FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The fractional change in the magnetic field intensity at a distance 'r' from centre on the axis of current carrying coil of radius 'a' to the magnetic field intensity at the centre of the same coil is : (Take r < a)

(1) $\frac{3}{2}\frac{a^2}{r^2}$ (2) $\frac{2}{3}\frac{a^2}{r^2}$

(3) $\frac{2}{3}\frac{r^2}{a^2}$ (4) $\frac{3}{2}\frac{r^2}{a^2}$

Official Ans. by NTA (4)

Sol. $B_{axis} = \frac{\mu_0 i R^2}{2\left(R^2+x^2\right)^{3/2}}$

$B_{centre} = \frac{\mu_0 i}{2R}$

$\therefore B_{centre} = \frac{\mu_0 i}{2a}$

$\therefore B_{axis} = \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}$

$\therefore$ fractional change in magnetic field =

$$\frac{\frac{\mu_0 i}{2a} - \frac{\mu_0 i a^2}{2\left(a^2+r^2\right)^{3/2}}}{\frac{\mu_0 i}{2a}} = 1 - \frac{1}{\left[1+\left(\frac{r^2}{a^2}\right)\right]^{3/2}}$$

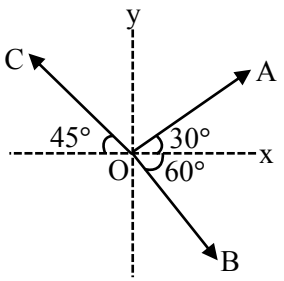
$$\approx 1-\left[1-\frac{3}{2}\frac{r^2}{a^2}\right] = \frac{3}{2}\frac{r^2}{a^2}$$

Note : $\left(1+\frac{r^2}{a^2}\right)^{-3/2} \approx \left(1-\frac{3}{2}\frac{r^2}{a^2}\right)$

[True only if r << a]

Hence option (4) is the most suitable option

2. The magnitude of vectors $\overrightarrow{OA}$, $\overrightarrow{OB}$ and $\overrightarrow{OC}$ in the given figure are equal. The direction of $\overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$ with x-axis will be :-



(1) $\tan^{-1}\frac{(1-\sqrt{3}-\sqrt{2})}{(1+\sqrt{3}+\sqrt{2})}$

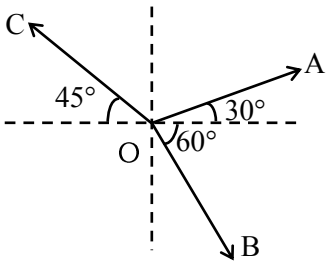
(2) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1+\sqrt{3}-\sqrt{2}\right)}$

(3) $\tan^{-1}\frac{\left(\sqrt{3}-1+\sqrt{2}\right)}{\left(1-\sqrt{3}+\sqrt{2}\right)}$

(4) $\tan^{-1}\frac{\left(1+\sqrt{3}-\sqrt{2}\right)}{\left(1-\sqrt{3}-\sqrt{2}\right)}$

Official Ans. by NTA (1)

Sol.



Let magnitude be equal to λ.

$$\overrightarrow{OA} = \lambda\left[\cos 30°\hat{i} + \sin 30\hat{j}\right] = \lambda\left[\frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}\right]$$

$$\overrightarrow{OB} = \lambda\left[\cos 60°\hat{i} - \sin 60\hat{j}\right] = \lambda\left[\frac{1}{2}\hat{i} - \frac{\sqrt{3}}{2}\hat{j}\right]$$

$$\overrightarrow{OC} = \lambda\left[\cos 45°\left(-\hat{i}\right) + \sin 45\hat{j}\right] = \lambda\left[-\frac{1}{\sqrt{2}}\hat{i} + \frac{1}{\sqrt{2}}\hat{j}\right]$$

$$\therefore \overrightarrow{OA}+\overrightarrow{OB}-\overrightarrow{OC}$$

$$= \lambda\left[\left(\frac{\sqrt{3}+1}{2}+\frac{1}{\sqrt{2}}\right)\hat{i} + \left(\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}\right)\hat{j}\right]$$

$\therefore$ Angle with x-axis

$$\tan^{-1}\left[\frac{\frac{1}{2}-\frac{\sqrt{3}}{2}-\frac{1}{\sqrt{2}}}{\frac{\sqrt{3}}{2}+\frac{1}{2}+\frac{1}{\sqrt{2}}}\right]=\tan^{-1}\left[\frac{\sqrt{2}-\sqrt{6}-2}{\sqrt{6}+\sqrt{2}+2}\right]$$

$$=\tan^{-1}\left[\frac{1-\sqrt{3}-\sqrt{2}}{\sqrt{3}+1+\sqrt{2}}\right]$$

Hence option (1)

3. Car B overtakes another car A at a relative speed of 40 ms^{-1}. How fast will the image of car B appear to move in the mirror of focal length 10 cm fitted in car A, when the car B is 1.9 m away from the car A?

(1) 4 ms^{-1} (2) 0.2 ms^{-1}

(3) 40 ms^{-1} (4) 0.1 ms^{-1}

Official Ans. by NTA (4)

Sol. V_B B V_A A

Mirror used is convex mirror (rear-view mirror)

$\therefore V_{I/m}=-m^2V_{O/m}$

Given,

$V_{O/m}=40m/s$

$$m=\frac{f}{f-u}=\frac{10}{10+190}=\frac{10}{200}$$

$$\therefore V_{I/m}=-\frac{1}{400}\times 40=-0.1m/s$$

$\therefore$ Car will appear to move with speed 0.1 m/s.

Hence option (4)

4. Inside a uniform spherical shell :

(a) the gravitational field is zero

(b) the gravitational potential is zero

(c) the gravitational field is same everywhere

(d) the gravitation potential is same everywhere

(e) all of the above

Choose the most appropriate answer from the options given below :

(1) (a), (c) and (d) only

(2) (e) only

(3) (a), (b) and (c) only

(4) (b), (c) and (d) only

Official Ans. by NTA (1)

Sol. Inside a spherical shell, gravitational field is zero and hence potential remains same everywhere

Hence option (1)

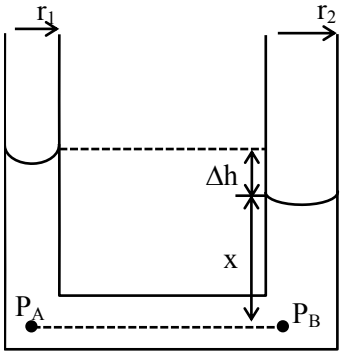
5. Two narrow bores of diameter 5.0 mm and 8.0 mm are joined together to form a U-shaped tube open at both ends. If this U-tube contains water, what is the difference in the level of two limbs of the tube. [Take surface tension of water T = 7.3 × 10^{-2} Nm^{-1}, angle of contact = 0, g = 10 ms^{-2} and density of water = 1.0 × 10^3 kg m^{-3}]

(1) 3.62 mm (2) 2.19 mm

(3) 5.34 mm (4) 4.97 mm

Official Ans. by NTA (2)

Sol.



We have $P_A = P_B$. [Points A & B at same horizontal level]

$$\therefore P_{atm}-\frac{2T}{r_1}+\rho g(x+\Delta h)=P_{atm}-\frac{2T}{r_2}+\rho gx$$

$$\therefore \rho g\Delta h=2T\left[\frac{1}{r_1}-\frac{1}{r_2}\right]$$

$$=2\times 7.3\times 10^{-2}\left[\frac{1}{2.5\times 10^{-3}}-\frac{1}{4\times 10^{-3}}\right]$$

$$\therefore \Delta h=\frac{2\times 7.3\times 10^{-2}\times 10^3}{10^3\times 10}\left[\frac{1}{2.5}-\frac{1}{4}\right]$$

= 2.19 × 10^{-3}m = 2.19 mm

Hence option (2)

6. An electric appliance supplies 6000 J/min heat to the system. If the system delivers a power of 90W. How long it would take to increase the internal energy by 2.5 × 10^3 J ?

(1) 2.5 × 10^2 s (2) 4.1 × 10^1s

(3) 2.4 × 10^3 s (4) 2.5 × 10^1s

Official Ans. by NTA (1)

Sol. $\Delta Q = \Delta U + \Delta W$

$$\frac{\Delta Q}{\Delta t} = \frac{\Delta U}{\Delta t} + \frac{\Delta W}{\Delta t}$$

$$\frac{6000}{60}\frac{J}{sec} = \frac{2.5\times10^3}{\Delta t} + 90$$

$\Delta t = 250$ sec

Option (1)

7. An inductor coil stores 64 J of magnetic field energy and dissipates energy at the rate of 640 W when a current of 8A is passed through it. If this coil is joined across an ideal battery, find the time constant of the circuit in seconds :

(1) 0.4 (2) 0.8 (3) 0.125 (4) 0.2

Official Ans. by NTA (4)

Sol. $U = \frac{1}{2}Li^2 = 64 \Rightarrow L = 2$

$i^2R = 640$

$$R = \frac{640}{(8)^2} = 10$$

$$\tau = \frac{L}{R} = \frac{1}{5} = 0.2$$

Option (4)

8. A series LCR circuit driven by 300 V at a frequency of 50 Hz contains a resistance R = 3 kΩ, an inductor of inductive reactance X_L = 250 πΩ and an unknown capacitor. The value of capacitance to maximize the average power should be : (Take $\pi^2 = 10$)

(1) 4 μF (2) 25 μF (3) 400 μF (4) 40 μF

Official Ans. by NTA (1)

Sol. For maximum average power

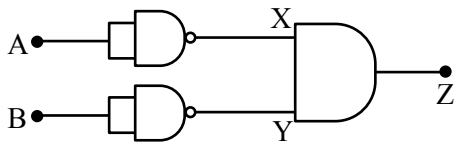
$$X_L = X_C$$

$$250\pi = \frac{1}{2\pi(50)C}$$

$$C = 4\times10^{-6}$$

Option (1)

9. Identify the logic operation carried out by the given circuit :-



(1) OR (2) AND (3) NOR (4) NAND

Official Ans. by NTA (3)

Sol.

A	B	X	Y	Z
1	1	0	0	0
1	0	0	1	0
0	1	1	0	0
0	0	1	1	1

Option (3)

10. A particular hydrogen like ion emits radiation of frequency 2.92×10^{15} Hz when it makes transition from n = 3 to n = 1. The frequency in Hz of radiation emitted in transition from n = 2 to n = 1 will be :

(1) 0.44×10^{15} (2) 6.57×10^{15}

(3) 4.38×10^{15} (4) 2.46×10^{15}

Official Ans. by NTA (4)

Sol. $$nf_1 = k\left(\frac{1}{1} - \frac{1}{3^2}\right)$$

$$nf_2 = k\left(1 - \frac{1}{2^2}\right)$$

$$\frac{f_1}{f_2} = \frac{8/9}{3/4} \Rightarrow f_2 = 2.46\times10^{15}$$

Option (4)

11. In a photoelectric experiment ultraviolet light of wavelength 280 nm is used with lithium cathode having work function ϕ = 2.5 eV. If the wavelength of incident light is switched to 400 nm, find out the change in the stopping potential. ($h = 6.63 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1})

(1) 1.3 V (2) 1.1 V (3) 1.9 V (4) 0.6 V

Official Ans. by NTA (1)

Sol. $$KE_{max} = eV_S = \frac{hc}{\lambda} - \phi$$

$$\Rightarrow eV_S = \frac{1240}{280} - 2.5 = 1.93eV$$

$$\rightarrow V_{S_1} = 1.93V \ \ldots (i)$$

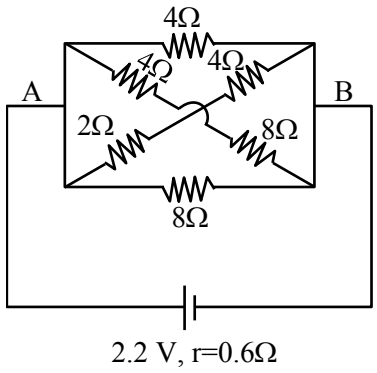
$$\rightarrow eV_{S_2} = \frac{1240}{400} - 2.5 = 0.6eV$$

$$\Rightarrow V_{S_2} = 0.6V \ \ldots (ii)$$

$$\Delta V = V_{S_1} - V_{S_2} = 1.93 - 0.6 = 1.33V$$

Option (1)

12. In the given figure, the emf of the cell is 2.2 V and if internal resistance is 0.6Ω. Calculate the power dissipated in the whole circuit :

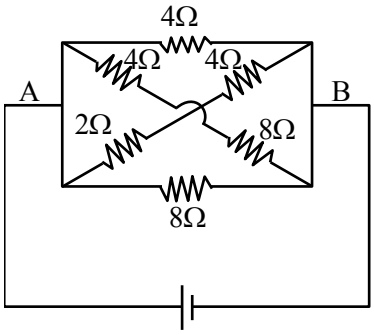


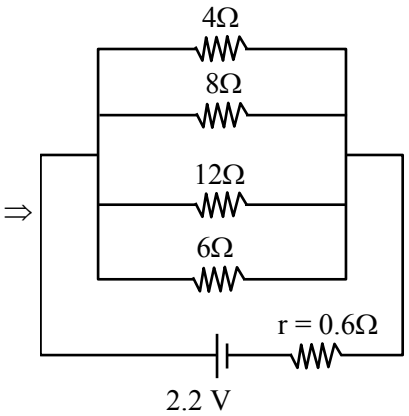
(1) 1.32 W (2) 0.65 W

(3) 2.2 W (4) 4.4 W

Official Ans. by NTA (3)

Sol.





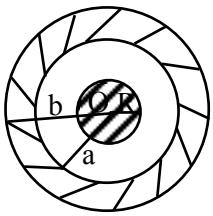
$$\frac{1}{R_{eq}}=\frac{1}{4}+\frac{1}{8}+\frac{1}{12}+\frac{1}{6}=\frac{6+3+2+4}{24}=\frac{15}{24}$$

$$R_{eq}=\frac{24}{15}=1.6\Rightarrow R_T=1.6+0.6=2.2\Omega$$

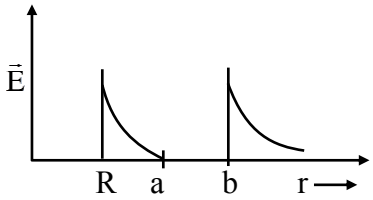
$$P=\frac{V^2}{R_T}=\frac{(2.2)^2}{2.2}=2.2W$$

Option (3)

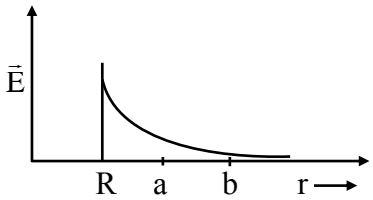
13. A solid metal sphere of radius R having charge q is enclosed inside the concentric spherical shell of inner radius a and outer radius b as shown in figure. The approximate variation electric field $\vec{E}$ as a function of distance r from centre O is given by



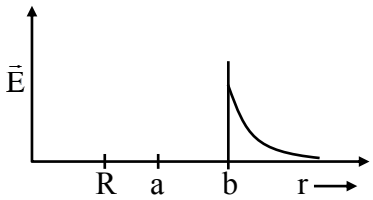
(1)



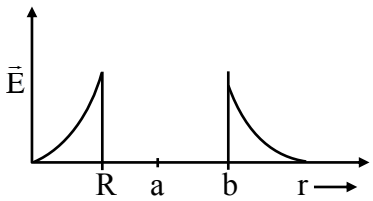
(2)



(3)



(4)



Official Ans. by NTA (1)

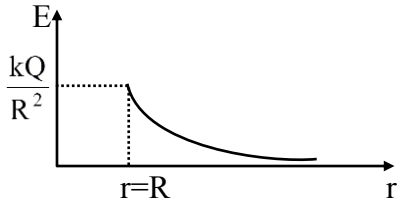
Official Ans. by ALLEN (1 or 2)

Sol. Considering outer spherical shell is non-conducting

Electric field inside a metal sphere is zero.

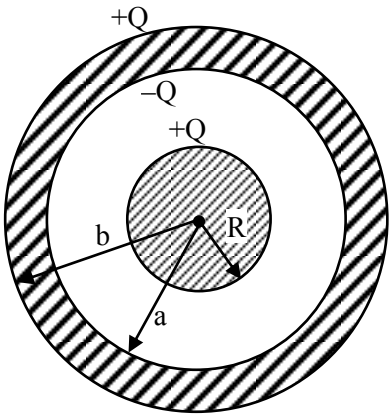
$r < R \Rightarrow E = 0$

$$r > R \Rightarrow E = \frac{kQ}{r^2}$$



Option (2)

Considering outer spherical shell is conducting

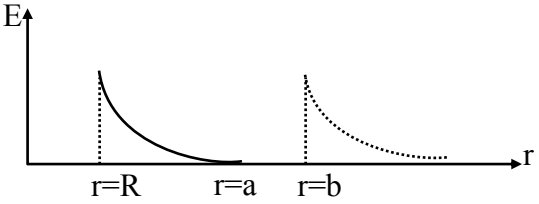


$r < R$, $E = 0$

$R \le r < a$ $\quad E = \dfrac{kQ}{r^2}$

$a \le r < b$, $\quad E = 0$

$r \ge b$ $\quad E = \dfrac{kQ}{r^2}$



Option (1)

14. The rms speeds of the molecules of Hydrogen, Oxygen and Carbondioxide at the same temperature are V_H, V_O and V_C respectively then :

(1) $V_H > V_O > V_C$ (2) $V_C > V_O > V_H$

(3) $V_H = V_O > V_C$ (4) $V_H = V_O = V_C$

Official Ans. by NTA (1)

Sol. $V_{RMS} = \sqrt{\dfrac{3RT}{M_W}}$

At the same temperature $V_{RMS} \propto \dfrac{1}{\sqrt{M_w}}$

$\Rightarrow V_H > V_O > V_C$

Option (1)

15. In a Screw Gauge, fifth division of the circular scale coincides with the reference line when the ratchet is closed. There are 50 divisions on the circular scale, and the main scale moves by 0.5 mm on a complete rotation. For a particular observation the reading on the main scale is 5 mm and the 20th division of the circular scale coincides with reference line. Calculate the true reading.

(1) 5.00 mm (2) 5.25 mm

(3) 5.15 mm (4) 5.20 mm

Official Ans. by NTA (3)

Sol. Least count (L.C) $= \dfrac{0.5}{50}$

$$\text{True reading} = 5 + \frac{0.5}{50} \times 20 - \frac{0.5}{50} \times 5$$

$$= 5 + \frac{0.5}{50}(15) = 5.15\text{mm}$$

Option (3)

16. What equal length of an iron wire and a copper-nickel alloy wire, each of 2 mm diameter connected parallel to give an equivalent resistance of 3Ω?

(Given resistivities of iron and copper-nickel alloy wire are 12 μΩ cm and 51 μΩ cm respectively)

(1) 82 m (2) 97 m

(3) 110 m (4) 90 m

Official Ans. by NTA (2)

Sol. $\dfrac{R_1R_2}{R_1+R_2} = 3$

$$\frac{\left(12\times10^{-6}\times10^{-2}\right)\ell\times4}{\pi(2)^2\times10^{-6}} \times \frac{\left(51\times10^{-6}\times10^{-2}\right)\ell\times4}{\pi(2)^2\times10^{-6}}$$

$$\frac{63\times10^{-6}\times10^{-2}\times\ell\times4}{\pi(2)^2\times10^{-6}}$$

$\Rightarrow \ell = 97\text{m}$

Option (2)

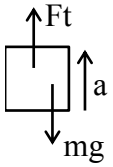
17. The initial mass of a rocket is 1000 kg. Calculate at what rate the fuel should be burnt so that the rocket is given an acceleration of 20 ms^{-2}. The gases come out at a relative speed of 500 ms^{-1} with respect to the rocket :[Use g = 10 m/s^2]

(1) 6.0×10^2 kg s^{-1} (2) 500 kg s^{-1}

(3) 10 kg s^{-1} (4) 60 kg s^{-1}

Official Ans. by NTA (4)

Sol.



$$F_{thrust} = \left(\frac{dm}{dt} \cdot V_{rel}\right)$$

$$\left(\frac{dm}{dt} V_{rel} - mg\right) = ma$$

$$\Rightarrow \left(\frac{dm}{dt}\right) \times 500 - 10^3 \times 10 = 10^3 \times 20$$

$$\frac{dm}{dt} = (60 kg/s)$$

Option (4)

18. If E, L, M and G denote the quantities as energy, angular momentum, mass and constant of gravitation respectively, then the dimensions of P in the formula $P = EL^2M^{-5}G^{-2}$ are :-

(1) $[M^0 L^1 T^0]$ (2) $[M^{-1} L^{-1} T^2]$

(3) $[M^1 L^1 T^{-2}]$ (4) $[M^0 L^0 T^0]$

Official Ans. by NTA (4)

Sol. $E = ML^2T^{-2}$

$L = ML^2T^{-1}$

$m = M$

$G = M^{-1}L^{+3}T^{-2}$

$$P = \frac{EL^2}{M^5G^2}$$

$$[P] = \frac{(ML^2T^{-2})(M^2L^4T^{-2})}{M^5(M^{-2}L^6T^{-4})} = M^0L^0T^0$$

Option (4)

19. The material filled between the plates of a parallel plate capacitor has resistivity 200 Ωm. The value of capacitance of the capacitor is 2 pF. If a potential difference of 40 V is applied across the plates of the capacitor, then the value of leakage current flowing out of the capacitor is : (given the value of relative permitivity of material is 50)

(1) 9.0 μA (2) 9.0 mA

(3) 0.9 mA (4) 0.9 μA

Official Ans. by NTA (3)

Sol. $\rho = 200\ \Omega m$

$C = 2 \times 10^{-12}\ F$

$V = 40\ V$

$K = 56$

$$i = \frac{q}{\rho k\varepsilon_0} = \frac{q_0}{\rho k\varepsilon_0} e^{-\frac{t}{\rho k\varepsilon_0}}$$

$$i_{max} = \frac{2\times10^{-12}\times40}{200\times50\times8.85\times10^{-12}}$$

$$= \frac{80}{10^4\times8.85} = 903\mu A = 0.9mA$$

Option (3)

20. **Statement-I :** By doping silicon semiconductor with pentavalent material, the electrons density increases.

Statement-II : The n-type semiconductor has net negative charge.

In the light of the above statements, choose the most appropriate answer from the options given below :

(1) Statement-I is true but Statement-II is false.

(2) Statement-I is false but Statement-II is true.

(3) Both Statement-I and Statement-II are true.

(4) Both Statement-I and Statement-II are false.

Official Ans. by NTA (1)

Sol. Pentavalent activities have excess free e^-

So e^- density increases but overall semiconductor is neutral.

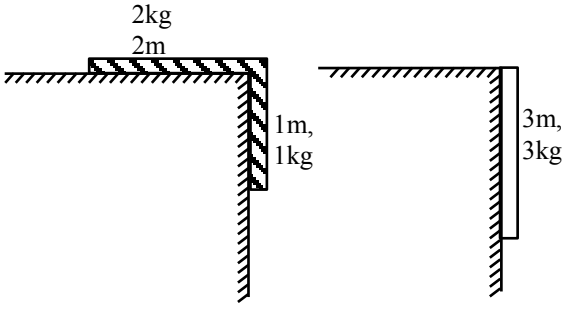
Option (1)

SECTION-B

1. A uniform chain of length 3 meter and mass 3 kg overhangs a smooth table with 2 meter laying on the table. If k is the kinetic energy of the chain in joule as it completely slips off the table, then the value of k is ..... . (Take g = 10 m/s^2)

Official Ans. by NTA (40)

Sol.



From energy conservation

$K_i + U_i = k_f + U_f$

$$0+\left(-1\times10\times\frac{1}{2}\right)=k_f+\left(-3\times10\times\frac{3}{2}\right)$$

$-5 = k_f - 45$

$k_f = 40$ J

Ans. 40.00

2. The electric field in a plane electromagnetic wave is given by

$$\vec{E}=200\cos\left[\left(\frac{0.5\times10^3}{m}\right)x-\left(1.5\times10^{11}\frac{rad}{s}\times t\right)\right]\frac{V}{m}\hat{j}$$

If this wave falls normally on a perfectly reflecting surface having an area of 100 cm^2. If the radiation pressure exerted by the E.M. wave on the surface during a 10 minute exposure is $\frac{x}{10^9}\frac{N}{m^2}$. Find the value of x.

Official Ans. by NTA (354)

Sol. $E_0 = 200$

$$I=\frac{1}{2}\varepsilon_0E_0^2\cdot C$$

Radiation pressure

$$P=\frac{2I}{C}$$

$$=\left(\frac{2}{C}\right)\left(\frac{1}{2}\varepsilon_0E_0^2C\right)$$

$$=\varepsilon_0E_0^2$$

$$=8.85\times10^{-12}\times200^2$$

$$=8.85\times10^{-8}\times4$$

$$=\frac{354}{10^9}$$

Ans. 354.0

3. A source and a detector move away from each other in absence of wind with a speed of 20 m/s with respect to the ground. If the detector detects a frequency of 1800 Hz of the sound coming from the source, then the original frequency of source considering speed of sound in air 340 m/s will be ..... Hz.

Official Ans. by NTA (2025)

Sol.

$$f'=f\left(\frac{C-V_0}{C+V_s}\right)$$

$$1800=f\left(\frac{340-20}{340+20}\right)$$

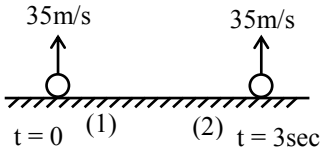
$f = 2025$ Hz

Ans. 2025

4. Two spherical balls having equal masses with radius of 5 cm each are thrown upwards along the same vertical direction at an interval of 3s with the same initial velocity of 35 m/s, then these balls collide at a height of ..... m. (Take g = 10 m/s^2)

Official Ans. by NTA (50)

Sol.



When both balls will collied

$y_1 = y_2$

$$35t-\frac{1}{2}\times10\times t^2=35(t-3)-\frac{1}{2}\times10\times(t-3)^2$$

$$35t-\frac{1}{2}\times10\times t^2=35t-105-\frac{1}{2}\times10\times t^2$$

$$-\frac{1}{2}\times10\times3^2+\frac{1}{2}\times10\times6t$$

$0 = 150 - 30\,t$

$t = 5$ sec

$\therefore$ Height at which both balls will collied

$$h=35t-\frac{1}{2}\times10\times t^2$$

$$=35\times5-\frac{1}{2}\times10\times5^2$$

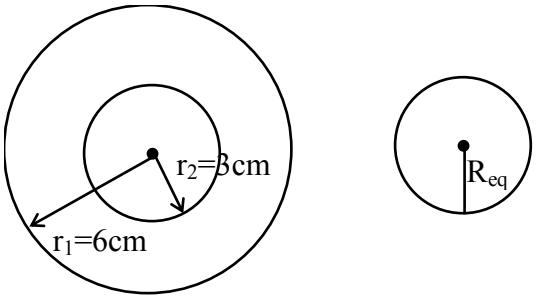
$h = 50$ m

Ans. 50.00

5. A soap bubble of radius 3 cm is formed inside the another soap bubble of radius 6 cm. The radius of an equivalent soap bubble which has the same excess pressure as inside the smaller bubble with respect to the atmospheric pressure is ..... cm.

Official Ans. by NTA (2)

Sol.



Excess pressure inside the smaller soap bubble

$$\Delta P = \frac{4S}{r_1} + \frac{4S}{r_2} \quad \text{.... (i)}$$

The excess pressure inside the equivalent soap bubble

$$\Delta P = \frac{4S}{R_{eq}} \quad \text{... (ii)}$$

From (i) & (ii)

$$\frac{4S}{R_{eq}} = \frac{4S}{r_1} + \frac{4S}{r_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{r_1} + \frac{1}{r_2}$$

$$= \frac{1}{6} + \frac{1}{3}$$

$R_{eq} = 2$ cm

Ans. 2.00

6. An amplitude modulated wave is represented by

$C_m(t) = 10(1 + 0.2\cos 12560t)\sin(111 \times 10^4 t)$ volts.

The modulating frequency in kHz will be ..... .

Official Ans. by NTA (2)

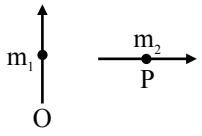
Sol. $W_m = 12560 = 2\pi f_m$

$$f_m = \frac{12560}{2\pi}$$

= 2000 Hz

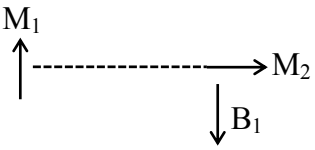
Ans. 2.00

7. Two short magnetic dipoles m_1 and m_2 each having magnetic moment of 1 Am^2 are placed at point O and P respectively. The distance between OP is 1 meter. The torque experienced by the magnetic dipole m_2 due to the presence of m_1 is ..... $\times 10^{-7}$ Nm.



Official Ans. by NTA (1)

Sol.



$$\vec{\tau} = \vec{M}_2 \times \vec{B}_1$$

$$\tau = M_2 B_1 \sin 90°$$

$$= 1 \times \frac{\mu_0}{4\pi}\frac{M_1}{(1)^3}1$$

$= 10^{-7}$ N.m

Ans. 1.00

8. Two travelling waves produces a standing wave represented by equation,

$y = 1.0$ mm $\cos(1.57\ cm^{-1}) \times \sin(78.5\ s^{-1})t$.

The node closest to the origin in the region $x > 0$ will be at x = ... cm.

Official Ans. by NTA (1)

Sol. For node

$$\cos\left(1.57 cm^{-1}\right)x = 0$$

$$\left(1.57 cm^{-1}\right)x = \frac{\pi}{2}$$

$$x = \frac{\pi}{2(1.57)} cm = 1cm$$

Ans. 1.00

9. White light is passed through a double slit and interference is observed on a screen 1.5 m away. The separation between the slits is 0.3 mm. The first violet and red fringes are formed 2.0 mm and 3.5 mm away from the central white fringes. The difference in wavelengths of red and voilet light is ..... nm.

Official Ans. by NTA (300)

Sol. Position of bright fringe $y = n\dfrac{D\lambda}{d}$

y_1 of red $= \dfrac{D\lambda_r}{d} = 3.5\text{mm}$

$\lambda_r = 3.5\times10^{-3}\dfrac{d}{D}$

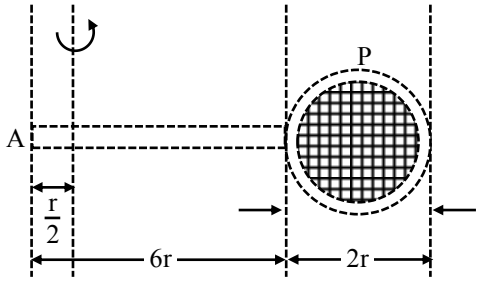
Similarly $\lambda_v = 2\times10^{-3}\dfrac{d}{D}$

$\lambda_r - \lambda_v = \left(1.5\times10^{-3}\right)\left(\dfrac{0.3\times10^{-3}}{1.5}\right)$

$= 3 \times 10^{-7} = 300$ nm

Ans. 300.0

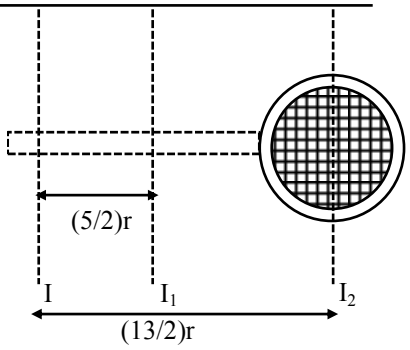
10. Consider a badminton racket with length scales as shown in the figure.



If the mass of the linear and circular portions of the badminton racket are same (M) and the mass of the threads are negligible, the moment of inertia of the racket about an axis perpendicular to the handle and in the plane of the ring at, $\dfrac{r}{2}$ distance from the end A of the handle will be ..... Mr^2.

Official Ans. by NTA (52)

Sol.



$$I = \left[I_1 + M\left(\frac{5}{2}r\right)^2\right] + \left[I_2 + M\left(\frac{13r}{2}\right)^2\right]$$

$$= \left[\frac{M\left(36r^2\right)}{12} + \frac{M\left(25r^2\right)}{4}\right] + \left[\frac{Mr^2}{2} + \frac{169Mr^2}{4}\right]$$

$= 52\ Mr^2$

Ans. 52.00

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Which one of the following complexes is violet in colour?

(1) $[Fe(CN)_6]^{4-}$ (2) $[Fe(SCN)_6]^{4-}$

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O$ (4) $[Fe(CN)_5NOS]^{4-}$

Official Ans. by NTA (4)

Sol. (1) $[Fe(CN)_6]^{4-} \rightarrow$ Pale yellow solution

(2) $[Fe(SCN)_6]^{4-} \rightarrow$ Blood red colour

(3) $Fe_4[Fe(CN_6)]_3 \cdot H_2O \rightarrow$ Prussian blue

(4) $[Fe(CN)_5NOS]^{4-} \rightarrow$ Violet colour

2. Which one of the following is correct for the adsorption of a gas at a given temperature on a solid surface?

(1) $\Delta H > 0, \Delta S > 0$ (2) $\Delta H > 0, \Delta S < 0$

(3) $\Delta H < 0, \Delta S < 0$ (4) $\Delta H < 0, \Delta S > 0$

Official Ans. by NTA (3)

Sol. (i) Adsorption of gas at metal surface is an exothermic process so $\Delta H < 0$

(ii) As the adsorption of gas on metal surface reduces the free movement of gas molecules thus restricting its randomness hences $\Delta S < 0$

3. Which one of the following when dissolved in water gives coloured solution in nitrogen atmosphere?

(1) $CuCl_2$ (2) AgCl

(3) $ZnCl_2$ (4) Cu_2Cl_2

Official Ans. by NTA (1)

Sol. (1) $CuCl_2 + nH_2O \rightarrow Cu^{+2}_{(aq.)}$

blue colour

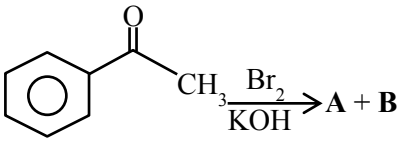
(2) $AgCl + nH_2O \rightarrow$ Insoluble

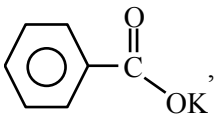
(3) $ZnCl_2 + nH_2O \rightarrow Zn^{+2}_{(aq.)}$

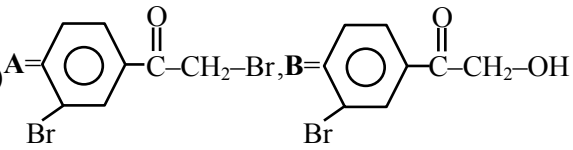
Colourless

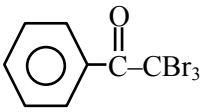
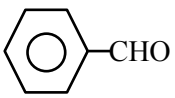
(4) $Cu_2Cl_2 + nH_2O \rightarrow$ Insoluble

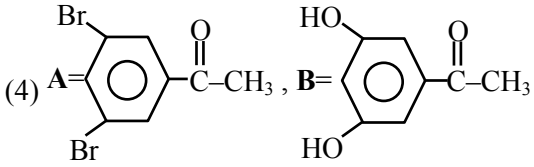
4. The major products formed in the following reaction sequence **A** and **B** are :



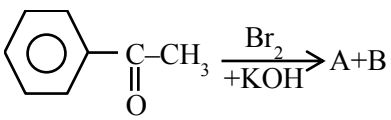
(1) **A** =  , **B**=$CHBr_3$

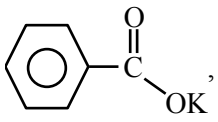
(2) **A**=  , **B**=

(3) **A**=  , **B**= 

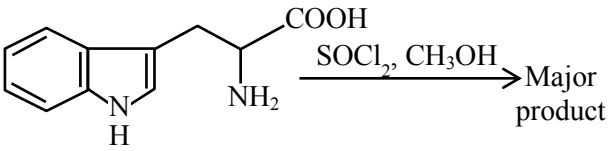
(4) 

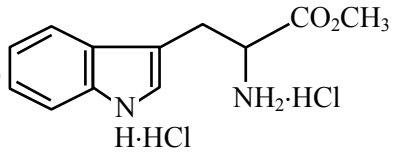
Official Ans. by NTA (1)

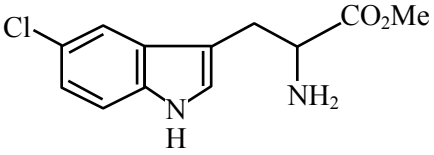
Sol. 

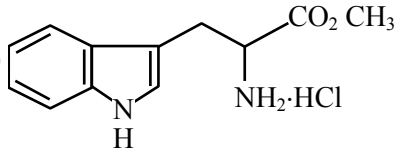
(1)  , $B=CHBr_3$

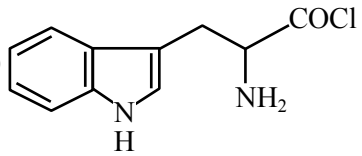
5. The major product formed in the following reaction is :



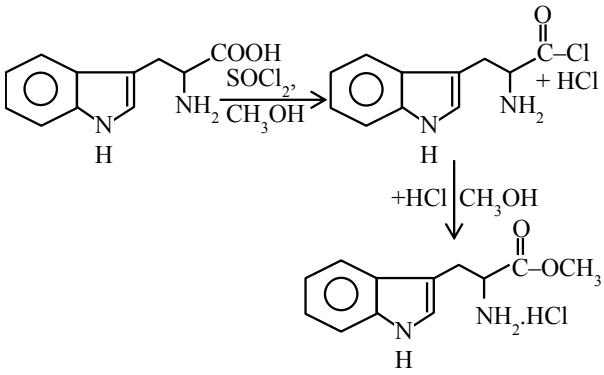
(1) 

(2) 

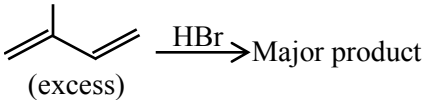
(3) 

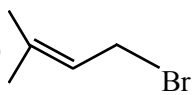
(4) 

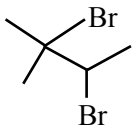
Official Ans. by NTA (3)

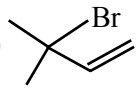
Sol. 

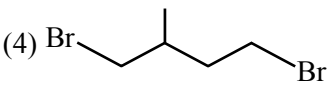
6. The major product formed in the following reaction is :



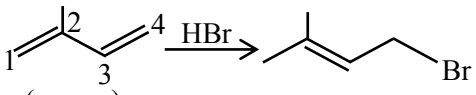
(1) 

(2) 

(3) 

(4) 

Official Ans. by NTA (1)

Sol. 

7. The polymer formed on heating Novolac with formaldehyde is :

(1) Bakelite (2) Polyester

(3) Melamine (4) Nylon 6,6

Official Ans. by NTA (1)

Sol. Novolac + formaldehyde → Bakelite

8. Given below are two statements :

Statement I : The limiting molar conductivity of KCl (strong electrolyte) is higher compared to that of CH_3COOH (weak electrolyte).

Statement II : Molar conductivity decreases with decrease in concentration of electrolyte.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(1) **Statement I** is true but **Statement II** is false.

(2) **Statement I** is false but **Statement II** is true.

(3) Both **Statement I** and **Statement II** are true.

(4) Both **Statement I** and **Statement II** are false.

Official Ans. by NTA (4)

Sol.

Ion	H^+	K^+	Cl^-	CH_3COO^-
Λ^∞_{m} Scm^2/mole	349.8	73.5	76.3	40.9

So $\Lambda^\infty_{m\ CH_3COOH} = \Lambda^\infty_{m\ (H^+)} + \Lambda^\infty_{m\ CH_3COO^-}$

$= 349.8 + 40.9$

$= 390.7$ Scm^2/mole

$\Lambda^\infty_{m\ KCl} = \Lambda^\infty_{m\ (K^+)} + \Lambda^\infty_{m\ (Cl^-)}$

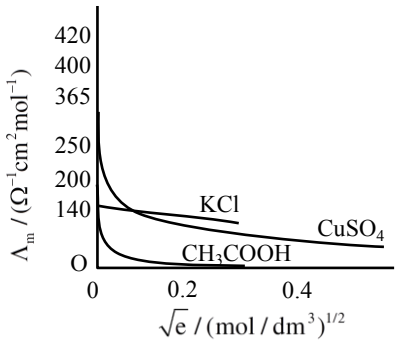
$= 73.5 + 76.3$

$= 149.3$ Scm^2/mole

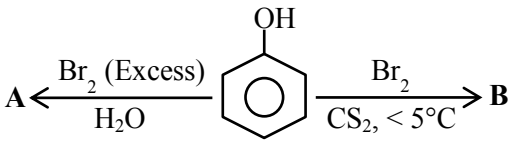
So statement-I is wrong or False.

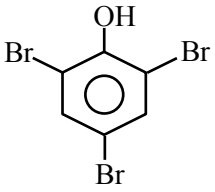
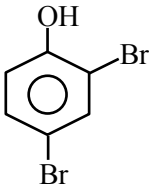
As the concentration decreases, the dilution increases which increases the degree of dissociation, thus increasing the no. of ions, which increases the molar conductance.

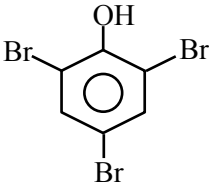
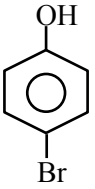
So statement-II is false.

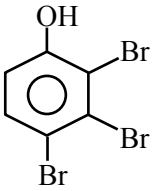
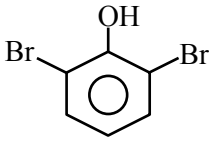


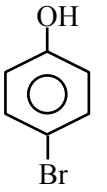
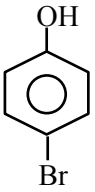
9. The correct options for the products **A** and **B** of the following reactions are :



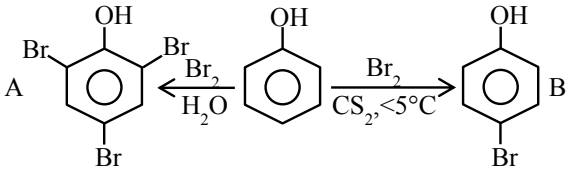
(1) **A=**  , **B=** 

(2) **A=**  , **B=** 

(3) **A=**  , **B=** 

(4) **A=**  , **B=** 

Official Ans. by NTA (2)

Sol. 

10. The conversion of hydroxyapatite occurs due to presence of F^- ions in water. The correct formula of hydroxyapatite is:

(1) $[3Ca_3(PO_4)_2 \cdot Ca(OH)_2]$

(2) $[3Ca(OH)_2 \cdot CaF_2]$

(3) $[Ca_3(PO_4)_2 \cdot CaF_2]$

(4) $[3Ca_3(PO_4)_2 \cdot CaF_2]$

Official Ans. by NTA (1)

Sol. The $F^{\ominus}$ ions make the enamel on teeth much harder by converting hydroxyapatite,$[3(Ca_3(PO_4)_2].Ca(OH)_2]$, the enamel on the surface of the teeth into much harder fluroappatite. $[3Ca_3(PO_4)_2.CaF_2]$

11. Given below are two statements.

Statement I: In the titration between strong acid and weak base methyl orange is suitable as an indicator.

Statement II: For titration of acetic acid with NaOH phenolphthalein is not a suitable indicator.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) **Statement I** is false but **Statement II** is true

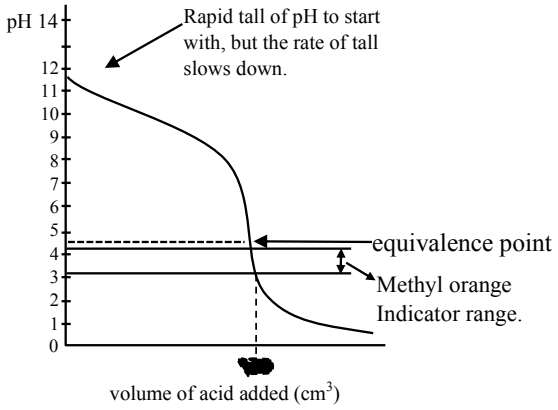
(2) **Statement I** is true but **Statement II** is false

(3) Both **Statement I** and **Statement II** are true

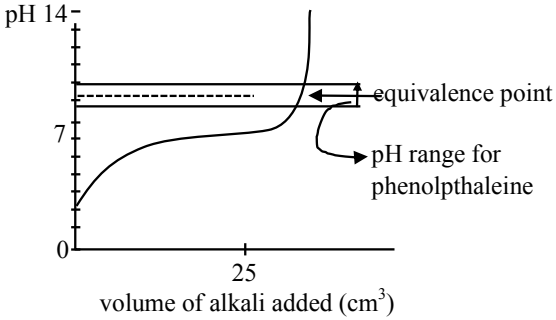
(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (2)

Sol. Titration curve for strong acid and weak base initially a buffer of weak base and conjugate acid is :



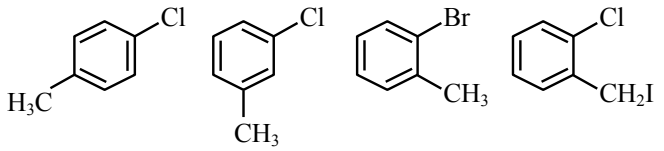
Formed, thus pH falls slowly and after equivalence point, so the pH falls sharply so methyl arrange, having pH range of 3.2 to 4.4 will weak as indicator. So statement-I is correct.



Titration curve for weak acid and strong base (NaOH)

Initially weak acid will form a buffer so pH increases slowly but after equivalence point. it rises sharply covering range of phenolphthalein so it will be suitable indicator so statement-II is false.

12. Among the following compounds I-IV, which one forms a yellow precipitate on reacting sequentially with (i) NaOH (ii) dil. HNO_3 (iii) $AgNO_3$?

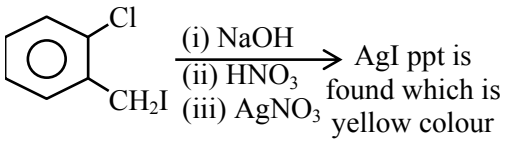


I II III IV

(1) II (2) IV

(3) I (4) III

Official Ans. by NTA (2)

Sol. 

Other compounds halide can't be removed because corresponding C^+ is highly unstable.

13. Which one of the following methods is most suitable for preparing deionized water?

(1) Synthetic resin method

(2) Clark's method

(3) Calgon's method

(4) Permutit method

Official Ans. by NTA (1)

Sol. Pure demineralised (de-ionized) water free from all soluble mineral salts is obtained by passing water successively through a cation exchange (in the H^+ form) and an anion exchange (in the OH^- form) resins.

14. Given below are two statements.

Statement I: The choice of reducing agents for metals extraction can be made by using Ellingham diagram, a plot of ΔG vs temperature.

Statement II: The value of ΔS increases from left to right in Ellingham diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are true

(2) **Statement I** is false but **Statement II** is true

(3) Both **Statement I** and **Statement II** are false

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (4)

Sol. Given statement-I is true as in a number of processes, one element is used to reduce the oxide of another metal. Any element will reduce the oxide of other metal which lie above it in the Ellingham diagram because the free energy change will become more negative.

Given statement-II is false as the value of ΔS is decreases from left to right in Ellingham diagram.

15. What are the products formed in sequence when excess of CO_2 is passed in slaked lime?

(1) $Ca(HCO_3)_2$, $CaCO_3$

(2) $CaCO_3$, $Ca(HCO_3)_2$

(3) CaO, $Ca(HCO_3)_2$

(4) CaO, $CaCO_3$

Official Ans. by NTA (2)

Sol. $Ca(OH)_2 + CO_2 \longrightarrow CaCO_3\downarrow + H_2O$

$CaCO_3\downarrow + CO_2 + H_2O \rightarrow Ca(HCO_3)_2$

16. Given below are two statements.

Statement I: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in positive charges on the nucleus as there is no strong hold on the electron by the nucleus.

Statement II: According to Bohr's model of an atom, qualitatively the magnitude of velocity of electron increases with decrease in principal quantum number.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(1) Both **Statement I** and **Statement II** are false

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is false but **Statement II** is true

(4) **Statement I** is true but **Statement II** is false

Official Ans. by NTA (3)

Sol. Velocity of electron in Bohr's atom is given by

$$V \propto \frac{Z}{n}$$

Z = atomic number of atom, corresponds to +ve charge so as Z increase velocity increases so statement-I is wrong.

and as 'n' decreases velocity increases so statement-II is correct.

17. The correct sequential addition of reagents in the preparation of 3-nitrobenzoic acid from benzene is:

(1) $Br_2/AlBr_3$, HNO_3/H_2SO_4, Mg/ether, CO_2, H_3O^+

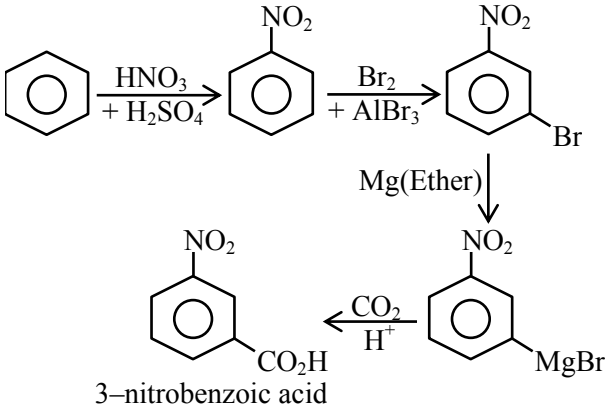
(2) $Br_2/AlBr_3$, NaCN, H_3O^+, HNO_3/H_2SO_4

(3) $Br_2/AlBr_3$, HNO_3/H_2SO_4, NaCN, H_3O^+

(4) HNO_3/H_2SO_4, $Br_2/AlBr_3$, Mg/ether, CO_2, H_3O^+

Official Ans. by NTA (4)

Sol.



3–nitrobenzoic acid

18. Given below are two statements.

Statement I: Frenkel defects are vacancy as well as interstitial defects.

Statement II: Frenkel defect leads to colour in ionic solids due to presence of F-centres.

Choose the **most appropriate** answer for the statements from the options given below:

(1) **Statement I** is false but **Statement II** is true

(2) Both **Statement I** and **Statement II** are true

(3) **Statement I** is true but **Statement II** is false

(4) Both **Statement I** and **Statement II** are false

Official Ans. by NTA (3)

Sol. Theory based.

19. The **incorrect** statement is:

(1) Cl_2 is more reactive than ClF.

(2) F_2 is more reactive than ClF.

(3) On hydrolysis ClF froms HOCl and HF.

(4) F_2 is a stronger oxidizing agent than Cl_2 in aqueous solution

Official Ans. by NTA (1)

Sol. (i) Reactivity order :

$F_2 > ClF$ (inter halogen) $> Cl_2$

(ii) $ClF + H_2O \rightarrow HOCl + HF$

(iii) Oxidizing power in aqueous solution

$F_2 > Cl_2 > Br_2 > I_2$

20. Excess of isobutane on reaction with Br_2 in presence of light at 125°C gives which one of the following, as the major product?

(1) $CH_3-\underset{\displaystyle CH_3}{\overset{\displaystyle Br}{\overset{|}{\underset{|}{C}}}}-CH_2-Br$

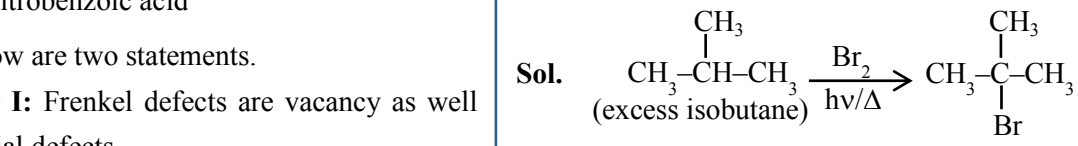
(2) $CH_3-\underset{\displaystyle CH_2Br}{\overset{}{\underset{|}{CH}}}-CH_2Br$

(3) $CH_3-\underset{\displaystyle CH_3}{\overset{}{\underset{|}{CH}}}-CH_2Br$

(4) $CH_3-\underset{\displaystyle CH_3}{\overset{\displaystyle CH_3}{\overset{|}{\underset{|}{C}}}}-Br$

Official Ans. by NTA (4)

Sol.

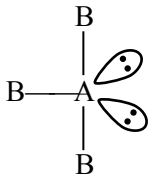


SECTION-B

1. AB_3 is an interhalogen T-shaped molecule. The number of lone pairs of electrons on A is _____. (Integer answer)

Official Ans. by NTA (2)

Sol. T-shaped molecule means 3 sigma bond and 2 lone pairs of electron on central atom.



2. These are physical properties of an element

(A) Sublimation enthalpy

(B) Ionisation enthalpy

(C) Hydration enthalpy

(D) Electron gain enthalpy

The total number of above properties that affect the reduction potential is _____ (Integer answer)

Official Ans. by NTA (3)

Sol. Sublimation enthalpy, Ionisation enthalpy and hydration enthalpy affect the reduction potential.

3. Of the following four aqueous solutions, total number of those solutions whose freezing point is lower than that of 0.10 M C_2H_5OH is _____ (Integer answer)

(i) 0.10 M $Ba_3(PO_4)_2$

(ii) 0.10 M Na_2SO_4

(iii) 0.10 M KCl

(iv) 0.10 M Li_3PO_4

Official Ans. by NTA (4)

Sol. As 0.1 M C_2H_5OH is non-dissociative and rest all salt given are electrolyte so in each case effective molarity > 0.1 so each will have lower freezing point.

4. The OH^- concentration in a mixture of 5.0 mL of 0.0504 M NH_4Cl and 2 mL of 0.0210 M NH_3 solution is $x \times 10^{-6}$ M. The value of x is _____. (Nearest integer)

[Given $K_w = 1 \times 10^{-14}$ and $K_b = 1.8 \times 10^{-5}$]

Official Ans. by NTA (3)

Sol. $\left[NH_4^+\right] = 0.0504$ & $\left[NH_3\right] = 0.0210$

$$\text{So } K_b = \frac{[NH_4^+][HO^-]}{[NH_3]}$$

$$[HO^-] = \frac{K_b \times [NH_3]}{[NH_4^+]} = 1.8\times10^{-5} \times \frac{2}{5} \times \frac{210}{504}$$

$$= 3 \times 10^{-6}$$

5. The number of 4*f* electrons in the ground state electronic configuration of Gd^{2+} is _____.

[Atomic number of Gd = 64]

Official Ans. by NTA (7)

Sol. The electronic configuration of

$_{64}Gd$: [Xe] $4f^7\ 5d^1\ 6s^2$

So the electronic configuration of

$_{64}Gd^{2+}$: [Xe] $4f^7\ 5d^1\ 6s^0$

i.e. the number of 4f electrons in the ground state electronic configuration of Gd^{2+} is 7.

6. The ratio of number of water molecules in Mohr's salt and potash alum is _____ $\times 10^{-1}$. (Integer answer)

Official Ans. by NTA (5)

Sol. **(5)**

Mohr's salt : $(NH_4)_2\ Fe(SO_4)_2.6H_2O$

The number of water molecules in Mohr's salt = 6

Potash alum : $KAl(SO_4)_2.12H_2O$

The number of water molecules in potash alum = 12

So ratio of number of water molecules in Mohr's salt and potash alum = $\frac{6}{12}$

$$= \frac{1}{2}$$

$$= 0.5$$

$$= 5 \times 10^{-1}$$

7. The following data was obtained for chemical reaction given below at 975 K.

$$2NO_{(g)} + 2H_{2(g)} \rightarrow N_{2(g)} + 2H_2O_{(g)}$$

	[NO]	$[H_2]$	Rate
	mol L^{-1}	mol L^{-1}	mol $L^{-1}s^{-1}$
(A)	8×10^{-5}	8×10^{-5}	7×10^{-9}
(B)	24×10^{-5}	8×10^{-5}	2.1×10^{-8}
(C)	24×10^{-5}	32×10^{-5}	8.4×10^{-8}

The order of the reaction with respect to NO is _____. [Integer answer]

Official Ans. by NTA (1)

Sol. $7 \times 10^{-9} = K \times (8 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ .....(1)

$2.1 \times 10^{-8} = K \times (24 \times 10^{-5})^x\ (8 \times 10^{-5})^y$ .....(2)

$$\frac{1}{3} = \left(\frac{1}{3}\right)^x \Rightarrow x = 1$$

8. The Born-Haber cycle for KCl is evaluated with the following data :

$\Delta_f H^\ominus$ for $KCl = -436.7\ kJ\ mol^{-1}$;

$\Delta_{sub} H^\ominus$ for K = 89.2 kJ mol^{-1};

$\Delta_{ionization} H^\ominus$ for K = 419.0 kJ mol^{-1}; $\Delta_{electron\ gain} H^\ominus$ for $Cl_{(g)}$ = –348.6 kJ mol^{-1}; $\Delta_{bond} H^\ominus$ for Cl_2 = 243.0 kJ mol^{-1}

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is _____ (Nearest integer)

Official Ans. by NTA (718)

Sol. $\Delta_f H^\ominus_{KCl} = \Delta_{sub} H^\ominus_{(K)} + \Delta_{ionization} H^\ominus_{(K)} + \frac{1}{2}\Delta_{bond} H^\ominus_{(Cl_2)}$

$$+ \Delta_{\text{electron gain}}H^{\ominus}_{(Cl)} + \Delta_{\text{lattice}}H^{\ominus}_{(KCl)}$$

$$\Rightarrow -436.7 = 89.2 + 419.0 + \frac{1}{2}(243.0) + \{-348.6\}$$

$$+ \Delta_{\text{lattice}}H^{\ominus}_{(KCl)}$$

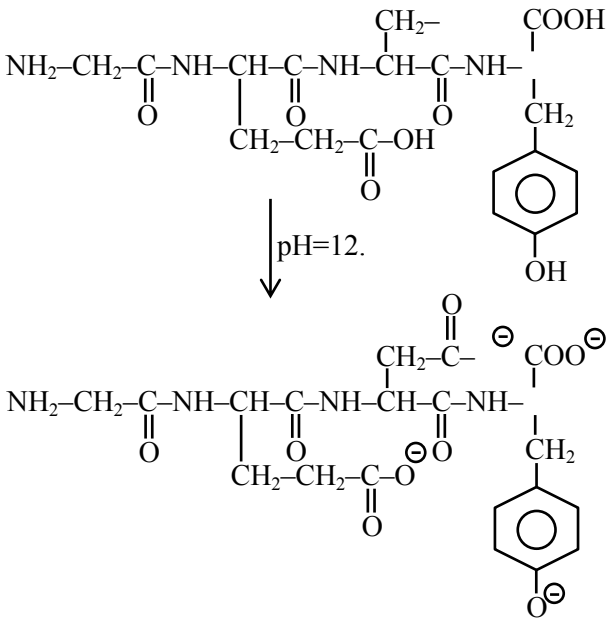
$$\Rightarrow \Delta_{\text{lattice}}H^{\ominus}_{(KCl)} = -717.8 \text{ kJ mol}^{-1}$$

The magnitude of lattice enthalpy of KCl in kJ mol^{-1} is 718 (Nearest integer).

9. The total number of negative charge in the tetrapeptide, Gly-Glu-Asp-Tyr, at pH 12.5 will be _____. (Integer answer)

Official Ans. by NTA (4)

Sol.



Total negative charge produced = 4.

10. An aqueous KCl solution of density 1.20 g mL^{-1} has a molality of 3.30 mol kg^{-1}. The molarity of the solution in mol L^{-1} is _____ (Nearest integer)

[Molar mass of KCl = 74.5]

Official Ans. by NTA (3)

Sol. 1000 kg solvent has 3.3 moles of KCl

1000 kg solvent $\longrightarrow$ 3.3 × 74.5 gm KCl

$\longrightarrow$ 245.85

Weight of solution = 1245.85 gm

Volume of solution = $\frac{1245.85}{1.2}$ ml

So molarity = $\frac{3.3\times1.2}{1245.85}\times1000 = 3.17$

FINAL JEE–MAIN EXAMINATION – AUGUST, 2021

(Held On Thursday 26th August, 2021) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The sum of solutions of the equation $\frac{\cos x}{1+\sin x} = |\tan 2x|,\ x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) - \left\{\frac{\pi}{4}, -\frac{\pi}{4}\right\}$ is :

(1) $-\frac{11\pi}{30}$ (2) $\frac{\pi}{10}$

(3) $-\frac{7\pi}{30}$ (4) $-\frac{\pi}{15}$

Official Ans. by NTA (1)

Sol. $\frac{\cos x}{1+\sin x} = |\tan 2x|$

$\Rightarrow \frac{\cos^2 x/2 - \sin^2 x/2}{(\cos x/2 + \sin x/2)} = |\tan 2x|$

$\Rightarrow \tan^2\left(\frac{\pi}{4} - \frac{x}{2}\right) = \tan^2 2x$

$\Rightarrow 2x = n\pi \pm \left(\frac{\pi}{4} - \frac{x}{2}\right)$

$\Rightarrow x = \frac{-3\pi}{10}, \frac{-\pi}{6}, \frac{\pi}{10}$

or sum $= \frac{-11\pi}{6}$.

2. The mean and standard deviation of 20 observations were calculated as 10 and 2.5 respectively. It was found that by mistake one data value was taken as 25 instead of 35. If α and $\sqrt{\beta}$ are the mean and standard deviation respectively for correct data, then (α, β) is :

(1) (11, 26) (2) (10.5, 25)

(3) (11, 25) (4) (10.5, 26)

Official Ans. by NTA (4)

Sol. Given :

Mean $(\bar{x}) = \frac{\Sigma x_i}{20} = 10$

or $\Sigma x_i = 200$ (incorrect)

or $200 - 25 + 35 = 210 = \Sigma x_i$ (Correct)

Now correct $\bar{x} = \frac{210}{20} = 10.5$

again given S.D = 2.5 (σ)

$\sigma^2 = \frac{\Sigma x_i^2}{20} - (10)^2 = (2.5)^2$

or $\Sigma x_i^2 = 2125$ (incorrect)

or $\Sigma x_i^2 = 2125 - 25^2 + 35^2$

$= 2725$ (Correct)

$\therefore$ correct $\sigma^2 = \frac{2725}{20} - (10.5)^2$

$\underline{\sigma}^2 = 26$

or $\sigma = 26$

$\therefore \underline{\alpha} = 10.5, \beta = 26$

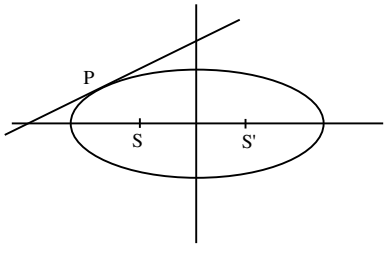
3. On the ellipse $\frac{x^2}{8} + \frac{y^2}{4} = 1$ let P be a point in the second quadrant such that the tangent at P to the ellipse is perpendicular to the line $x + 2y = 0$. Let S and S' be the foci of the ellipse and e be its eccentricity. If A is the area of the triangle SPS' then, the value of $(5 - e^2)$. A is :

(1) 6 (2) 12

(3) 14 (4) 24

Official Ans. by NTA (1)

Sol.



Equation of tangent : $y = 2x + 6$

at P

$\therefore$ P(–8/3, 2/3)

$e = \frac{1}{\sqrt{2}}$

S & S' = (–2, 0) & (2, 0)

Area of $\Delta SPS' = \frac{1}{2} \times 4 \times \frac{2}{3}$

$A = \frac{4}{3}$

$\therefore (5 - e^2)A = (5 - \frac{1}{2})\frac{4}{3} = 6$

4. Let $y = y(x)$ be a solution curve of the differential equation $(y + 1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$, $x \in \left(0, \frac{\pi}{2}\right)$. If $\lim\limits_{x \to 0+} xy(x) = 1$, then the value of $y\left(\frac{\pi}{4}\right)$ is :

(1) $-\frac{\pi}{4}$ (2) $\frac{\pi}{4} - 1$

(3) $\frac{\pi}{4} + 1$ (4) $\frac{\pi}{4}$

Official Ans. by NTA (4)

Sol. $(y + 1)\tan^2 x\, dx + \tan x\, dy + y\, dx = 0$

or $\frac{dy}{dx} + \frac{\sec^2 x}{\tan x}.y = -\tan x$

$IF = e^{\int \frac{\sec^2 x}{\tan x} dx} = e^{\ln \tan x} = \tan x$

$\therefore y \tan x = -\int \tan^2 x dx$

or $y \tan x = -\tan x + x + C$

or $y = -1 + \frac{x}{\tan x} + \frac{C}{\tan x}$

or $\lim\limits_{x \to 0} xy = -x + \frac{x^2}{\tan x} + \frac{Cx}{\tan x} = 1$

or $C = 1$

$y(x) = \cot x + x \cot x - 1$

$y\left(\frac{\pi}{4}\right) = \frac{\pi}{4}$

5. Let A and B be independent events such that $P(A) = p$, $P(B) = 2p$. The largest value of p, for which P (exactly one of A, B occurs) $= \frac{5}{9}$, is :

(1) $\frac{1}{3}$ (2) $\frac{2}{9}$

(3) $\frac{4}{9}$ (4) $\frac{5}{12}$

Official Ans. by NTA (4)

Sol. P(Exactly one of A or B)

$= P\left(A \cap \bar{B}\right) + P\left(\bar{A} \cap B\right) = \frac{5}{9}$

$= P(A)P(\bar{B}) + P(\bar{A})P(B) = \frac{5}{9}$

$\Rightarrow P(A)(1 - P(B)) + (1 - P(A))P(B) = \frac{5}{9}$

$\Rightarrow p(1 - 2p) + (1 - p)\, 2p = \frac{5}{9}$

$\Rightarrow 36p^2 - 27\, p + 5 = 0$

$\Rightarrow p = \frac{1}{3}$ or $\frac{5}{12}$

$p_{max} = \frac{5}{12}$

6. Let $\theta \in \left(0, \frac{\pi}{2}\right)$. If the system of linear equations

$(1 + \cos^2\theta)x + \sin^2\theta\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + (1 + \sin^2\theta)\, y + 4 \sin 3\theta\, z = 0$

$\cos^2\theta\, x + \sin^2\theta\, y + (1 + 4 \sin 3\theta)\, z = 0$

has a non–trivial solution, then the value of θ is :

(1) $\frac{4\pi}{9}$ (2) $\frac{7\pi}{18}$ (3) $\frac{\pi}{18}$ (4) $\frac{5\pi}{18}$

Official Ans. by NTA (2)

Sol. **Case–I**

$$\begin{vmatrix} 1+\cos^2\theta & \sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & 1+\sin^2\theta & 4\sin 3\theta \\ \cos^2\theta & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2$

$$\begin{vmatrix} 2 & \sin^2\theta & 4\sin 3\theta \\ 2 & 1+\sin^2\theta & 4\sin 3\theta \\ 1 & \sin^2\theta & 1+4\sin 3\theta \end{vmatrix} = 0$$

$R_1 \to R_1 - R_2,\ R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & -1 & 0 \\ 1 & 1 & -1 \\ 1 & \sin^2\theta & 1+4\sin^3\theta \end{vmatrix} = 0$$

or $4 \sin 3\theta = -2$

$\sin 3\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{18}$

7. Let $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$, $0 < x < 1$. Then :

(1) $(1-x)^2 f'(x) - 2(f(x))^2 = 0$

(2) $(1+x)^2 f'(x) + 2(f(x))^2 = 0$

(3) $(1-x)^2 f'(x) + 2(f(x))^2 = 0$

(4) $(1+x)^2 f'(x) - 2(f(x))^2 = 0$

Official Ans. by NTA (3)

Sol. $f(x) = \cos\left(2\tan^{-1}\sin\left(\cot^{-1}\sqrt{\frac{1-x}{x}}\right)\right)$

$\cot^{-1}\sqrt{\frac{1-x}{x}} = \sin^{-1}\sqrt{x}$

or $f(x) = \cos(2\tan^{-1}\sqrt{x})$

$= \cos\tan^{-1}\left(\frac{2\sqrt{x}}{1-x}\right)$

$f(x) = \frac{1-x}{1+x}$

Now $f'(x) = \frac{-2}{(1+x)^2}$

or $f'(x)(1-x)^2 = -2\left(\frac{1-x}{1+x}\right)^2$

or $(1-x)^2 f'(x) + 2(f(x))^2 = 0$.

8. The sum of the series

$\frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots + \frac{2^{100}}{x^{2^{100}}+1}$ when $x = 2$ is :

(1) $1 + \frac{2^{101}}{4^{101}-1}$ (2) $1 + \frac{2^{100}}{4^{101}-1}$

(3) $1 - \frac{2^{100}}{4^{100}-1}$ (4) $1 - \frac{2^{101}}{4^{101}-1}$

Official Ans. by NTA (4)

Allen Ans. (BONUS)

Sol. $S = \frac{1}{x+1} + \frac{2}{x^2+1} + \frac{2^2}{x^4+1} + \ldots \frac{2^{100}}{x^{2^{100}}+1}$

$S + \frac{1}{1-x} = \frac{1}{1-x} + \frac{1}{x+1} + \ldots = \frac{2}{1-x^2} + \frac{2}{1+x^2} + \ldots$

$S + \frac{1}{1-x} = \frac{2^{101}}{1-x^{2^{101}}}$

Put $x = 2$

$S = 1 - \frac{2^{101}}{2^{2^{101}}-1}$

Not in option (BONUS)

9. If ${}^{20}C_r$ is the co-efficient of x^r in the expansion of $(1+x)^{20}$, then the value of $\sum_{r=0}^{20} r^2\,{}^{20}C_r$ is equal to :

(1) 420×2^{19} (2) 380×2^{19}

(3) 380×2^{18} (4) 420×2^{18}

Official Ans. by NTA (4)

Sol. $\sum_{r=0}^{20} r^2 .{}^{20}C_r$

$\sum\left(4(r-1)+r\right).{}^{20}C_r$

$\sum r(r-1).\frac{20\times 19}{r(r-1)}.{}^{18}C_r + r.\frac{20}{r}.\sum {}^{19}C_{r-1}$

$\Rightarrow 20 \times 19.2^{18} + 20.2^{19}$

$\Rightarrow 420 \times 2^{18}$

10. Out of all the patients in a hospital 89% are found to be suffering from heart ailment and 98% are suffering from lungs infection. If K% of them are suffering from both ailments, then K can not belong to the set :

(1) {80, 83, 86, 89} (2) {84, 86, 88, 90}

(3) {79, 81, 83, 85} (4) {84, 87, 90, 93}

Official Ans. by NTA (3)

Sol. $n(A \cup B) \geq n(A) + n(B) - n(A \cap B)$

$100 \geq 89 + 98 - n(A \cup B)$

$n(A \cup B) \geq 87$

$87 \leq n(A \cup B) \leq 89$

Option (3)

11. The equation $\arg\left(\frac{z-1}{z+1}\right)=\frac{\pi}{4}$ represents a circle with:

(1) centre at (0, –1) and radius $\sqrt{2}$

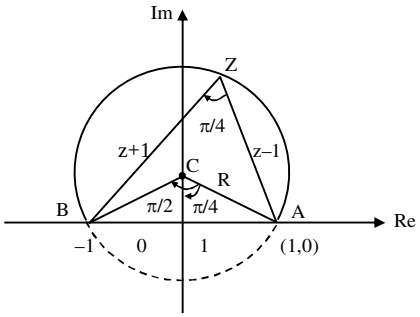
(2) centre at (0, 1) and radius $\sqrt{2}$

(3) centre at (0,0) and radius $\sqrt{2}$

(4) centre at (0,1) and radius 2

Official Ans. by NTA (2)

Sol.



In ΔOAC

$$\sin\left(\frac{\pi}{4}\right)=\frac{1}{AC}$$

$$\Rightarrow AC=\sqrt{2}$$

Also, $\tan\frac{\pi}{4}=\frac{OA}{OC}=\frac{1}{OC}$

$\Rightarrow OC = 1$

$\therefore$ centre (0, 1) ; Radius = $\sqrt{2}$

12. Let $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=\hat{j}-\hat{k}$. If $\vec{c}$ is a vector such that $\vec{a}\times\vec{c}=\vec{b}$ and $\vec{a}.\vec{c}=3$, then $\vec{a}.(\vec{b}\times\vec{c})$ is equal to :

(1) –2 (2) –6 (3) 6 (4) 2

Official Ans. by NTA (1)

Sol. $|\vec{a}|=\sqrt{3};\ \vec{a}.\vec{c}=3;\ \vec{a}\times\vec{b}=-2\hat{i}+\hat{j}+\hat{k},\ \vec{a}\times\vec{c}=\vec{b}$

Cross with $\vec{a}$.

$$\vec{a}\times(\vec{a}\times\vec{c})=\vec{a}\times\vec{b}$$

$$\Rightarrow (\vec{a}.\vec{c})\vec{a}-a^2\vec{c}=\vec{a}\times\vec{b}$$

$$\Rightarrow 3\vec{a}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow 3\hat{i}+3\hat{j}+3\hat{k}-3\vec{c}=-2\hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow \vec{c}=\frac{5\hat{i}}{3}+\frac{2\hat{j}}{3}+\frac{2\hat{k}}{3}$$

$$\therefore \vec{a}.(\vec{b}\times\vec{c})=(\vec{a}\times\vec{b}).\vec{c}=\frac{-10}{3}+\frac{2}{3}+\frac{2}{3}=-2$$

13. If a line along a chord of the circle $4x^2 + 4y^2 + 120x + 675 = 0$, passes through the point (–30, 0) and is tangent to the parabola $y^2 = 30x$, then the length of this chord is :

(1) 5 (2) 7 (3) $5\sqrt{3}$ (4) $3\sqrt{5}$

Official Ans. by NTA (4)

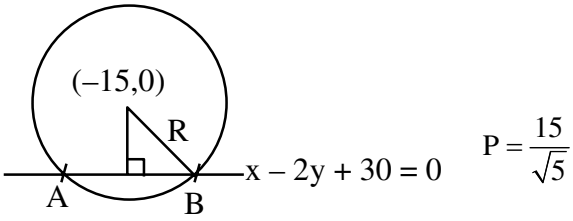
Sol. Equation of tangent to $y^2 = 30\,x$

$$y = mx + \frac{30}{4m}$$

Pass thru (–30, 0) : $a = -30m + \frac{30}{4m} \Rightarrow m^2 = 1/4$

$$\Rightarrow m=\frac{1}{2} \text{ or } m=-\frac{1}{2}$$

At $m=\frac{1}{2}$: $y=\frac{x}{2}+15 \Rightarrow x-2y+30=0$



$$\ell_{AB}=2\sqrt{R^2-P^2}=2\sqrt{\frac{225}{4}-\frac{225}{5}}$$

$$\Rightarrow \ell_{AB}=30.\sqrt{\frac{1}{20}}=\frac{15}{\sqrt{5}}=3\sqrt{5}$$

14. The value of $\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}\right)^2+\left(\frac{x-1}{x+1}\right)^2-2\right)^{1/2}dx$ is:

(1) $\log_e 4$ (2) $\log_e 16$

(3) $2\log_e 16$ (4) $4\log_e(3+2\sqrt{2})$

Official Ans. by NTA (2)

Sol. $$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left(\left(\frac{x+1}{x-1}-\frac{x-1}{x+1}\right)^2\right)^{1/2}dx$$

$$I=\int_{-1/\sqrt{2}}^{1/\sqrt{2}}\left|\frac{4x}{x^2-1}\right|dx \Rightarrow I=2.4\int_{0}^{1/\sqrt{2}}\left|\frac{x}{x^2-1}\right|dx$$

$$\Rightarrow I=-4\int_{0}^{1/\sqrt{2}}\frac{2x}{x^2-1}dx \Rightarrow I=-4\ln\left|x^2-1\right|_0^{1/\sqrt{2}}$$

$\Rightarrow I = 4 \ln 2 \Rightarrow I = \ln 16$

15. A plane P contains the line

$x + 2y + 3z + 1 = 0 = x - y - z - 6$,

and is perpendicular to the plane $-2x + y + z + 8 = 0$.

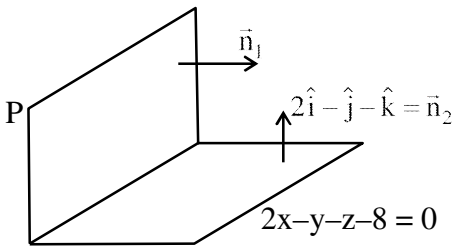
Then which of the following points lies on P ?

(1) $(-1, 1, 2)$ (2) $(0, 1, 1)$

(3) $(1, 0, 1)$ (4) $(2, -1, 1)$

Official Ans. by NTA (2)

Sol. Equation of plane P can be assumed as



$P : x + 2y + 3z + 1 + \lambda (x - y - z - 6) = 0$

$\Rightarrow P : (1 + \lambda)x + (2 - \lambda)y + (3 - \lambda)z + 1 - 6\lambda = 0$

$\Rightarrow \vec{n}_1 = (1+\lambda)\hat{i} + (2-\lambda)\hat{j} + (3-\lambda)\hat{k}$

$\therefore \vec{n}_1 . \vec{n}_2 = 0$

$\Rightarrow 2(1 + \lambda) - (2 - \lambda) - (3 - \lambda) = 0$

$\Rightarrow 2 + 2\lambda - 2 + \lambda - 3 + \lambda = 0 \Rightarrow \lambda = \frac{3}{4}$

$\Rightarrow P : \frac{7x}{4} + \frac{5}{4}y + \frac{9z}{4} - \frac{14}{4} = 0$

$\Rightarrow 7x + 5y + 9z = 14$

$(0, 1, 1)$ lies on P

16. If $A = \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$, $B = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}$, $i = \sqrt{-1}$, and

$Q = A^TBA$, then the inverse of the matrix $A\,Q^{2021}\,A^T$ is equal to :

(1) $\begin{pmatrix} \frac{1}{\sqrt{5}} & -2021 \\ 2021 & \frac{1}{\sqrt{5}} \end{pmatrix}$ (2) $\begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

(3) $\begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$ (4) $\begin{pmatrix} 1 & -2021i \\ 0 & 1 \end{pmatrix}$

Official Ans. by NTA (2)

Sol. $AA^T = \begin{pmatrix} \frac{1}{5} & \frac{2}{\sqrt{5}} \\ \frac{-2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix} \begin{pmatrix} \frac{1}{\sqrt{5}} & \frac{-2}{\sqrt{5}} \\ \frac{2}{\sqrt{5}} & \frac{1}{\sqrt{5}} \end{pmatrix}$

$AA^T = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$

$Q^2 = A^TBA\ A^TBA = A^TBIBA$

$\Rightarrow Q^2 = A^T B^2 A$

$Q^3 = A^TB^2AA^TBA \Rightarrow Q^3 = A^TB^3A$

Similarly : $Q^{2021} = A^TB^{2021}A$ ..... (1)

Now $B^2 = \begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}$

$B^3 = \begin{pmatrix} 1 & 0 \\ 2i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ i & 1 \end{pmatrix} \Rightarrow B^3 = \begin{pmatrix} 1 & 0 \\ 3i & 1 \end{pmatrix}$

Similarly $B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore AQ^{2021}A^T = AA^T B^{2021} AA^T = IB^{2021}I$

$\Rightarrow AQ^{2021}A^T = B^{2021} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}$

$\therefore (AQ^{2021}A^T)^{-1} = \begin{pmatrix} 1 & 0 \\ 2021i & 1 \end{pmatrix}^{-1} = \begin{pmatrix} 1 & 0 \\ -2021i & 1 \end{pmatrix}$

17. If the sum of an infinite GP a, ar, ar^2, ar^3,... is 15 and the sum of the squares of its each term is 150, then the sum of ar^2, ar^4, ar^6, ... is :

(1) $\frac{5}{2}$ (2) $\frac{1}{2}$

(3) $\frac{25}{2}$ (4) $\frac{9}{2}$

Official Ans. by NTA (2)

Sol. Sum of infinite terms :

$\frac{a}{1-r} = 15$ .....(i)

Series formed by square of terms:

$a^2, a^2r^2, a^2r^4, a^2r^6$ .....

Sum = $\frac{a^2}{1-r^2} = 150$

$\Rightarrow \frac{a}{1-r} . \frac{a}{1+r} = 150 \Rightarrow 15 . \frac{a}{1+r} = 150$

$\Rightarrow \frac{a}{1+r} = 10$ ..... (ii)

by (i) and (ii) $a = 12$; $r = \frac{1}{5}$

Now series : ar^2, ar^4, ar^6

$$\text{Sum} = \frac{ar^2}{1-r^2} = \frac{12.\left(1/25\right)}{1-1/25} = \frac{1}{2}$$

18. The value of $\lim\limits_{n\to\infty}\frac{1}{n}\sum\limits_{r=0}^{2n-1}\frac{n^2}{n^2+4r^2}$ is:

(1) $\frac{1}{2}\tan^{-1}(2)$ (2) $\frac{1}{2}\tan^{-1}(4)$

(3) $\tan^{-1}(4)$ (4) $\frac{1}{4}\tan^{-1}(4)$

Official Ans. by NTA (2)

Sol. $L = \lim\limits_{n\to\infty}\frac{1}{n}.\sum\limits_{r=0}^{2n-1}\frac{1}{1+4\left(\frac{r}{n}\right)^2}$

$\Rightarrow L = \int_0^2 \frac{1}{1+4x^2}dx$

$\Rightarrow L = \frac{1}{2}\tan^{-1}(2x)\Big|_0^2 \Rightarrow L = \frac{1}{2}\tan^{-1}4$

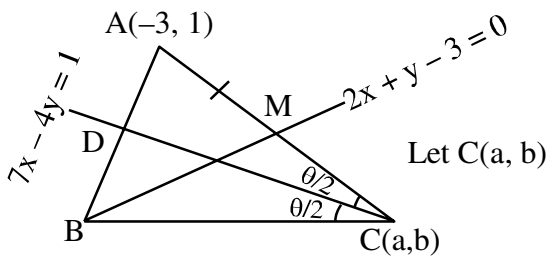
19. Let ABC be a triangle with A(–3, 1) and $\angle ACB = \theta$, $0 < \theta < \frac{\pi}{2}$. If the equation of the median through B is $2x + y - 3 = 0$ and the equation of angle bisector of C is $7x - 4y - 1 = 0$, then $\tan\theta$ is equal to:

(1) $\frac{1}{2}$ (2) $\frac{3}{4}$

(3) $\frac{4}{3}$ (4) 2

Official Ans. by NTA (3)

Sol.



$\therefore$ $M\left(\frac{a-3}{2}, \frac{b+1}{2}\right)$ lies on $2x + y - 3 = 0$

$\Rightarrow 2a + b = 11$ ..... (i)

$\because$ C lies on $7x - 4y = 1$

$\Rightarrow 7a - 4b = 1$ .....(ii)

$\therefore$ by (i) and (ii) : $a = 3$, $b = 5$

$\Rightarrow$ C(3, 5)

$\therefore$ $m_{AC} = 2/3$

Also, $m_{CD} = 7/4$

$$\Rightarrow \tan\frac{\theta}{2} = \left|\frac{\frac{2}{3}-\frac{4}{4}}{1+\frac{14}{12}}\right| \Rightarrow \tan\frac{\theta}{2} = \frac{1}{2}$$

$$\Rightarrow \tan\theta = \frac{2.\frac{1}{2}}{1-\frac{1}{4}} = \frac{4}{3}$$

20. If the truth value of the Boolean expression $\left((p\vee q)\wedge(q\to r)\wedge(\sim r)\right)\to(p\wedge q)$ is false, then the truth values of the statements p, q, r respectively can be:

(1) T F T (2) F F T

(3) T F F (4) F T F

Official Ans. by NTA (3)

Sol.

p	q	r	$\underbrace{p\vee q}_{a}$	$\underbrace{q\to r}_{b}$	$a\wedge b$	$\sim r$	$\underbrace{a\wedge b\wedge(\sim r)}_{c}$	$\underbrace{p\wedge q}_{d}$	$c\to d$
T	F	T	T	T	T	F	F	F	T
F	F	T	F	T	F	F	F	F	T
T	F	F	T	T	T	T	T	F	F
F	T	F	T	F	F	T	F	F	T

SECTION-B

1. Let $z = \frac{1-i\sqrt{3}}{2}$, $i = \sqrt{-1}$. Then the value of $21+\left(z+\frac{1}{z}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+\left(z^3+\frac{1}{z^3}\right)^3+...+\left(z^{21}+\frac{1}{z^{21}}\right)^3$ is _____.

Official Ans. by NTA (13)

Sol. $Z=\frac{1-\sqrt{3}i}{2}=e^{-i\frac{\pi}{3}}$

$$z^r+\frac{1}{z^r}=2\cos\left(-\frac{\pi}{3}\right)r=2\cos\frac{r\pi}{3}$$

$$\Rightarrow\ 21+\sum_{r=1}^{21}\left(z^r+\frac{1}{z^r}\right)^3=8\left(\cos^3\frac{r\pi}{3}\right)=2\left(\cos r\pi+3\cos\frac{r\pi}{3}\right)$$

$$\Rightarrow\ 21+\left(z+\frac{1}{2}\right)^3+\left(z^2+\frac{1}{z^2}\right)^3+.....\left(z^{21}+\frac{1}{z^{21}}\right)^3$$

$$=21+\sum_{r=1}^{21}\left(z^r+\frac{1}{z^r}\right)^3$$

$$=21+\sum_{r=1}^{21}\left(2\cos r\pi+6\cos\frac{r\pi}{3}\right)$$

$$=21-2-6$$

$$=13$$

2. The sum of all integral values of k ($k \neq 0$) for which the equation $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$ in x has no real roots, is _____.

Official Ans. by NTA (66)

Sol. $\frac{2}{x-1}-\frac{1}{x-2}=\frac{2}{k}$

$x \in R-\{1, 2\}$

$\Rightarrow\ k(2x-4-x+1)=2(x^2-3x+2)$

$\Rightarrow\ k(x-3)=2(x^2-3x+2)$

for $x \neq 3$, $\quad k=2\left(x-3+\frac{2}{x-3}+3\right)$

$$x-3+\frac{2}{x-3}\geq 2\sqrt{2},\ \forall x>3$$

$$\&\ x-3+\frac{2}{x-3}\leq -2\sqrt{2},\ \forall x<-3$$

$$\Rightarrow 2\left(x-3+\frac{2}{x-3}+3\right)\in\left(-\infty,6-4\sqrt{2}\right]\cup\left[6+4\sqrt{2},\infty\right)$$

for no real roots

$k \in (6-4\sqrt{2},6+4\sqrt{2})-\{0\}$

Integral $k \in \{1, 2..... 11\}$

Sum of k = 66

3. Let the line L be the projection of the line

$$\frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$$

in the plane $x - 2y - z = 3$. If d is the distance of the point (0, 0, 6) from L, then d^2 is equal to _____.

Official Ans. by NTA (26)

Sol. $L_1: \frac{x-1}{2}=\frac{y-3}{1}=\frac{z-4}{2}$

for foot of $\perp r$ of (1, 3, 4) on $x - 2y - z - 3 = 0$

$(1 + t) - 2(3 - 2t) - (4 - t) - 3 = 0$

$\Rightarrow\ t = 2$

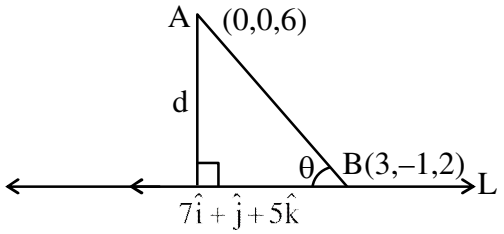
So foot of $\perp r \hat{=} (3, -1, 2)$

& point of intersection of L_1 with plane

is (–11, –3, –8)

dr's of L is <14, 2, 10>

$\cong$ <7, 1, 5>



$$d=AB\sin\theta=\left|\frac{\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\3&-1&-4\\7&1&5\end{vmatrix}}{\sqrt{7^2+1^2+5^2}}\right|$$

$$\Rightarrow d^2=\frac{1^2+(43)^2+(10)^2}{49+1+25}=26$$

4. If ${}^1P_1+2\cdot{}^2P_2+3\cdot{}^3P_3+...+15\cdot{}^{15}P_{15}={}^qP_r-s,\ 0\leq s\leq 1$, then ${}^{q+s}C_{r-s}$ is equal to _____.

Official Ans. by NTA (136)

Sol. $^{1}P_{1} + 2.^{2}P_{2} + 3.^{3}P_{3} + ... + 15 \cdot {}^{15}P_{15}$

$= 1! + 2.2! + 3.3! + ...15 \times 15!$

$= \sum_{r=1}^{15} (r+1-1)r!$

$= \sum_{r=1}^{15} (r+1)!-(r)!$

$= 16! - 1$

$= {}^{16}P_{16} - 1$

$\Rightarrow q = r = 16, s = 1$

${}^{q+s}C_{r-s} = {}^{17}C_{15} = 136$

5. A wire of length 36 m is cut into two pieces, one of the pieces is bent to form a square and the other is bent to form a circle. If the sum of the areas of the two figures is minimum, and the circumference of the circle is k (meter), then $\left(\frac{4}{\pi}+1\right)k$ is equal to _____.

Official Ans. by NTA (36)

Sol. Let x + y = 36

x is perimeter of square and y is perimeter of circle

side of square = x/4

radius of circle = $\frac{y}{2\pi}$

Sum Areas = $\left(\frac{x}{4}\right)^2 + \pi\left(\frac{y}{2\pi}\right)^2$

$= \frac{x^2}{16} + \frac{(36-x)^2}{4\pi}$

For min Area :

$x = \frac{144}{\pi+4}$

$\Rightarrow$ Radius = $y = 36 - \frac{144}{\pi+4}$

$\Rightarrow k = \frac{36\pi}{\pi+4}$

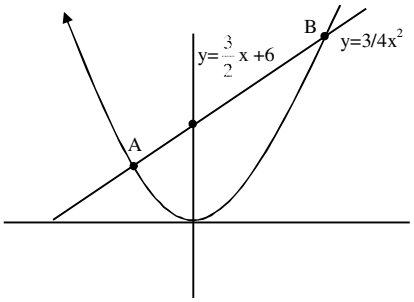
$\left(\frac{4}{\pi}+1\right)k = 36$

6. The area of the region

$S = \{(x, y) : 3x^2 \le 4y \le 6x + 24\}$ is _____.

Official Ans. by NTA (27)

Sol.



For A & B

$3x^2 = 6x + 24 \Rightarrow x^2 - 2x - 8 = 0$

$\Rightarrow x = -2, 4$

Area = $\int_{-2}^{4} \left(\frac{3}{2}x + 6 - \frac{3}{4}x^2\right) dx$

$= \left[\frac{3x^2}{4} + 6x - \frac{x^3}{4}\right]_{-2}^{4} = 27$

7. The locus of a point, which moves such that the sum of squares of its distances from the points (0, 0), (1, 0), (0, 1) (1, 1) is 18 units, is a circle of diameter d. Then d^2 is equal to _____.

Official Ans. by NTA (16)

Sol. Let P(x, y)

$x^2 + y^2 + x^2 + (y-1)^2 + (x-1)^2 + y^2 + (x-1)^2 + (y-1)^2$;

$\Rightarrow 4(x^2 + y^2) - 4y - 4x = 14$

$\Rightarrow x^2 + y^2 - x - y - \frac{7}{2} = 0$

$d = 2\sqrt{\frac{1}{4} + \frac{1}{4} + \frac{7}{2}}$

$\Rightarrow d^2 = 16$

8. If y = y(x) is an implicit function of x such that $\log_e(x + y) = 4xy$, then $\frac{d^2y}{dx^2}$ at x = 0 is equal to _____.

Official Ans. by NTA (40)

Sol. $\ln(x + y) = 4xy \qquad (\text{At } x = 0, y = 1)$

$x + y = e^{4xy}$

$$\Rightarrow 1+\frac{dy}{dx}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)$$

At $x = 0$ $\quad \boxed{\frac{dy}{dx}=3}$

$$\frac{d^2y}{dx^2}=e^{4xy}\left(4x\frac{dy}{dx}+4y\right)^2+e^{4xy}\left(4x\frac{d^2y}{dx^2}+4y\right)$$

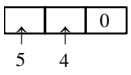
At $x = 0$, $\frac{d^2y}{dx^2} = e^0(4)^2 + e^0(24)$

$$\Rightarrow \frac{d^2y}{dx^2}=40$$

9. The number of three-digit even numbers, formed by the digits 0, 1, 3, 4, 6, 7 if the repetition of digits is not allowed, is _____.

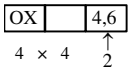
Official Ans. by NTA (52)

Sol. (i) When '0' is at unit place



Number of numbers = 20

(ii) When 4 or 6 are at unit place



Number of numbers = 32

So number of numbers = 52

10. Let $a, b \in \mathbf{R}$, $b \neq 0$, Define a function

$$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & \text{for } x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & \text{for } x>0. \end{cases}$$

If f is continuous at x = 0, then 10 – ab is equal to _____.

Official Ans. by NTA (14)

Sol. $$f(x)=\begin{cases} a\sin\frac{\pi}{2}(x-1), & x\le 0 \\ \frac{\tan 2x-\sin 2x}{bx^3}, & x>0 \end{cases}$$

For continuity at '0'

$$\lim_{x\to 0^+} f(x) = f(0)$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\tan 2x-\sin 2x}{bx^3}=-a$$

$$\Rightarrow \lim_{x\to 0^+}\frac{\frac{8x^3}{3}+\frac{8x^3}{3!}}{bx^3}=-a$$

$$\Rightarrow 8\left(\frac{1}{3}+\frac{1}{3!}\right)=-ab$$

$\Rightarrow 4 = -ab$

$\Rightarrow 10 - ab = 14$