FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. If $Z = \frac{A^2B^3}{C^4}$, then the relative error in Z will be :

(A) $\frac{\Delta A}{A} + \frac{\Delta B}{B} + \frac{\Delta C}{C}$

(B) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} - \frac{4\Delta C}{C}$

(C) $\frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}$

(D) $\frac{\Delta A}{A} + \frac{\Delta B}{B} - \frac{\Delta C}{C}$

Official Ans. by NTA (C)

Sol. $Z = \frac{A^2B^3}{C^4}$

In case of error

$$\frac{dZ}{Z} = \frac{2dA}{A} + \frac{3dB}{B} + \frac{4dC}{C}$$

$$\boxed{\frac{\Delta Z}{Z} = \frac{2\Delta A}{A} + \frac{3\Delta B}{B} + \frac{4\Delta C}{C}}$$

2. $\vec{A}$ is a vector quantity such that $|\vec{A}| =$ non-zero constant. Which of the following expressions is true for $\vec{A}$?

(A) $\vec{A}.\vec{A} = 0$

(B) $\vec{A} \times \vec{A} < 0$

(C) $\vec{A} \times \vec{A} = 0$

(D) $\vec{A} \times \vec{A} > 0$

Official Ans. by NTA (C)

Sol. $|\vec{A}| \neq 0$

$$\vec{A} \times \vec{A} = |\vec{A}||\vec{A}|\sin 0°\hat{n} = 0$$

3. Which of the following relations is true for two unit vectors $\hat{A}$ and $\hat{B}$ making an angle θ to each other?

(A) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}|\tan\frac{\theta}{2}$

(B) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}|\tan\frac{\theta}{2}$

(C) $|\hat{A}+\hat{B}| = |\hat{A}-\hat{B}|\cos\frac{\theta}{2}$

(D) $|\hat{A}-\hat{B}| = |\hat{A}+\hat{B}|\cos\frac{\theta}{2}$

Official Ans. by NTA (B)

Sol. $|\hat{A}+\hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 + 2|\hat{A}||\hat{B}|\cos\theta}$

$$= \sqrt{1+1+2\cos\theta}$$

$$= \sqrt{2(1+\cos\theta)}$$

$$= \sqrt{2\times 2\cos^2\frac{\theta}{2}}$$

$$= 2\cos\frac{\theta}{2}$$

$$|\hat{A}-\hat{B}| = \sqrt{|\hat{A}|^2 + |\hat{B}|^2 - 2|\hat{A}||\hat{B}|\cos\theta}$$

$$= \sqrt{2-2\cos\theta}$$

$$= 2\sin\frac{\theta}{2}$$

$$\frac{|\hat{A}+\hat{B}|}{|\hat{A}-\hat{B}|} = \cot\frac{\theta}{2}$$

4. If force $\vec{F} = 3\hat{i} + 4\hat{j} - 2\hat{k}$ acts on a particle having position vector $2\hat{i} + \hat{j} + 2\hat{k}$ then, the torque about the origin will be :-

(A) $3\hat{i} + 4\hat{j} - 2\hat{k}$

(B) $-10\hat{i} + 10\hat{j} + 5\hat{k}$

(C) $10\hat{i} + 5\hat{j} - 10\hat{k}$

(D) $10\hat{i} + \hat{j} - 5\hat{k}$

Official Ans. by NTA (B)

Sol. $\vec{\tau} = \vec{r} \times \vec{F}$

$$= \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & 2 \\ 3 & 4 & -2 \end{vmatrix}$$

$$= \hat{i}(-2-8) - \hat{j}(-4-6) + \hat{k}(8-3)$$

$$= -10\hat{i} + 10\hat{j} + 5\hat{k}$$

5. The height of any point P above the surface of earth is equal to diameter of earth. The value of acceleration due to gravity at point P will be : (Given g = acceleration due to gravity at the surface of earth)

(A) g/2

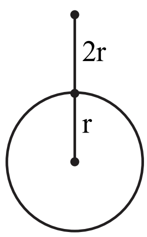
(B) g/4

(C) g/3

(D) g/9

Official Ans. by NTA (D)

Sol. $g = \dfrac{Gm}{r^2}$



$$g' = \frac{Gm}{(3r)^2}$$

$$g' = \frac{Gm}{9r^2}$$

$$g' = \frac{g}{9}$$

6. The terminal velocity (v_t) of the spherical rain drop depends on the radius (r) of the spherical rain drop as:-

(A) $r^{1/2}$ (B) r

(C) r^2 (D) r^3

Official Ans. by NTA (C)

Sol. $v_t = \dfrac{2}{9}\dfrac{gr^2(\rho_p - \rho_l)}{\eta}$; $v_t \propto r^2$

7. The relation between root mean square speed (v_{rms}) and most probable speed (v_p) for the molar mass M of oxygen gas molecule at the temperature of 300 K will be :-

(A) $v_{rms} = \sqrt{\dfrac{2}{3}}v_p$ (B) $v_{rms} = \sqrt{\dfrac{3}{2}}v_p$

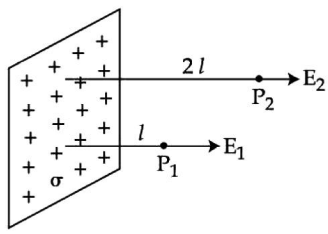
(C) $v_{rms} = v_p$ (D) $v_{rms} = \sqrt{\dfrac{1}{3}}v_p$

Official Ans. by NTA (B)

Sol. $v_{rms} = \sqrt{\dfrac{3RT}{M}}$ and $v_{mp} = \sqrt{\dfrac{2RT}{M}}$

Thus $v_{rms} = \sqrt{\dfrac{3}{2}}v_{mp}$

8. In the figure, a very large plane sheet of positive charge is shown. P_1 and P_2 are two points at distance l and $2l$ from the charge distribution. If σ is the surface charge density, then the magnitude of electric fields E_1 and E_2 at P_1 and P_2 respectively are :



(A) $E_1 = \sigma/\varepsilon_0$, $E_2 = \sigma/2\varepsilon_0$

(B) $E_1 = 2\sigma/\varepsilon_0$, $E_2 = \sigma/\varepsilon_0$

(C) $E_1 = E_2 = \sigma/2\varepsilon_0$

(D) $E_1 = E_2 = \sigma/\varepsilon_0$

Official Ans. by NTA (C)

Sol. As the sheet is very large $\vec{E}$ is independent of distance from it.

Thus $E_1 = E_2 = \frac{\sigma}{2\varepsilon_0}$

9. Match List-I with List-II

List-I	List-II
(A) AC generator	(I) Detects the presence of current in the circuit
(B) Galvanometer	(II) Converts mechanical energy into electrical energy
(C) Transformer	(III) Works on the principle of resonance in AC circuit
(D) Metal detector	(IV) Changes an alternating voltage for smaller or greater value

Choose the **correct answer** from the options given below :-

(A) (A)–(II), B–(I), (C)–(IV), (D)–(III)

(B) (A)–(II), B–(I), (C)–(III), (D)–(IV)

(C) (A)–(III), B–(IV), (C)–(II), (D)–(I)

(D) (A)–(III), B–(I), (C)–(II), (D)–(IV)

Official Ans. by NTA (A)

Sol. AC generator converts mechanical energy into electrical energy. Galvanometer shows deflection when current passes through it so it is used to show presence of current in any wire. Transformer is used to step up or step down the voltage. Metals detectors contain inductor coils and use principle of induction and resonance in AC circuit.

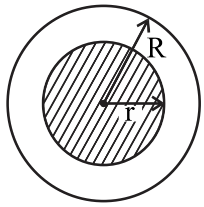
10. A long straight wire with a circular cross-section having radius R, is carrying a steady current I. The current I is uniformly distributed across this cross-section. Then the variation of magnetic field due to current I with distance r (r < R) from its centre will be :-

(A) $B \propto r^2$ (B) $B \propto r$

(C) $B \propto \frac{1}{r^2}$ (D) $B \propto \frac{1}{r}$

Official Ans. by NTA (B)

Sol. Use Ampere's law



$$B.2\pi r = \mu_0 . \frac{I}{\pi R^2} . \pi r^2$$

Thus $B \propto r$

11. If wattless current flows in the AC circuit, then the circuit is

(A) Purely Resistive circuit

(B) Purely Inductive circuit

(C) LCR series circuit

(D) RC series circuit only

Official Ans. by NTA (B)

Sol. Purely Inductive circuit

$$\theta = \frac{\pi}{2}$$

$$\cos\frac{\pi}{2} = 0$$

Average power = 0

12. The electric field in an electromagnetic wave is given by $E = 56.5 \sin \omega(t - x/c)\, NC^{-1}$. Find the intensity of the wave if it is propagating along x-axis in the free space. (Given $\varepsilon_0 = 8.85 \times 10^{-12} C^2N^{-1}m^{-2}$)

(A) 5.65 Wm^{-2} (B) 4.24 Wm^{-2}

(C) 1.9 × 10^{-7} Wm^{-2} (D) 56.5 Wm^{-2}

Official Ans. by NTA (B)

Sol. $I = \frac{1}{2}\varepsilon_0 E_0^2 c$

$$I = \frac{1}{2} \times (8.85 \times 10^{-12})(56.5)^2 \times (3 \times 10^8)$$

= 4.24 Wm^{-2}.

13. The two light beams having intensities I and 9I interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\frac{\pi}{2}$ at point P and π at point Q. Then the difference between the resultant intensities at P and Q will be :

(A) 2 I (B) 6 I

(C) 5 I (D) 7 I

Official Ans. by NTA (B)

Sol. $I_P = I + 9I + 2\sqrt{I \times 9I}\cos\frac{\pi}{2}$

$I_P = 10\ I$

$I_Q = I + 9I + 2\sqrt{I \times 9I}\cos\pi$

$= 10\ I - 6I = 4I$

$\therefore I_P - I_Q = 10I - 4I = 6I$

14. A light wave travelling linearly in a medium of dielectric constant 4, incident on the horizontal interface separating medium with air. The angle of incidence for which the total intensity of incident wave will be reflected back into the same medium will be (Given : relative permeability of medium $\mu_r = 1$)

(A) 10° (B) 20°

(C) 30° (D) 60°

Official Ans. by NTA (D)

Sol. For total internal reflection, $i > \theta_C$

$\Rightarrow \sin i > \sin\theta_C$

$\Rightarrow \sin i > \frac{\mu_R}{\mu_D}$ ....(1)

Also $\mu = \sqrt{\mu_r \in_r}$

$\frac{\mu_R}{\mu_D} = \frac{\sqrt{1 \times 1}}{\sqrt{4 \times 1}} = \frac{1}{2}$

From (1), $\sin i > \frac{1}{2} \Rightarrow i > 30°$, i = 60°

15. Given below are two statements :-

Statement I : Davisson-Germer experiment establishes the wave nature of electrons.

Statement II : If electrons have wave nature, they can interfere and show diffraction.

In the light of the above statements choose the **correct answer** from the options given below:-

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (A)

Sol. In Davisson-Germer experiment the electrons exhibit diffraction there by proving that electrons have wave nature. Hence both statement are correct.

Sol. Both the options are correct by concept.

16. The ratio for the speed of the electron in the 3rd orbit of He^+ to the speed of the electron in the 3rd orbit of hydrogen atom will be :-

(A) 1 : 1 (B) 1 : 2

(C) 4 : 1 (D) 2 : 1

Official Ans. by NTA (D)

Sol. $v \propto \frac{Z}{n} \propto Z$ (n = constant)

$\Rightarrow \frac{v_{He^+}}{v_H} = \frac{Z_{He^+}}{Z_H} = \frac{2}{1}$

17. The photodiode is used to detect the optiocal signals. These diodes are preferably operated in reverse biased mode because.

(A) fractional change in majority carriers produce higher forward bias current

(B) fractional change in majority carriers produce higher reverse bias current

(C) fractional change in minority carriers produce higher forward bias current

(D) fractional change in minority carriers produce higher reverse bias current

Official Ans. by NTA (D)

Sol. Very small change in minority charge carriers produces high value of reverse bias current.

18. A signal of 100 THz frequency can be transmitted with maximum efficiency by :

(A) Coaxial cable

(B) Optical fibre

(C) Twisted pair of copper wires

(D) Water

Official Ans. by NTA (B)

Sol. Optical fibre frequency range is 1 THz to 1000 THz.

19. The difference of speed of light in the two media A and B ($v_A - v_B$) is 2.6×10^7 m/s. If the refractive index of medium B is 1.47, then the ratio of refractive index of medium B to medium A is : (Given : speed of light in vacuum $c = 3 \times 10^8\ ms^{-1}$)

(A) 1.303

(B) 1.318

(C) 1.13

(D) 0.12

Official Ans. by NTA (C)

Sol. $v = \frac{c}{\mu}$

$$\Rightarrow v_B = \frac{3\times10^8}{1.47} = 2.04\times10^8 = 20.4\times10^7\ m/s$$

$$\because v_A - v_B = 2.6\times10^7\ m/s$$

$$\therefore v_A = (20.4+2.6)\times10^7 = 23\times10^7\ m/s$$

$$\therefore \frac{\mu_B}{\mu_A} = \frac{v_A}{v_B} = \frac{23\times10^7}{20.4\times10^7} = 1.13$$

20. A teacher in his physics laboratory allotted an experiment to determine the resistance (G) of a galvanometer. Students took the observations for $\frac{1}{3}$ deflection in the galvanometer. Which of the below is **true** for measuring value of G?

(A) $\frac{1}{3}$ deflection method cannot be used for determining the resistance of the galvanometer.

(B) $\frac{1}{3}$ deflection method can be used and in this case the G equals to twice the value of shunt resistance(s).

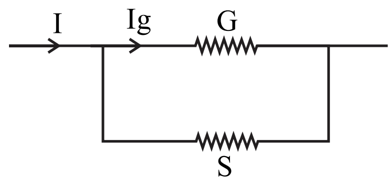
(C) $\frac{1}{3}$ deflection method can be used and in this case, the G equals to three times the value of shunt resistance(s)

(D) $\frac{1}{3}$ deflection method can be used and in this case the G value equals to the shunt resistance(s).

Official Ans. by NTA (B)

Sol. In galvanometer

$$\Rightarrow (I - I_g)S = I_g G$$



$$\frac{I_g}{I} = \frac{S}{S+G}$$

$$\Rightarrow \frac{1}{3} = \frac{S}{S+G} \Rightarrow S+G = 3S \Rightarrow G = 2S$$

SECTION-B

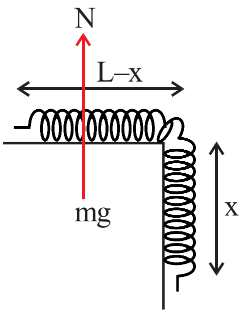
1. A uniform chain of 6 m length is placed on a table such that a part of its length is hanging over the edge of the table. The system is at rest. The co-efficient of static friction between the chain and the surface of the table is 0.5, the maximum length of the chain hanging from the table is __________m.

Official Ans. by NTA 2

Sol. Mass per unit length = λ

$$N = mg = \lambda(L - x)g$$

$$fs_{max} = \mu_s N$$



$$fs_{max} = (0.5)(\lambda)(L - x)g$$

And also $fs_{max} = m_x g$

$$0.5\lambda(L - x)g = \lambda xg$$

$$\frac{L - x}{2} = x$$

$$\frac{L}{2} = \frac{3x}{2} \Rightarrow x = \frac{L}{3} = \frac{6}{3} = 2m$$

2. A 0.5 kg block moving at a speed of 12 ms^{-1} compresses a spring through a distance 30 cm when its speed is halved. The spring constant of the spring will be ___________ Nm^{-1}.

Official Ans. by NTA 600

Sol. $U_i + K_i = U_f + K_f$

$$\Rightarrow 0 + \frac{1}{2}m(12)^2 = \frac{1}{2}K(0.3)^2 + \frac{1}{2}m(6)^2$$

$$\Rightarrow 0.5(12^2 - 6^2) = K(0.3)^2$$

K = 600 N/m

3. The velocity of upper layer of water in a river is 36 kmh^{-1}. Shearing stress between horizontal layers of water is 10^{-3} Nm^{-2}. Depth of the river is __________m. (Co-efficiency of viscosity of water is 10^{-2} Pa.s)

Official Ans. by NTA 100

Sol. $F = \eta A \frac{\Delta v_x}{\Delta y}$

$$\frac{F}{A} = \eta \frac{\Delta v_x}{\Delta y}$$

$$\Rightarrow 10^{-3} = 10^{-2} \times \frac{36 \times 1000}{h \times 3600}$$

$$\Rightarrow h = 10^{-2} \times \frac{36 \times 1000}{10^{-3} \times 3600} = 100 \text{ m}$$

4. A steam engine intakes 50g of steam at 100°C per minute and cools it down to 20°C. If latent heat of vaporization of steam is 540 cal g^{-1}, then the heat rejected by the steam engine per minute is ______ × 10^3 cal.

Official Ans. by NTA 31

Sol. Heat rejected = $mL_f + mS\Delta T$

= (50 × 540) + 50 (1) (100 – 20)

= 31000 Cal

= 31 × 10^3 Cal

5. The first overtone frequency of an open organ pipe is equal to the fundamental frequency of a closed organ pipe. If the length of the closed organ pipe is 20 cm. The length of the open organ pipe is _____ cm.

Official Ans. by NTA 80

Sol. $f_1 = \frac{2v}{2l_1}$

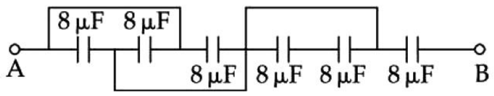
$$f_2 = \frac{v}{4l_2}$$

$f_1 = f_2$

$$= \frac{2v}{2l_1} = \frac{v}{4l_2}$$

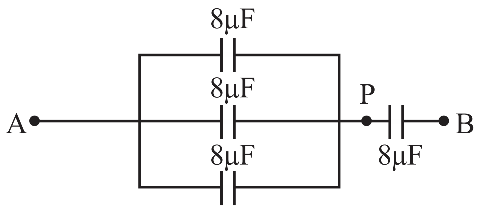
$l_1 = 4l_2 = 80$ cm

6. The equivalent capacitance between points A and B in below shown figure will be _____μF.



Official Ans. by NTA 6

Sol. Two capacitors are short circuited



Finally equivalent capacitance

$$=\frac{24\times 8}{24+8}=\frac{24\times 8}{32}=6\mu F$$

7. A resistor develops 300 J of thermal energy in 15s, when a current of 2A is passed through it. If the current increases to 3A, the energy developed in 10s is _________ J.

Official Ans. by NTA 450

Sol. $H = i^2Rt$

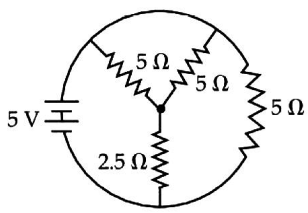
$300 = 2^2 \times R \times 15$

$$\Rightarrow R=\frac{300}{60}=5\Omega$$

Now, for $i = 3A$, $t = 10s$, $R = 5\Omega$

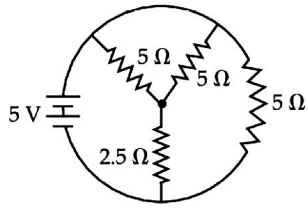
$H = 3^2 \times 5 \times 10 = 450$ J

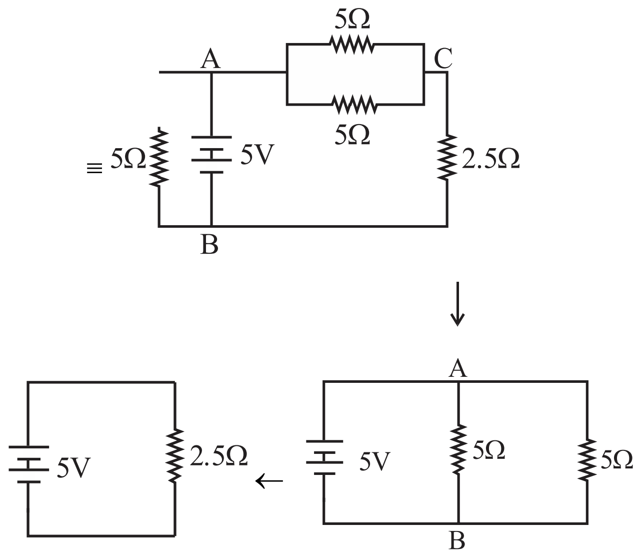
8. The total current supplied to the circuit as shown in figure by the 5V battery is __________A



Official Ans. by NTA 2

Sol.





Current supplied by 5V battery

$$=\frac{5V}{2.5\Omega}=2A$$

9. The current in a coil of self inductance 2.0 H is increasing according to $I = 2\sin(t^2)A$. The amount of energy spent during the period when current changes from 0 to 2A is _______ J.

Official Ans. by NTA 4

Sol. $I=2\sin(t^2)\Rightarrow dI=4t\sin(t^2)dt$

If $I=0\Rightarrow t=0$

and $I=2\Rightarrow 2=2\sin t^2$

$$\Rightarrow t=\sqrt{\frac{\pi}{2}}$$

$$E=\int LI\,dI$$

$$=\int 2\times 2\sin(t^2)\times 4t\cos(t^2)dt$$

$$=8\int_0^{\sqrt{\pi/2}} t\sin(2t^2)dt$$

$$=2\left[-\cos(2t^2)\right]_0^{\sqrt{\pi/2}}$$

$$=2\left[-\cos\pi+\cos 0\right]=4$$

10. A force on an object of mass 100g is $(10\hat{i}+5\hat{j})$ N. The position of that object at t = 2s is $(a\hat{i}+b\hat{j})$ m after starting from rest. The value of $\frac{a}{b}$ will be ________

Official Ans. by NTA 2

Sol. $\vec{F}=10\hat{i}+5\hat{j}$

m = 100 g = 0.1 kg

$$\vec{a}=\frac{\vec{F}}{m}=100\hat{i}+50\hat{j}$$

$$\vec{S}=\vec{u}t+\frac{1}{2}\vec{a}t^2=\frac{1}{2}\vec{a}t^2(\text{as } \vec{u}=0)$$

$$=\frac{1}{2}(100\hat{i}+50\hat{j})2^2$$

$$=200\hat{i}+100\hat{j}$$

$$=a\hat{i}+b\hat{j}$$

$$a=200,\ b=100$$

$$\therefore \frac{a}{b}=2$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Bonding in which of the following diatomic molecule(s) become(s) stronger, on the basis of MO Theory, by removal of an electron ?

(A) NO (B) N_2

(C) O_2 (D) C_2

(E) B_2

Choose the most appropriate answer from the options given below :-

(A) (A), (B), (C) only (B) (B), (C), (E) only

(C) (A), (C) only (D) (D) only

Official Ans. by NTA (C)

Sol. Bond strength $\propto$ Bond order

removal of electron from antibonding MO increases B.O.

NO & O_2 has valence e⁻ in π^*orbital.

2. Incorrect statement for Tyndall effect is :-

(A) The refractive indices of the dispersed phase and the dispersion medium differ greatly in magnitude.

(B) The diameter of the dispersed particles is much smaller than the wavelength of the light used.

(C) During projection of movies in the cinemas hall, Tyndall effect is noticed.

(D) It is used to distinguish a true solution from a colloidal solution.

Official Ans. by NTA (B)

Sol. The diameter of dispersed particle should be somewhat below or near the wavelength of light.

3. The pair, in which ions are isoelectronic with Al^{3+} is :-

(A) Br^- and Be^{2+} (B) Cl^- and Li^+

(C) S^{2-} and K^+ (D) O^{2-} and Mg^{2+}

Official Ans. by NTA (D)

Sol. Isoelectronic species have same no. of electrons Al^{+3}, O^{2-}, Mg^{+2} all have 10 electrons.

4. Leaching of gold with dilute aqueous solution of NaCN in presence of oxygen gives complex [A], which on reaction with zinc forms the elemental gold and another complex [B]. [A] and [B], respectively are :-

(A) $[Au(CN)_4]^-$ and $[Zn(CN)_2(OH)_2]^{2-}$

(B) $[Au(CN)_2]^-$ and $[Zn(OH)_4]^{2-}$

(C) $[Au(CN)_2]^-$ and $[Zn(CN)_4]^{2-}$

(D) $[Au(CN)_4]^{2-}$ and $[Zn(CN)_6]^{4-}$

Official Ans. by NTA (C)

Sol. $Au + NaCN \rightarrow Na[Au(CN)_2]$

$Zn + Na[Au(CN)_2] \rightarrow Na_2[Zn(CN)_4] + Au$

5. Number of electron deficient molecules among the following

PH_3, B_2H_6, CCl_4, NH_3, LiH and BCl_3 is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Sol. Electron deficient species have less than 8 electrons (or two electrons for H) in their valence (incomplete octet)

B_2H_6, BCl_3 have incomplete octet.

6. Which one of the following alkaline earth metal ions has the highest ionic mobility in its aqueous solution?

(A) Be^{2+} (B) Mg^{2+}

(C) Ca^{2+} (D) Sr^{2+}

Official Ans. by NTA (D)

Sol. Highest ionic mobility corresponds to lowest extent of hydration and highest size of gaseous ion.

Hence Sr^{2+} has the highest ionic mobility in its aqueous solution

7. White precipitate of AgCl dissolves in aqueous ammonia solution due to formation of :

(A) $[Ag(NH_3)_4]Cl_2$ (B) $[Ag(Cl)_2(NH_3)_2]$

(C) $[Ag(NH_3)_2]Cl$ (D) $[Ag(NH_3)Cl]Cl$

Official Ans. by NTA (C)

Sol. $AgCl + 2NH_3 \rightarrow \underset{\text{soluble}}{[Ag(NH_3)_2]^+Cl^-}$

8. Cerium (IV) has a noble gas configuration. Which of the following is correct statement about it?

(A) It will not prefer to undergo redox reactions.

(B) It will prefer to gain electron and act as an oxidizing agent

(C) It will prefer to give away an electron and behave as reducing agent

(D) It acts as both, oxidizing and reducing agent.

Official Ans. by NTA (B)

Sol. Cerium exists in two different oxidation state +3, +4

$Ce^{+4} + e^- \rightarrow Ce^{3+}$ $E^0 = +1.61$ V

$Ce^{+3} + 3e^- \rightarrow Ce$ $E^0 = -2.336$ V

It shows Ce^{+4} acts as a strong oxidising agent & accepts electron.

9. Among the following, which is the strongest oxidizing agent ?

(A) Mn^{3+} (B) Fe^{3+}

(C) Ti^{3+} (D) Cr^{3+}

Official Ans. by NTA (A)

Sol. Strongest oxidising agent have highest reduction potential value

$E^0_{Mn^{+3}/Mn^{+2}} = 1.51V$ (highest)

10. The eutrophication of water body results in :

(A) loss of Biodiversity

(B) breakdown of organic matter

(C) increase in biodiversity

(D) decrease in BOD.

Official Ans. by NTA (A)

Sol. Eutrophication of water body results in loss of Biodiversity.

11. Phenol on reaction with dilute nitric acid, gives two products. Which method will be most effective for large scale separation ?

(A) Chromatographic separation

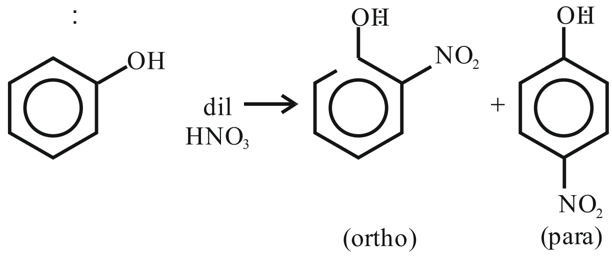
(B) Fractional Crystallisation

(C) Steam distillation

(D) Sublimation

Official Ans. by NTA (C)

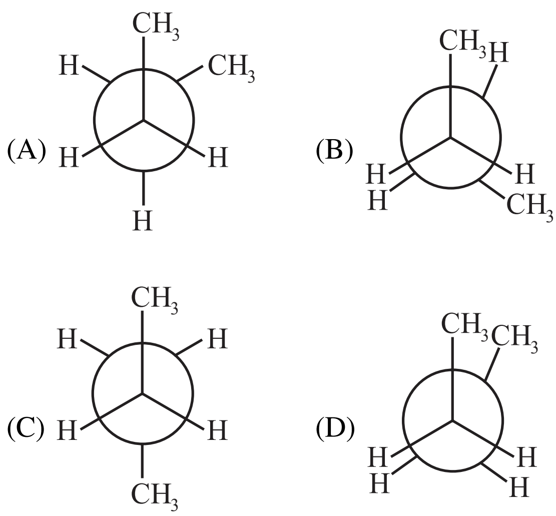
Sol.



Para product has higher boiling point than ortho as intermolecular H-bond is possible in former, where as intramolecular H-bond is possible in ortho product.

Steam distillation can separate them as ortho product is steam volatile.

12. In the following structures, which one is having staggered conformation with maximum dihedral angle?

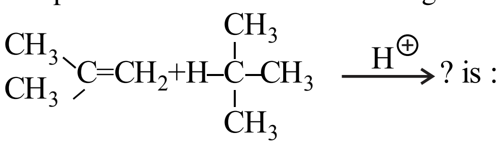


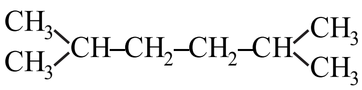
Official Ans. by NTA (C)

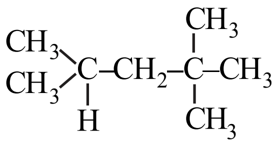
Sol. Dihedral angle : It's the angle b/w 2 specified groups ($-CH_3$ here)

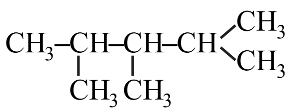
Staggered form is Given in option (C) & the angle is 180^0

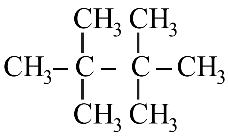
13. The products formed in the following reaction.



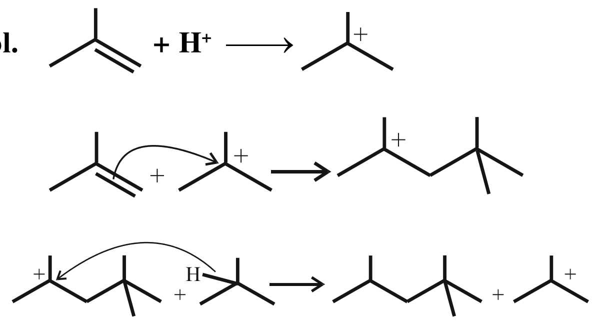
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. 

14. The IUPAC name of ethylidene chloride is :-

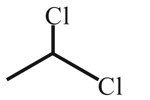
(A) 1-Chloroethene

(B) 1-Chloroethyne

(C) 1,2-Dichloroethane

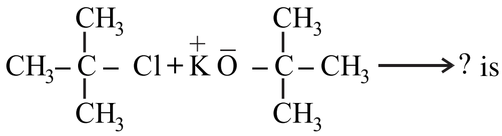
(D) 1,1-Dichloroethane

Official Ans. by NTA (D)

Sol. 

"1, 1-Dichloroethane is Ethylidene chloride"

15. The major product in the reaction



(A) t-Butyl ethyl ether

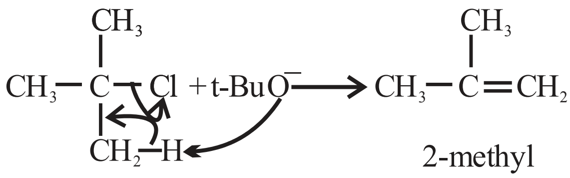
(B) 2,2-Dimethyl butane

(C) 2-Methyl pent-1-ene

(D) 2-Methyl prop-1-ene

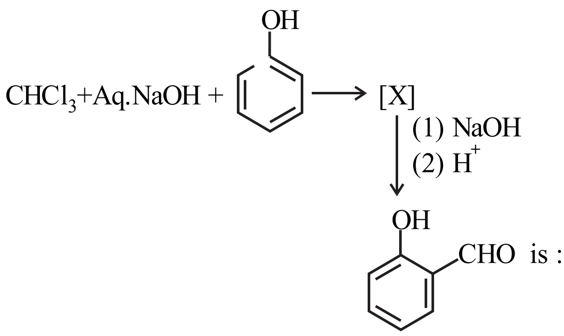
Official Ans. by NTA (D)

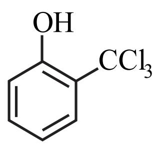
Sol. We have been given a bulky base, hence elimination will take place & not substitution.

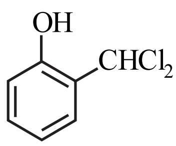


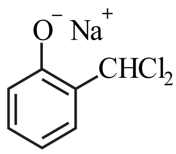
2-methyl prop-1-ene

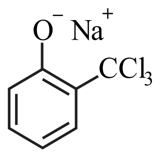
16. The intermediate X, in the reaction



(A) 

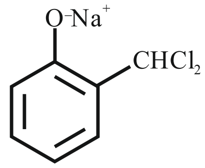
(B) 

(C) 

(D) 

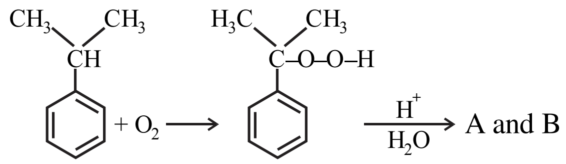
Official Ans. by NTA (C)

Sol. It's a classic Reimer-Tiemann reaction.

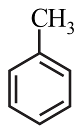


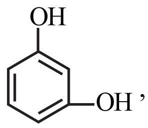
Will be the intermediate formed.

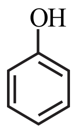
17. In the following reaction :

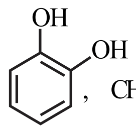


The compounds A and B respectively are :-

(A)  , CH_3COOH

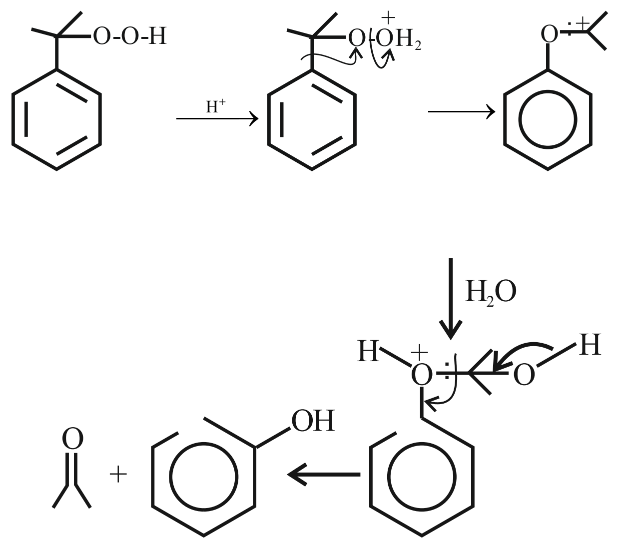
(B)  , CH_3COOH

(C)  , CH_3COCH_3

(D)  , CH_3COCH_3

Official Ans. by NTA (C)

Sol. Given reaction is cumene-Peroxide method for the preparation of phenol.
In this reaction



Acetone Phenol

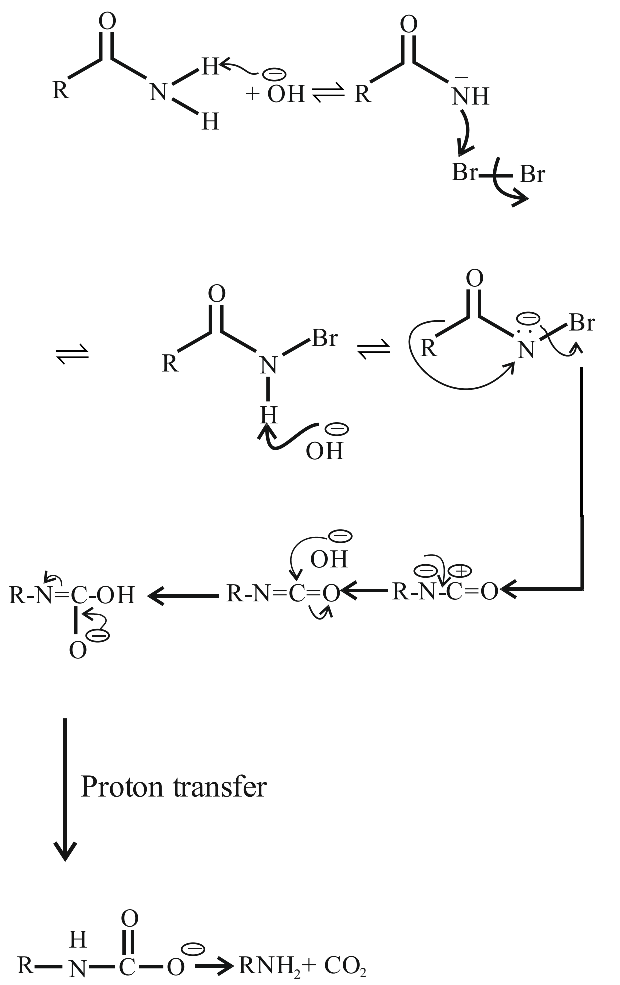
18. The reaction of $\mathrm{R{-}\underset{\underset{O}{\|}}{C}{-}NH_2}$ with bromine and KOH gives RNH_2 as the end product. Which one of the following is the intermediate product formed in this reaction ?

(A) $\mathrm{R{-}\underset{\underset{O}{\|}}{C}{-}NH{-}Br}$ (B) R–NH–Br

(C) R–N=C=O (D) $\mathrm{R{-}\underset{\underset{O}{\|}}{C}{-}NBr_2}$

Official Ans. by NTA (C)

Sol. The given reaction is Hoffmann-Bromide degradation method.



19. Using very little soap while washing clothes, does not serve the purpose of cleaning of clothes because

(A) soap particles remain floating in water as ions

(B) the hydrophobic part of soap is not able to take away grease

(C) the micelles are not formed due to concentration of soap, below its CMC value

(D) colloidal structure of soap in water is completely disturbed.

Official Ans. by NTA (C)

Sol. Micelle formation only takes place above CMC.

20. Which one of the following is an example of artificial sweetner ?

(A) Bithional (B) Alitame

(C) Salvarsan (D) Lactose

Official Ans. by NTA (B)

Sol. Alitame is a second generation dipeptide sweetner that is 200 times sweeter than sucrose.

SECTION-B

1. The number of N atoms is 681 g of $C_7H_5N_3O_6$ is $x \times 10^{21}$. The value of x is ______ ($N_A = 6.02 \times 10^{23}\ mol^{-1}$) (Nearest Integer)

Official Ans. by NTA (5418)

Sol. M.M. of $C_7H_5N_3O_6$ is 84 + 5 + 42 + 96 = 227

$$n_{C_7H_5N_3O_6} = \frac{681}{227} = 3$$

$$n_N = \frac{681}{227} \times 3 = 9 \text{ mol}$$

no. of N atoms = 9 × 6.02×10^{23}

$$= 5418 \times 10^{21}$$

$\therefore$ The answer is 5418.

2. The distance between Na^+ and Cl^- ions in solid NaCl of density 43.1 g cm^{-3} is ________ × 10^{-10}m. (Nearest Integer)

(Given : $N_A = 6.02 \times 10^{23}\ mol^{-1}$)

Official Ans. by NTA (1)

Sol. Unit cell formula – Na_4Cl_4

Mass per unit cell = $\dfrac{Z \times M.M.}{N_A}$g

$$= \frac{4 \times 58.5}{N_A} g$$

$$d_{unit\ cell} = \frac{m}{V} = \frac{m}{a^3}$$

$$\Rightarrow \frac{4 \times 58.5}{N_A \cdot a^3} = 43.1$$

$$\Rightarrow a^3 = 9.02 \times 10^{-24} cm^3$$

$$\Rightarrow a = 2.08 \times 10^{-8} cm$$

$$\Rightarrow a = 2.08 \times 10^{-10} m$$

Also $a = 2\left(r_{Na^+} + r_{Cl^-}\right)$

$$\Rightarrow r_{Na^+} + r_{Cl^-} = 1.04 \times 10^{-10} m$$

$\therefore$ The answer is 1

3. The longest wavelength of light that can be used for the ionisation of lithium atom (Li) in its ground state is x × 10^{-8} m. The value of x is ____________. (Nearest Integer)

(Given : Energy of the electron in the first shell of the hydrogen atom is -2.2×10^{-18} J; $h = 6.63 \times 10^{-34}$ Js and $c = 3 \times 10^8\ ms^{-1}$)

Official Ans. by NTA (4)

Sol. We can not calculate I.E. of lithium atom.

4. The standard entropy change for the reaction $4Fe(s) + 3O_2(g) \rightarrow 2Fe_2O_3(s)$ is $-550\ JK^{-1}$ at 298 K.

[Given : The standard enthalpy change for the reaction is $-165\ kJ\ mol^{-1}$]. The temperature in K at which the reaction attains equilibrium is _________. (Nearest Integer)

Official Ans. by NTA (300)

Sol. $\Delta G = \Delta H - T\Delta S = 0$ at equilibrium

$\Rightarrow -165\times10^3 - T\times(-505) = 0$

$\Rightarrow T = 300K$

The answer is 300

5. 1 L aqueous solution of H_2SO_4 contains 0.02 m mol H_2SO_4. 50% of this solution is diluted with deionized water to give 1 L solution (A). In solution (A), 0.01 m mol of H_2SO_4 are added. Total m mols of H_2SO_4 in the final solution is ___________ $\times$ 10^3 m mols.

Official Ans. by NTA (0)

Sol. $n_{H_2SO_4}$ in Sol^n A = 50% of original solution

$= 0.01$ m mol.

$n_{H_2SO_4}$ in Final solution $= 0.01 + 0.01$

$= 0.02$ mmol

$= 0.00002 \times 10^3$ mmol

The answer 0

6. The standard free energy change ($\Delta G°$) for 50% dissociation of N_2O_4 into NO_2 at 27°C and 1 atm pressure is –x J mol^{-1}. The value of x is _________. (Nearest Integer)

[Given : R = 8.31 J K^{-1} mol^{-1}, log 1.33 = 0.1239 ln 10 = 2.3]

Official Ans. by NTA (710)

Sol.

	N_2O_4	$\rightleftharpoons$	$2NO_2$
t = 0	1 mol		
t = t	(1-0.5) mol		0.5×2 mol
	= 0.5 mol		1 mol

$$k_P = \frac{\left(\frac{1}{1.5}\times1\right)^2}{\left(\frac{0.5}{1.5}\times1\right)} = \frac{1}{0.75} = \frac{100}{75}$$

$= 1.33$

$\Delta G^0 = -RT\ell n k_P$

$= -8.31\times300\times\ell n(1.33) = -710.45$ J / mol

$= -710$ J/mol.

7. In a cell, the following reactions take place

$Fe^{2+} \rightarrow Fe^{3+} e^-$ $\qquad E^o_{Fe^{3+}/Fe^{2+}} = 0.77$ V

$2I^- \rightarrow I_2 + 2e^-$ $\qquad E^o_{I_2/I^-} = 0.54$ V

The standard electrode potential for the spontaneous reaction in the cell is x × 10^{-2}V 298 K. The value of x is __________ (Nearest Integer)

Official Ans. by NTA (23)

Sol. $\underset{\text{Cathode}}{Fe^{+3}} + \underset{\text{anode}}{I^-} \longrightarrow I_2 + Fe^{+2}$

$E^0_{Cell} = E^0_{cathode} - E^0_{anode}$

=0.77–0.54

=0.23

$= 23\times10^{-2}$ V

8. For a given chemical reaction

$$\gamma_1A + \gamma_2B \rightarrow \gamma_3C + \gamma_4D$$

Concentration of C changes from 10 mmol dm^{-3} to 20 mmol dm^{-3} in 10 seconds. Rate of appearance of D is 1.5 times the rate of disappearance of B which is twice the rate of disappearance A. The rate of appearance of D has been experimentally determined to be 9 mmol dm^{-3} s^{-1}. Therefore the rate of reaction is ________mmol dm^{-3} s^{-1}. (Nearest Integer)

Official Ans. by NTA (1)

Sol. $\gamma_1A + \gamma_2B \longrightarrow \gamma_3C + \gamma_4D$

Given : $+\frac{d[D]}{dt} = \frac{-3}{2}\frac{d[B]}{dt}$

$\Rightarrow \frac{-1}{2}\frac{d[B]}{dt} = \frac{+1}{3}\frac{d[D]}{dt}$

$$-\frac{d[B]}{dt}=-2\frac{d[A]}{dt}\Rightarrow-\frac{1}{2}\frac{d[B]}{dt}=\frac{-d(A)}{dt}$$

$$+\frac{d[B]}{dt}=9\text{ mmol dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{20-10}{10}=1\text{ mmol dm}^{-3}\text{s}^{-1}$$

$$\frac{+d[C]}{dt}=\frac{1}{9}\times\frac{+d[D]}{dt}$$

$$1A+2B\longrightarrow\frac{1}{3}C+3D$$

$$\Rightarrow 3A+6B\longrightarrow C+9D$$

Rate of reaction = $\frac{+d[C]}{dt}=1\text{ mmol dm}^{-3}\text{ s}^{-1}$

9. If $[Cu(H_2O)_4]^{2+}$ absorbs a light of wavelength 600 nm for d–d transition, then the value of octahedral crystal field splitting energy for $[Cu(H_2O)_6]^{2+}$ will be___________$\times 10^{-21}$ J. (Nearest Integer)

(Given : $h = 6.63 \times 10^{-34}$Js

and $c = 3.08 \times 10^{8}$ ms^{-1})

Official Ans. by NTA (746)

Sol. $$\Delta_t=\frac{hc}{\lambda}=\frac{6.63\times10^{-34}\times3.08\times10^{8}}{600\times10^{-9}}$$

$$=\frac{6.63\times3.08\times10^{-17}}{600}$$

$$=0.034034\times10^{-17}$$

$$=340.34\times10^{-21}\text{ J}$$

$$\Delta_0=\frac{9}{4}\Delta_t$$

$$=\frac{9}{4}\times340.34\times10^{-21}$$

$$=765.765\times10^{-21}\text{ J}$$

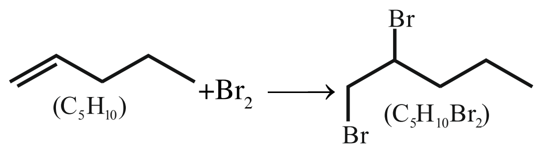
$$\approx766\times10^{-21}\text{ J}$$

Answer = 766

10. Number of grams of bromine that will completely react with 5.0g of pent-1-ene is _______ $\times$ 10^{-2}g. (Atomic mass of Br = 80 g/mol) [Nearest Integer)

Official Ans. by NTA (1143)

Sol.



moles of Br_2 = moles of C_5H_{10}

$$\Rightarrow\frac{w}{160}=\frac{5}{70}$$

$$\Rightarrow w=\frac{5\times160}{70}g$$

$$=11.428\text{ g}$$

$$=1142.8\times10^{-2}\text{ g}\approx1143\times10^{-2}\text{ g}$$

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Saturday 25th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let a circle C touch the lines $L_1 : 4x - 3y + K_1 = 0$ and $L_2 : 4x - 3y + K_2 = 0$, $K_1, K_2 \in R$. If a line passing through the centre of the circle C intersects L_1 at (–1, 2) and L_2 at (3, – 6), then the equation of the circle C is

(A) $(x - 1)^2 + (y - 2)^2 = 4$

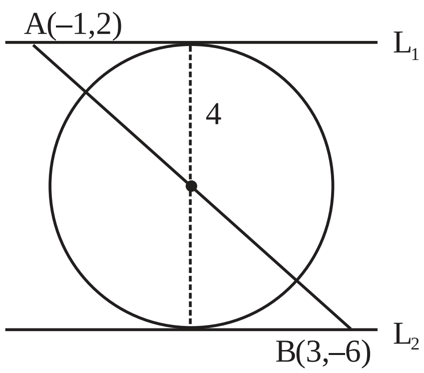
(B) $(x + 1)^2 + (y - 2)^2 = 4$

(C) $(x - 1)^2 + (y + 2)^2 = 16$

(D) $(x - 1)^2 + (y - 2)^2 = 16$

Official Ans. by NTA (C)

Sol.



$L_1 : 4x - 3y + K_1 = 0$

$L_2 : 4x - 3y + K_2 = 0$

now

$-4 - 6 + K_1 = 0 \Rightarrow K_1 = 10$

$12 + 18 + K_2 = 0 \Rightarrow K_2 = -30$

$\Rightarrow$ Tangent to the circle are

$4x - 3y + 10 = 0$

$4x - 3y - 30 = 0$

Length of diameter $2r = \dfrac{|10+30|}{5} = 8$

$\Rightarrow r = 4$

Now centre is mid point of A & B

$x = 1, y = -2$

Equation of circle

$(x - 1)^2 + (y + 2)^2 = 16$ **Ans.**

2. The value of $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ is equal to

(A) $\frac{\pi^2}{4}$ (B) $\frac{\pi^2}{2}$

(C) $\frac{\pi}{4}$ (D) $\frac{\pi}{2}$

Official Ans. by NTA (C)

Sol. $\int_0^{\pi} \frac{e^{\cos x}\sin x}{(1+\cos^2 x)(e^{\cos x}+e^{-\cos x})}dx$ ....(1)

Use King's property

$I = \int_0^{\pi} \frac{e^{-\cos x}\sin x}{(1+\cos^2 x)(e^{-\cos x}+e^{\cos x})}dx$ ....(2)

On adding equation (1) and (2), we get

$2I = \int_0^{\pi} \frac{\sin x}{1+\cos^2 x}dx = 2\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}dx$

On putting cos x = t, we get

$I = \int_0^{1} \frac{dt}{1+t^2} = \left(\tan^{-1} t\right)_0^1 = \frac{\pi}{4}$

3. Let a, b and c be the length of sides of a triangle ABC such that $\frac{a+b}{7} = \frac{b+c}{8} = \frac{c+a}{9}$. If r and R are the radius of incircle and radius of circumcircle of the triangle ABC, respectively, then the value of $\frac{R}{r}$ is equal to

(A) $\frac{5}{2}$ (B) 2

(C) $\frac{3}{2}$ (D) 1

Official Ans. by NTA (A)

Sol. $\frac{a+b}{7}=\frac{b+c}{8}=\frac{c+a}{9}=\lambda$

$a + b = 7\lambda,\ b + c = 8\lambda,\ a + c = 9\lambda$

$\Rightarrow\ a + b + c = 12\lambda$

Now $a = 4\lambda,\ b = 3\lambda,\ c = 5\lambda$

$\because\ c^2 = b^2 + a^2$

$\angle C = 90°$

$$\Delta = \frac{1}{2}ab\sin C = \frac{1}{2}ab$$

$$\frac{R}{r} = \frac{c}{2\sin C}\times\frac{s}{\Delta} = \frac{c}{2}\times\frac{6\lambda}{\frac{1}{2}ab} = \frac{c}{ab}\times 6\lambda = \frac{5}{2}$$

4. Let $f : N \to R$ be a function such that $f(x+y)=2f(x)f(y)$ for natural numbers x and y. If $f(1) = 2$, then the value of α for which

$$\sum_{k=1}^{10} f(\alpha+k) = \frac{512}{3}(2^{20}-1)$$

holds, is

(A) 2 (B) 3

(C) 4 (D) 6

Official Ans. by NTA (C)

Sol. $f : N \to R,\ f(x + y) = 2f(x)f(y)$ ....(1)

$f(1) = 2,$

$$\sum_{k=1}^{10} f(\alpha+k) = 2f(\alpha)\sum_{k=1}^{10} f(k)$$

$$= 2f(\alpha)\left(f(1)+f(2)+.....+f(10)\right) \quad ....(2)$$

From (1)

$f(2) = 2f^2(1) = 2^3$

$f(3) = 2f(2)f(1) = 2^5$

$\vdots \qquad \vdots$

$f(10) = 2^9 f^{10}(1) = 2^{19}$

$f(\alpha) = 2^{2\alpha-1}$; $\alpha \in N$

from (2)

$$\sum_{k=1}^{10} f(\alpha+k) = 2\left(2^{2\alpha-1}\right)\left(2+2^3+2^5+....+2^{19}\right)$$

$$\frac{512}{3}(2^{20}-1) = 2^{2\alpha}\left(2\frac{(2^{20}-1)}{3}\right)$$

Hence $\alpha = 4$

5. Let A be a 3 × 3 real matrix such that

$$A\begin{pmatrix}1\\1\\0\end{pmatrix}=\begin{pmatrix}1\\1\\0\end{pmatrix}; A\begin{pmatrix}1\\0\\1\end{pmatrix}=\begin{pmatrix}-1\\0\\1\end{pmatrix} \text{ and } A\begin{pmatrix}0\\0\\1\end{pmatrix}=\begin{pmatrix}1\\1\\2\end{pmatrix}.$$

If $X = (x_1, x_2, x_3)^T$ and I is an identity matrix of order 3, then the system $(A - 2I)X = \begin{pmatrix}4\\1\\1\end{pmatrix}$ has

(A) no solution

(B) infinitely many solutions

(C) unique solution

(D) exactly two solutions

Official Ans. by NTA (B)

Sol. $A = \begin{bmatrix}a_1 & b_1 & c_1\\ a_2 & b_2 & c_2\\ a_3 & b_3 & c_3\end{bmatrix}$

$$A\begin{bmatrix}0\\0\\1\end{bmatrix}=\begin{bmatrix}c_1\\c_2\\c_3\end{bmatrix}=\begin{bmatrix}1\\1\\2\end{bmatrix}$$

$\Rightarrow\ c_1 = 1,\ c_2 = 1,\ c_3 = 2$

$$A\begin{bmatrix}1\\0\\1\end{bmatrix}=\begin{bmatrix}c_1+a_1\\c_2+a_2\\c_3+a_3\end{bmatrix}=\begin{bmatrix}-1\\0\\1\end{bmatrix}$$

$\Rightarrow\ a_1 = -2,\ a_2 = -1,\ a_3 = -1$

$$A\begin{bmatrix}1\\1\\0\end{bmatrix}=\begin{bmatrix}a_1+b_1\\a_2+b_2\\a_3+b_3\end{bmatrix}=\begin{bmatrix}1\\1\\0\end{bmatrix}$$

$\Rightarrow\ b_1 = 3,\ b_2 = 2,\ b_3 = 1$

$$\Rightarrow\ A = \begin{bmatrix}-2 & 3 & 1\\ -1 & 2 & 1\\ -1 & 1 & 2\end{bmatrix}$$

$$\Rightarrow\ A - 2I = \begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}$$

$|A - 2I| = 0$

Now, $\begin{bmatrix}-4 & 3 & 1\\ -1 & 0 & 1\\ -1 & 1 & 0\end{bmatrix}\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}=\begin{bmatrix}4\\1\\1\end{bmatrix}$

$-4x_1 + 3x_2 + x_3 = 4$ ....(1)

$-x_1 + x_3 = 1$ .....(2)

$-x_1 + x_2 = 1$ ....(3)

$(1) - [(2) + 3(3)]$

$0 = 0 \Rightarrow$ infinite solutions

6. Let $f : R \to R$ be defined as $f(x) = x^3 + x - 5$. If $g(x)$ is a function such that $f(g(x)) = x$, $\forall x \in R$, then $g'(63)$ is equal to ______.

(A) $\frac{1}{49}$ (B) $\frac{3}{49}$

(C) $\frac{43}{49}$ (D) $\frac{91}{49}$

Official Ans. by NTA (A)

Sol. $f(x) = x^3 + x - 5$

$\Rightarrow f'(x) = 3x^2 + 1 \Rightarrow$ increasing function

$\Rightarrow$ invertible

$\Rightarrow$ $g(x)$ is inverse of $f(x)$

$\Rightarrow g(f(x)) = x$

$\Rightarrow g'(f(x))f'(x) = 1$

$f(x) = 63$

$\Rightarrow x^3 + x - 5 = 63$

$\Rightarrow x = 4$

put $x = 4$

$g'(f(4))f'(4) = 1$

$g'(63) \times 49 = 1 \quad \{f'(4) = 49\}$

$g'(63) = \frac{1}{49}$

7. Consider the following two propositions:

P1 : $\sim(p \to \sim q)$

P2 : $(p \wedge \sim q) \wedge ((\sim p) \vee q)$

If the proposition $p \to ((\sim p) \vee q)$ is evaluated as FALSE, then:

(A) P1 is TRUE and P2 is FALSE

(B) P1 is FALSE and P2 is TRUE

(C) Both P1 and P2 are FALSE

(D) Both P1 and P2 are TRUE

Official Ans. by NTA (C)

Sol.

p	q	$\sim p$	$\sim q$	$\sim p \vee q$	$p \to (\sim p \vee q)$	$p \to \sim q$	$\sim(p \to \sim q)$	$p \wedge \sim q$	p_2
T	T	F	F	T	T	F	T	F	F
T	F	F	T	F	F	T	F	T	F
F	T	T	F	T	T	T	F	F	F
F	F	T	T	T	T	T	F	F	F

$p \to (\sim p \vee q)$ is F when p is true q is false

From table

P1 & P2 both are false

8. If $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \dots \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$, then the remainder when K is divided by 6 is

(A) 1 (B) 2

(C) 3 (D) 5

Official Ans. by NTA (D)

Sol. $\frac{1}{2\cdot 3^{10}} + \frac{1}{2^2\cdot 3^9} + \frac{1}{2^3\cdot 3^8} + \dots + \frac{1}{2^{10}\cdot 3} = \frac{K}{2^{10}\cdot 3^{10}}$

$K = 2^9 + 2^8 \cdot 3 + 2^7 \cdot 3^2 + \dots + 3^9$

$$= \frac{2^9\left(\left(\frac{3}{2}\right)^{10} - 1\right)}{\frac{3}{2} - 1} = 3^{10} - 2^{10}$$

Now, $3^{10} - 2^{10} = (3^5 - 2^5)(3^5 + 2^5)$

$= (211)(275)$

$= (35 \times 6 + 1)(45 \times 6 + 5)$

$= 6\lambda + 5$

Remainder is 5.

9. Let $f(x)$ be a polynomial function such that $f(x) + f'(x) + f''(x) = x^5 + 64$. Then, the value of $\lim_{x\to 1} \frac{f(x)}{x-1}$

(A) – 15 (B) – 60

(C) 60 (D) 15

Official Ans. by NTA (A)

Sol. $\underset{x\to 1}{\text{Lt}} \frac{f(x)}{x-1} = f'(1) (\text{and } f(1) = 0)$

$f(x) + f'(x) + t''(x) = x^5 + 64$

$f'(x) + f''(x) + f'''(x) = 5x^4$

$f''(x) + f'''(x) + f^{iv}(x) = 20x^3$

$f'''(x) + f^{iv}(x) + f^{v}(x) = 60x^2$

$\therefore f^{v}(x) - f''(x) = 60x^2 - 20x^3$

$\Rightarrow 120 - f''(1) = 40 \Rightarrow f''(1) = 80$

Also $f(1) + f'(1) + f''(1) = 65 \Rightarrow f'(1) = -15$. **Ans.**

10. Let E_1 and E_2 be two events such that the conditional probabilities $P(E_1|E_2)=\frac{1}{2}$, $P(E_2|E_1)=\frac{3}{4}$ and $P(E_1\cap E_2)=\frac{1}{8}$. Then:

(A) $P(E_1\cap E_2)=P(E_1)\cdot P(E_2)$

(B) $P(E'_1\cap E'_2)=P(E'_1)\cdot P(E_2)$

(C) $P(E_1\cap E'_2)=P(E_1)\cdot P(E_2)$

(D) $P(E'_1\cap E_2)=P(E_1)\cdot P(E_2)$

Official Ans. by NTA (C)

Sol.

(A) $P(E_1)\cdot P(E_2)=\frac{1}{6}\cdot\frac{1}{4}=\frac{1}{24}\neq P(E_1\cap E_2)$

(B) $P(E'_1\cap E'_2)=1-P(E_1\cup E_2)$

$=1-\left(P(E_1)+P(E_2)-P(E_1\cap E_2)\right)$

$=1-\left(\frac{1}{6}+\frac{1}{4}-\frac{1}{8}\right)=\frac{17}{24}$

$P(E'_1)P(E_2)=\frac{5}{6}\times\frac{1}{4}=\frac{5}{24}$

(C) $P(E_1\cap E'_2)=P(E_1)-P(E_1\cap E_2)=\frac{1}{6}-\frac{1}{8}=\frac{1}{24}$

(D) $P(E'_1\cap E_2)=P(E_2)-P(E_1\cap E_2)=\frac{1}{4}-\frac{1}{8}=\frac{1}{8}$

11. Let $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$. If M and N are two matrices given by $M=\sum_{k=1}^{10}A^{2k}$ and $N=\sum_{k=1}^{10}A^{2k-1}$ then MN^2 is

(A) a non-identity symmetric matrix

(B) a skew-symmetric matrix

(C) neither symmetric nor skew-symmetric matrix

(D) an identify matrix

Official Ans. by NTA (A)

Sol. $A=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}$

$A^2=\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}\begin{bmatrix}0 & -2\\ 2 & 0\end{bmatrix}=\begin{bmatrix}-4 & 0\\ 0 & -4\end{bmatrix}=-4I$

$A^3=-4A$

$A^4=(-4I)(-4I)=(-4)^2I$

$A^5=(-4)^2A,\quad A^6=(-4)^3I$

$M=\sum_{k=1}^{10}A^{2k}=A^2+A^4+\ldots+A^{20}$

$=[-4+(-4)^2+(-4)^3+\ldots+(-4)^{20}]I$

$=-4\lambda I$

$\Rightarrow$ M is symmetric matrix

$N=\sum_{k=1}^{10}A^{2k-1}=A+A^3+\ldots+A^{19}$

$=A[1+(-4)+(-4)^2+\ldots+(-4)^9]$

$=\lambda A\Rightarrow$ skew symmetric

$\Rightarrow$ N^2 is symmetric matrix

$\Rightarrow$ MN^2 is non identity symmetric matrix

12. Let $g:(0,\infty)\to R$ be a differentiable function such that

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c,$$

for all $x>0$, where c is an arbitrary constant. Then.

(A) g is decreasing in $\left(0,\frac{\pi}{4}\right)$

(B) g' is increasing in $\left(0,\frac{\pi}{4}\right)$

(C) g + g' is increasing in $\left(0,\frac{\pi}{2}\right)$

(D) g – g' is increasing in $\left(0,\frac{\pi}{2}\right)$

Official Ans. by NTA (D)

Sol.

$$\int\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)dx=\frac{xg(x)}{e^x+1}+c$$

On differentiating both sides w.r.t. x, we get

$$\left(\frac{x(\cos x-\sin x)}{e^x+1}+\frac{g(x)(e^x+1-xe^x)}{(e^x+1)^2}\right)$$

$$=\frac{(e^x+1)\big(g(x)+xg'(x)\big)-e^x\cdot x\cdot g(x)}{\left(e^x+1\right)^2}$$

$(e^x + 1)x(\cos x - \sin x) + g(x)(e^x + 1 - xe^x)$

$= (e^x+1)\big(g(x)+xg'(x)\big)-e^x\cdot x\cdot g(x)$

$\Rightarrow g'(x) = \cos x - \sin x$

$\Rightarrow g(x) = \sin x + \cos x + C$

$g(x)$ is increasing in $(0, \pi/4)$

$g''(x) = -\sin x - \cos x < 0$

$\Rightarrow g'(x)$ is decreasing function

let $h(x) = g(x) + g'(x) = 2\cos x + C$

$\Rightarrow h'(x) = g'(x) + g''(x) = -2\sin x < 0$

$\Rightarrow$ h is decreasing

let $\phi(x) = g(x) - g'(x) = 2\sin x + C$

$\Rightarrow \phi'(x) = g'(x) - g''(x) = 2\cos x > 0$

$\Rightarrow \phi$ is increasing

Hence option D is correct.

13. Let $f : R \to R$ and $g : R \to R$ be two functions defined by $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$ and $g(x)=\dfrac{1-2e^{2x}}{e^x}$. Then, for which of the following range of α, the inequality

$f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$ holds?

(A) (2, 3) (B) (–2, – 1)

(C) (1, 2) (D) (–1, 1)

Official Ans. by NTA (A)

Sol. $f(x) = \log_e(x^2 + 1) - e^{-x} + 1$

$\Rightarrow f'(x) = \dfrac{2x}{x^2+1}+e^{-x} > 0 \quad \forall x \in R$

$\Rightarrow$ f is strictly increasing

$g(x) = \dfrac{1-2e^{2x}}{e^x} = e^{-x} - 2e^x$

$\Rightarrow g'(x) = -\left(2e^x+e^{-x}\right)<0 \quad \forall x \in R$

$\Rightarrow$ g is decreasing

Now $f\left(g\left(\frac{(\alpha-1)^2}{3}\right)\right)>f\left(g\left(\alpha-\frac{5}{3}\right)\right)$

$\Rightarrow g\left(\frac{(\alpha-1)^2}{3}\right)>g\left(\alpha-\frac{5}{3}\right)$

$\Rightarrow \frac{(\alpha-1)^2}{3}<\alpha-\frac{5}{3}$

$\Rightarrow \alpha^2 - 5\alpha + 6 < 0$

$\Rightarrow (\alpha - 2)(\alpha - 3) < 0$

$\Rightarrow \alpha \in (2, 3)$

14. Let $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ $a_i > 0$, $i = 1, 2, 3$ be a vector which makes equal angles with the coordinates axes OX, OY and OZ. Also, let the projection of $\vec{a}$ on the vector $3\hat{i}+4\hat{j}$ be 7. Let $\vec{b}$ be a vector obtained by rotating $\vec{a}$ with 90°. If $\vec{a}, \vec{b}$ and x-axis are coplanar, then projection of a vector $\vec{b}$ on $3\hat{i}+4\hat{j}$ is equal to

(A) $\sqrt{7}$ (B) $\sqrt{2}$

(C) 2 (D) 7

Official Ans. by NTA (B)

Sol. $\vec{a}=a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$

$\vec{a}=\lambda\left(\frac{1}{\sqrt{3}}\hat{i}+\frac{1}{\sqrt{3}}\hat{j}+\frac{1}{\sqrt{3}}\hat{k}\right) = \frac{\lambda}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)$

Now projection of $\vec{a}$ on $\vec{b}=7$

$\Rightarrow \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|} = 7$

$\dfrac{\frac{\lambda}{\sqrt{3}}\left(\hat{i}+\hat{j}+\hat{k}\right)\cdot\left(3\hat{i}+4\hat{j}\right)}{5} = 7$

$\lambda = 5\sqrt{3}$

$\vec{a} = 5(\hat{i} + \hat{j} + \hat{k})$

now $\vec{b} = 5\alpha(\hat{i}+\hat{j}+\hat{k}) + \beta(\hat{i})$

$\vec{a}\cdot\vec{b} = 0$

$\Rightarrow 25\alpha(3) + 5\beta = 0$

$\Rightarrow 15\alpha + \beta = 0 \quad \Rightarrow \quad \beta = -15\alpha$

$\vec{b} = 5\alpha(-2\hat{i}+\hat{j}+\hat{k})$

$\left|\vec{b}\right| = 5\sqrt{3}$

$\Rightarrow \alpha = \pm\dfrac{1}{\sqrt{2}}$

$\vec{b} = \pm\dfrac{5}{\sqrt{2}}(-2\hat{i}+\hat{j}+\hat{k})$

Projection of $\vec{b}$ on $3\hat{i}+4\hat{j}$ is

$$\frac{\vec{b}\cdot(3\hat{i}+4\hat{j})}{5} = \pm\frac{5}{\sqrt{2}}\left(\frac{-6+4}{5}\right) = \pm\sqrt{2}$$

15. Let $y = y(x)$ be the solution of the differential equation $(x + 1)y' - y = e^{3x}(x + 1)^2$, with $y(0) = \frac{1}{3}$. Then, the point $x = -\frac{4}{3}$ for the curve $y = y(x)$ is:

(A) not a critical point

(B) a point of local minima

(C) a point of local maxima

(D) a point of inflection

Official Ans. by NTA (B)

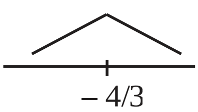
Sol. $(x + 1)dy - y\,dx = e^{3x}(x + 1)^2$

$$\frac{(x+1)dy - ydx}{(x+1)^2} = e^{3x}$$

$$d\left(\frac{y}{x+1}\right) = e^{3x} \quad \Rightarrow \quad \frac{y}{x+1} = \frac{e^{3x}}{3} + C$$

$$\left(0, \frac{1}{3}\right) \quad \Rightarrow \quad C = 0 \quad \Rightarrow \quad y = \frac{(x+1)e^{3x}}{3}$$

$$\frac{dy}{dx} = \frac{1}{3}\left((x+1)3e^{3x} + e^{3x}\right) = \frac{3^{3x}}{3}(3x+4)$$



Clearly, $x = \dfrac{-4}{3}$ is point of local minima

16. If $y = m_1x + c_1$ and $y = m_2x + c_2$, $m_1 \neq m_2$ are two common tangents of circle $x^2 + y^2 = 2$ and parabola $y^2 = x$, then the value of $8|m_1m_2|$ is equal to

(A) $3+4\sqrt{2}$ (B) $-5+6\sqrt{2}$

(C) $-4+3\sqrt{2}$ (D) $7+6\sqrt{2}$

Official Ans. by NTA (C)

Sol. C_1: $x^2 + y^2 = 2$

C_2 : $y^2 = x$

Let tangent to parabola be $y = mx + \dfrac{1}{4m}$.

It is also a tangent of circle so distance from centre of circle (0, 0) will be $\sqrt{2}$.

$$\left|\frac{\frac{1}{4m}}{\sqrt{1+m^2}}\right| = \sqrt{2} \quad \Rightarrow \quad 1 = 32m^2 + 32m^4$$

by solving

$$m^2 = \frac{3\sqrt{2}-4}{8}, \quad m^2 = \frac{-3\sqrt{2}-4}{8} \text{ (rejected)}$$

$$m = \pm\sqrt{\frac{3\sqrt{2}-4}{8}}$$

so, $8\,|m_1m_2| = 3\sqrt{2}-4$

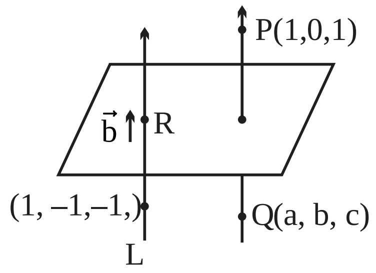
17. Let Q be the mirror image of the point P(1, 0, 1) with respect to the plane S : $x + y + z = 5$. If a line L passing through (1, –1, –1), parallel to the line PQ meets the plane S at R, then QR^2 is equal to:

(A) 2 (B) 5

(C) 7 (D) 11

Official Ans. by NTA (B)

Sol.



Let parallel vector of L = $\vec{b}$

mirror image of Q on given plane x+y+z=5

$$\frac{a-1}{1}=\frac{b-0}{1}=\frac{c-1}{1}=\frac{-2(2-5)}{3}$$

a = 3, b = 2, c = 3

Q≡(3, 2, 3)

$\because \quad \vec{b} \,||\, \overrightarrow{PQ}$

so, $\vec{b} = (1,1,1)$

Equation of line

$$L : \frac{x-1}{1}=\frac{y+1}{1}=\frac{z+1}{1}$$

Let point R, $(\lambda+1, \lambda-1, \lambda-1)$

lying on plane x + y + z = 5,

so, 3λ – 1 = 5

$\Rightarrow \quad \lambda = 2$

Point R is (3, 1, 1)

QR^2 = 5 **Ans.**

18. If the solution curve y = y(x) of the differential equation $y^2dx + (x^2 - xy + y^2)dy = 0$, which passes through the point (1, l) and intersects the line y = $\sqrt{3}$ x at the point (α, $\sqrt{3}$ α), then value of $\log_e(\sqrt{3}\,\alpha)$ is equal to

(A) $\frac{\pi}{3}$ (B) $\frac{\pi}{2}$

(C) $\frac{\pi}{12}$ (D) $\frac{\pi}{6}$

Official Ans. by NTA (C)

Sol. $y^2dx - xy\,dy = -(x^2 + y^2)dy$

$y(y\,dx - x\,dy) = -(x^2 + y^2)\,dy$

$-y(x\,dx - y\,dx) = -(x^2 + y^2)dy$

$$\frac{xdy-ydx}{x^2}=\left(1+\frac{y^2}{x^2}\right)\frac{dy}{y}$$

$$\Rightarrow \frac{d(y/x)}{1+\frac{y^2}{x^2}}=\frac{dy}{y}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = ln\, y + C$$

$$\left(\alpha, \sqrt{3}\alpha\right) \Rightarrow \frac{\pi}{3}=\ln\left(\sqrt{3}\alpha\right)+\frac{\pi}{4}$$

$$\therefore \quad ln\left(\sqrt{3}\alpha\right)=\frac{\pi}{12}$$

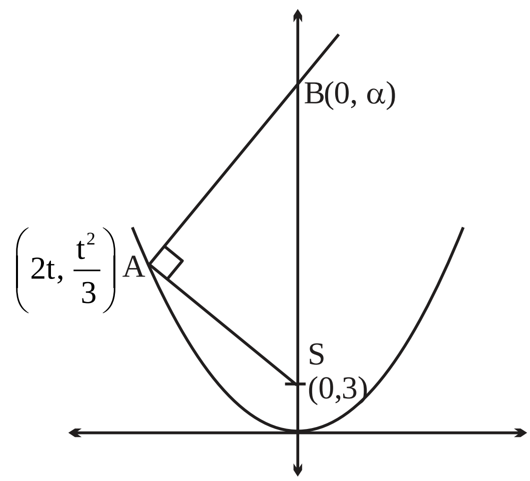
19. Let x = 2t, $y=\frac{t^2}{3}$ be a conic. Let S be the focus and B be the point on the axis of the conic such that SA ⊥ BA, where A is any point on the conic. If k is the ordinate of the centroid of ΔSAB, then $\lim\limits_{t\to 1} k$ is equal to

(A) $\frac{17}{18}$ (B) $\frac{19}{18}$

(C) $\frac{11}{18}$ (D) $\frac{13}{18}$

Official Ans. by NTA (D)

Sol.



parabola $x^2 = 12y$

SA ⊥ SB

so, $m_{AS} \cdot m_{AB} = -1$

$$\frac{\left(3-\frac{t^2}{3}\right)}{(0-2t)} \cdot \frac{\left(\alpha-\frac{t^2}{3}\right)}{(0-2t)} = -1$$

by solving

$$3\alpha = \frac{27t^2+t^4}{t^2-9}$$

ordinate of centriod of $\Delta SAB = K = \dfrac{\alpha+\frac{t^2}{3}+3}{3}$

$$k = \frac{9+3\alpha+t^2}{9}$$

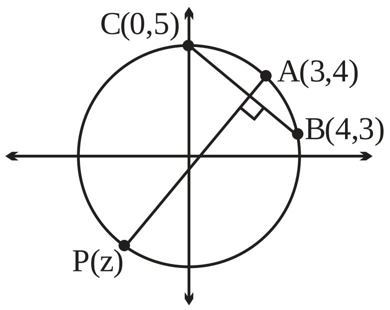
$$\lim_{t\to 1} k = \lim_{t\to 1} \frac{1}{9}\left(9+t^2+\frac{27t^2+t^4}{(t^2-9)}\right) = \frac{13}{18}$$

20. Let a circle C in complex plane pass tltrough the points $z_1 = 3 + 4i$, $z_2 = 4 + 3i$ and $z_3 = 5i$. If $z(\neq z_1)$ is a point on C such that the line through z and z_1 is perpendicular to the line through z_2 and z_3, then arg(z) is equal to :

(A) $\tan^{-1}\left(\frac{2}{\sqrt{5}}\right)-\pi$ (B) $\tan^{-1}\left(\frac{24}{7}\right)-\pi$

(C) $\tan^{-1}(3)-\pi$ (D) $\tan^{-1}\left(\frac{3}{4}\right)-\pi$

Sol.



Slope of BC $= \frac{3-5}{4-0} = -\frac{1}{2}$

Slope of AP = 2

equation of AP : $y - 4 = 2(x - 3)$

$\Rightarrow y = 2(x - 1)$

P lies on circle $x^2 + y^2 = 25$

$\Rightarrow x^2 + (2(x - 1))^2 = 25$

$\Rightarrow x = -\frac{7}{5}$ and $y = -\frac{24}{5}$

$\Rightarrow \arg(z) = \tan^{-1}\left(\frac{24}{7}\right)-\pi$

SECTION-B

1. Let C_r denote the binomial coefficient of x^r in the expansion of $(1 + x)^{10}$. If $\alpha, \beta \in R$. $C_1 + 3\cdot2C_2 + 5\cdot3C_3 + \ldots$ upto 10 terms

$$= \frac{\alpha\times 2^{11}}{2^\beta-1}\left(C_0+\frac{C_1}{2}+\frac{C_2}{3}+\text{.....upto 10 terms}\right)$$

then the value of $\alpha + \beta$ is equal to

Official Ans. by NTA (286)

(BONUS)

Sol. $(1 + x)^{10} = C_0 + C_1x + C_2x^2 + \ldots\ldots + C_{10}x^{10}$

Differentiating

$10(1 + x)^9 = C_1 + 2C_2x + 3C_3x^2 + \ldots. + 10C_{10}x^9$

replace $x \to x^2$

$10(1+x^2)^9 = C_1 + 2C_2x^2 + 3C_3x^4 + \ldots. + 10C_{10}x^{18}$

$10\cdot x(1+x^2)^9 = C_1x + 2C_2x^3 + 3C_3x^5 + \ldots. + 10C_{10}x^{19}$

Differentiating

$$10\left((1+x^2)^9\cdot 1+x\cdot 9(1+x^2)^8 2x\right)$$

$= C_1x+2\,C_2\cdot 3x^3 + 3\cdot 5\cdot C_3x^4 + \ldots. + 10\cdot 19C_{10}x^{18}$

putting x = 1

$10(2^9+18\cdot 2^8)$

$= C_1 + 3\cdot2\cdot C_2 + 5\cdot3\cdot C_3 + \ldots. + 19\cdot10\cdot C_{10}$

$C_1 + 3\cdot2\cdot C_2 + \ldots\ldots + 19\cdot10\cdot C_{10}$

$= 10\cdot 2^9\cdot 10 = 100\cdot 2^9$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots..+\underset{\substack{\uparrow\\ 10^{th}\text{ term}}}{\frac{C_9}{11}}+\underset{\substack{\uparrow\\ 11^{th}\text{ term}}}{\frac{C_{10}}{11}}=\frac{2^{11}-1}{11}$$

$$C_0+\frac{C_1}{2}+\frac{C_2}{3}+\ldots..+\frac{C_9}{11}=\frac{2^{11}-2}{11}$$

Now, $100\cdot 2^9 = \dfrac{\alpha\cdot 2^{11}}{2^\beta-1}\left(\dfrac{2^{11}-2}{11}\right)$

Eqn. of form $y = k(2^x - 1)$.

It has infinite solutions even if we take $x, y \in N$.

2. The number of 3-digit odd numbers, whose sum of digits is a multiple of 7, is ______.

Official Ans. by NTA (63)

Sol. x y z $\leftarrow$ odd number

z = 1,3,5,7,9

x+y+ z = 7, 14, 21 [sum of digit multiple of 7]

$\underset{1to9}{x} + \underset{0to9}{y} = 6, 4, 2, 13, 11, 9, 7, 5, 20, 18, 16, 14, 12$

$x + y = 6 \Rightarrow (1,5), (2, 4), (3, 3), (4, 2), (5, 1), (6, 0)$

$\rightarrow$ T.N. = 6

$x + y = 4 \Rightarrow (1,3), (2, 2), (3, 1), (4,0)$

$\rightarrow$ T.N = 4

$x + y = 2 \Rightarrow (1,1), (2,0)$

$\rightarrow$ T.N. = 2

$x + y = 13 \Rightarrow (4,9), (5,8), (6,7), (7,6), (8,5),(9,4)$

$\rightarrow$ T.N. = 6

$x + y = 11 \Rightarrow (2,9), (3,8), (4,7), (5,6), (6,5), (6,5), (7,4), (8,3), (9,2)$

$\rightarrow$ T.N. = 8

$x + y = 9 \Rightarrow (1,8),(2,7),(3,8),(4,5),(5,4),....(8,1),(9,0)$

$\rightarrow$ T.N. = 9

$x + y = 7 \Rightarrow (1,8),(2,5),(3,4),.... (8, 1), (7,0)$

$\rightarrow$ T.N. = 7

$x + y = 5 \Rightarrow (1,4),(2,3), (3, 2), (4,1),(5,0)$

$\rightarrow$ T.N. = 5

$x + y = 20 \Rightarrow$ Not possible

$x + y = 18 \Rightarrow (9,9)$ $\rightarrow$ T.N. = 1

$x + y = 16 \Rightarrow (7,9),(8,8),(9,7)$

$\rightarrow$ T.N. = 3

$x + y = 14 \Rightarrow (5,9),(6,8),(7,7),(8,6),(9,5)$

$\rightarrow$ T.N. = 5

$x + y = 12 \Rightarrow (3,9),(4,8),(5,7),(6,6)....(9,3)$

$\rightarrow$ T.N. = 7

3. Let θ be the angle between the vectors $\vec{a}$ and $\vec{b}$, where $|\vec{a}| = 4$, $|\vec{b}| = 3$ $\theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$. Then

$\left|\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right)\right|^2 + 4\left(\vec{a} \cdot \vec{b}\right)^2$ is equal to ____

Official Ans. by NTA (576)

Sol. $|\vec{a}| = 4, |\vec{b}| = 3 \quad \theta \in \left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$$\left|\left(\vec{a} - \vec{b}\right) \times \left(\vec{a} + \vec{b}\right)\right|^2 + 4\left(\vec{a} \cdot \vec{b}\right)^2$$

$$\left|\vec{a} \times \vec{b} - \vec{b} \times \vec{a}\right|^2 + 4a^2b^2\cos^2\theta$$

$$2\left|\vec{a} \times \vec{b}\right|^2 + 4a^2b^2\cos^2\theta$$

$$4a^2b^2\sin^2\theta + 4a^2b^2\cos^2\theta$$

$$4a^2b^2 = 4 \times 16 \times 9 = 576$$

4. Let the abscissae of the two points P and Q be the roots of $2x^2 - rx + p = 0$ and the ordinates of P and Q be the roots of $x^2 - sx - q = 0$. If the equation of the circle described on PQ as diameter is $2(x^2 + y^2) - 11x - 14y - 22 = 0$, then $2r + s - 2q + p$ is equal to

Official Ans. by NTA (7)

Sol. $2x^2 - rx + p = 0 \begin{cases} x_1 \\ x_2 \end{cases}$

$y^2 - sy - q = 0 \begin{cases} y_1 \\ y_2 \end{cases}$

Equation of the circle with PQ as diameter is $2(x^2 + y^2) - rx - 2sy + p - 2q = 0$

on comparing with the given equation

r = 11, s = 7

p – 2q = – 22

$\therefore 2r + s - 2q + p = 22 + 7 - 22 = 7$

5. The number of values of x in the interval $\left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$ for which $14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$ holds, is ________

Official Ans. by NTA (4)

Sol. $x \in \left(\frac{\pi}{4}, \frac{7\pi}{4}\right)$

$$14\text{cosec}^2x - 2\sin^2x = 21 - 4\cos^2x$$

$$= 21 - 4\left(1 - \sin^2x\right)$$

$$= 17 + 4\sin^2x$$

$$14\text{cosec}^2x - 6\sin^2x = 17$$

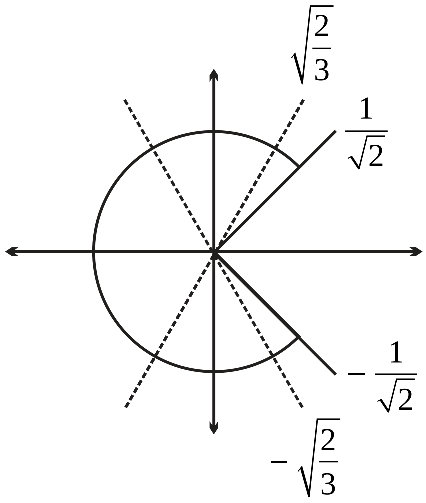
$$\text{let} \sin^2x = p$$

$$\frac{14}{p}-6p=17 \Rightarrow 14-6p^2=17p$$

$$6p^2+17p-14=0$$

$$p=-3.5, \frac{2}{3} \quad \Rightarrow \quad \sin^2 x = \frac{2}{3}$$

$$\Rightarrow \quad \sin x = \pm\sqrt{\frac{2}{3}}$$



$\therefore$ Total 4 solutions

6. For a natural number n, let $a_n = 19^n - 12^n$. Then, the value of $\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}$ is

Official Ans. by NTA (4)

Sol. $a_n = 19^n - 12^n$

$$\frac{31\alpha_9-\alpha_{10}}{57\alpha_8}=\frac{31\left(19^9-12^9\right)-\left(19^{10}-12^{10}\right)}{57\alpha_8}$$

$$=\frac{19^9(31-19)-12^9(31-12)}{57\alpha_8}$$

$$=\frac{19^9 \cdot 12-12^{19} \cdot 19}{57\alpha_8}$$

$$=\frac{12 \cdot 19\left(19^8-12^8\right)}{57\alpha_8} = 4$$

7. Let f : R $\rightarrow$ R be a function defined by $f(x)=\left(2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right)^{\frac{1}{50}}$. If the function $g(x)=f(f(f(x)))+f(f(x))$, the the greatest integer less than or equal to g (1) is ______

Official Ans. by NTA (2)

Sol. $f(x)=\left[2\left(1-\frac{x^{25}}{2}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$

$$f(x)=\left[\left(2-x^{25}\right)\left(2+x^{25}\right)\right]^{\frac{1}{50}}$$

$$= (4 - x^{50})^{1/50}$$

$$f(f(x))=\left(4-\left(\left(4-x^{50}\right)^{1/50}\right)^{50}\right)^{1/50} = x$$

$$g(x) = f(f(f(x)))+f(f(x))$$

$$= f(x) + x$$

$$g(1) = f(1) + 1 = 3^{1/50} + 1$$

$$[g(1)] = [3^{1/50} + 1] = 2$$

8. Let the lines

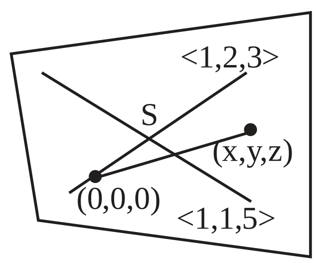
$L_1 : \vec{r}=\lambda(\hat{i}+2\hat{j}+3\hat{k}), \lambda \in R$

$L_2 : \vec{r}=(\hat{i}+3\hat{j}+\hat{k})+\mu(\hat{i}+\hat{j}+5\hat{k}); \mu \in R$

intersect at the point S. If a plane ax + by – z + d = 0 passes through S and is parallel to both the lines L_1 and L_2, then the value of a + b + d is equal to _______

Official Ans. by NTA (5)

Sol. Both the lines lie in the same plane



$\therefore$ equation of the plane

$$\begin{vmatrix} x & y & z \\ 1 & 2 & 3 \\ 1 & 1 & 5 \end{vmatrix}=0$$

$\Rightarrow 7x - 2y - z = 0$

$\therefore$ a + b + d = 5

9. Let A be a 3×3 matrix having entries from the set $\{-1, 0, 1\}$. The number of all such matrices A having sum of all the entries equal to 5, is ________

Official Ans. by NTA (414)

Sol. **Case-I:** $1 \to 7$ times

and $-1 \to 2$ times

number of possible matrix $= \dfrac{9!}{7!\,2!} = 36$

Case-II: $1 \to 6$ times,

$-1 \to 1$ times

and $0 \to 2$ times

number of possible matrix $= \dfrac{9!}{6!\,2!} = 252$

Case-III: $1 \to 5$ times,

and $0 \to 4$ times

number of possible matrix $= \dfrac{9!}{5!\,4!} = 126$

Hence total number of all such matrix A $= 414$

10. The greatest integer less than or equal to the sum of first 100 terms of the sequence $\dfrac{1}{3}, \dfrac{5}{9}, \dfrac{19}{27}, \dfrac{65}{81}, \ldots\ldots$ is equal to

Official Ans. by NTA (98)

Sol. $\dfrac{1}{3} + \dfrac{5}{9} + \dfrac{19}{27} + \dfrac{65}{81} + \ldots$

$$\left(1-\frac{2}{3}\right)+\left(1-\frac{4}{9}\right)+\left(1-\frac{8}{27}\right)+\left(1-\frac{16}{81}\right)\ldots\text{100 terms}$$

$$100-\left[\frac{2}{3}+\left(\frac{2}{3}\right)^2+\ldots\right]$$

$$100-\frac{\frac{2}{3}\left(1-\left(\frac{2}{3}\right)^{100}\right)}{1-\frac{2}{3}}$$

$$100-2\left(1-\left(\frac{2}{3}\right)^{100}\right)$$

$$S=98+2\left(\frac{2}{3}\right)^{100}$$

$$\Rightarrow [S]=98$$