25^{th} Feb. 2021 | Shift - 2
PHYSICS

SECTION - A

1. Match List I with List II.

	List I	**List II**
(a)	Rectifier	(i) Used either for stepping up or stepping down the a.c. Voltage
(b)	Stabilizer	(ii) Used to convert a.c. voltage into d.c. voltage
(c)	Transformer	(iii) Used to remove any ripple in the rectified output voltage
(d)	Filter	(iv) Used for constant output voltage even when the input voltage or load current change

Choose the correct answer form the options given below:

(1) (a)-(ii), (b)- (i), (c)-(iv), (d)-(iii)
(2) (a)-(ii), (b)- (iv), (c)-(i), (d)-(iii)
(3) (a)-(ii), (b)- (i), (c)-(iii), (d)-(iv)
(4) (a)-(iii), (b)- (iv), (c)-(i), (d)-(ii)

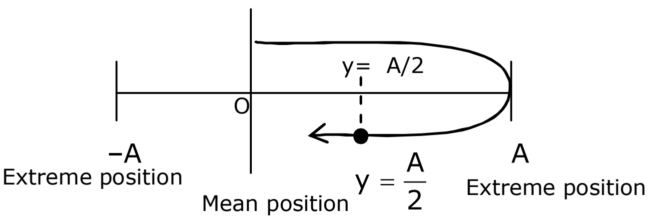
Sol. **2**

(a)Rectifier:- used to convert a.c voltage into d.c. Voltage.
(b) Stabilizer:- used for constant output voltage even when the input voltage or load current change
(c) Transformer:- used either for stepping up or stepping down the a.c. voltage.
(d) Filter:- used to remove any ripple in the rectified output voltage.

2. $Y = A \sin(\omega t + \phi_0)$ is the time – displacement equation of a SHM, At t = 0 the displacement of the particle is $Y = \frac{A}{2}$ and it is moving along negative x-direction. Then the initial phase angle ϕ_0 will be.

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{3}$ (3) $\frac{2\pi}{3}$ (4) $\frac{5\pi}{6}$

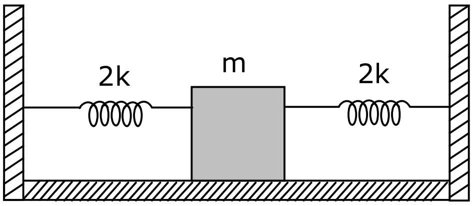
Sol. **4**



The initial phase angle $\phi_0 = \pi - \frac{\pi}{6}$

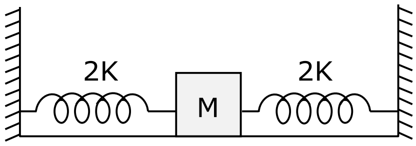
$= \frac{5\pi}{6}$

3. Two identical spring of spring constant '2K' are attached to a block of mass m and to fixed support (see figure). When the mass is displaced from equilibrium position on either side, it executes simple harmonic motion. Then time period of oscillations of this system is:



(1) $\pi\sqrt{\frac{m}{k}}$ (2) $\pi\sqrt{\frac{m}{2k}}$ (3) $2\pi\sqrt{\frac{m}{k}}$ (4) $2\pi\sqrt{\frac{m}{2k}}$

Sol. **1**



Dut to parallel combination K_{eff} = 2k + 2k

= 4k

$$\because T = 2\pi\sqrt{\frac{m}{k_{eff}}}$$

$$= 2\pi\sqrt{\frac{m}{4k}}$$

$$T = \pi\sqrt{\frac{m}{k}}$$

4. The wavelength of the photon emitted by a hydrogen atom when an electron makes a transition from n = 2 to n = 1 state is:

(1) 194.8 nm (2) 490.7 nm (3) 913.3 nm (4) 121.8 nm

Sol. **4**

ΔE = 10.2 eV

$\frac{hc}{\lambda}$ = 10.2 ev

$$\lambda = \frac{hc}{(10.2)e}$$

$$= \frac{12400}{10.2} \text{Å}$$

= 121.56 nm

$\simeq$ 121.8 nm

5. In a ferromagnetic material, below the curie temperature, a domain is defined as:

(1) a macroscopic region with consecutive magnetic diploes oriented in opposite direction.

(2) a macroscopic region with zero magnetization.

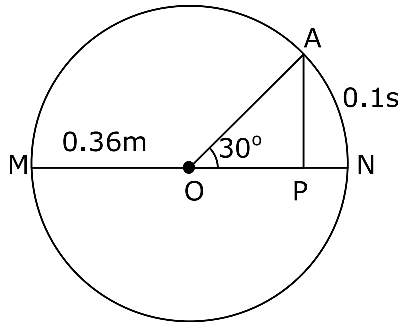
(3) a macroscopic region with saturation magnetization.

(4) a macroscopic region with randomly oriented magnetic dipoles.

Sol. **3**

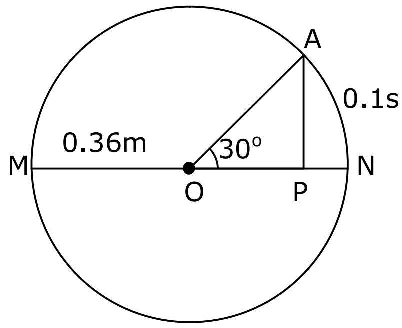
In a ferromagnetic material, below the curie temperature a domain is defined as a macroscopic region with saturation magnetization.

6. The point A moves with a uniform speed along the circumference of a circle of radius 0.36m and cover 30^o in 0.1s. The perpendicular projection 'P' form 'A' on the diameter MN represents the simple harmonic motion of 'P'. The restoration force per unit mass when P touches M will be:



(1) 100 N (2) 50 N (3) 9.87 N (4) 0.49 N

Sol. **3**



The point a covers 30^o in 0.1 sec.

Means $\frac{\pi}{6} \longrightarrow 0.1 \text{ sec}.$

$1 \longrightarrow \frac{0.1}{\frac{\pi}{6}}$

$2\pi = \longrightarrow \frac{0.1 \times 6}{\pi} \times 2\pi$

T = 1.2 sec.

We know that $\omega = \frac{2\pi}{T}$

$$\omega = \frac{2\pi}{1.2}$$

Restoration force (F) = $m\omega^2 A$

Then Restoration force per unit mass $\left(\frac{F}{m}\right) = \omega^2 A$

$$\left(\frac{F}{m}\right) = \left(\frac{2\pi}{1.2}\right)^2 \times 0.36$$

$$\cong 9.87 \text{ N}$$

7. The stopping potential for electrons emitted from a photosensitive surface illuminated by light of wavelength 491 nm is 0.710 V. When the incident wavelength is changed to a new value, the stopping potential is 1.43V. The new wavelength is:

(1) 400 NM (2) 382 nm (3) 309 nm (4) 329 nm

Sol. **2**

From the photoelectric effect equation

$$\frac{hc}{\lambda} = \phi + ev_s$$

so $ev_{s_1} = \frac{hc}{\lambda_1} - \phi$ ......(i)

$ev_{s_2} = \frac{hc}{\lambda_2} - \phi$ .....(ii)

Subtract equation (i) from equation (ii)

$$ev_{s_1} - ev_{s_2} = \frac{hc}{\lambda_1} - \frac{hc}{\lambda_2}$$

$$v_{s_1} - v_{s_2} = \frac{hc}{e}\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right)$$

$$(0.710 - 1.43) = 1240\left(\frac{1}{491} - \frac{1}{\lambda_2}\right)$$

$$\frac{-0.72}{1240} = \frac{1}{491} - \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{1}{491} + \frac{0.72}{1240}$$

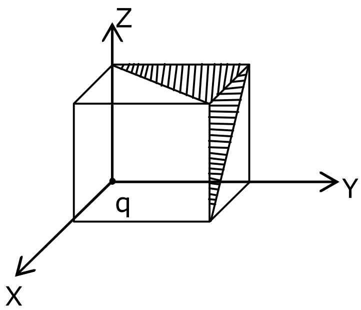
$$\frac{1}{\lambda_2} = 0.00203 + 0.00058$$

$\frac{1}{\lambda_2} = 0.00261$

$\lambda_2 = 383.14$

$\lambda_2 \simeq 382\text{nm}$

8. A charge 'q' is placed at one corner of a cube as shown in figure. The flux of electrostatic field $\vec{E}$ though the shaded area is:



(1) $\frac{q}{48\varepsilon_0}$ (2) $\frac{q}{8\varepsilon_0}$ (3) $\frac{q}{24\varepsilon_0}$ (4) $\frac{q}{4\varepsilon_0}$

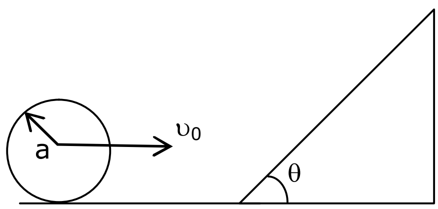
Sol. **3**

$\phi = \frac{q}{24\varepsilon_0}$

$\phi_T = \left(\frac{q}{24\varepsilon_0} + \frac{q}{24\varepsilon_0}\right) \times \frac{1}{2}$

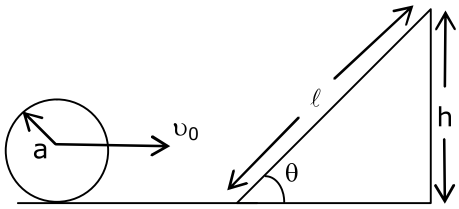
$\phi_T = \frac{q}{24\varepsilon_0}$

9. A sphere of radius 'a' and mass 'm' rolls along horizontal plane with constant speed υ_0. It encounters an inclined plane at angle θ and climbs upward. Assuming that it rolls without slipping how far up the sphere will travel ?



(1) $\frac{2}{5}\frac{\upsilon_0^2}{g\sin\theta}$ (2) $\frac{10\upsilon_0^2}{7g\sin\theta}$ (3) $\frac{\upsilon_0^2}{5g\sin\theta}$ (4) $\frac{\upsilon_0^2}{2g\sin\theta}$

Sol. **Bonus, our answer $\left(\frac{7v_0^2}{10g\sin\theta}\right)$, NTA answer (2)**



From energy conservation

$$mgh = \frac{1}{2}m\upsilon_0^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}m\upsilon_0^2 + \frac{1}{2}\times\frac{2}{5}ma^2\times\frac{\upsilon_0^2}{a^2}$$

$$gh = \frac{1}{2}\upsilon_0^2 + \frac{1}{5}\upsilon_0^2$$

$$gh = \frac{7}{10}\upsilon_0^2$$

$$h = \frac{7}{10}\frac{\upsilon_0^2}{g}$$

from triangle, $\sin\theta = \frac{h}{\ell}$

then $h = \ell\sin\theta$

$$\ell\sin\theta = \frac{7}{10}\frac{\upsilon_0^2}{g}$$

$$\ell = \frac{7}{10}\frac{\upsilon_0^2}{g\sin\theta}$$

10. Consider the diffraction pattern obtained from the sunlight incident on a pinhole of diameter 0.1 μm. If the diameter of the pinhole is slightly increased, it will affect the diffraction pattern such that:

(1) its size decreases, but intensity increases
(2) its size increases, but intensity decreases
(3) its size increases, and intensity increases
(4) its size decreases, and intensity decreases

Sol. **1**

$$\sin\theta = \frac{1.22\lambda}{D}$$

If D is increased, then $\sin\theta$ will decreased

$\therefore$ size of circular fringe will decrease but intensity increases

11. An electron of mass m_e and a proton of mass m_p = 1836 m_e are moving with the same speed. The ratio of their de Broglie wavelength $\frac{\lambda_{electron}}{\lambda_{Proton}}$ will be:

(1) 918 (2) 1836 (3) $\frac{1}{1836}$ (4) 1

Sol. **2**

Given mass of electron = m_e

Mass of proton = m_p

$\therefore$ given $m_p = 1836\ m_e$

From de-Broglie wavelength

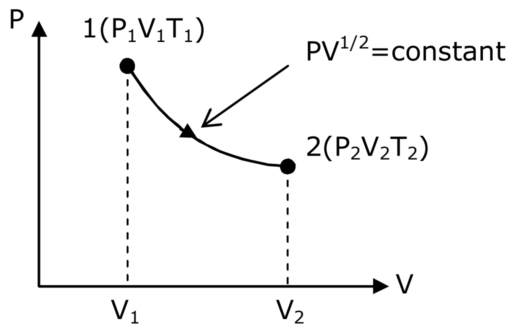
$\lambda = \frac{h}{p} = \frac{h}{mv}$

$\frac{\lambda_e}{\lambda_p} = \frac{m_p}{m_e}$

$= \frac{1836 m_e}{m_e}$

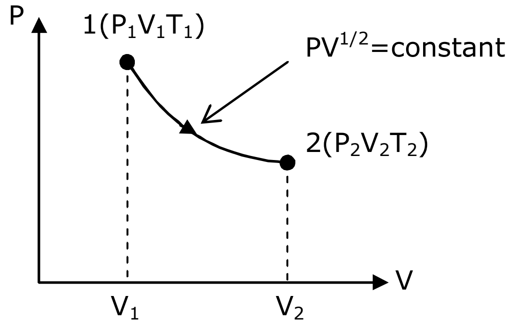
$\frac{\lambda_e}{\lambda_p} = 1836$

12. thermodynamic process is shown below on a P-V diagram for one mole of an ideal gas. If $V_2 = 2V_1$ then the ratio of temperature T_2/T_1 is:



(1) $\frac{1}{\sqrt{2}}$ (2) $\frac{1}{2}$ (3) 2 (4) $\sqrt{2}$

Sol. **4**



From p-v diagram,

Given $Pv^{1/2}$ = constant .....(i)

We know that

Pv = nRT

$P \propto \left(\frac{T}{v}\right)$

Put in equation (i)

$\left(\frac{T}{v}\right)(v)^{1/2}$ = constant

$T \propto v^{1/2}$

$\frac{T_2}{T_1} = \sqrt{\frac{v_2}{v_1}}$

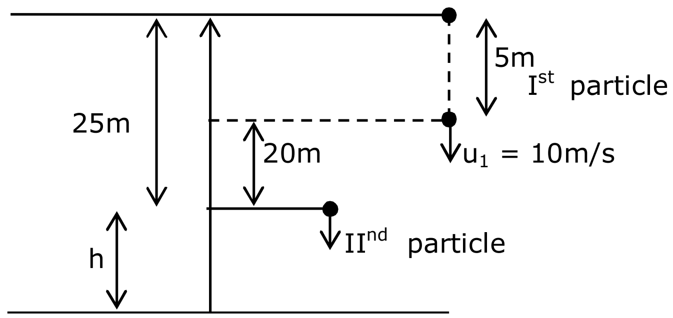
$\frac{T_2}{T_1} = \sqrt{\frac{2v_1}{v_1}}$

$\frac{T_2}{T_1} = \sqrt{2}$

13. A stone is dropped from the top of a building. When it crosses a point 5m below the top, another stone starts to fall from a point 25m below the top, Both stones reach the bottom of building simultaneously. The height of the building is:

(1) 45 m (2) 35 m (3) 25 m (4) 50 m

Sol. **1**



For particle (1)

$20+h = 10t + \frac{1}{2} gt^2$ .....(i)

For particle (2)

$h = \frac{1}{2} gt^2$ .....(ii)

put equation (ii) in equation (i)

$20 + \frac{1}{2}gt^2 = 10t + \frac{1}{2}gt^2$

t = 2sec.

Put in equation (ii)

$h = \frac{1}{2} gt^2$

$= \frac{1}{2} \times 10 \times 2^2$

h = 20m

the height of the building = 25 + 20 = 45m

14. if a message signal of frequency 'f_m' is amplitude modulated with a carrier signal of frequency 'f_c' and radiated through an antenna, the wavelength of the corresponding signal in air is:

(1) $\frac{c}{f_c + f_m}$ (2) $\frac{c}{f_c - f_m}$ (3) $\frac{c}{f_m}$ (4) $\frac{c}{f_c}$

Sol. 4

Given frequency of massage signal $= f_m$

frequency of carrier signal $= f_c$

the wavelength of the corresponding signal in air is $\Rightarrow \lambda = \frac{c}{f_c}$

15. Given below are two statements:

Statement I: In a diatomic molecule, the rotational energy at a given temperature obeys Maxwell's distribution.

Statement II: in a diatomic molecule, the rotational energy at a given temperature equals the translational kinetic energy for each molecule.

In the light of the above statements, choose the correct answer from the options given below:

(1) Both statement I and statement II are false.

(2) Both statement I and statement II are true.

(3) Statement I is false but statement II is true

(4) Statement I is true but statement II is false.

Sol. **4**

The translational kinetic energy & rotational kinetic energy both obey Maxwell's distribution independent of each other.

T.K.E of diatomic molecules = $\frac{3}{2}kT$

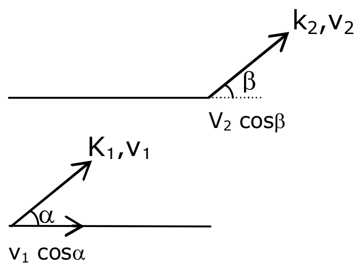
R.K.E. of diatomic molecules = $\frac{2}{2}kT$

So statement I is true but statement II is false.

16. An electron with kinetic energy K_1 enters between parallel plates of a capacitor at an angle 'α' with the plates. It leaves the plates at angle 'β' with kinetic energy K_2. Then the ratio of kinetic energies $K_1 : K_2$ will be:

(1) $\frac{\sin^2\beta}{\cos^2\alpha}$ (2) $\frac{\cos^2\beta}{\cos^2\alpha}$ (3) $\frac{\cos\beta}{\sin\alpha}$ (4) $\frac{\cos\beta}{\cos\alpha}$

Sol. **2**



$\because$ $v_1 \cos\alpha = v_2\cos\beta$

$$\frac{v_1}{v_2} = \frac{\cos\beta}{\cos\alpha}$$

Then the ratio of kinetic energies

$$\frac{k_1}{k_2} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_2^2} = \left(\frac{v_1}{v_2}\right)^2 = \left(\frac{\cos\beta}{\cos\alpha}\right)^2$$

$$\frac{k_1}{k_2} = \frac{\cos^2\beta}{\cos^2\alpha}$$

17. An LCR circuit contains resistance of 110Ω and a supply of 220 V at 300 rad/s angular frequency. If only capacitance is removed from the circuit, current lags behind the voltage by 45°. If on the other hand, only inductor is removed the current leads by 45° with the applied voltage. The rms current flowing in the circuit will be:

(1) 2.5A (2) 2A (3) 1A (4) 1.5 A

Sol. **2**

Since ϕ remain same, circuit is in resonance

$\therefore I_{rms} = \frac{V_{rms}}{z}$

$= \frac{220}{110}$

$I_{rms} = 2A$

18. For extrinsic semiconductors: when doping level is increased;

(1) Fermi–level of p and n-type semiconductors will not be affected.

(2) Fermi–level of p-type semiconductors will go downward and Fermi–level of n-type semiconductor will go upward.

(3) Fermi–level of both p–type and n–type semiconductors will go upward for $T > T_F$ K and downward for $T < T_F$K, where T_F is Fermi temperature.

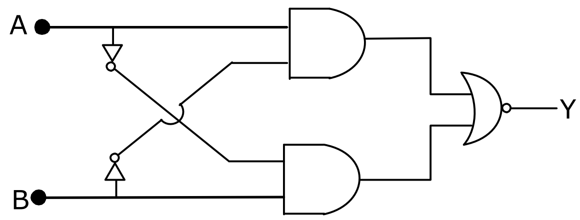
(4) Fermi–level of p-type semiconductor will go upward and Fermi–level of n–type semiconductors will go downward.

Sol. **2**

In n-type semiconductor pentavalent impurity is added. Each pentavalent impurity donates a free electron. So the Fermi-level of n-type semiconductor will go upward .

& In p-type semiconductor trivalent impurity is added. Each trivalent impurity creates a hole in the valence band. So the Fermi-level of p-type semiconductor will go downward.

19. The truth table for the following logic circuit is:



(1)

A	B	Y
0	0	1
0	1	0
1	0	1
1	1	0

(2)

A	B	Y
0	0	0
0	1	1
1	0	0
1	1	1

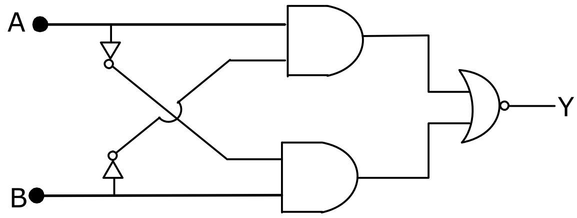
(3)

A	B	Y
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	Y
0	0	1
0	1	0
1	0	0
1	1	1

Sol. **4**



If A = B = 0 then output y = 1
If A = B = 1 then output y = 1

20. If e is the electronic charged, c is the speed of light in free space and h is planck's constant, the quantity $\frac{1}{4\pi\varepsilon_0}\frac{|e|^2}{hc}$ has dimensions of :

(1) $[LC^{-1}]$ (2) $[M^0 L^0 T^0]$ (3) $[M L T^0]$ (4) $[M L T^{-1}]$

Sol. **2**

Given
e = electronic charge
c = speed of light in free space
h = planck's constant

$$\frac{1}{4\pi\varepsilon_0}\frac{e^2}{hc} = \frac{ke^2}{hc}\times\frac{\lambda^2}{\lambda^2}$$

$$= \frac{F\times\lambda}{E}$$

$$= \frac{E}{E}$$

= dimensionless
= $[M^0 L^0 T^0]$

SECTION – B

1. The percentage increase in the speed of transverse waves produced in a stretched string if the tension is increased by 4% will be___________%.

Sol. **2**

Speed of transverse wave is

$$V = \sqrt{\frac{T}{\mu}}$$

$$\ell n\ v = \frac{1}{2}\ell nT - \frac{1}{2}\ell n\mu$$

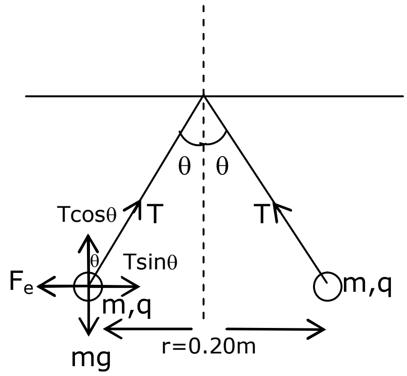
$$\frac{\Delta v}{v} = \frac{1}{2}\frac{\Delta T}{T}$$

$$= \frac{1}{2}\times 4$$

$$\frac{\Delta v}{v} = 2\%$$

2. Two small spheres each of mass 10 mg are suspended from a point by threads 0.5 m long. They are equally charged and repel each other to a distance of 0.20 m . Then charge on each of the sphere is $\frac{a}{21}\times 10^{-8}C$. The value of 'a' will be________.

Sol. **20**



$$T\sin\theta = \frac{kq^2}{r^2}$$

$$T\cos\theta = mg$$

$$\tan\theta = \frac{kq^2}{mgr^2}$$

$$q^2 = \frac{\tan\theta\, mgr^2}{k}$$

$$\because \tan\theta = \frac{0.1}{0.5} = \frac{1}{5}$$

$$q^2 = \frac{1}{5}\times\frac{10\times10^{-6}\times10\times0.2\times0.2}{9\times10^9}$$

$$q = \frac{2\sqrt{2}}{3}\times10^{-8}$$

after comparison from the given equation

a = 20

3. The peak electric field produced by the radiation coming from the 8 W bulb at a distance of 10 m is $\frac{x}{10}\sqrt{\frac{\mu_0 c}{\pi}}\frac{V}{m}$. The efficiency of the bulb is 10% and it is a point source. The value of x is ____.

Sol. **2**

$$I = \frac{1}{2}c\in_0 E_0^2$$

$$\frac{8}{4\pi\times10^2} = \frac{1}{2}\times c\times\frac{1}{\mu_0 c^2}\times E_0^2$$

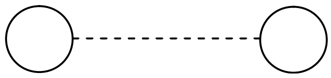
$$E_0 = \frac{2}{10}\sqrt{\frac{\mu_0 c}{\pi}}$$

$\Rightarrow$ x = 2

4. Two identical conducting spheres with negligible volume have 2.1nC and -0.1nC charges, respectively. They are brought into contact and then separated by a distance of 0.5 m. The electrostatic force acting between the spheres is _______ $\times 10^{-9}$ N.

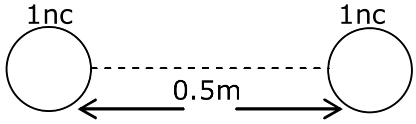
[Given : $4\pi\varepsilon_0 = \frac{1}{9\times10^9}$ SI unit]

Sol. **36**



When they are brought into contact & then separated by a distance = 0.5 m

Then charge distribution will be



The electrostatic force acting b/w the sphere is

$$F_e = \frac{kq_1q_2}{r^2}$$

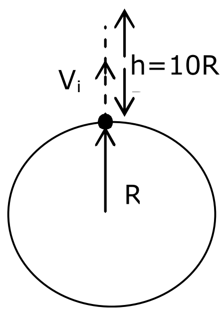
$$= \frac{9\times10^9 \times 1\times10^{-9} \times 1\times10^{-9}}{(0.5)^2}$$

$$= \frac{900}{25}\times10^{-9}$$

$F_e = 36 \times 10^{-9}$ N

5. The initial velocity υ_i required to project a body vertically upward from the surface of the earth to reach a height of 10R, where R is the radius of the earth, may be described in terms of escape velocity υ_e such that $\upsilon_i = \sqrt{\frac{x}{y}} \times \upsilon_e$. The value of x will be_______.

Sol. **10**



Here R = radius of the earth

From energy conservation

$$\frac{-Gm_e m}{R}+\frac{1}{2}mv_i^2=\frac{-Gm_e m}{11R}+0$$

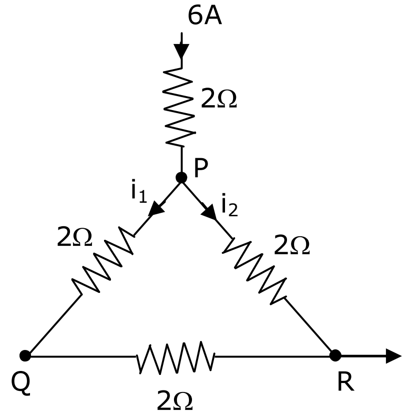
$$\frac{1}{2}mv_i^2=\frac{10}{11}\frac{Gm_e m}{R}$$

$$V_i=\sqrt{\frac{20}{11}\frac{Gm_e}{R}}$$

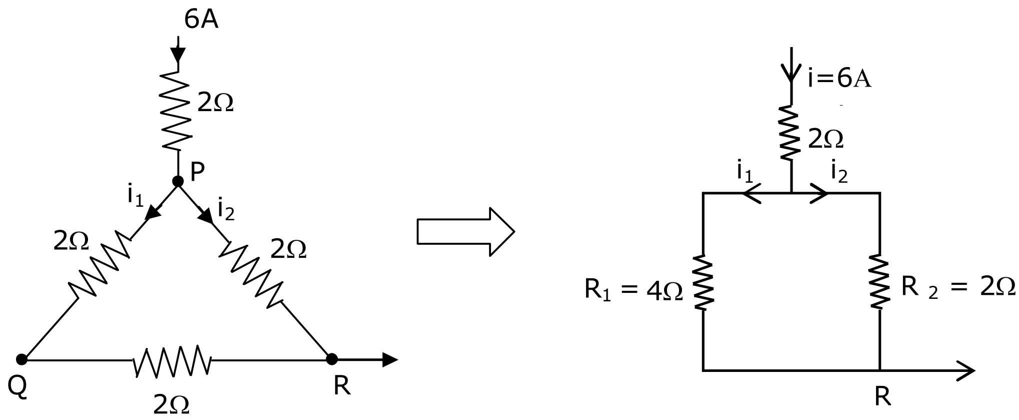
$$V_i=\sqrt{\frac{10}{11}}v_e \qquad \left\{\because \text{escape velocity } v_e=\sqrt{\frac{2Gm_e}{R}}\right\}$$

Then the value of x = 10

6. A current of 6A enters one corner P of an equilateral triangle PQR having 3 wires of resistance 2Ω each and leaves by the corner R. The currents i_1 in ampere is______.



Sol. **2**



The current $i_1 = \left(\frac{R_2}{R_1 + R_2}\right)i$

$= \left(\frac{2}{4+2}\right) \times 6$

$i_1 = 2A$

7. The wavelength of an X-ray beam is 10 Å. The mass of a fictitious particle having the same energy as that of the X – ray photons is $\frac{x}{3}h$ kg. The value of x is ________.

Sol. **10**

Given wavelength of an x-ray beam = 10Å

$\because \quad E = \frac{hc}{\lambda} = mc^2$

$m = \frac{h}{c\lambda}$

The mass of a fictitious particle having the same energy as that of the x-ray photons = $\frac{x}{3}hkg$

$\frac{x}{3}h = \frac{h}{c\lambda}$

$x = \frac{3}{c\lambda}$

$= \frac{3}{3 \times 10^8 \times 10 \times 10^{-10}}$

$x = 10$

8. A reversible heat engine converts one- fourth of the heat input into work. When the temperature of the sink is reduced by 52K, its efficiency is doubled. The temperature in Kelvin of the source will be_______.

Sol. 208

$\because\ n = \frac{w}{Q_{in}} = \frac{1}{4}$

$\frac{1}{4} = 1 - \frac{T_1}{T_2}$

$\frac{T_1}{T_2} = \frac{3}{4}$

When the temperature of the sink is reduced by 52k then its efficiency is doubled.

$\frac{1}{2} = 1 - \frac{(T_1 - 52)}{T_2}$

$\frac{T_1 - 52}{T_2} = \frac{1}{2}$

$\frac{T_1}{T_2} \frac{-52}{T_2} = \frac{1}{2}$

$\frac{3}{4} - \frac{52}{T_2} = \frac{1}{2}$

$\frac{52}{T_2} = \frac{1}{4}$

$T_2 = 208$ k

9. Two particles having masses 4g and 16g respectively are moving with equal kinetic energies. The ratio of the magnitudes of their linear momentum is n:2. The value of n will be_____.

Sol. 1

$\because$ relation b/w kinetic energy & momentum is

$P = \sqrt{2mKE}$ ($\because$ KE = same)

$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}}$

$\frac{n}{2} = \sqrt{\frac{4}{16}}$

n = 1

10. If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$, the angle between $\vec{P}$ and $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$. The value of '$\theta$' will be____.

Sol. **180**

If $\vec{P} \times \vec{Q} = \vec{Q} \times \vec{P}$

Only if $\vec{P} = 0$

Or $\vec{Q} = 0$

The angle b/w $\vec{P}$ & $\vec{Q}$ is $\theta(0^\circ < \theta < 360^\circ)$

So $\theta = 180^\circ$

25th Feb. 2021 | Shift - 2
CHEMISTRY

Section -A

1. Given below are two statements :

Statement I :

The identification of Ni^{2+} is carried out by dimethyl glyoxime in the presence of NH_4OH

Statement II :

The dimethyl glyoxime is a bidentate neutral ligand.

In the light of the above statements, choose the correct answer from the options given below :

(1) Both statement I and statement II are true

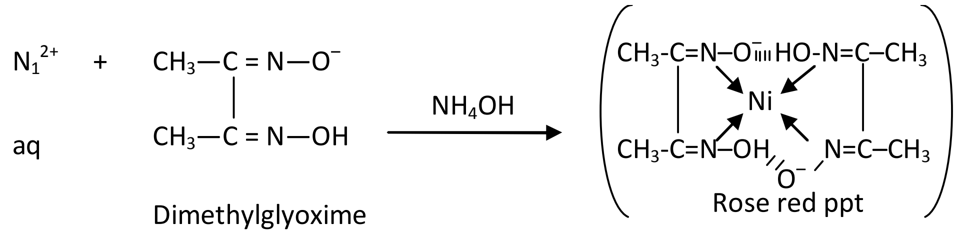
(2) Both statement I and statement II are false

(3) Statement I is false but statement II is true

(4) Statement I is true but statement II is false

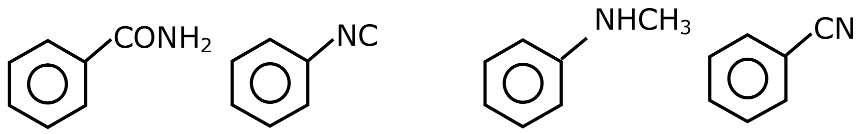
Ans. 4

Sol.



Dimethyl glyoxime is a negative bidentate legend.

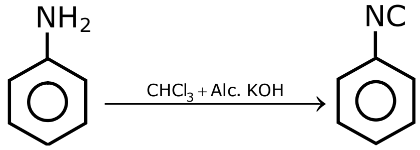
2. Carbylamine test is used to detect the presence of primary amino group in an organic compound. Which of the following compound is formed when this test is performed with aniline ?



(1) (2) (3) (4)

Ans. 2

Sol.



3. The correct order of bond dissociation enthalpy of halogen is :

(1) $F_2>Cl_2>Br_2>I_2$

(2) $Cl_2>F_2>Br_2>I_2$

(3) $Cl_2>Br_2>F_2>I_2$

(4) $I_2>Br_2>Cl_2>F_2$

Ans. 3

Sol. Fact based

F_2 has F — F, F_2 involves repulsion of non-bonding electrons & more over its size is small & hence due to high repulsion its bond dissociation energy in very low.

4. Which one of the following statements is FALSE for hydrophilic sols ?

(1) These sols are reversible in nature

(2) The sols cannot be easily coagulated

(3) They do not require electrolytes for stability.

(4) Their viscosity is of the order of that of H_2O

Ans. **4**

Sol. Fact base

5. Water does not produce CO on reacting with :

(1) C_3H_8

(2) C

(3) CH_4

(4) CO_2

Ans. **4**

Sol. $H_2O + CO_2 \rightarrow H_2CO_3$

6. What is 'X' in the given reaction ?

$$\begin{matrix} CH_2OH \\ | \\ CH_2OH \end{matrix} + \text{oxalic acid} \xrightarrow{210^0C} \underset{\text{(major product)}}{x}$$

(1) $\begin{matrix} CH_2 \\ \| \\ CH_2 \end{matrix}$ (2) $\begin{matrix} CH{-}OH \\ \| \\ CH_2 \end{matrix}$ (3) $\begin{matrix} CH_2OH \\ | \\ CHO \end{matrix}$ (4) $\begin{matrix} CHO \\ | \\ CHO \end{matrix}$

Ans. **1**

Sol.

$$\begin{matrix} CH_2-OH \\ | \\ CH_2-OH \end{matrix} + oxalic\ acid \xrightarrow{210^0 C} CH_2 = CH_2$$

7. If which of the following order the given complex ions are arranged correctly with respect to their decreasing spin only magnetic moment ?

(i) $[FeF_6]^{3-}$ (ii) $[Co(NH_3)_6]^{3+}$

(iii) $[NiCl_4]^{2-}$ (iv) $[Cu(NH_3)_4]^{2+}$

(1) (ii)>(i)>(iii)>(iv) (2) (iii)>(iv)>(ii)>(i)

(3) (ii)>(iii)>(i)>(iv) (4) (i)>(iii)>(iv)>(ii)

Ans. **4**

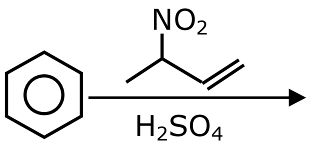
Sol. $[FeF_6]^{3-}$ Fe^{3+} $3d^5 \rightarrow$ 5-unpaired electrons as F^- is weal field legend

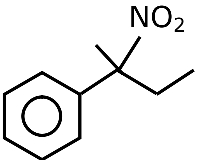
$[Co(NH_3)_6]^{3+}$ Co^{3+} $3d^6 \rightarrow$ No-unpaired electron as NH_3 is strong field light and causes pairing

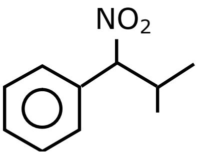
$[NiCl4]^{2-}$ Ni^{2+} $3d^8 \rightarrow$ 2-unpaired electrons

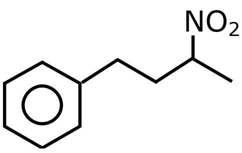
$[Cu(NH_3)_4]^{2+}$ Cu^{2+} $3d^9 \rightarrow$ 1-unpaired electrons

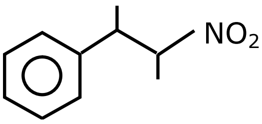
8. The major product of the following reaction is :



(1) 

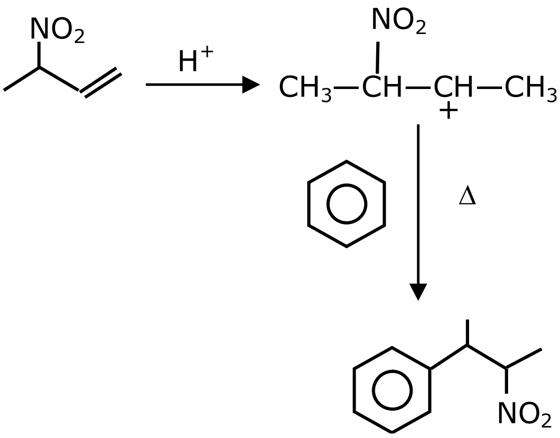
(2) 

(3) 

(4) 

Ans. **4**

Sol.



9. The correct sequence of reagents used in the preparation of 4-bromo-2-nitroethyl benzene from benezene is :

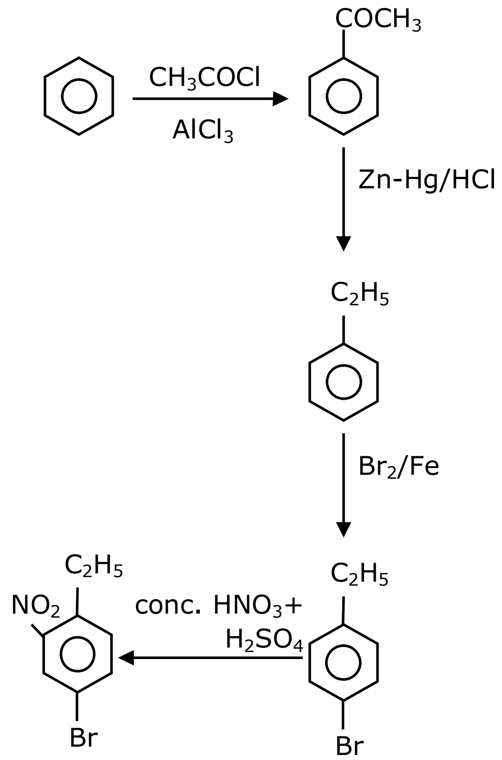
(1) $CH_3COCl/AlCl_3$, $Br_2/AlBr_3$, HNO_3/H_2SO_4, Zn/HCl

(2) $CH_3COCl/AlCl_3$, Zn-Hg/HCl, $Br_2/AlBr_3$, HNO_3/H_2SO_4

(3) $Br_2/AlBr_3$, $CH_3COCl/AlCl_3$, HNO_3/H_2SO_4, Zn/HCl

(4) HNO_3/H_2SO_4, $Br_2/AlCl_3$, $CH_3COCl/AlCl_3$, Zn-Hg/HCl

Ans. **2**



Sol.

10. The major components of German Silver are :

(1) Cu, Zn and Ag (2) Ge, Cu and Ag

(3) Zn, Ni and Ag (4) Cu, Zn and Ni

Ans. **4**

Sol. Fact

German silver is alloy which does not have silver.

Cu-50%; Ni-30%; Zn-20%

11. The method used for the purification of Indium is :

(1) van Arkel method (2) vapour phase refining

(3) zone refining (4) Liquation

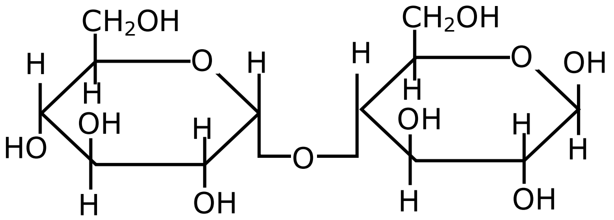
Ans. **3**

Sol. Fact

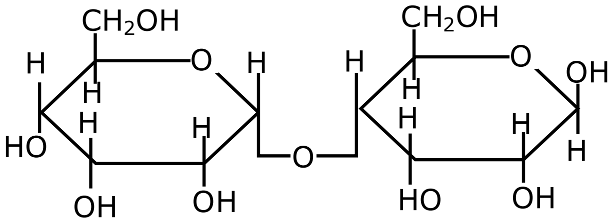
Ga, In, Si, Ge are refined by zone refining or vaccume refining.

12. Which of the following is correct structure of α-anomer of maltose :

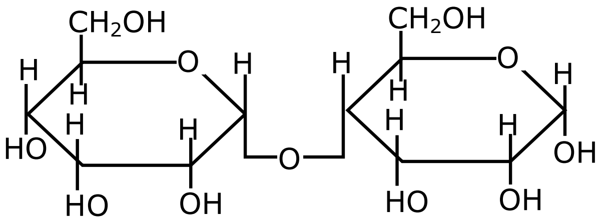
(1)



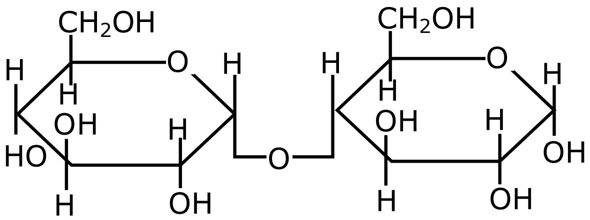
(2)



(3)

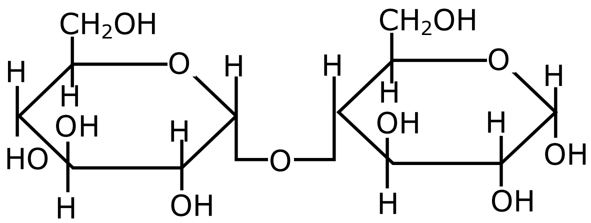


(4)



Ans. **4**

Sol.



[α-Anomer of maltose]

13. The major product of the following reaction is :

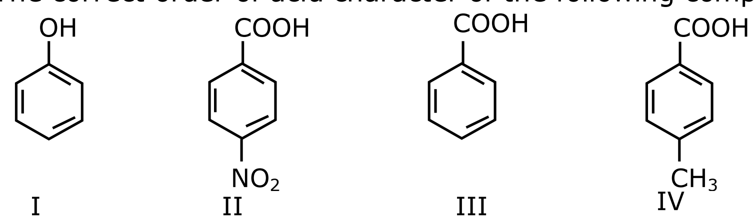
$$CH_3CH_2CH{=}CH_2 \xrightarrow[\text{Rh catalyst}]{H_2/CO}$$

(1) $CH_3CH_2CH_2CHO$
(2) $CH_3CH_2CH{=}CH{-}CHO$
(3) $CH_3CH_2CH_2CH_2CHO$
(4) $CH_3CH_2C(CHO){=}CH_2$

Ans. **3**

Sol. $CH_3-CH_2-CH=CH_2 \xrightarrow[Rh\ catalyst]{H_2/CO} CH_3CH_2CH_2CHO$

14. The correct order of acid character of the following compounds is :

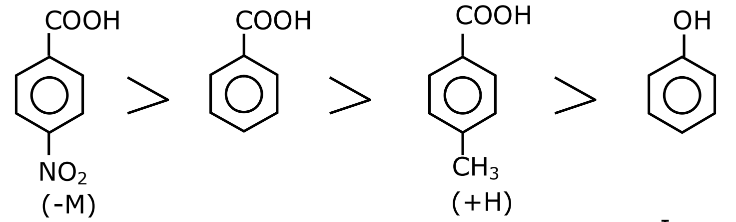


I II III IV

(1) II>III>IV>I
(2) III>II>I>IV
(3) IV>III>II>I
(4) I>II>III>IV

Ans. **1**

Sol. Acidity of carboxylic acid $\propto$ -R>-H>-I $\propto \dfrac{1}{+R > +H > +I}$

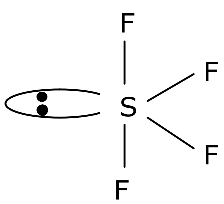


II III IV I

15. Which among the following species has unequal bond lengths ?

(1) XeF_4 (2) SiF_4 (3) BF_4^- (4) SF_4

Ans. **4**



Sp^3d Hybridisation Sea-saw shape & axial bond length is more than equitorial bond length

Sol.

16. Given below are two statements :

Statement I :

α and β forms of sulphur can change reversibly between themselves with slow heating or slow cooling.

Statement II :

At room temperature the stable crystalline form of sulphur is monoclinic sulphur.

In the light of the above statements, choose the correct answer from the options given below.

(1) Both statement I and statement II are false

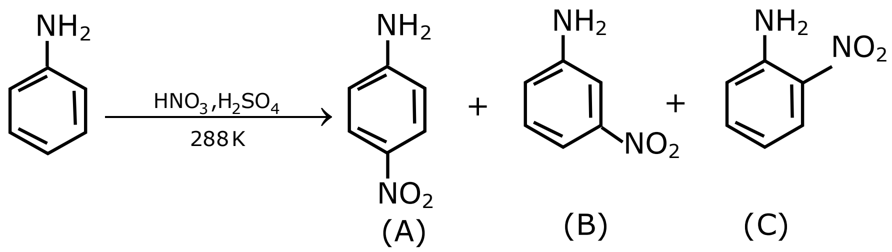
(2) Statement I is true but statement II is false

(3) Both statement I and statement II are true

(4) Statement I is false but statement II is true

Ans. **2**

Sol.
$$\underset{\alpha-sulphur}{S_{Rhambic}} \underset{95.6^0 C}{\rightleftharpoons} \underset{\beta-sulphur}{S_{Monoclinic}}$$

17. 

Correct statement about the given chemical reaction is :

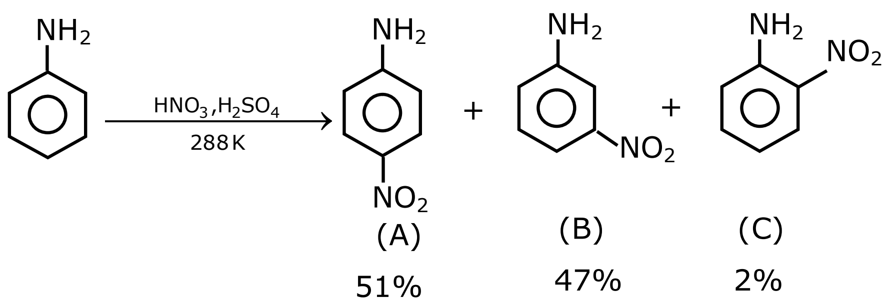
(1) Reaction is possible and compound (A) will be major product.

(2) The reaction will form sulphonated product instead of nitration.

(3) $-\ddot{N}H_2$ group is ortho and para directive, so product (B) is not possible.

(4) Reaction is possible and compound (B) will be the major product.

Ans. **1**

Sol. 

18. Which of the following compound is added to the sodium extract before addition of silver nitrate for testing of halogens ?

(1) Nitric acid (2) Sodium hydroxide

(3) Hydrochloric acid (4) Ammonia

Ans. **1**

Sol. $NaCN + HNO_3 \rightarrow NaNO_3 + HCN \uparrow$

$Na_2S + HNO_3 \rightarrow NaNO_3 + H_2S \uparrow$

Nilnic acid decomposed NaCN & Na_2S, else they precipitate in test & misquite the resolve

19. Given below are two statements :

Statement I :

The pH of rain water is normally ~5.6.

Statement II :

If the pH of rain water drops below 5.6, it is called acid rain.

In the light of the above statements, choose the correct answer from the option given below.

(1) Statement I is false but Statement II is true

(2) Both statement I and statement II are true

(3) Both statement I and statement II are false

(4) Statement I is true but statement II is false

Ans. **2**

Sol. Both statements are correct

20. The solubility of $Ca(OH)_2$ in water is :

[Given : The solubility product of $Ca(OH)_2$ in water = 5.5×10^{-6}]

(1) 1.11×10^{-6} (2) 1.77×10^{-6}

(3) 1.77×10^{-2} (4) 1.11×10^{-2}

Ans. **4**

$$Ca(OH)_2 \rightleftharpoons \underset{s}{Ca^{+2}} + \underset{(2s+10^{-7})}{2OH^-}$$

$s(2s+10^{-7})^2 = 55\times10^{-7}$

$4s^3 = 55\times10^{-7}$

$s^3 = \frac{5500}{4}\times10^{-9}$

$s = \left(\frac{2250}{2}\right)^{1/3}\times10^{-3}$

$s = (1125)^{1/3}\times10^{-3}$

$s = 1.11\times10^{-2}$

Section -B

1. If a compound AB dissociates to the extent of 75% in an aqueous solution, the molality of the solution which shows a 2.5 K rise in the boiling point of the solution is ______molal.
(Rounded-off to the nearest integer)
[K_b=0.52 K kg mol^{-1}]

Ans. **3**

$AB \rightarrow A^+ + B^-$

$1-\alpha \quad \alpha \quad \alpha$

α=3/4

N = 2

i = [1+(2-1)α]

2.5 = [1+(2-1)3/4]×0.52 ×m

m = $\dfrac{2.5}{0.52\times 7/4} = \dfrac{10}{3.64} = 2.747$

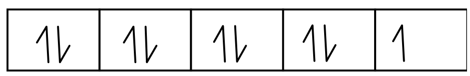
m= 2.747 $\simeq 3$ mol/kg

2. The spin only magnetic moment of a divalent ion in aqueous solution (atomic number 29) is ____BM.

Ans. **2**

Sol. ${}_{29}Cu^{+2} \rightarrow [Ar]^{18}\underline{3d^9}$

d^9 →



No. of unpaired e^- = 1

Magnetic moment = $\mu = \sqrt{n(n+2)}$

$\mu = \sqrt{(1)(1+2)} = \sqrt{3}B.M.$

= 1.73 Ans.

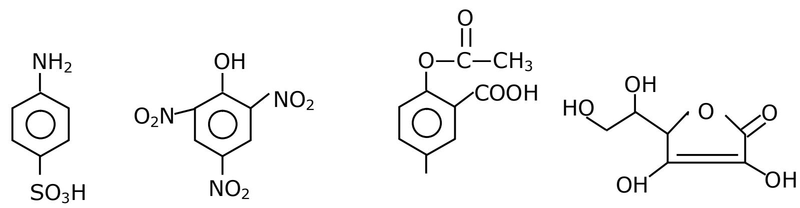
3. The number of compound/s given below which contain/s —COOH group is ______.

(1) Sulphanilic acid (2) Picric acid

(3) Aspirin (4) Ascorbic acid

Ans. **1**

Sol.



(A) (B) (C) (D)

4. The unit cell of copper corresponds to a face centered cube of edge length 3.596 Å with one copper atom at each lattice point. The calculated density of copper in kg/m^3 is ______.

[Molar mass of Cu : 63.54 g; Avogadro number = 6.022×10^{23}]

Ans. **9077**

Sol. a = 3.596 Å

$$d = \frac{Z\times GMM}{N_A\times a^3}$$

$$d = \frac{4\times63.54\times10^{-3}}{6.022\times10^{23}\times(3.596\times10^{-10})^3}$$

$d = 0.9076 \times 10^4 = 9076.2\ kg/m^3$

5. Consider titration of NaOH solution versus 1.25 M oxalic acid solution. At the end point following burette readings were obtained.

(i) 4.5 ml. (ii) 4.5 ml. (iii) 4.4 ml. (iv) 4.4 ml (v) 4.4 ml

If the volume of oxalic acid taken was 10.0 ml. then the molarity of the NaOH solution is ____M. (Rounded-off to the nearest integer)

Ans. **6**

Eq. of NaOH = Eq. of oxalic acid

$$[NaOH]\times1\times4.4 = \frac{5}{4}\times2\times10$$

$$[NaOH] = \frac{100}{4\times4.4} = \frac{25}{4.4} = 5.68$$

Nearest integer = 6M Ans.

6. Electromagnetic radiation of wavelength 663 nm is just sufficient to ionize the atom of metal A. The ionization energy of metal A in kJ mol^{-1} is____. (Rounded off to the nearest integer)

[$h=6.63\times10^{-34}Js$, $c = 3.00\times10^8 ms^{-1}$, $N_A=6.02\times10^{23}\ mol^{-1}$]

Ans. **180**

Sol. Energy req. to ionize an atom of metal 'A' = $\frac{hc}{\lambda} = \frac{hc}{663nm}$

for 1 mole atoms of 'A'

Total energy required = $N_A\times\frac{hc}{\lambda}$

$$= \frac{6.023\times10^{23}\times6.63\times10^{-34}\times3\times10^8}{663\times10^{-9}}$$

$= 6.023\times3\times10^{23-34+8+7}$

$= 18.04\times10^4$ J/mol

= 180.4 KJ/mol

Nearest Integer = 180 KJ/Mol.

7. The rate constant of a reaction increases by five times on increase in temperature from 27^0 C to 52^0C. The value of activation energy in kJ mol^{-1} is ______. (Rounded off to the nearest integer) [R=8.314 J K^{-1} mol^{-1}]

Ans. **52**

$$\frac{K_{52^0 C}}{K_{27^0 C}} = 5$$

$$\ln\left\{\frac{k_{T_2}}{k_{T_1}}\right\} = \frac{E_a}{R}\left\{\frac{1}{T_1} - \frac{1}{T_2}\right\}$$

$$\ln(5) = \frac{E_a}{R}\left\{\frac{1}{300} - \frac{1}{325}\right\}$$

$$\frac{2.303 \times 0.7 \times 8.314 \times 300 \times 325}{25} = E_a$$

E_a = 51524.96 J/mol

E_a = 51.524 KJ/mol

52 Ans.

8. Copper reduces NO_3^- into NO and NO_2 depending upon the concentration of HNO_3 in solution. (Assuming fixed [Cu^{2+}] and $P_{NO}=P_{NO_2}$), the HNO_3 concentration at which the thermodynamic tendency for reduction of NO_3^- into NO and NO_2 by copper is same is 10^x M. The value of 2x is ______. (Rounded-off to the nearest integer)

[Given : $E^0_{Cu^{2+}/Cu} = 0.34V, E^0_{NO_3^-/NO} = 0.96V, E^0_{NO_3^-/NO_2} = 0.79V$ and at 298 K, $\frac{RT}{F}(2.303) = 0.059$]]

Ans. **1**

Sol. Anode

Cu(s) $\rightarrow$ Cu^{+2} + $2e^-$

Cathode (1)

$$\frac{3e^- + 4H^+ + NO_3^- \rightarrow NO + 2H_2O}{8H^- + 2N0_3^- + 3Cu(s) \rightarrow 3Cu^{+2} + 2NO + 4H_2O}$$

$$Q = \frac{\left[Cu^{+2}\right]^3 \times (p_{NO})^2}{\left[NO_3^-\right]^2 \left[H^+\right]^8}$$

$$\in^0_{cell} = 1.3$$

$$\in_{cell} = 1.3 - \frac{0.059}{6}\log\frac{\left(Cu^{+2}\right)^3 (p_{NO})^2}{\left(NO_3^-\right)^2 \times \left(H^+\right)^8} \quad ......(1)$$

Anode Cu(s) $\rightarrow$ Cu^{+2} + $2e^-$

Cathode $\dfrac{e^- + 2n^+ + NO_3^- \rightarrow NO_2 + H_2O}{Cu(s) + 4H^+ + 2NO_3^- \rightarrow 2NO_2 + 2H_2O + Cu^{+2}}$

$\in^0_{cell} = 1.13$

$$Q = \frac{(Cu^{+2})(p_{NO_2})^2}{\left(NO_3^-\right)^2 \left(H^+\right)^4}$$

$$\in_{cell} = 1.13 - \frac{0.059}{2} \log \frac{\left(Cu^{+2}\right)\left(p_{NO_2}\right)^2}{\left(NO_3^-\right)^2 \left(H^+\right)^4}$$

$\in_{cell_T} = \in_{cell_2}$

$$1.3 - \frac{0.059}{6} \log\left(Q_1\right) = 1.13 - \frac{0.059}{2} \log\left(Q_2\right)$$

$$0.17 = \frac{0.059}{6} \left\{ \log(Q_1) - 3\log(Q_2) \right\}$$

$$= \frac{0.059}{6} \left\{ \log \frac{\left(Cu^{+2}\right)^3 \times \left(p_{NO}\right)^2 \times \left(NO_3^-\right)^6 \left(H^+\right)^{12}}{\left(NO_3^-\right)^2 \left(H^+\right)^8 \times \left(Cu^{+2}\right)^3 \times \left(p_{NO_2}\right)^6} \right\}$$

$$= \frac{0.059}{6} \left\{ \log \frac{\left[NO_3^-\right]^4 \left[H^+\right]^4}{\left(P_{NO_2}\right)^4} \right\}$$

$$0.17 = \frac{0.059}{6} \times 8 \log\left(HNO_3\right)$$

log (HNO_3) = 2.16

[HNO_3] = $10^{2.16}$ = 10^x

x = 2.16 $\Rightarrow$ 2x = 4.32 $\approx$ 4

9. Five moles of an ideal gas at 293 K is expanded isothermally from an initial pressure of 2.1 MPa to 1.3 MPa against at constant external 4.3 MPa. The heat transferred in this process is ____kJ mol^{-1}. (Rounded-off of the nearest integer)

[Use R = 8.314 J mol^{-1} K^{-1}]

Ans. **15**

Sol. Moles (n) = 5

T = 293k

Process = IsoT. $\rightarrow$ Irreversible

P_{ini} = 2.1 M Pa

P_t = 1.3 M Pa

P_{ext} = 4.3 mPa

Work = - P_{ext} Δv

$$= -4.3 \times \left(\frac{5 \times 293R}{1.3} - \frac{5 \times 293}{2.1} \right)$$

$$= -5 \times 293 \times 8.314 \times 43 \left(\frac{1}{13} - \frac{1}{21} \right)$$

$$= \frac{5 \times 293 \times 8.314 \times 43 \times 8}{21 \times 13}$$

= -15347.7049 J

= - 15.34 KJ

Isothermal process, so $\Delta U = 0$

w = - Q

Q = 15.34 KJ / mol

So answer is 15

10. Among the following, number of metal/s which can be used as electrodes in the photoelectric cell is _____(Integer answer).

(A) Li (B) Na (C) Rb (D) Cs

Ans. **1**

Sol. Cs is used in photoelectric cell due to its very low ionization potential.

25^{th} Feb. 2021 | Shift - 2
MATHEMATICS

SECTION-A

1. A plane passes through the points A(1, 2, 3), B(2, 3, 1) and C(2, 4, 2). If O is the origin and P is (2, −1, 1), then the projection of $\overrightarrow{OP}$ on this plane is of length:

(1) $\sqrt{\frac{2}{5}}$

(2) $\sqrt{\frac{2}{3}}$

(3) $\sqrt{\frac{2}{11}}$

(4) $\sqrt{\frac{2}{7}}$

Ans. **(3)**

Sol. A(1, 2, 3), B(2, 3, 1), C(2, 4, 2), O(0, 0, 0)

Equation of plane passing through A, B, C will be

$$\begin{vmatrix} x-1 & y-2 & z-3 \\ 2-1 & 3-2 & 1-3 \\ 2-1 & 4-2 & 2-3 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-1 & y-2 & z-3 \\ 1 & 1 & -2 \\ 1 & 2 & -1 \end{vmatrix} = 0$$

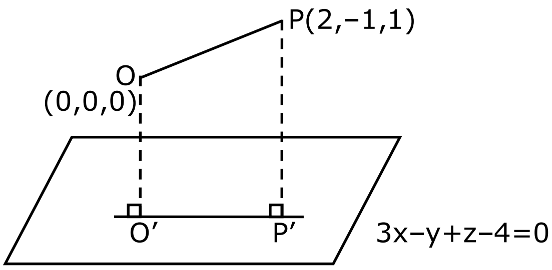
$\Rightarrow$ (x − 1)(−1 + 4) − (y − 2)(−1 + 2) + (z − 3)(2 − 1) = 0

$\Rightarrow$ (x − 1)(3) − (y − 2)(1) + (z − 3)(1) = 0

$\Rightarrow$ 3x − 3 − y + 2 + z − 3 = 0

$\Rightarrow$ 3x − y + z − 4 = 0, is the required plane.

Now, given O(0, 0, 0) & P(2, −1, 1)



Plane is 3x − y + z − 4 = 0

O′ & P′ are foot of perpendiculars.

for O'

$$\frac{x-0}{3}=\frac{y-0}{-1}=\frac{z-0}{1}=\frac{-(0-0+0-4)}{9+1+1}$$

$$\frac{x}{3}=\frac{y}{-1}=\frac{z}{1}=\frac{4}{11}$$

$$\Rightarrow \quad O'\left(\frac{12}{11},\frac{-4}{11},\frac{4}{11}\right)$$

for P'

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\frac{-(3(2)-(-1)+1-4)}{9+1+1}$$

$$\frac{x-2}{3}=\frac{y+1}{-1}=\frac{z-1}{1}=\left(\frac{-4}{11}\right)$$

$$P'\left(\frac{-12}{11}+2,\ \frac{4}{11}-1,\ \frac{-4}{11}+1\right)$$

$$\Rightarrow \quad P'\left(\frac{10}{11},\frac{-7}{11},\frac{7}{11}\right)$$

$$O'P'=\sqrt{\left(\frac{10}{11}-\frac{12}{11}\right)^2+\left(\frac{-7}{11}+\frac{4}{11}\right)^2+\left(\frac{7}{11}-\frac{4}{11}\right)^2}$$

$$\Rightarrow \quad O'P'=\frac{1}{11}\sqrt{4+9+9}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{22}}{11}$$

$$\Rightarrow \quad O'P'=\frac{\sqrt{2}\times\sqrt{11}}{11}$$

$$\Rightarrow \quad O'P'=\sqrt{\frac{2}{11}}$$

2. The contrapositive of the statement "If you will work, you will earn money" is:

(1) If you will not earn money, you will not work

(2) You will earn money, if you will not work

(3) If you will earn money, you will work

(4) To earn money, you need to work

Ans. **(1)**

Sol. Contrapositive of $p \rightarrow q$ is $\sim q \rightarrow \sim p$

$p \rightarrow$ you will work

$q \rightarrow$ you will earn money

$\sim q \rightarrow$ you will not earn money

$\sim p \rightarrow$ you will not work

$\sim q \rightarrow \sim p \Rightarrow$ if you will not earn money, you will not work.

3. If $\alpha, \beta \in R$ are such that $1 - 2i$ (here $i^2 = -1$) is a root of $z^2 + \alpha z + \beta = 0$, then $(\alpha - \beta)$ is equal to:

(1)7

(2) −3

(3) 3

(4)−7

Ans. **(4)**

Sol. $(1-2i)^2 + \alpha(1-2i) + \beta = 0$

$1 - 4 - 4i + \alpha - 2i\alpha + \beta = 0$

$(\alpha + \beta - 3) - i(4 + 2\alpha) = 0$

$\alpha + \beta - 3 = 0$ & $4 + 2\alpha = 0$

$\alpha = -2 \quad \beta = 5$

$\alpha - \beta = -7$

4. If $I_n = \int_{\pi/4}^{\pi/2} \cot^n x\, dx$, then:

(1) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in G.P.

(2) $\frac{1}{I_2+I_4}, \frac{1}{I_3+I_5}, \frac{1}{I_4+I_6}$ are in A.P.

(3) $I_2 + I_4$, $I_3 + I_5$, $I_4 + I_6$ are in A.P.

(4) $I_2 + I_4$, $(I_3 + I_5)^2$, $I_4 + I_6$ are in G.P.

Ans. (2)

Sol.

$$I_{n+2} + I_n = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cot^n x . \operatorname{cosec}^2 x\, dx = \left[\frac{-(\cot x)^{n+1}}{n+1}\right]_{\frac{\pi}{4}}^{\frac{\pi}{2}}$$

$$I_{n+2} + I_n = \frac{1}{n+1}$$

$$I_2 + I_4 = \frac{1}{3}, I_3 + I_5 = \frac{1}{4}, I_4 + I_6 = \frac{1}{5}$$

5. If for the matrix, $A = \begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}$, $AA^T = I_2$, then the value of $\alpha^4 + \beta^4$ is:

(1) 1

(2) 3

(3) 2

(4) 4

Ans. (1)

Sol. $\begin{bmatrix} 1 & -\alpha \\ \alpha & \beta \end{bmatrix}\begin{bmatrix} 1 & \alpha \\ -\alpha & \beta \end{bmatrix} = \begin{bmatrix} 1+\alpha^2 & \alpha-\alpha\beta \\ \alpha-\alpha\beta & \alpha^2+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$1 + \alpha^2 = 1$

$\alpha^2 = 0$

$\alpha^2 + \beta^2 = 1$

$\beta^2 = 1$

$\alpha^4 = 0$

$\beta^4 = 1$

$\alpha^4 + \beta^4 = 1$

6. Let x denote the total number of one-one functions from a set A with 3 elements to a set B with 5 elements and y denote the total number of one-one functions from the set A to the set A × B. Then:

(1) $y = 273x$

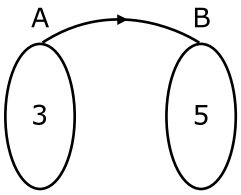
(2) $2y = 91x$

(3) $y = 91x$

(4) $2y = 273x$

Ans. (2)

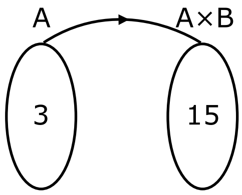
Sol. Number of elements in A = 3
Number of elements in B = 5
Number of elements in A × B = 15



Number of one-one function

$x = 5 \times 4 \times 3$

$x = 60$



Number of one-one function

$y = 15 \times 14 \times 13$

$y = 15 \times 4 \times \frac{14}{4} \times 13$

$y = 60 \times \frac{7}{2} \times 13$

$2y = (13)(7x)$

$2y = 91x$

7. If the curve $x^2 + 2y^2 = 2$ intersects the line $x + y = 1$ at two points P and Q, then the angle subtended by the line segment PQ at the origin is:

(1) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{4}\right)$

(2) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{4}\right)$

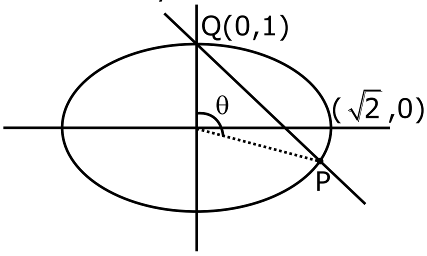
(3) $\frac{\pi}{2} + \tan^{-1}\left(\frac{1}{3}\right)$

(4) $\frac{\pi}{2} - \tan^{-1}\left(\frac{1}{3}\right)$

Ans. **(1)**

Sol. Ellipse $\frac{x^2}{2}+\frac{y^2}{1}=1$

Line $x + y = 1$



Using homogenisation

$x^2 + 2y^2 = 2(1)^2$

$x^2 + 2y^2 = 2(x + y)^2$

$x^2 + 2y^2 = 2x^2 + 2y^2 + 4xy$

$x^2 + 4xy = 0$

for $ax^2 + 2hxy + by^2 = 0$

$\tan\theta = \left|\frac{2\sqrt{h^2-ab}}{a+b}\right|$

$\tan\theta = \left|\frac{2\sqrt{(2)^2-0}}{1+0}\right|$

$\tan\theta = -4$

$\cot\theta = -\frac{1}{4}$

$\theta = \cot^{-1}\left(-\frac{1}{4}\right)$

$\theta = \pi - \cot^{-1}\left(\frac{1}{4}\right)$

$\theta = \pi - \left(\frac{\pi}{2}-\tan^{-1}\left(\frac{1}{4}\right)\right)$

$\theta = \frac{\pi}{2}+\tan^{-1}\left(\frac{1}{4}\right)$

8. The integral $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$, x > 0, is equal to:

(where c is a constant of integration)

(1) $\log_e|x^2 + 5x - 7| + c$

(2) $\frac{1}{4}\log_e|x^2 + 5x - 7| + c$

(3) $4\log_e|x^2 + 5x - 7| + c$

(4) $\log_e\sqrt{x^2 + 5x - 7} + c$

Ans. **(3)**

Sol. $\int \frac{e^{3\log_e 2x} + 5e^{2\log_e 2x}}{e^{4\log_e x} + 5e^{3\log_e x} - 7e^{2\log_e x}} dx$

$= \int \frac{8x^3 + 5(4x^2)}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x^3 + 20x^2}{x^4 + 5x^3 - 7x^2}$

$= \int \frac{8x + 20}{x^2 + 5x - 7}$

$= \int \frac{4(2x+5)}{x^2 + 5x - 7}$ $\left\{\begin{matrix}\text{Let } x^2 + 5x - 7 = t \\ (2x+5)dx = dt\end{matrix}\right\}$

$= \int \frac{4dt}{t}$

$= 4\ln|t| + C$

$= 4\ln|(x^2 + 5x - 7)| + c$

9. A hyperbola passes through the foci of the ellipse $\frac{x^2}{25}+\frac{y^2}{16}=1$ and its transverse and conjugate axes coincide with major and minor axes of the ellipse, respectively. If the product of their eccentricities is one, then the equation of the hyperbola is:

(1) $\frac{x^2}{9}-\frac{y^2}{4}=1$

(2) $\frac{x^2}{9}-\frac{y^2}{16}=1$

(3) $x^2 - y^2 = 9$

(4) $\frac{x^2}{9}-\frac{y^2}{25}=1$

Ans. **(2)**

$e_1 = \sqrt{1-\frac{16}{25}} = \frac{3}{5}$ foci ($\pm ae$, 0)

Foci $=(\pm 3, 0)$

Let equation of hyperbola be $\frac{x^2}{A^2}-\frac{y^2}{B^2}=1$

Passes through (± 3, 0)

Sol. $A^2 = 9, A = 3, e_2 = \frac{5}{3}$

$e_2{}^2 = 1+\frac{B^2}{A^2}$

$\frac{25}{9} = 1+\frac{B^2}{9} \Rightarrow B^2 = 16$

Ans $\frac{x^2}{9}-\frac{y^2}{16}=1$

10. $\lim_{x\to\infty}\left[\frac{1}{n}+\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+......+\frac{n}{(2n-1)^2}\right]$ is equal to:

(1) 1

(2) $\frac{1}{3}$

(3) $\frac{1}{2}$

(4) $\frac{1}{4}$

Ans. **(3)**

Sol.

$$\underset{x\to\infty}{\mathrm{Lim}}\sum_{r=0}^{n-1}\frac{n}{(n+r)^2} = \underset{x\to\infty}{\mathrm{Lim}}\sum_{r=0}^{n-1}\frac{n^2}{n^2\left(1+\frac{r}{n}\right)^2} = \int_0^1\frac{dx}{(1+x)^2}$$

$$= -\left[\frac{1}{1+x}\right]_0^1 \Rightarrow -\left[\frac{1}{2}-1\right] = \frac{1}{2}$$

11. In a group of 400 people, 160 are smokers and non-vegetarian; 100 are smokers and vegetarian and the remaining 140 are non-smokers and vegetarian. Their chances of getting a particular chest disorder are 35%, 20% and 10% respectively. A person is chosen from the group at random and is found to be suffering from the chest disorder. The probability that the selected person is a smoker and non-vegetarian is:

(1) $\frac{7}{45}$

(2) $\frac{8}{45}$

(3) $\frac{14}{45}$

(4) $\frac{28}{45}$

Ans. (4)

Sol. Based on Baye's theorem

$$\text{Probability} = \frac{\left(160 \times \frac{35}{100}\right)}{\left(160 \times \frac{35}{100}\right) + \left(100 \times \frac{20}{100}\right) + \left(140 \times \frac{10}{100}\right)}$$

$$= \frac{5600}{9000}$$

$$= \frac{28}{45}$$

12. The following system of linear equations

$3x + 3y + 2z = 9$

$3x + 2y + 2z = 9$

$x - y + 4z = 8$

(1) does not have any solution

(2) has a unique solution

(3) has a solution (α, β, γ) satisfying $\alpha + \beta^2 + \gamma^3 = 12$

(4) has infinitely many solutions

Ans. (2)

Sol. $\Delta = \begin{vmatrix} 2 & 3 & 2 \\ 3 & 2 & 2 \\ 1 & -1 & 4 \end{vmatrix} = -20 \neq 0$ $\quad\therefore$ unique solution

$\Delta_x = \begin{vmatrix} 9 & 3 & 2 \\ 9 & 2 & 2 \\ 8 & -1 & 4 \end{vmatrix} = 0$

$\Delta_y = \begin{vmatrix} 2 & 9 & 2 \\ 3 & 9 & 2 \\ 1 & 8 & 4 \end{vmatrix} = -20$

$\Delta_z = \begin{vmatrix} 2 & 3 & 9 \\ 3 & 2 & 9 \\ 1 & -1 & 8 \end{vmatrix} = -40$

$\therefore \quad x = \frac{\Delta_x}{\Delta} = 0$

$y = \frac{\Delta_y}{\Delta} = 1$

$z = \frac{\Delta_z}{\Delta} = 2$

Unique solution: (0, 1, 2)

13. The minimum value of $f(x) = a^{a^x} + a^{1-a^x}$, where $a, x \in R$ and $a > 0$, is equal to:

(1) $a + \frac{1}{a}$

(2) a + 1

(3) 2a

(4) $2\sqrt{a}$

Ans. (4)

Sol. AM $\geq$ GM

$$\frac{a^{ax} + \frac{a}{a^{ax}}}{2} \geq \left(a^{ax}.\frac{a}{a^{ax}}\right)^{1/2} \Rightarrow a^{ax} + a^{1-ax} \geq 2\sqrt{a}$$

14. A function f(x) is given by $f(x) = \frac{5^x}{5^x + 5}$, then the sum of the series

$f\left(\frac{1}{20}\right) + f\left(\frac{2}{20}\right) + f\left(\frac{3}{20}\right) + \ldots\ldots + f\left(\frac{39}{20}\right)$

is equal to:

(1) $\frac{19}{2}$

(2) $\frac{49}{2}$

(3) $\frac{39}{2}$

(4) $\frac{29}{2}$

Ans. **(3)**

Sol.

$f(x) = \frac{5^x}{5^x + 5}$ ....(i)

$f(2-x) = \frac{5^{2-x}}{5^{2-x} + 5}$

$f(2-x) = \frac{5}{5^x + 5}$......(ii)

Adding equation (i) and (ii)

$f(x) + f(2-x) = 1$

$f\left(\frac{1}{20}\right) + f\left(\frac{39}{20}\right) = 1$

$f\left(\frac{2}{20}\right) + f\left(\frac{38}{20}\right) = 1$

:

:

$f\left(\frac{19}{20}\right) + f\left(\frac{21}{20}\right) = 1$

and $f\left(\frac{20}{20}\right) = f(1) = \frac{1}{2}$

$\Rightarrow 19 + \frac{1}{2} \Rightarrow \frac{39}{2}$

15. Let α and β be the roots of $x^2 - 6x - 2 = 0$. If $a_n = \alpha^n - \beta^n$ for $n \geq 1$, then the value of $\frac{a_{10} - 2a_8}{3a_9}$ is:

(1) 4

(2) 1

(3) 2

(4) 3

Ans. **(3)**

Sol. $x^2 - 6x - 2 = 0$ with roots α, β: $\alpha + \beta = 6$, $\alpha\beta = -2$

and $\alpha^2 - 6\alpha - 2 = 0 \Rightarrow \alpha^2 - 2 = 6\alpha$

$\beta^2 - 6\beta - 2 = 0 \Rightarrow \beta^2 - 2 = 6\beta$

Now

$$\frac{a_{10} - 2a_8}{3a_9} = \frac{\left(\alpha^{10} - \beta^{10}\right) - 2\left(\alpha^8 - \beta^8\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\left(\alpha^{10} - 2\alpha^8\right) - \left(\beta^{10} - 2\beta^8\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8\left(\alpha^2 - 2\right) - \beta^8\left(\beta^2 - 2\right)}{3(\alpha^9 - \beta^9)}$$

$$= \frac{\alpha^8(6\alpha) - \beta^8(6\beta)}{3(\alpha^9 - \beta^9)} = \frac{6\left(\alpha^9 - \beta^9\right)}{3(\alpha^9 - \beta^9)} = \frac{6}{3} = 2$$

16. Let A be a 3×3 matrix with det(A) = 4. Let R_i denote the i^{th} row of A. If a matrix B is obtained by performing the operation $R_2 \to 2R_2 + 5R_3$ on 2A, then det(B) is equal to:

(1) 64

(2) 16

(3) 80

(4) 128

Ans. **(1)**

Sol.

$$A = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}$$

$$2A = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 2R_{21} & 2R_{22} & 2R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$R_2 \to 2R_2 + 5R_3$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} + 10R_{31} & 4R_{22} + 10R_{32} & 4R_{23} + 10R_{33} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$R_2 \to R_2 - 5R_3$

$$B = \begin{bmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{bmatrix}$$

$$|B| = \begin{vmatrix} 2R_{11} & 2R_{12} & 2R_{13} \\ 4R_{21} & 4R_{22} & 4R_{23} \\ 2R_{31} & 2R_{32} & 2R_{33} \end{vmatrix}$$

$$|B| = 2 \times 2 \times 4 \begin{vmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{vmatrix}$$

$= 16 \times 4$

$= 64$

17. The shortest distance between the line $x - y = 1$ and the curve $x^2 = 2y$ is:

(1) $\frac{1}{2}$

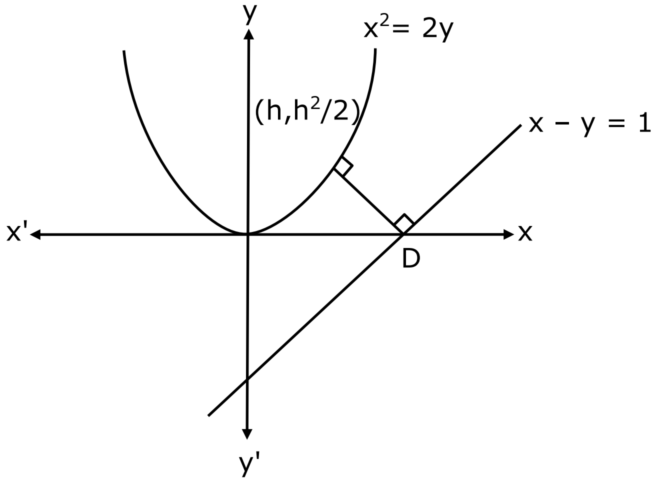
(2) 0

(3) $\frac{1}{2\sqrt{2}}$

(4) $\frac{1}{\sqrt{2}}$

Ans. **(3)**

Sol. Shortest distance must be along common normal



m_1 (slope of line x−y = 1) = 1 $\Rightarrow$ slope of perpendicular line =−1

$m_2 = \frac{2x}{2} = x \Rightarrow m_2 = h \Rightarrow$ slope of normal $-\frac{1}{h}$

$-\frac{1}{h} = -1 \Rightarrow h = 1$

so point is $\left(1, \frac{1}{2}\right)$

$D = \left|\frac{1 - \frac{1}{2} - 1}{\sqrt{1+1}}\right| = \frac{1}{2\sqrt{2}}$

18. Let A be a set of all 4-digit natural numbers whose exactly one digit is 7. Then the probability that a randomly chosen element of A leaves remainder 2 when divided by 5 is:

(1) $\frac{1}{5}$

(2) $\frac{2}{9}$

(3) $\frac{97}{297}$

(4) $\frac{122}{297}$

Ans. **(3)**

Sol. Total cases

(4 × 9 × 9 × 9) − (3 × 9 × 9)

Probability = $\frac{(3 \times 9 \times 9) - (2 \times 9) + (8 \times 9 \times 9)}{(4 \times 9^3) - (3 \times 9^2)}$

$= \frac{97}{217}$

19. $\text{cosec}\left[2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right]$ is equal to:

(1) $\frac{75}{56}$

(2) $\frac{65}{56}$

(3) $\frac{56}{33}$

(4) $\frac{65}{33}$

Ans. **(2)**

Sol. $\text{cosec}\left(2\cot^{-1}(5)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$\text{cosec}\left(2\tan^{-1}\left(\frac{1}{5}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$=\text{cosec}\left(\tan^{-1}\left(\frac{2\left(\frac{1}{5}\right)}{1-\left(\frac{1}{5}\right)^2}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

$=\text{cosec}\left(\tan^{-1}\left(\frac{5}{12}\right)+\cos^{-1}\left(\frac{4}{5}\right)\right)$

Let $\tan^{-1}(5/12)=\theta \Rightarrow \sin\theta=\frac{5}{13}, \cos\theta=\frac{12}{13}$

and $\cos^{-1}\left(\frac{4}{5}\right)=\phi \Rightarrow \cos\phi=\frac{4}{5}$ and $\sin\phi=\frac{3}{5}$

$=\text{cosec}(\theta+\phi)$

$=\frac{1}{\sin\theta\cos\phi+\cos\theta\sin\phi}$

$=\frac{1}{\frac{5}{13}.\frac{4}{5}+\frac{12}{13}.\frac{3}{5}}=\frac{65}{56}$

20. If $0 < x, y < \pi$ and $\cos x + \cos y - \cos(x + y) = \frac{3}{2}$, then $\sin x + \cos y$ is equal to:

(1) $\frac{1+\sqrt{3}}{2}$

(2) $\frac{1-\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{2}$

(4) $\frac{1}{2}$

Ans. (1)

Sol.

$$2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)-\left[2\cos^2\left(\frac{x+y}{2}\right)-1\right]=\frac{3}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[\cos\left(\frac{x-y}{2}\right)-\cos\left(\frac{x+y}{2}\right)\right]=\frac{1}{2}$$

$$2\cos\left(\frac{x+y}{2}\right)\left[2\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)\right]=\frac{1}{2}$$

$$\cos\left(\frac{x+y}{2}\right).\sin\left(\frac{x}{2}\right).\sin\left(\frac{y}{2}\right)=\frac{1}{8}$$

Possible when $\frac{x}{2}=30^\circ$ & $\frac{y}{2}=30^\circ$

$x = y = 60^\circ$

$$\sin x+\cos y=\frac{\sqrt{3}}{2}+\frac{1}{2}=\frac{\sqrt{3}+1}{2}$$

SECTION-B

1. If $\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$ exists and is equal to b, then the value of a – 2b is ______.

Ans. (5)

$$\lim_{x\to 0}\frac{ax-\left(e^{4x}-1\right)}{ax\left(e^{4x}-1\right)}$$

Applying L' Hospital Rule

$$\lim_{x\to 0}\frac{a-4e^{4x}}{a\left(e^{4x}-1\right)+ax\left(4e^{4x}\right)} \quad \text{So } a=4$$

Sol. Applying L' Hospital Rule

$$\lim_{x\to 0}\frac{-16e^{4x}}{a\left(4e^{4x}\right)+a\left(4e^{4x}\right)+ax\left(16e^{4x}\right)}$$

$$\frac{-16}{4a+4a}=\frac{-16}{32}=-\frac{1}{2}=b$$

$$a-2b=4-2\left(\frac{-1}{2}\right)=4+1=5$$

2. A line is a common tangent to the circle $(x-3)^2+y^2=9$ and the parabola $y^2 = 4x$. If the two points of contact (a, b) and (c, d) are distinct and lie in the first quadrant, then 2(a+c) is equal to ______.

Ans. (9)

Sol. Circle: $(x-3)^2+y^2=9$

Parabola: $y^2 = 4x$

Let tangent $y = mx + \frac{a}{m}$

$$y = mx + \frac{1}{m}$$

$m^2x - my + 1 = 0$

the above line is also tangent to circle

$(x-3)^2+y^2=9$

$\therefore \perp$ from (3, 0) = 3

$$\left|\frac{3m^2-0+1}{\sqrt{m^2+m^4}}\right| = 3$$

$(3m^2+1)^2 = 9(m^2+m^4)$

$6m^2 + 1 + 9m^4 = 9m^2 + 9m^4$

$3m^2 = 1$

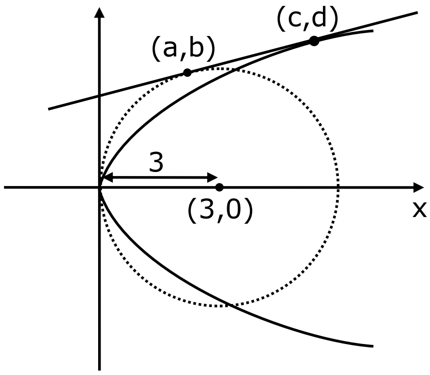
$m = \pm\frac{1}{\sqrt{3}}$

$\therefore$tangent is

$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ or $y = -\frac{1}{\sqrt{3}}x - \sqrt{3}$

(it will be used) (rejected)

$m = \frac{1}{\sqrt{3}}$



for Parabola $\left(\frac{a}{m^2}, \frac{2a}{m}\right) \equiv (3, 2\sqrt{3})$

(c, d)

for Circle $y = \frac{1}{\sqrt{3}}x + \sqrt{3}$ & $(x-3)^2 + y^2 = 9$

solving, $(x-3)^2 + \left(\frac{1}{\sqrt{3}}x + \sqrt{3}\right)^2 = 9$

$x^2 + 9 - 6x + \frac{1}{3}x^2 + 3 + 2x = 9$

$\frac{4}{3}x^2 - 4x + 3 = 0$

$4x^2 - 12x + 9 = 0$

$4x^2 - 6x - 6x + 9 = 0$

$2x(2x-3) - 3(2x-3) = 0$

$(2x-3)(2x-3) = 0$

$x = \frac{3}{2}$

$\therefore \quad y = \frac{1}{\sqrt{3}}\left(\frac{3}{2}\right) + \sqrt{3}$

$y = \frac{\sqrt{3}}{2} + \sqrt{3}$

$y = \frac{3\sqrt{3}}{2}$

$(a, b) \equiv \left(\frac{3}{2}, \ \frac{3\sqrt{3}}{2}\right)$

$2(a + c) = 2\left(\frac{3}{2} + 3\right)$

$= 2\left(\frac{3}{2} + \frac{6}{2}\right)$

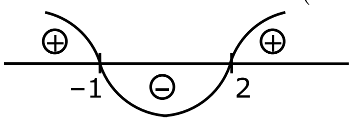
$= 9$

3. The value of $\int_{-2}^{2} \left|3x^2 - 3x - 6\right| \, dx$ is _______.

Ans. **(19)**

Sol. $3\int_{-2}^{2} \left|x^2 - x - 2\right| dx \qquad x^2 - x - 2$

$= (x-2)(x+1)$



$= 3\left\{\int_{-2}^{-1} \left(x^2 - x - 2\right) dx + \int_{-1}^{2} \left(-x^2 + x + 2\right) dx\right\}$

$= 3\left[\left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-2}^{-1} - \left(\frac{x^3}{3} - \frac{x^2}{2} - 2x\right)_{-1}^{2}\right]$

$= 19$

4. If the remainder when x is divided by 4 is 3, then the remainder when $(2020+x)^{2022}$ is divided by 8 is ______.

Ans. **(1)**

Sol. Let x = 4k + 3

$(2020 + x)^{2022}$

$= (2020 + 4k + 3)^{2022}$

$= (4(505) + 4k + 3)^{2022}$

$= (4P + 3)^{2022}$

$= (4P + 4 - 1)^{2022}$

$= (4A - 1)^{2022}$

${}^{2022}C_0(4A)^0(-1)^{2022} + {}^{2022}C_1(4A)^1(-1)^{2021} + \ldots\ldots$

$1 + 8\lambda$

Reminder is 1.

5. A line 'ℓ' passing through origin is perpendicular to the lines

$$\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$$

$$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$$

If the co-ordinates of the point in the first octant on 'ℓ_2' at the distance of $\sqrt{17}$ from the point of intersection of 'ℓ' and 'ℓ_1' are (a, b, c), then 18(a+b+c) is equal to ______.

Ans. (44)

Sol. $\ell_1 : \vec{r} = (3+t)\hat{i} + (-1+2t)\hat{j} + (4+2t)\hat{k}$

$\ell_1 : \dfrac{x-3}{1} = \dfrac{y+1}{2} = \dfrac{z-4}{2} \quad \Rightarrow \quad$ D.R. of ℓ_1 = 1, 2, 2

$\ell_2 : \vec{r} = (3+2s)\hat{i} + (3+2s)\hat{j} + (2+s)\hat{k}$

$\ell_2 : \dfrac{x-3}{2} = \dfrac{y-3}{2} = \dfrac{z-2}{1} \quad \Rightarrow \quad$ D.R. of ℓ_2 = 2, 2, 1

D.R. of ℓ is $\perp$ to $\ell_1 \& \ell_2$

$\therefore$ D.R. of $\ell \,||\, (\ell_1 \times \ell_2) \quad \Rightarrow \quad \langle -2,\ 3,\ -2 \rangle$

$\therefore$ Equation of $\ell : \dfrac{x}{2} = \dfrac{y}{-3} = \dfrac{z}{2}$

Solving ℓ & ℓ_1

$(2\lambda, -3\lambda, 2\lambda) = (\mu + 3, 2\mu - 1, 2\mu + \mu)$

$\Rightarrow$ $2\lambda = \mu + 3$

$-3\lambda = 2\mu - 1$

$2\lambda = 2\mu + 4$

$\Rightarrow$ $\mu + 3 = 2\mu + 4$

$\mu = -1$

$\lambda = 1$

P(2, –3, 2) {intersection point}

Let, $Q(2\nu + 3, 2\nu + 3, \nu + 2)$ be point on ℓ_2

Now, $PQ = \sqrt{(2\nu+3-2)^2+(2\nu+3+3)^2+(\nu+2-2)^2} = \sqrt{17}$

$\Rightarrow$ $(2\nu + 1)^2 + (2\nu + 6)^2 + (\nu)^2 = 17$

$\Rightarrow$ $9\nu^2 + 28\nu + 36 + 1 - 17 = 0$

$\Rightarrow$ $9\nu^2 + 28\nu + 20 = 0$

$\Rightarrow$ $9\nu^2 + 18\nu + 10\nu + 20 = 0$

$\Rightarrow$ $(9\nu + 10)(\nu + 2) = 0$

$\Rightarrow$ $\nu = -2$ (rejected), $-\frac{10}{9}$ (accepted)

$Q\left(3-\frac{20}{9},\ 3-\frac{20}{9},\ 2-\frac{10}{9}\right)$

$\left(\frac{7}{9},\ \frac{7}{9},\ \frac{8}{9}\right)$

$\therefore$ $18(a + b + c)$

$= 18\left(\frac{7}{9}+\frac{7}{9}+\frac{8}{9}\right)$

$= 44$

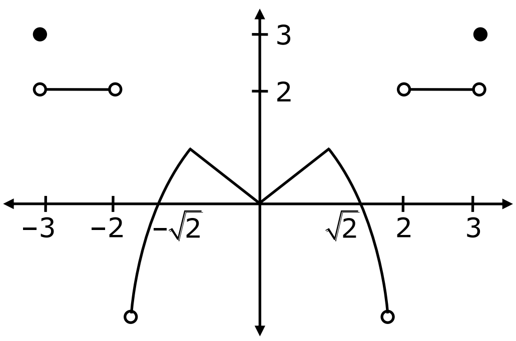
6. A function f is defined on [−3,3] as

$$f(x)=\begin{cases}\min\{|x|,2-x^2\}, & -2\le x\le 2\\ [|x|] & ,2<|x|\le 3\end{cases}$$

where [x] denotes the greatest integer $\le x$. The number of points, where f is not differentiable in (−3,3) is _______.

Ans. **(5)**

Sol.



Points of non-differentiability in (−3, 3) are at x = −2, −1, 0, 1, 2.

i.e. 5 points.

7. If the curves $x = y^4$ and $xy = k$ cut at right angles, then $(4k)^6$ is equal to _______.

Ans. **4**

Sol. $4y^3\dfrac{dy}{dx}=1$ & $x\dfrac{dy}{dx}+y=0$

$m_1=\dfrac{1}{4y^3}$ $\dfrac{dy}{dx}=\dfrac{-y}{x}=m_2$

$m_1m_2=-1$

$\dfrac{1}{4.y^3}\times\dfrac{-y}{x}=-1 \quad \because x=y^4$

$\dfrac{1}{4.y^6}=1$ and $xy=k$

$y^6=\dfrac{1}{4}$ $\Rightarrow k=y^5$

$\Rightarrow k^6=y^{30}$

$\Rightarrow k^6=\left(\dfrac{1}{4}\right)^5$

$\therefore (4k)^6=4^6\times k^6=4$

8. The total number of two digit numbers 'n', such that 3^n+7^n is a multiple of 10, is ______.

Ans. (45)

Sol. $\because 7^n = (10-3)^n = 10K + (-3)^n$

$\therefore 7^n + 3^n = 10K + (-3)^n + 3^n$ → 10K if n = odd

→ $10K + 2.3^n$ if n = even

Let n = 2t; $t \in N$

$\therefore 3^n = 3^{2t} = (10-1)^t$

$= 10p + (-1)^t$

$= 10p \pm 1$

$\therefore$ if n = even then $7^n + 3^n$ will not be multiply of 10

So if n is odd then only $7^n + 3^n$ will be multiply of 10

$\therefore$ n = 11, 13, 15, ..., 99

$\therefore$ Ans 45

9. Let $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$ and $\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $8\sqrt{3}$ square units, then $\vec{a}\cdot\vec{b}$ is equal to ______.

Ans. (2)

Sol. $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

Area of parallelogram = $|\vec{a} \times \vec{b}|$

$= \left|(\hat{i} + \alpha\hat{j} + 3\hat{k}) \times (3\hat{i} - \alpha\hat{j} + \hat{k})\right|$

$8\sqrt{3} = \left|(4\alpha)\hat{i} + 8\hat{j} - (4\alpha)\hat{k}\right|$

$(64)(3) = 16\alpha^2 + 64 + 16\alpha^2$

$(64)(3) = 32\alpha^2 + 64$

$6 = \alpha^2 + 2$

$\alpha^2 = 4$

$\therefore$ $\vec{a} = \hat{i} + \alpha\hat{j} + 3\hat{k}$

$\vec{b} = 3\hat{i} - \alpha\hat{j} + \hat{k}$

$\vec{a}\cdot\vec{b} = 3 - \alpha^2 + 3$

$= 6 - \alpha^2$

$= 6 - 4$

$= 2$

10. If the curve y = y(x) represented by the solution of the differential equation $(2xy^2 - y)dx + xdx = 0$, passes through the intersection of the lines, 2x − 3y=1 and 3x+2y=8, then |y(1)| is equal to ______.

Ans. 1

Sol. Given,

$(2xy^2 - y)dx + xdx = 0$

$\Rightarrow \frac{dy}{dx} + 2y^2 - \frac{y}{x} = 0$

$\Rightarrow -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\left(\frac{1}{x}\right) = 2$

$\frac{1}{y} = z$

$-\frac{1}{y^2}\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow \frac{dz}{dx} + z\left(\frac{1}{x}\right) = 2$

$\text{I.F.} = e^{\int \frac{1}{x}dx} = x$

$\therefore \; z(x) = \int 2(x)\,dx = x^2 + c$

$\Rightarrow \frac{x}{y} = x^2 + c$

As it passes through P(2, 1)

[Point of intersection of 2x − 3y = 1 and 3x + 2y = 8]

$\therefore \frac{2}{1} = 4 + c$

$\Rightarrow c = -2$

$\Rightarrow \frac{x}{y} = x^2 - 2$

Put x = 1

$\frac{1}{y} = 1 - 2 = -1$

$\Rightarrow y(1) = -1$

$\Rightarrow |y(1)| = 1$