrt{3}(k^2 + 1)$

$x = 2\left(k + \frac{1}{k}\right)$ ....(3)

Substracting equation (1) & (2)

$y = 2\sqrt{3}\left(\frac{1}{k} - k\right)$ ....(4)

$\therefore \frac{x^2}{4} - \frac{y^2}{12} = 4$

$\frac{x^2}{16}-\frac{y^2}{48}=1$ Hyperbola

$\therefore$ $e^2 = 1 + \frac{48}{16}$

e = 2

4. If A = $\begin{bmatrix} 0 & -\tan\left(\frac{\theta}{2}\right) \\ \tan\left(\frac{\theta}{2}\right) & 0 \end{bmatrix}$ and $(I_2 + A)(I_2 - A)^{-1}$

$= \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$, then $13(a^2 + b^2)$ is equal to _______.

Ans. (13)

Sol. $A = \begin{bmatrix} 0 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 0 \end{bmatrix}$

$\Rightarrow I + A = \begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow I - A = \begin{bmatrix} 1 & \tan\frac{\theta}{2} \\ -\tan\frac{\theta}{2} & 1 \end{bmatrix}$ $\{\therefore |I - A| = \sec^2\theta/2\}$

$\Rightarrow (I - A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$\Rightarrow (I+A)(I-A)^{-1} = \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}\begin{bmatrix} 1 & -\tan\frac{\theta}{2} \\ \tan\frac{\theta}{2} & 1 \end{bmatrix}$

$= \frac{1}{\sec^2\frac{\theta}{2}}\begin{bmatrix} 1-\tan^2\frac{\theta}{2} & -2\tan\frac{\theta}{2} \\ 2\tan\frac{\theta}{2} & 1-\tan^2\frac{\theta}{2} \end{bmatrix}$

$a = \frac{1-\tan^2\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$

$$b = \frac{2\tan\frac{\theta}{2}}{\sec^2\frac{\theta}{2}}$$

$\therefore$ $a^2 + b^2 = 1$

5. Let f(x) be a polynomial of degree 6 in x, in which the coefficient of x^6 is unity and it has extrema at x = −1 and x = 1. If $\lim_{x\to 0}\frac{f(x)}{x^3} = 1$, then 5.f(2) is equal to __________

Ans. (144)

Sol. $f(x) = x^6 + ax^5 + bx^4 + x^3$

$\therefore$ $f'(x) = 6x^5 + 5ax^4 + 4bx^3 + 3x^2$

Roots 1 & − 1

$\therefore$ 6 + 5a + 4b + 3 = 0 & − 6 + 5a − 4b + 3 = 0 solving

$a = -\frac{3}{5}$ $\qquad$ $b = -\frac{3}{2}$

$\therefore$ $f(x) = x^6 - \frac{3}{5}x^5 - \frac{3}{2}x^4 + x^3$

$\therefore$ $5.f(2) = 5\left[64 - \frac{96}{5} - 24 + 8\right] = 144$

6. The number of points, at which the function f(x) = |2x + 1| −3|x+2|+|x^2 + x−2|, x $\in$ R is not differentiable, is _________.

Ans. (2)

Sol. f(x) = |2x + 1| − 3|x + 2| + |x^2 + x − 2|

$$f(x) = \begin{cases} x^2 - 7 \;; & x > 1 \\ -x^2 - 2x - 3 \;; & -\frac{1}{2} < x < 1 \\ -x^2 - 6x - 5 \;; & -2 < x < \frac{-1}{2} \\ x^2 + 2x + 3; & x < -2 \end{cases}$$

$$\therefore f'(x) = \begin{cases} 2x \;; & x > 1 \\ -2x - 3 \;; & -\frac{1}{2} < x < 1 \\ -2x - 6; & -2 < x < \frac{-1}{2} \\ 2x + 2; & x < -2 \end{cases}$$

Check at 1, −2 and $\frac{-1}{2}$

Non. Differentiable at x = 1 and $\frac{-1}{2}$

7. If the system of equations

$$kx + y + 2z = 1$$
$$3x - y - 2z = 2$$
$$-2x - 2y - 4z = 3$$

has infinitely many solutions, then k is equal to __________.

Ans. (21)

Sol. D = 0

$$\Rightarrow \begin{vmatrix} k & 1 & 2 \\ 3 & -1 & -2 \\ -2 & -2 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k (4 – 4)– 1(–12 –4) +2(–6 –2)

$\Rightarrow$ 16 – 16 = 0

Also. $D_1 = D_2 = D_3 = 0$

$$\Rightarrow D_2 = \begin{vmatrix} k & 1 & 2 \\ 3 & 2 & -2 \\ -2 & 3 & -4 \end{vmatrix} = 0$$

$\Rightarrow$ k(–8+6)–1(–12–4)+2(9+4)=0

$\Rightarrow$ –2k + 16 + 26 = 0

$\Rightarrow$ 2k = 42

$\Rightarrow$ k = 21

8. Let $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}, \vec{b} = \hat{i} - \hat{j}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$ and $\vec{r}.\vec{b} = 0$, then $\vec{r}.\vec{a}$ is equal to ________

Ans. (12)

Sol. $\vec{r} \times \vec{a} = \vec{c} \times \vec{a}$

$\vec{r} \times \vec{a} - \vec{c} \times \vec{a} = 0$

$(\vec{r} - \vec{c}) \times \vec{a} = 0$

$\therefore \vec{r} - \vec{c} = \lambda \vec{a}$

$\vec{r} = \lambda \vec{a} + \vec{c}$

$\vec{r}.\vec{b} = \lambda \vec{a}.\vec{b} + \vec{c}.\vec{b} = 0$

$\Rightarrow \lambda$ (1–2) + 2 = 0

$\Rightarrow \lambda$ = 2

$\therefore \vec{r} = 2\vec{a} + \vec{c}$

$\vec{r}.\vec{a} = 2|\vec{a}|^2 + \vec{a}.\vec{c}$

= 2(1 + 4 + 1) + (1 – 2 + 1)

= 12

9. Let $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$, where x, y and z are real numbers such that $x + y + z > 0$ and $xyz = 2$.

If $A^2 = I_3$, then the value of $x^3 + y^3 + z^3$ is _______.

Ans. (7)

Sol. $A = \begin{bmatrix} x & y & z \\ y & z & x \\ z & x & y \end{bmatrix}$ $\therefore$ $|A| = \left(x^3 + y^3 + z^3 - 3xyz\right)$

$A^2 = I_3$

$|A^2| = 1$

$\therefore (x^3 + y^3 + z^3 - 3xyz)^2 = 1$

$\Rightarrow x^3 + y^3 + z^3 - 3xyz = 1$ only as $(x+y+z>0)$

$\Rightarrow x^3 + y^3 + z^3 = 6 + 1 = 7$

10. The total number of numbers, lying between 100 and 1000 that can be formed with the digits 1, 2, 3, 4,5, if the repetition of digits is not allowed and numbers are divisible by either 3 or 5 is ________.

Ans. (32)

Sol. $\boxed{}\boxed{}\boxed{}$ divisible by $\rightarrow 3$

$12 \rightarrow 3, 4, 5 \rightarrow 3! = 6$

$15 \rightarrow 2, 3, 4 \rightarrow 3! = 6$

$24 \rightarrow 1, 3, 5 \rightarrow 3! = 6$

$42 \rightarrow 1, 2, 3 \rightarrow 3! = 6$

24

divisible by 5

$\boxed{}\boxed{}\boxed{5} = 12$

4×3

Required No. = 24 + 12 − 4 = 32