FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Two projectiles are thrown with same initial velocity making an angle of 45° and 30° with the horizontal respectively. The ratio of their respective ranges will be

(A) $1:\sqrt{2}$ (B) $\sqrt{2}:1$

(C) $2:\sqrt{3}$ (D) $\sqrt{3}:2$

Official Ans. by NTA (C)

Sol. Let projection speed is u

$$R_1=\frac{u^2\text{Sin}(90^\circ)}{g};R_2=\frac{u^2\sin(60^\circ)}{g}$$

$$\frac{R_1}{R_2}=\frac{2}{\sqrt{3}}$$

2. In a Vernier Calipers. 10 divisions of Vernier scale is equal to the 9 divisions of main scale. When both jaws of Vernier calipers touch each other, the zero of the Vernier scale is shifted to the left of zero of the main scale and 4^{th} Vernier scale division exactly coincides with the main scale reading. One main scale division is equal to 1 mm. While measuring diameter of a spherical body, the body is held between two jaws. It is now observed that zero of the Vernier scale lies between 30 and 31 divisions of main scale reading and 6^{th} Vernier scale division exactly. coincides with the main scale reading. The diameter of the spherical body will be :

(A) 3.02 cm (B) 3.06 cm

(C) 3.10 cm (D) 3.20 cm

Official Ans. by NTA (C)

Sol. 1 M.S.D = 1mm

9 M.S.D = 10 V.S.D

1 V.S.D = 0.9 M.S.D = 0.9 mm

L.C of vernier caliper = 1–0.9 = 0.1 mm = 0.01 cm

zero error = –(10–4)×0.1mm = – 0.6 mm

Reading = M.S.R + V.S.R – Zero error

= 3cm + 6 × 0.01 – [–0.06]

= 3 + 0.06 + 0.06

= 3.12 cm

Nearest given answer in the options is 3.10

3. A ball of mass 0.15 kg hits the wall with its initial speed of 12 ms^{-1} and bounces back without changing its initial speed. If the force applied by the wall on the ball during the contact is 100 N. calculate the time duration of the contact of ball with the wall.

(A) 0.018 s (B) 0.036 s

(C) 0.009 s (D) 0.072 s

Official Ans. by NTA (B)

Sol. $\vec{P}_i=0.15\times12(\hat{i})$

$\vec{P}_f=0.15\times12(-\hat{i})$

$|\overrightarrow{\Delta P}|=3.6\ \text{kg}-\text{m/s}$

$3.6=F\Delta t$

$3.6=100\ \Delta t$

$\Delta t=0.036$ sec

4. A body of mass 8 kg and another of mass 2 kg are moving with equal kinetic energy. The ratio of their respective momenta will be :

(A) 1:1 (B) 2:1 (C) 1:4 (D) 4:1

Official Ans. by NTA (B)

Sol. $\text{K.E}=\frac{P^2}{2m}$

$K_1=\frac{P_1^2}{2(8)}$; $K_2=\frac{P_2^2}{2(2)}$

$K_1=K_2$

So,

$4P_2^2=P_1^2$

$\frac{P_1}{P_2}=2$

5. Two uniformly charged spherical conductors A and B of radii 5 mm and 10 mm are separated by a distance of 2 cm. If the spheres are connected by a conducting wire, then in equilibrium condition, the ratio of the magnitudes of the electric fields at the surface of the sphere A and B will be :

(A) 1 : 2 (B) 2 : 1 (C) 1 : 1 (D) 1 : 4

Official Ans. by NTA (B)

Sol. $V_A = V_B$

$$\frac{KQ_A}{R_A} = \frac{KQ_B}{R_B}$$

$$\frac{Q_A}{Q_B} = \frac{R_A}{R_B} = \frac{1}{2}$$

$$E_A = \frac{KQ_A}{R_A^{\ 2}}; E_B = \frac{KQ_B}{R_B^{\ 2}}$$

$$\frac{E_A}{E_B} = \frac{Q_A}{Q_B} \times \frac{R_B^{\ 2}}{R_A^{\ 2}} = \frac{R_B}{R_A} = \frac{2}{1}$$

6. The oscillating magnetic field in a plane electromagnetic wave is given by $B_y = 5 \times 10^{-6}$ sin 1000π $(5x - 4 \times 10^8\, t)$T. The amplitude of electric field will be :

(A) 15×10^2 Vm^{-1} (B) 5×10^{-6} Vm^{-1}

(C) 16×10^{12} Vm^{-1} (D) 4×10^2 Vm^{-1}

Official Ans. by NTA (D)

Sol. $B_0 = 5 \times 10^{-6}$

$$v = \text{Speed of wave} = \frac{4 \times 10^8}{5} = 8 \times 10^7 \left[\therefore v = \frac{w}{k}\right]$$

$E_0 = vB_0 = 40 \times 10^1$

$= 4 \times 10^2$ V/m

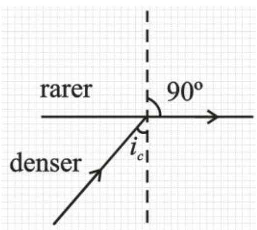
7. Light travels in two media M_1 and M_2 with speeds 1.5×10^8 ms^{-1} and 2.0×10^8 ms^{-1} respectively. The critical angle between them is:

(A) $\tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$ (B) $\tan^{-1}\left(\frac{2}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{4}\right)$ (D) $\sin^{-1}\left(\frac{2}{3}\right)$

Official Ans. by NTA (A)

Sol.



$$v = \frac{c}{n}$$

$n_d \sin i_c = n_r \sin 90°$

$$\sin i_c = \frac{n_r}{n_d} = \frac{v_d}{v_r}$$

$$\sin i_c = \frac{1.5 \times 10^8}{2 \times 10^8} = \frac{1.5}{2}$$

$$\sin i_c = \frac{3}{4}$$

$$\tan i_c = \frac{3}{\sqrt{4^2 - 3^2}} \Rightarrow \frac{3}{\sqrt{7}}$$

$$i_c = \tan^{-1}\left(\frac{3}{\sqrt{7}}\right)$$

8. A body is projected vertically upwards from the surface of earth with a velocity equal to one third of escape velocity. The maximum height attained by the body will be:

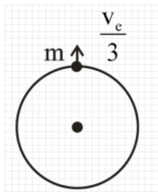
(Take radius of earth = 6400 km and g=10 ms^{-2})

(A) 800 km (B) 1600 km

(C) 2133 km (D) 4800 km

Official Ans. by NTA (A)

Sol.



$$V_e = \sqrt{\frac{2Gm}{R}}$$

$$\frac{-GMm}{R} + \frac{1}{2}m\frac{V_e^{\ 2}}{9} = -\frac{GMm}{R+h}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{V_e^{\ 2}}{18}$$

$$\frac{GM}{R+h} = \frac{GM}{R} - \frac{GM}{9R}$$

$$\frac{GM}{R+h} = \frac{8GM}{9R}$$

$$\frac{1}{R+h} = \frac{8}{9R}$$

$9R = 8R + 8h$

$$h = \frac{R}{8} \Rightarrow \frac{6400}{8} \Rightarrow 800\text{km}$$

9. The maximum and minimum voltage of an amplitude modulated signal are 60 V and 20 V respectively. The percentage modulation index will be :

(A) 0.5% (B) 50% (C) 2% (D) 30%

Official Ans. by NTA (B)

Sol. $V_{max} = 60$

$V_{min} = 20$

% modulation =

$$\left(\frac{V_{max} - V_{min}}{V_{max} + V_{min}}\right)100 \Rightarrow \left(\frac{60-20}{60+20}\right)100 \Rightarrow \left(\frac{40}{80}\right)100$$

$\Rightarrow$ 50%

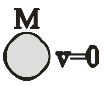
10. A nucleus of mass M at rest splits into two parts having masses $\frac{M'}{3}$ and $\frac{2M'}{3}$ $(M' < M)$. The ratio of de Broglie wavelength of two parts will be :

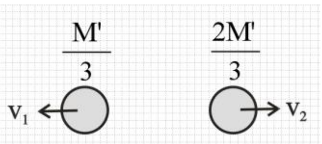
(A) 1 : 2 (B) 2 : 1

(C) 1 : 1 (D) 2 : 3

Official Ans. by NTA (C)

Sol.





$|\vec{P}_1| = |\vec{P}_2|$

Here $\vec{P}$ is momentum

So $\lambda = \frac{h}{P}$

Hence both will have same de broglie wavelength.

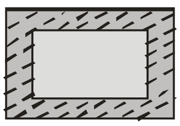
11. An ice cube of dimensions 60 cm × 50 cm × 20 cm is placed in an insulation box of wall thickness 1 cm. The box keeping the ice cube at 0°C of temperature is brought to a room of temperature 40°C. The rate of melting of ice is approximately: (Latent heat of fusion of ice is 3.4×10^5 J kg^{-1} and thermal conducting of insulation wall is $0.05\ Wm^{-1}°C^{-1}$)

(A) 61×10^{-1} kg s^{-1} (B) 61×10^{-5} kg s^{-1}

(C) 208 kg s^{-1} (D) 30×10^{-5} kg s^{-1}

Official Ans. by NTA (B)

Sol.



$$\frac{dQ}{dt} = \frac{KA\Delta T}{\ell}$$

$A = 2\,(0.6 \times 0.5 + 0.5 \times 0.2 + 0.2 \times 0.6)$

$= 2(0.3 + 0.1 + 0.12)$

$= 2(0.4 + 0.12)$

$= 2(0.52)$

$= 1.04\ m^2$

$$R_{th} = \frac{\ell}{KA} \Rightarrow \frac{1\times10^{-2}}{0.05\times1.04} \Rightarrow \frac{10^{-2}}{0.052}$$

$$\frac{dQ}{dt} = \frac{\Delta T}{R_{th}} \Rightarrow \frac{40\times0.052}{10^{-2}} \Rightarrow 2.08\times10^2\ J/s$$

$2.08 \times 10^2 = m \times 3.4 \times 10^5$

$$m = \frac{2.08}{3.4\times10^3} \Rightarrow 0.61\times10^{-3}\ \text{kg/s}$$

$= 61 \times 10^{-5}$ Kg/s

12. A gas has n degrees of freedom. The ratio of specific heat of gas at constant volume to the specific heat of gas at constant pressure will be :

(A) $\frac{n}{n+2}$ (B) $\frac{n+2}{n}$

(C) $\frac{n}{2n+2}$ (D) $\frac{n}{n-2}$

Official Ans. by NTA (A)

Sol. $C_v = \frac{nR}{2}$ $\quad C_p = \frac{(n+2)R}{2}$

$$\frac{C_v}{C_p} = \frac{n}{n+2}$$

13. A transverse wave is represented by y = 2sin $(\omega t - kx)$ cm. The value of wavelength (in cm) for which the wave velocity becomes equal to the maximum particle velocity, will be ;

(A) 4π (B) 2π

(C) π (D) 2

Official Ans. by NTA (A)

Sol. $y = 2\sin(\omega t - kx)$

Maximum particle velocity = $A\,\omega$

Wave velocity = $\frac{\omega}{k}$

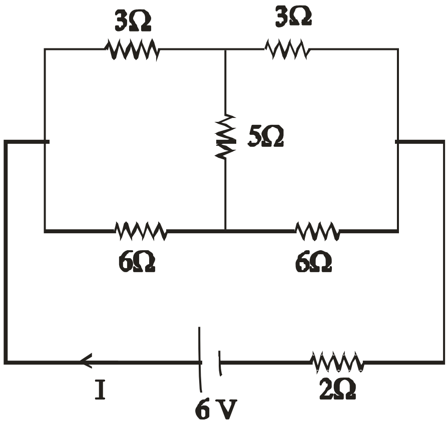
$$\frac{\omega}{k} = A\,\omega$$

$$k = \frac{1}{A} = \frac{2\pi}{\lambda}$$

$\lambda = 2\pi A$

$= 4\,\pi$ cm

14. A battery of 6 V is connected to the circuit as shown below. The current I drawn from the battery is :

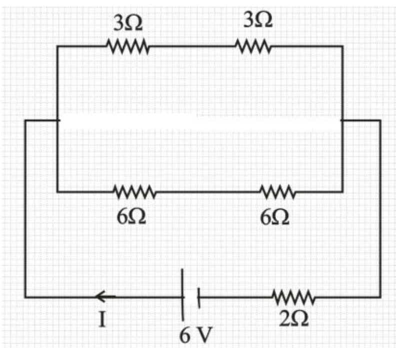


(A) 1A (B) 2A

(C) $\frac{6}{11}$A (D) $\frac{4}{3}$A

Official Ans. by NTA (A)

Sol. Balanced wheat stone bridge in circuit so there is no current in 5 Ω resistor so it can be removed from the circuit.



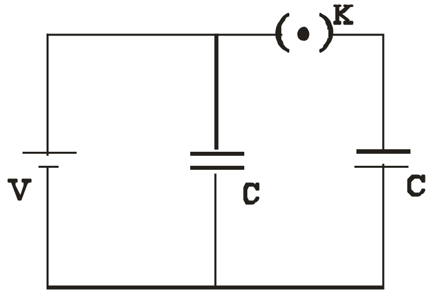
$$R_{eq} = \frac{6\times12}{6+12}+2$$

$$= \frac{6\times12}{18}+2$$

$$R_{eq} = 6\Omega$$

$$I = \frac{V}{R_{eq}} = \frac{6}{6} = 1 \text{ Amp.}$$

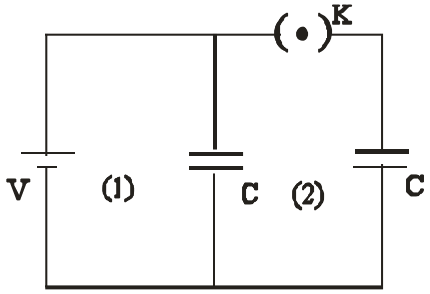
15. A source of potential difference V is connected to the combination of two identical capacitors as shown in the figure. When key 'K' is closed, the total energy stored across the combination is E_1. Now key 'K' is opened and dielectric of dielectric constant 5 is introduced between the plates of the capacitors. The total energy stored across the combination is now E_2. The ratio E_1/E_2 will be :



(A) $\frac{1}{10}$ (B) $\frac{2}{5}$

(C) $\frac{5}{13}$ (D) $\frac{5}{26}$

Official Ans. by NTA (C)



Sol.

(1) Switch is closed

$C_{eq} = 2C$

Energy $E_1 = \frac{1}{2}C_{eq}V^2$

$= \frac{1}{2}2C\times V^2$

$E_1 = CV^2$

(ii) When switch is opened charge on right capacitor remain CV while potential on left capacitor remain same

Dielectric K = 5

$C' = KC$

$C' = 5C$

$$E_2 = \frac{1}{2}(5C)V^2 + \frac{(CV)^2}{2(5C)}$$

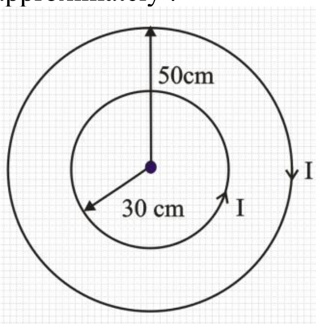
$$E_2 = \frac{5CV^2}{2} + \frac{CV^2}{10}$$

$$E_2 = \frac{13CV^2}{5}$$

$$\frac{E_1}{E_2} = \frac{CV^2}{\frac{13CV^2}{5}} = \frac{5}{13}$$

$$\frac{E_1}{E_2} = \frac{5}{13}$$

16. Two concentric circular loops of radii r_1=30 cm and r_2=50 cm are placed in X-Y plane as shown in the figure. A current I = 7A is flowing through them in the direction as shown in figure. The net magnetic moment of this system of two circular loops is approximately :

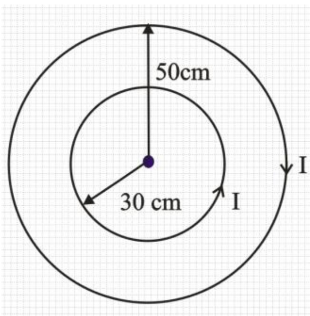


(A) $\frac{7}{2}\hat{k}\ Am^2$ (B) $-\frac{7}{2}\hat{k}\ Am^2$

(C) $7\hat{k}\ Am^2$ (D) $-7\hat{k}\ Am^2$

Official Ans. by NTA (B)

Sol.



Magnetic moment

$$\vec{M} = -i\pi(0.5)^2\hat{k} + i\pi(0.3)^2\hat{k}$$

$$\vec{M} = -7\times\frac{22}{7}\left(\frac{25}{100} - \frac{9}{100}\right)\hat{k}$$

$$= -22\left(\frac{16}{100}\right)\hat{k}$$

$$\vec{M} = -3.52\hat{k}\ \ Am^2$$

$$= -\frac{7}{2}\hat{k}\ \ Am^2$$

17. A velocity selector consists of electric field $\vec{E} = E\hat{k}$ and magnetic field $\vec{B} = B\hat{j}$ with B=12 mT. The value E required for an electron of energy 728 eV moving along the positive x-axis to pass undeflected is :

(Given, mass of electron = 9.1×10^{-31}kg)

(A) 192 kVm^{-1} (B) 192 m Vm^{-1}

(C) 9600 kVm^{-1} (D) 16 kVm^{-1}

Official Ans. by NTA (A)

Sol. $\vec{E} = E\,\hat{k}$ B = 12 mT

$\vec{B} = B\,\hat{j}$ Energy = 728 eV

$$\text{Energy} = \frac{1}{2}mv^2$$

$$728eV = \frac{1}{2}\times 9.1\times10^{-31}\times v^2$$

$$728\times1.6\times10^{-19} = \frac{1}{2}\times9.1\times10^{-31}\times v^2$$

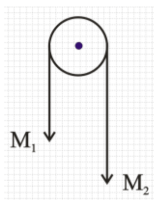
$v = 16 \times 10^6$ m/s

E = vB

$E = 16\times10^6 \times 12 \times 10^{-3}$

$E = 192 \times 10^3$ V/m

18. Two masses M_1 and .M_2 are tied together at the two ends of a light inextensible string that passes over a frictionless pulley. When the mass M_2 is twice that of M_1. the acceleration of the system is a_1. When the mass M_2 is thrice that of M_1. The acceleration of The system is a_2. The ratio $\frac{a_1}{a_2}$ will be:



(A) $\frac{1}{3}$ (B) $\frac{2}{3}$

(C) $\frac{3}{2}$ (D) $\frac{1}{2}$

Official Ans. by NTA (B)

Sol. $a = \frac{m_2g - m_1g}{m_1 + m_2}$

Case 1 $M_2 = 2m_1$

$$a_1 = \frac{2m_1g - m_1g}{3m_1}$$

$a_1 = g/3$

Case -2

$M_2 = 3m_1$

$$a_2 = \frac{3m_1g - m_1g}{4m_1}$$

$$a_2 = \frac{g}{2}$$

$$\frac{a_1}{a_2} = \frac{\frac{g}{3}}{\frac{g}{2}} = \frac{2}{3}$$

19. Mass numbers of two nuclei are in the ratio of 4:3. Their nuclear densities will be in the ratio of

(A) 4:3 (B) $\left(\frac{3}{4}\right)^{\frac{1}{3}}$

(C) 1 : 1 (D) $\left(\frac{4}{3}\right)^{\frac{1}{3}}$

Official Ans. by NTA (C)

Sol. Radius of nucleus $R = R_0 A^{\frac{1}{3}}$

$$\text{Density of nucleus} = \frac{\text{Mass of nucleus}}{\text{volume of nucleus}}$$

$$\rho = \frac{m \times A}{\frac{4}{3}\pi R^3}$$ Where m : mass of proton or neutron

$$\rho = \frac{m \times A}{\frac{4}{3}\pi R_0^3 A}$$

$\rho \propto A^0$

Hence density of nucleus is independent of mass number

20. The area of cross section of the rope used to lift a load by a crane is 2.5 × 10^{-4}m^2. The maximum lifting capacity of the crane is 10 metric tons. To increase the lifting capacity of the crane to 25 metric tons, the required area of cross section of the rope should be :
(take g =10 ms^{-2})
(A) 6.25 × 10^{-4}m^2
(B) 10 × 10^{-4}m^2
(C) 1 × 10^{-4}m^2
(D) 1.67 × 10^{-4}m^2

Official Ans. by NTA (A)

Sol. Since breaking stress (Maximum lifting capacity) is the property of material so it will remain same.

$$\text{breaking stress} = \frac{\text{Maximum lifting capacity}}{\text{Area of cross section of rope}}$$

$$\frac{10}{2.5 \times 10^{-4}} = \frac{25}{A}$$

$A = 625 \times 10^{-6}$

$= 6.25 \times 10^{-4}\ m^2$

SECTION-B

1. If $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})$ m and $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})$ m. The magnitude of component of vector $\vec{A}$ along vector $\vec{B}$ will be________ m.

Official Ans. by NTA (2)

Sol. $\vec{A} = (2\hat{i} + 3\hat{j} - \hat{k})$m and $\vec{B} = (\hat{i} + 2\hat{j} + 2\hat{k})$m

Component of $\vec{A}$ along $\vec{B} = \vec{A} \cdot \hat{B}$

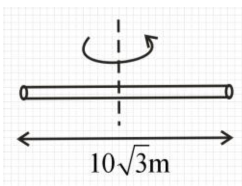
$$= \frac{\vec{A}.\vec{B}}{|\vec{B}|} = \frac{2+6-2}{\sqrt{1^2+2^2+2^2}}$$

$$= \frac{6}{3} = 2$$

2. The radius of gyration of a cylindrical rod about an axis of rotation perpendicular to its length and passing through the center will be_______ m. Given, the length of the rod is $10\sqrt{3}$ m.

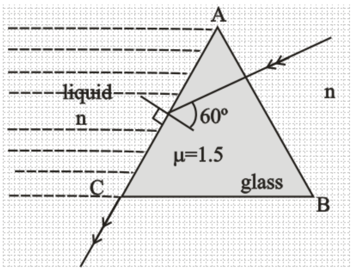
Official Ans. by NTA (5)

Sol.



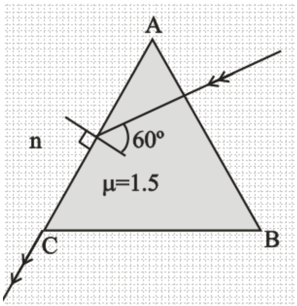
$$I = \frac{m\ell^2}{12} = mk^2 \Rightarrow k^2 = \frac{\ell^2}{12} \Rightarrow k = \frac{\ell}{\sqrt{12}} = \frac{\ell}{2\sqrt{3}} = \frac{10\sqrt{3}}{2\sqrt{3}} = 5$$

3. In the given figure, the face AC of the equilateral prism is immersed in a liquid of refractive index 'n'. For incident angle 60° at the side AC, the refracted light beam just grazes along face AC. The refractive index of the liquid $n = \frac{\sqrt{x}}{4}$. The value of x is_________ .
(Given refractive index of glass = 1.5)



Official Ans. by NTA (27)

Sol.



Using snell's law at face AC

$1.5 \sin 60° = n \times \sin 90°$

$$1.5 \times \frac{\sqrt{3}}{2} = n = \frac{\sqrt{x}}{4}$$

$$3\sqrt{3} = \sqrt{x}$$

$x = 27$

4. Two lighter nuclei combine to form a comparatively heavier nucleus by the relation given below:
$^{2}_{1}X + ^{2}_{1}X = ^{4}_{2}Y$
The binding energies per nucleon $^{2}_{1}X$ and $^{4}_{2}Y$ are 1.1 MeV and 7.6 MeV respectively. The energy released in this process is_______ . MeV.

Official Ans. by NTA (26)

Sol. Energy released in the given process = Binding energy of product – Binding energy of reactants

$= 7.6 \times 4 - (1.1 \times 2) \times 2$

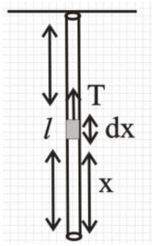
$= 30.4 - 4.4$

$= 26$ MeV

5. A uniform heavy rod of mass 20 kg. Cross sectional area 0.4 m^2 and length 20 m is hanging from a fixed support. Neglecting the lateral contraction, the elongation in the rod due to its own weight is $x \times 10^{-9}$ m. The value of x is_____.
:(Given. Young's modulus $Y = 2 \times 10^{11}$ Nm^{-2} and $g = 10$ ms^{-2})

Official Ans. by NTA (25)

Sol.



$$Y = \frac{T}{A}\frac{dx}{dy}$$

$m = 20$ kg

A = 0.4 m^2

$l = 20m$

let extension is dy in length dx

$$Y = \frac{\text{stress}}{\text{strain}}$$

$$Y = \frac{\frac{T}{A}}{\frac{dy}{dx}} = \frac{T}{A}.\frac{dx}{dy}$$

$$dy = \frac{Tdx}{AY}$$

Tension at a distance x from lower end $= \frac{mg}{\ell}x$

$$\text{So.} \int_0^{\Delta l} dy = \int_0^{\ell} \frac{mg}{\ell} x \frac{dx}{AY}$$

$$\Delta\ell = \frac{mg}{\ell AY}\left[\frac{x^2}{2}\right]_0^{\ell}$$

$$\Delta\ell = \frac{mg\ell}{2AY}$$

$$\Delta\ell = \frac{20 \times 10 \times 20}{2 \times 0.4 \times 2 \times 10^{11}}$$

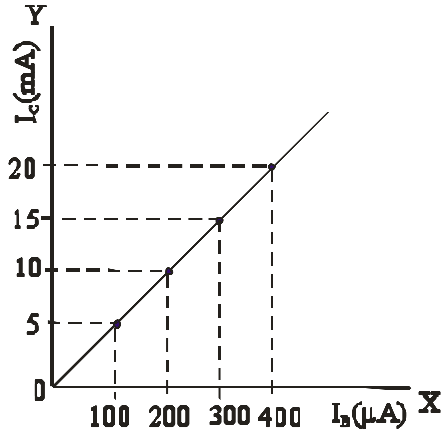
2500×10^{-11}

$\Delta\ell = 25 \times 10^{-9}$

$= x \times 10^{-9}$

$x = 25$

6. The typical transfer characteristic of a transistor in CE configuration is shown in figure. A load resistor of 2 $k\Omega$ is connected in the collector branch of the circuit used. The input resistance of the transistor is 0.50 $k\Omega$. The voltage gain of the transistor is



Official Ans. by NTA (200)

Sol. Current gain in C-E configuration

$$\Rightarrow \beta = \frac{\Delta I_C}{\Delta I_B}$$

$R_C = 2k\Omega$, R_B=0.50 $k\Omega$

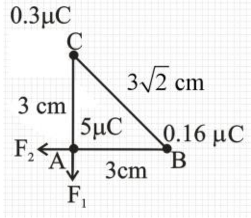
$$\text{Voltage gain} = \frac{\Delta I_C R_C}{\Delta I_B R_B} = \frac{5\times10^{-3}}{100\times10^{-6}} \times \frac{2}{0.5}$$

$$= \frac{10^{-2}}{5\times10^{-5}} = \frac{1000}{5} = 200$$

7. Three point charges of magnitude $5\mu C, 0.16\mu C$ and $0.3\mu C$ are located at the vertices A, B, C of a right angled triangle whose sides are AB = 3cm, $BC = 3\sqrt{2}$ cm and CA=3 cm and point A is the right angle corner. Charge at point A experiences _______ N of electrostatic force due to the other two charges.

Official Ans. by NTA (17)

Sol.



$$F_1 = \frac{k\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= \frac{9\times10^9\times5\times0.3\times10^{-12}}{9\times10^{-4}}$$

$$= 1.5\times10 = 15 \text{ N}$$

$$F_2 = \frac{9\times10^9\times5\times0.16\times10^{-12}}{9\times10^{-4}} = 8\text{N}$$

force experienced by charge at $A = \sqrt{F_1^2 + F_2^2}$

$$= \sqrt{15^2 + 8^2}$$

$$= \sqrt{289} = 17 \text{ N}$$

8. In a coil of resistance 8Ω, the magnetic flux due to an external magnetic field varies with time as $\phi = \frac{2}{3}(9 - t^2)$. The value of total heat produced in the coil, till the flux becomes zero, will be_______J.

Official Ans. by NTA (2)

Sol. $\phi = \frac{2}{3}(9 - t^2) = 0$

t = 3 sec

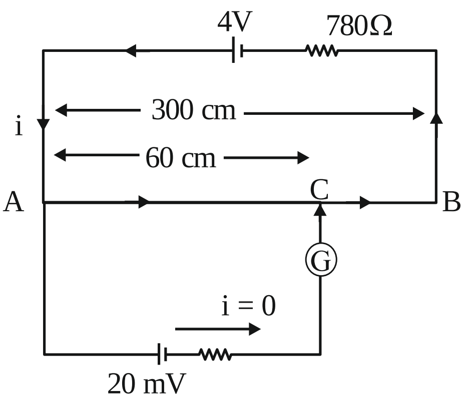
$$e = \frac{-d\phi}{dt} = -\frac{2}{3}(0 - 2t) = \frac{4t}{3}$$

Heat produced in 3 sec = $\int \frac{e^2}{r} dt = \int_0^3 \frac{16t^2}{9\times8} dt = 2J$

9. A potentiometer wire of length 300 cm is connected in series with a resistance 780 Ω and a standard cell of emf 4V. A constant current flows through potentiometer wire. The length of the null point for cell of emf 20 mV is found to be 60 cm. The resistance of the potentiometer wire is____ Ω .

Official Ans. by NTA (20)

Sol.



Let resistance of potentiometers wire is R

$$i = \frac{4}{R+780}$$

Potential difference across AB

$$= \frac{4R}{R+780}$$

Potential difference across AC

$$= \frac{4R \times 60}{(R+780)\times 300} = \frac{4R}{5(R+780)}$$

This should be equal to 20 mV

$$\frac{4R}{5(R+780)} = 20\times 10^{-3} = 2\times 10^{-2}$$

$$4R = 10^{-1}(R + 780)$$

$$4R = \frac{R}{10} + 78$$

$$4R - \frac{R}{10} = 78$$

$$\frac{39R}{10} = 78$$

$$\boxed{R = 20\Omega}$$

10. As per given figures, two springs of spring constants K and 2K are connected to mass m. If the period of oscillation in figure (a) is 3s, then the period of oscillation in figure (b) will be $\sqrt{x}$ s. The value of x is__________.

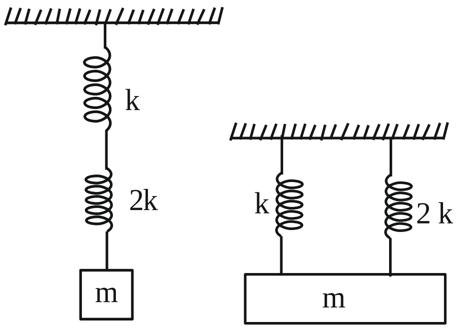


figure (a) figure (b)

Official Ans. by NTA (2)

Sol.

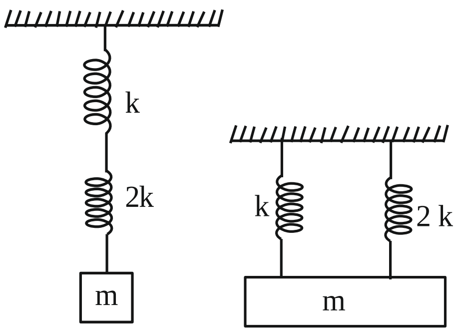


figure (a) figure (b)

For figure (a) :

$$K_{eq} = \frac{K\times 2K}{K+2K} = \frac{2K}{3}$$

$$T = 2\pi\sqrt{\frac{m}{K_{eq}}} = 2\pi\sqrt{\frac{m}{2K/3}} = 2\pi\sqrt{\frac{3m}{2K}}$$

For figure (b):

$$K_{eq} = 3K,\ T' = 2\pi\sqrt{\frac{m}{3K}}$$

$$\frac{T'}{T} = \sqrt{\frac{m\times 2K}{3K\times 3m}} = \frac{\sqrt{2}}{3}$$

$$T' = \sqrt{2}$$

$$x = 2$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. Hemoglobin contains 0.34% of iron by mass. The number of Fe atoms in 3.3 g of hemoglobin is : (Given : Atomic mass of Fe is 56 u, N_A in 6.022 × 10^{23} mol^{-1})

(A) 1.21×10^5 (B) 12.0×10^{16}

(C) 1.21×10^{20} (D) 3.4×10^{22}

Official Ans. by NTA (C)

Sol. No. of Fe atoms = $\frac{0.34}{100} \times \frac{3.3}{56} \times 6.022 \times 10^{23}$

$= 1.206 \times 10^{20}$

2. Arrange the following in increasing order of their covalent character.

(A) CaF_2 (B) $CaCl_2$

(C) $CaBr_2$ (D) CaI_2

Choose the correct answer from the options given below.

(A) B < A < C < D (B) A < B < C < D

(C) A < B < D < C (D) A < C < B < D

Official Ans. by NTA (B)

Sol. According to Fajan's rule,

Covalent character ∝ size of Anion

3. Class XII students were asked to prepare one litre of buffer solution of pH 8.26 by their chemistry teacher. The amount of ammonium chloride to be dissolved by the student in 0.2 M ammonia solution to make one litre of the buffer is (Given pK_b (NH_3) = 4.74; Molar mass of NH_3 = 17 g mol^{-1}; Molar mass of NH_4Cl = 53.5 g mol^{-1})

(A) 53.5 g (B) 72.3 g

(C) 107.0 g (D) 126.0 g

Official Ans. by NTA (C)

Sol. POH = 14 – 8.26

$= pK_b + \log\frac{[NH_4^+]}{[NH_3]}$

$= 5.74 = 4.74 + \log\frac{[NH_4^+]}{0.2} \Rightarrow [NH_4^+] = 2$

Hence

$NH_4Cl = 2 \times 53.5 = 107$ g

4. At 30°C, the half life for the decomposition of AB_2 is 200 s and is independent of the initial concentration of AB_2. The time required for 80% of the AB_2 to decompose is (Given: log 2 = 0.30; log 3 = 0.48)

(A) 200 s (B) 323 s

(C) 467 s (D) 532 s

Official Ans. by NTA (C)

Sol. $T_{1/2}$ = 200 s and 1st order reaction

$$K = \frac{2.303\log 2}{200} = \frac{2.303}{t}\log\frac{A_0}{0.2A_0}$$

$$\frac{\log 2}{200} = \frac{1}{t}\log 5$$

$$t = \frac{7}{3} \times 200 = 466.67\text{s} = 467 \text{ s}$$

5. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Finest gold is red in colour, as the size of the particles increases, it appears purple then blue and finally gold.

Assertion R : The colour of the colloidal solution depends on the wavelength of light scattered by the dispersed particles.

In the light of the above statements, choose the most appropriate answer from the options given below;

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (A)

6. The metal that has very low melting point and its periodic position is closer to a metalloid is :

(A) Al (B) Ga

(C) Se (D) In

Official Ans. by NTA (B)

Sol. **Melting point**

Al → 933 K

Ga → 303 K

In → 430 K

Se → 490 K

7. The metal that is not extracted from its sulphide ore is :

(A) Aluminium (B) Iron

(C) Lead (D) Zinc

Official Ans. by NTA (A)

Sol. Al is extracted from $Al_2O_3 \cdot 2H_2O$ i.e., Bauxite ore

8. The products obtained from a reaction of hydrogen peroxide and acidified potassium permanganate are

(A) Mn^{4+}, H_2O only (B) Mn^{2+}, H_2O only

(C) Mn^{4+}, H_2O, O_2 only (D) Mn^{2+}, H_2O, O_2 only

Official Ans. by NTA (D)

Sol. $6H^+ + 2MnO_4^- + 5H_2O_2 \longrightarrow 2Mn^{+2} + 8H_2O + 5O_2$

9. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : LiF is sparingly soluble in water.

Reason R : The ionic radius of Li^+ ion is smallest among its group members, hence has least hydration enthalpy.

In the light of the above statements, choose the most appropriate answer from the options given below .

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Due to high lattice energy LiF is sparingly soluble in water. Li^+ has high hydration energy among its group members due to smallest size.

10. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Boric acid is a weak acid

Reason R : Boric acid is not able to release H^+ ion on its own. It receives OH^- ion from water and releases H^+ ion.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A

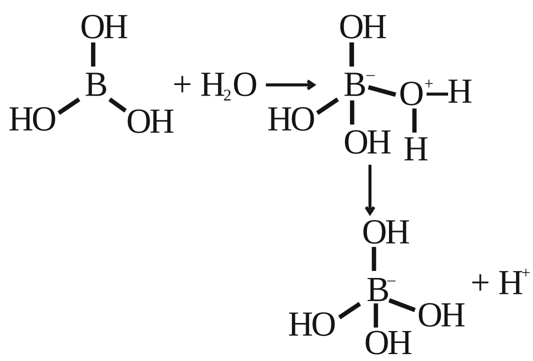
(B) Both A and R are correct but R is NOT the correct explanation of A

(C) A is correct but R is not correct

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol.



11. The metal complex that is diamagnetic is (Atomic number : Fe, 26; Cu, 29)

(A) $K_3[Cu(CN)_4]$

(B) $K_2[Cu(CN)_4]$

(C) $K_3[Fe(CN)_4]$

(D) $K_4[FeCl_6]$

Official Ans. by NTA (A)

Sol. $K_3[Cu(CN)_4]$

O.N. of copper is Cu^{+1}

$Cu^{+1} = [Ar]3d^{10} \Rightarrow$ Diamagnetic

12. Match List I with List II

List I **Pollutant**	**List II** **Source**
A. Microorganisms	I. Strip mining
B. Plant nutrients	II. Domestic sewage
C. Toxic heavy metals	III. Chemical fertilizer
D. Sediment	IV. Chemical factory

Choose the correct answer from the options given below :

(A) A-II, B-III, C-IV, D-I

(B) A-II, B-I, C-IV, D-III

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-IV, C-III, D-II

Official Ans. by NTA (A)

Sol.

List I Pollutant	List II Source
A. Microorganisms	Domestic sewage
B. Plant nutrients	Chemical fertilizer
C. Toxic heavy metals	Chemical factory
D. Sediment	Strip mining

13. The correct decreasing order of priority of functional groups in naming an organic compound as per IUPAC system of nomenclature is :

(A) $—COOH > —CONH_2 > —COCl > —CHO$

(B) $—SO_3H > —COCl > —CONH_2 > —CN$

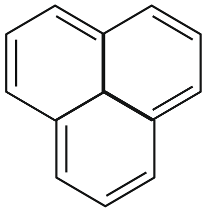
(C) $—COOR > —COCl > —NH_2 > \rangle C = o$

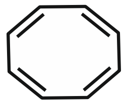
(D) $—COOH > —COOR > —CONH_2 > —COCl$

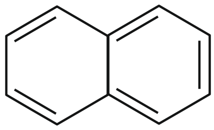
Official Ans. by NTA (B)

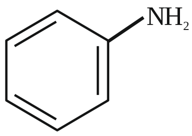
Sol. $—SO_3H > —COCl > —CONH_2 > —CN$

14. Which of the following is not an example of benzenoid compound ?

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

15. Hydrolysis of which compound will give carbolic acid ?

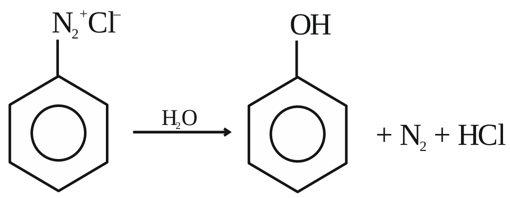
(A) Cumene

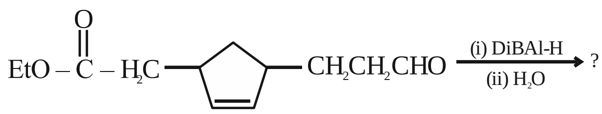
(B) Benzenediazonium chloride

(C) Benzal chloride

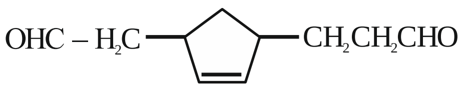
(D) Ethylene glycol ketal

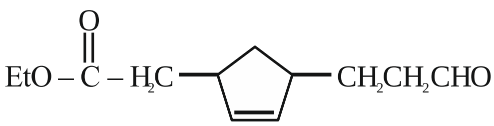
Official Ans. by NTA (B)

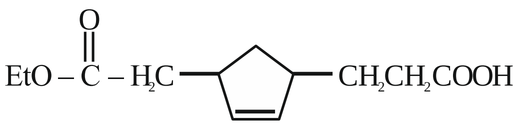
Sol. 

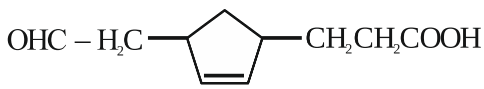
16. 

Consider the above reaction and predict the major product.

(A) 

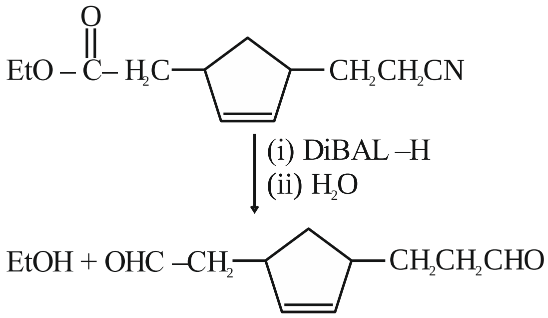
(B) 

(C) 

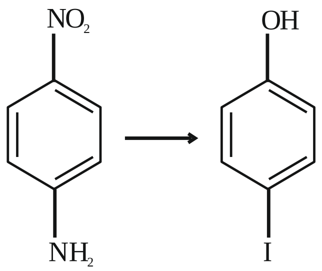
(D) 

Official Ans. by NTA (A)

Sol.



17. The correct sequential order of the reagents for the given reaction is :



(A) HNO_2, Fe/H^+, HNO_2, KI, H_2O/H^+

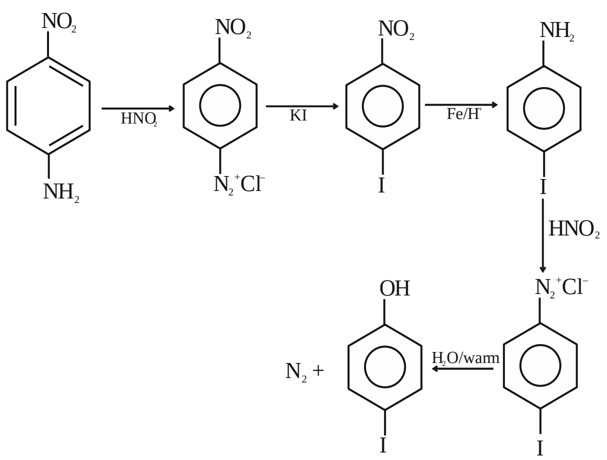
(B) HNO_2, KI, Fe/H^+, HNO_2, H_2O/warm

(C) HNO_2, KI, HNO_2, Fe/H^+, H_2O/H^+

(D) HNO_2, Fe/H^+, KI, HNO_2, H_2O/warm

Official Ans. by NTA (B)

Sol.



18. Vulcanization of rubber is carried out by heating a mixture of :

(A) isoprene and styrene

(B) neoprene and sulphur

(C) isoprene and sulphur

(D) neoprene and styrene

Official Ans. by NTA (C)

Sol. Vulcanization of rubber is carried out by heating a mixture of isoprene & sulphur

19. Animal starch is the other name of :

(A) amylose (B) maltose

(C) glycogen (D) amylopectin

Official Ans. by NTA (C)

Sol. Glycogen

20. Given below are two statements : one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Phenolphthalein is a pH dependent indicator, remains colourless in acidic solution and gives pink colour in basic medium

Reason R : Phenolphthalein is a weak acid. It doesn't dissociate in basic medium.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both A and R are true and R is the correct explanation of A

(B) Both A and R are true but R is NOT the correct explanation of A.

(C) A is true but R is false

(D) A is false but R is true

Official Ans. by NTA (C)

Sol. Phenolphthalein dissociate in basic medium

$$HPh(aq) \rightleftharpoons H^+ + Ph^-$$

(colourless) (Pink)

SECTION-B

1. A 10 g mixture of hydrogen and helium is contained in a vessel of capacity 0.0125 m^3 at 6 bar and 27°C. The mass of helium in the mixture is ________ g. (nearest integer)

Given : R = 8.3 $JK^{-1}mol^{-1}$ (Atomic masses of H and He are 1u and 4u, respectively)

Official Ans. by NTA (8)

Sol. $PV = n_{mix}RT$

$$n_{mix} = \frac{6 \times 12.5}{0.083 \times 300} \approx 3$$

Let mole of He = x

Mole of $H_2 = 3 - x$

$4x + 2(3 - x) = 10$

$\boxed{x = 2mol}$

Mass of He = 8g

2. Consider an imaginary ion ${}^{48}_{22}X^{3-}$. The nucleus contains 'a'% more neutrons than the number of electrons in the ion. The value of 'a' is ____. [nearest integer]

Official Ans. by NTA (4)

Sol. ${}^{48}_{22}X^{3-}$

No. of neutrons = 26

No. of electrons = 25

% of extra neutrons

than electrons = $\frac{26-25}{25}\times 100 = 4$

3. For the reaction

$H_2F_2(g) \rightarrow H_2(g) + F_2(g)$

$\Delta U = -59.6$ kJ mol^{-1} at 27°C.

The enthalpy change for the above reaction is (–) ___ kJ mol^{-1} [nearest integer] Given : R = 8.314 JK^{-1} mol^{-1}.

Official Ans. by NTA (57)

Sol. $\Delta H = \Delta U + \Delta n_g RT$

$\Delta H = -59.6 + 1 \times 8.314 \times 300 \times 10^{-3} = -57.10$

4. The elevation in boiling point for 1 molal solution of non-volatile solute A is 3K. The depression in freezing point for 2 molal solution of A in the same solvent is 6 K. The ratio of K_b and K_f i.e., K_b/K_f is 1 : X. The value of X is [nearest integer]

Official Ans. by NTA (1)

Sol. $\Delta T_b = iK_bm_1$ $\Delta T_f = iK_fm_2$

$$\frac{\Delta T_b}{\Delta T_f} = \frac{K_b \times 1}{K_f \times 2} \Rightarrow \frac{3}{6} = \frac{1}{2} = \frac{K_b}{K_f} \times \frac{1}{2}$$

$$\frac{K_b}{K_f} = \frac{1}{1} \Rightarrow x = 1$$

5. 20 mL of 0.02 M hypo solution is used for the titration of 10 mL of copper sulphate solution, in the presence of excess of KI using starch as an indicator. The molarity of Cu^{2+} is found to be _____ $\times 10^{-2}$ M [nearest integer]

Given : $2Cu^{2+} + 4I^{-} \rightarrow Cu_2I_2 + I_2$

$I_2 + 2S_2O_3^{2-} \rightarrow 2I^{-} + S_4O_6^{2-}$

Official Ans. by NTA (4)

Sol. $n_{eq.}$ of I_2 = n_{eq} of $Na_2S_2O_3$ = 20 × 0.002 × 1

2 × n_{mol} of I_2 = 0.4

n_{mol} of I_2 = 0.2 m mol

n_{mol} of Cu^{+2} = 0.2 × 2 × 10^{-3}

$$[Cu^{+2}] = \frac{0.4\times 10^{-3}}{10\times 10^{-3}} = 0.04 = 4\times 10^{-2}$$

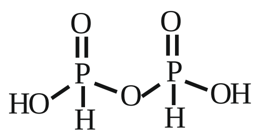
6. The number of non-ionisable protons present in the product B obtained from the following reaction is _____. $C_2H_5OH + PCl_3 \rightarrow C_2H_5Cl + A$

$A + PCl_3 \rightarrow B$

Official Ans. by NTA (2)

Sol. $C_2H_5OH + PCl_3 \longrightarrow C_2H_5Cl + H_3PO_3$

$H_3PO_3 + PCl_3 \longrightarrow H_4P_2O_5 + HCl$



7. The spin-only magnetic moment value of the compound with strongest oxidizing ability among MnF_4, MnF_3 and MnF_2 is ______ B.M. [nearest integer]

Official Ans. by NTA (5)

Sol.

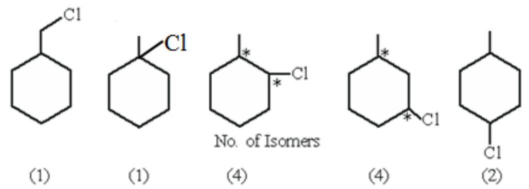
MnF_4	MnF_3	MnF_2
+4	+3	+2
E.C = [Ar]3d^3	[Ar]3d^4	[Ar]3d^5

Hence $MnF_3 \Rightarrow$ strongest O.A

$\mu = \sqrt{4(4+2)} = \sqrt{24} = 4.89 = 5$

8. Total number of isomers (including stereoisomers) obtain on monochlorination of methylcyclohexane is ________.

Official Ans. by NTA (12)

Sol. 

9. A 100 mL solution of CH_3CH_2MgBr on treatment with methanol produces 2.24 mL of a gas at STP. The weight of gas produced is _______ mg. [nearest integer]

Official Ans. by NTA (3)

Sol. $CH_3–CH_2–MgBr + CH_3OH \longrightarrow$

$CH_3–CH_3 + Mg\begin{smallmatrix} OCH_3 \\ Br \end{smallmatrix}$

$$n = \frac{2.24 \times 10^{-3}}{22.4} = 10^{-4}$$

$W = n \times M$

$= 10^{-4} \times 30 = 3$ mg

10. How many of the following drugs is/are example(s) of broad spectrum antibiotic ?
Ofloxacin, Penicillin G, Terpineol, Salvarsan

Official Ans. by NTA (1)

Sol. Ofloxacin

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Tuesday 26th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. The minimum value of the sum of the squares of the roots of $x^2+(3-a)x+1=2a$ is:

(A) 4 (B) 5

(C) 6 (D) 8

Official Ans. by NTA (C)

Sol. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

let $f(a) = (3 - a)^2 - 2(1 - 2a)$

$f(a) = a^2 - 2a + 7$

$f(a) = (a - 1)^2 + 6$

$f(a))_{min.} = 6$

2. If $z = x + iy$ satisfies $|z| - 2 = 0$ and $|z-i|-|z+5i|=0$, then

(A) $x + 2y - 4 = 0$ (B) $x^2 + y - 4 = 0$

(C) $x + 2y + 4 = 0$ (D) $x^2 - y + 3 = 0$

Official Ans. by NTA (C)

Sol. $|z-i|-|z+5i|=0$

$\Rightarrow |x + (y - 1)i| = |x + (y + 5)i|$

$x^2 + (y - 1)^2 = x^2 + (y + 5)^2$

$(y - 1)^2 - (y + 5)^2 = 0$

$(2y + 4)(-6) = 0$

$y = -2$

$\therefore x^2 + (-2)^2 = 4$

$x = 0$

$Z \equiv (0, -2)$, check options

3. Let $A = \begin{bmatrix}1\\1\\1\end{bmatrix}$ and $B = \begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}$, then the value of $A'BA$ is:

(A) 1224 (B) 1042 (C) 540 (D) 539

Official Ans. by NTA (D)

Sol. $A'BA = [1\ \ 1\ \ 1]\begin{bmatrix}9^2 & -10^2 & 11^2\\ 12^2 & 13^2 & -14^2\\ -15^2 & 16^2 & 17^2\end{bmatrix}\begin{bmatrix}1\\1\\1\end{bmatrix}$

$=[9^2+12^2-15^2\quad -10^2+13^2+16^2\quad 11^2-14^2+17^2]\begin{bmatrix}1\\1\\1\end{bmatrix}$

$= [9^2+12^2-15^2 -10^2+13^2+16^2 + 11^2-14^2+17^2]$

$= [539]$

4. $\sum_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\ {}^nC_j$ is equal to

(A) $2^{2n} - {}^{2n}C_n$ (B) $2^{2n-1} - {}^{2n-1}C_{n-1}$

(C) $2^{2n} - \frac{1}{2}\,{}^{2n}C_n$ (D) $2^{n-1} + {}^{2n-1}C_n$

Official Ans. by NTA (B)

Sol. $\sum_{\substack{i,j=0\\ i\neq j}}^{n} {}^nC_i\ {}^nC_j$

$= \sum_{i=0}^{n} {}^nC_i \cdot \sum_{j=0}^{n} {}^nC_j - \sum_{i=j=0}^{n} \left({}^nC_i\right)^2$

$= (2^n)(2^n) - {}^{2n}C_n$

$= 2^{2n} - {}^{2n}C_n$

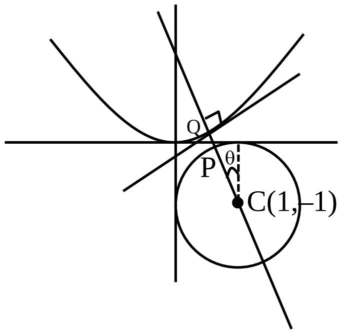
5. Let P and Q be any points on the curves $(x-1)^2+(y+1)^2=1$ and $y = x^2$, respectively. The distance between P and Q is minimum for some value of the abscissa of P in the interval

(A) $\left(0,\frac{1}{4}\right)$ (B) $\left(\frac{1}{2},\frac{3}{4}\right)$

(C) $\left(\frac{1}{4},\frac{1}{2}\right)$ (D) $\left(\frac{3}{4},1\right)$

Official Ans. by NTA (C)

Sol.



$Q = (t, t^2)$

$m_{CQ} = m_{normal}$

$\frac{t^2+1}{t-1} = -\frac{1}{2t}$

Let $f(t) = 2t^3 + 3t - 1$

$f\left(\frac{1}{4}\right) f\left(\frac{1}{3}\right) < 0 \Rightarrow t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$P \equiv (1 + \cos(90+\theta), -1 + \sin(90+\theta))$

$P = (1 - \sin\theta, -1 + \cos\theta)$

$m_{normal} = m_{CP} \Rightarrow -\frac{1}{2t} = \frac{\cos\theta}{-\sin\theta} \Rightarrow \tan\theta = 2t$

$x = 1 - \sin\theta = 1 - \frac{2t}{\sqrt{1+4t^2}} = g(t) \ \ (\text{let})$

$\Rightarrow g'(t) < 0$

$g(t) \downarrow$ function

$t \in \left(\frac{1}{4}, \frac{1}{3}\right)$

$\Rightarrow g(t) \in (0.44, 0.485) \in \left(\frac{1}{4}, \frac{1}{2}\right)$

6. If the maximum value of a, for which the function $f_a(x) = \tan^{-1} 2x - 3ax + 7$ is non-decreasing in $\left(-\frac{\pi}{6}, \frac{\pi}{6}\right)$, is $\bar{a}$, then $f_{\bar{a}}\left(\frac{\pi}{8}\right)$ is equal to

(A) $8 - \frac{9\pi}{4(9+\pi^2)}$ (B) $8 - \frac{4\pi}{9(4+\pi^2)}$

(C) $8\left(\frac{1+\pi^2}{9+\pi^2}\right)$ (D) $8 - \frac{\pi}{4}$

Official Ans. by NTA (A)

Sol. $f_a(x) = \tan^{-1} 2x - 3ax + 7$

$f_a'(x) = \frac{2}{1+4x^2} - 3a \ge 0$

$a \le \left(\frac{2}{3(1+4x^2)}\right)_{min.}$ at $x = \pm\frac{\pi}{6}$

$a_{max} = \bar{a} = \frac{6}{9+\pi^2}$

$f_{\bar{a}}(\frac{\pi}{8}) = \tan^{-1}\frac{\pi}{4} - 3\frac{6}{9+\pi^2}\frac{\pi}{8} + 7 = \tan^{-1}\frac{\pi}{4} - \frac{9\pi}{4(\pi^2+9)} + 7$

7. Let $\beta = \lim_{x\to 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$ for some $\alpha \in \mathbb{R}$. Then the value of $\alpha + \beta$ is :

(A) $\frac{14}{5}$ (B) $\frac{3}{2}$ (C) $\frac{5}{2}$ (D) $\frac{7}{2}$

Official Ans. by NTA (C)

Sol. $\beta = \lim_{x\to 0} \frac{\alpha x - (e^{3x} - 1)}{\alpha x (e^{3x} - 1)}$

$\beta = \lim_{x\to 0} \frac{1 + \alpha x - \left[1 + 3x + \frac{9x^2}{2!} + ...\right]}{(\alpha x)\frac{(e^{3x}-1)}{3x}3x}$

$\beta = \lim_{x\to 0} \frac{(\alpha x - 3x) - \frac{9x^2}{2!} - ....}{3\alpha x^2}$

For existence of limit $\alpha - 3 = 0$

$\alpha = 3$

Limit $\beta = \frac{-3}{2\alpha}$

$\beta = -\frac{1}{2}$

Now,

$\alpha + \beta = \frac{5}{2}$

8. The value of $\log_e 2 \frac{d}{dx}(\log_{\cos x} \operatorname{cosec} x)$ at $x = \frac{\pi}{4}$ is

(A) $-2\sqrt{2}$ (B) $2\sqrt{2}$ (C) -4 (D) 4

Official Ans. by NTA (D)

Sol. $\log_e 2 \frac{d}{dx}(\log_{\cos x} \text{cosec}x)$

Let,

$$y = \log_{\cos x} \text{cosec}x$$

$$y = -\frac{\ln(\sin x)}{\ln(\cos x)}$$

$$\frac{dy}{dx} = -\frac{[\cot x \cdot \ln(\cos x) + \tan x \cdot \ln(\sin x)]}{(\ln(\cos x))^2}$$

$$\left.\frac{dy}{dx}\right)_{x=\frac{\pi}{4}} = \frac{4}{\ln 2}$$

Now,

$$\Rightarrow \log_e 2 \cdot \frac{4}{\ln 2} = 4$$

9. $\int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$ is equal to :-

(A) $10(\pi+4)$ (B) $10(\pi+2)$

(C) $20(\pi-2)$ (D) $20(\pi+2)$

Official Ans. by NTA (D)

Sol. $I = \int_0^{20\pi} (|\sin x| + |\cos x|)^2 dx$; (Jack property)

$$I = 40\int_0^{\pi/2} (\sin x + \cos x)^2 dx$$

$$I = 40\int_0^{\pi/2} (1 + \sin 2x)dx$$

$I = 20[\pi + 2]$

10. Let the solution curve $y = f(x)$ of the differential equation $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}, x \in (-1,1)$ pass through the origin. Then $\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx$ is equal to

(A) $\frac{\pi}{3} - \frac{1}{4}$ (B) $\frac{\pi}{3} - \frac{\sqrt{3}}{4}$

(C) $\frac{\pi}{6} - \frac{\sqrt{3}}{4}$ (D) $\frac{\pi}{6} - \frac{\sqrt{3}}{2}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx} + \frac{xy}{x^2-1} = \frac{x^4+2x}{\sqrt{1-x^2}}$

I.F = $e^{\int \frac{x}{x^2-1}dx}$

I.F = $\sqrt{1-x^2}$

Solution of D.E.

$$y \cdot \sqrt{1-x^2} = \int \frac{x^4+2x}{\sqrt{1-x^2}} \cdot \sqrt{1-x^2}dx$$

$$y \cdot \sqrt{1-x^2} = \int (x^4+2x)dx$$

$$y \cdot \sqrt{1-x^2} = \frac{x^5}{5} + x^2 + C$$

At $x = 0, y = 0$, get $C = 0$

$$y = \frac{x^5}{5\sqrt{1-x^2}} + \frac{x^2}{\sqrt{1-x^2}}$$

Now,

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^5}{5\sqrt{1-x^2}}dx + \int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = 0 + 2\int_0^{\frac{\sqrt{3}}{2}} \frac{x^2}{\sqrt{1-x^2}}dx$$

$$\int_{-\frac{\sqrt{3}}{2}}^{\frac{\sqrt{3}}{2}} f(x)dx = \frac{\pi}{3} - \frac{\sqrt{3}}{4}$$

11. The acute angle between the pair of tangents drawn to the ellipse $2x^2 + 3y^2 = 5$ from the point (1,3) is

(A) $\tan^{-1}\left(\frac{16}{7\sqrt{5}}\right)$ (B) $\tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$

(C) $\tan^{-1}\left(\frac{32}{7\sqrt{5}}\right)$ (D) $\tan^{-1}\left(\frac{3+8\sqrt{5}}{35}\right)$

Official Ans. by NTA (B)

Sol. Equation of tangent to the ellipse $2x^2 + 3y^2 = 5$ is

$$y = mx \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

It pass through (1, 3)

$$3 = m \pm \sqrt{\frac{5}{2}m^2 + \frac{5}{3}}$$

$$3m^2 + 12m - \frac{44}{3} = 0$$

Let θ be the angle between the tangents

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$\tan\theta = \left|\frac{3\sqrt{320}}{-35}\right|$$

$$\theta = \tan^{-1}\left(\frac{24}{7\sqrt{5}}\right)$$

12. The equation of a common tangent to the parabolas $y = x^2$ and $y = -(x-2)^2$ is

(A) y = 4(x-2) (B) y = 4 (x-1)

(C) y = 4 (x+1) (D) y = 4 (x+2)

Official Ans. by NTA (B)

Sol. Equation of tangent of $y = x^2$ be

$tx = y + at^2$ ....(1)

$$y = tx - \frac{t^2}{4}$$

Solve with $y = -(x-2)^2$

$$tx - \frac{t^2}{4} = -(x-2)^2$$

$$x^2 + x(t-4) - \frac{t^2}{4} + 4 = 0$$

D = 0

$$(t-4)^2 - 4\cdot\left(4 - \frac{t^2}{4}\right) = 0$$

$t^2 - 4t = 0$

t = 0 or t = 4

From eq. (1), required common tangent is

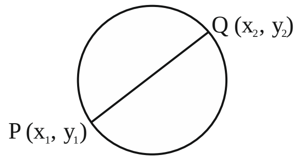
y = 4 (x-1)

13. Let the abscissae of the two points P and Q on a circle be the roots of $x^2 - 4x - 6 = 0$ and the ordinates of P and Q be the roots of $y^2 + 2y - 7 = 0$. If PQ is a diameter of the circle $x^2 + y^2 + 2ax + 2by + c = 0$, then the value of (a+b-c) is

(A) 12 (B) 13 (C) 14 (D) 16

Official Ans. by NTA (A)

Sol.



Equation of circle diameter form

$(x - x_1)(x - x_2) + (y - y_1)(y - y_2) = 0$

(where x_1, x_2 are the roots of $x^2 - 4x - 6 = 0$ and y_1, y_2 are the roots of $y^2 + 2y - 7 = 0$)

$x^2 + y^2 - 4x + 2y - 13 = 0$

Now,

Compare it with the given equation, we get

a = –2, b = 1, c = –13

Now

a + b – c = 12

14. If the line x-1 = 0, is a directrix of the hyperbola $kx^2 - y^2 = 6$, then the hyperbola passes through the point

(A) $\left(-2\sqrt{5}, 6\right)$ (B) $\left(-\sqrt{5}, 3\right)$

(C) $\left(\sqrt{5}, -2\right)$ (D) $\left(2\sqrt{5}, 3\sqrt{6}\right)$

Official Ans. by NTA (C)

Sol. $\frac{x^2}{6/k} - \frac{y^2}{6} = 1$ ......(1)

$$e^2 = 1 + \frac{6}{6/k}$$

$$e = \sqrt{1+k}$$

$$a = \sqrt{\frac{6}{k}}$$

Eq. of directrix $x = \frac{a}{e} \Rightarrow x = \sqrt{\frac{6}{k(k+1)}}$

$$\frac{6}{k(k+1)} = 1$$

k = 2

From eq. (1), we get $2x^2 - y^2 = 6$

Check options

15. A vector $\vec{a}$ is parallel to the line of intersection of the plane determined by the vectors $\hat{i}, \hat{i}+\hat{j}$ and the plane determined by the vectors $\hat{i}-\hat{j}, \hat{i}+\hat{k}$. The obtuse angle between $\vec{a}$ and the vector $\vec{b}=\hat{i}-2\hat{j}+2\hat{k}$ is

(A) $\frac{3\pi}{4}$ (B) $\frac{2\pi}{3}$

(C) $\frac{4\pi}{5}$ (D) $\frac{5\pi}{6}$

Official Ans. by NTA (A)

Sol. $\vec{n}_1 = \hat{i}\times(\hat{i}+\hat{j}) = \hat{k}$

$\vec{n}_2 = (\hat{i}+\hat{k})\times(\hat{i}-\hat{j})$

$= \hat{i}+\hat{j}-\hat{k}$

Line of intersection along $\vec{n}_1 \times \vec{n}_2$

$= \hat{k}\times(\hat{i}+\hat{j}-\hat{k}) = -\hat{i}+\hat{j}$

D.R of $\vec{a} = -\hat{i}+\hat{j}$

D.R of $\vec{b} = \hat{i}-2\hat{j}+2\hat{k}$

$\vec{a}\cdot\vec{b} = -3$ and $(\vec{a}\wedge\vec{b}) = \theta$

$\cos\ \theta = \frac{-3}{\sqrt{2}\times 3}$

$\theta = \frac{3\pi}{4}$

16. If $0 < x < \frac{1}{\sqrt{2}}$ and $\frac{\sin^{-1}x}{\alpha} = \frac{\cos^{-1}x}{\beta}$, then a value of $\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right)$ is

(A) $4\sqrt{(1-x^2)}(1-2x^2)$

(B) $4x\sqrt{(1-x^2)}(1-2x^2)$

(C) $2x\sqrt{(1-x^2)}(1-4x^2)$

(D) $4\sqrt{(1-x^2)}(1-4x^2)$

Official Ans. by NTA (B)

Sol. $\frac{\sin^{-1}x}{\alpha} = \frac{\cos^{-1}x}{\beta} = k$

$\sin^{-1}x = k\alpha$

$\cos^{-1}x = k\beta$

$k = \frac{\pi}{2(\alpha+\beta)}$ ....(i)

$\sin\left(\frac{2\pi\alpha}{\alpha+\beta}\right) = \sin(4\sin^{-1}x)$

$= 2\sin(2\sin^{-1}x)\cos(2\sin^{-1}x)$

$= 4x\sqrt{1-x^2}(1-2x^2)$

17. Negation of the Boolean expression $p \Leftrightarrow (q \Rightarrow p)$ is

(A) $(\sim p)\wedge q$ (B) $p\wedge(\sim q)$

(C) $(\sim p)\vee(\sim q)$ (D) $(\sim p)\wedge(\sim q)$

Official Ans. by NTA (D)

Sol. $\sim(p\leftrightarrow(q\rightarrow p))$

$\sim(p\leftrightarrow q) = (p\wedge\sim q)\vee(q\wedge\sim p)$

$\sim(p\leftrightarrow(q\rightarrow p)) = (p\wedge\sim(q\rightarrow p))\vee((q\rightarrow p)\wedge\sim p)$

$(p\wedge\sim(q\rightarrow p)) = p\wedge(q\wedge\sim p) = (p\wedge\sim p)\wedge q = c$

$(q\rightarrow p)\wedge\sim p = (\sim q\vee p)\wedge\sim p = \sim p\wedge(\sim q\vee p)$

$= (\sim p\wedge\sim q)\vee(\sim p\wedge p) = \sim p\wedge\sim q$

$\sim(p\leftrightarrow(q\rightarrow p)) = c\vee(\sim p\wedge\sim q) = \sim p\wedge\sim q$

18. Let X be a binomially distributed random variable with mean 4 and variance $\frac{4}{3}$. Then 54 P(X$\le$ 2) is equal to

(A) $\frac{73}{27}$ (B) $\frac{146}{27}$

(C) $\frac{146}{81}$ (D) $\frac{126}{81}$

Official Ans. by NTA (B)

Sol. np = 4

npq = 4/3

n = 6, p = 2/3, q = 1/3

54(P(X = 2) + P (X = 1) + P(X = 0))

$54\left({}^6C_2\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^4 + {}^6C_1\left(\frac{2}{3}\right)^1\left(\frac{1}{3}\right)^5 + {}^6C_0\left(\frac{2}{3}\right)^0\left(\frac{1}{3}\right)^6\right)$

$= \frac{146}{27}$

19. The integral $\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$ is equal to

(A) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|+C$

(B) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\left(\frac{x}{2}+\frac{\pi}{3}\right)}\right|+C$

(C) $\log_e\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}\right|+C$

(D) $\frac{1}{2}\log_e\left|\frac{\tan\left(\frac{x}{2}-\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}-\frac{\pi}{6}\right)}\right|+C$

Official Ans. by NTA (A)

Sol. $I=\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(1+\frac{2}{\sqrt{3}}\sin 2x\right)}dx$

$$\frac{\sqrt{3}}{2}\int \frac{\left(1-\frac{1}{\sqrt{3}}\right)(\cos x-\sin x)}{\left(\frac{\sqrt{3}}{2}+\sin 2x\right)}dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2}-\frac{1}{2}\right)(\cos x-\sin x)}{\sin 60^\circ+\sin 2x}dx$$

$$\int \frac{\left(\frac{\sqrt{3}}{2}\cos x-\frac{1}{2}\cos x-\frac{\sqrt{3}}{2}\sin x+\frac{1}{2}\sin x\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\int \frac{\left(\cos\left(x-\frac{\pi}{6}\right)-\sin\left(x+\frac{\pi}{6}\right)\right)}{2\sin\left(x+\frac{\pi}{6}\right)\cos\left(x-\frac{\pi}{6}\right)}dx$$

$$\frac{1}{2}\left(\int\frac{dx}{\sin\left(x+\frac{\pi}{6}\right)}-\int\frac{dx}{\cos\left(x-\frac{\pi}{6}\right)}\right)$$

$$\frac{1}{2}\ln\left|\frac{\tan\left(\frac{x}{2}+\frac{\pi}{12}\right)}{\tan\left(\frac{x}{2}+\frac{\pi}{6}\right)}\right|$$

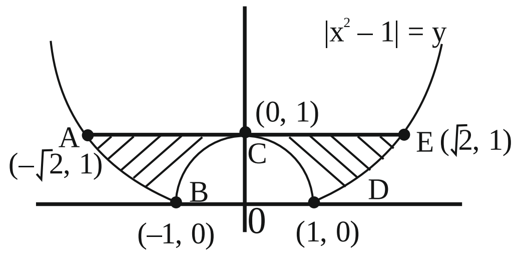
20. The area bounded by the curves $y=|x^2-1|$ and $y=1$ is

(A) $\frac{2}{3}\left(\sqrt{2}+1\right)$ (B) $\frac{4}{3}\left(\sqrt{2}-1\right)$

(C) $2\left(\sqrt{2}-1\right)$ (D) $\frac{8}{3}\left(\sqrt{2}-1\right)$

Official Ans. by NTA (D)

Sol. $y=|x^2-1|$



Area = ABCDEA

$$=2\left(\int_0^1\left(1-(1-x^2)\right)dx+\int_1^{\sqrt{2}}\left(1-(x^2-1)\right)dx\right)$$

$$=\frac{8}{3}(\sqrt{2}-1)$$

SECTION-B

1. Let A = {1,2,3,4,5,6,7} and B = {3,6,7,9}. Then the number of elements in the set $\{C\subseteq A : C\cap B\neq\phi\}$ is_______

Official Ans. by NTA (112)

Sol. A = {1,2,3,4,5,6,7} and

B = {3,6,7,9}

Total subset of A = 2^7 = 128

$C\cap B=\phi$ when set C contains the element 1, 2, 4, 5

$\therefore S = \{C \subseteq A; C \cap B \neq \phi\}$

$= \text{Total} - (C \cap B = \phi)$

$= 128 - 2^4 = 112$

2. The largest value of a, for which the perpendicular distance of the plane containing the lines $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$ and $\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$ from the point (2,1,4) is $\sqrt{3}$, is________.

Official Ans. by NTA (20)

Sol. $\vec{r} = (\hat{i} + \hat{j}) + \lambda(\hat{i} + a\hat{j} - \hat{k})$

$\vec{r} = (\hat{i} + \hat{j}) + \mu(-\hat{i} + \hat{j} - a\hat{k})$

D.R's of plane containing these lines is

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & a & -1 \\ -1 & 1 & -a \end{vmatrix} = \hat{i}(1 - a^2) - \hat{j}(-a - 1) + \hat{k}(1 + a)$$

$\vec{n} = (1 - a)\hat{i} + \hat{j} + \hat{k}$

One point in plane : (1, 1, 0)

$\therefore$ equation of plane is

$(1 - a)(x - 1) + (y - 1) + (z - 0) = 0$

$(1 - a)x + y + z + a - 2 = 0$

$$\therefore D = \frac{|(1 - a)2 + 1 + 4 + a - 2|}{\sqrt{(1 - a)^2 + 1 + 1}}$$

$\Rightarrow |5 - a| = \sqrt{3}.\sqrt{a^2 - 2a + 3}$

$\Rightarrow a^2 + 2a - 8 = 0$

$\Rightarrow a = 2, -4$

$\therefore$ largest value of a = 2

3. Numbers are to be formed between 1000 and 3000, which are divisible by 4, using the digits 1,2,3,4,5 and 6 without repetition of digits. Then the total number of such numbers is ____________.

Official Ans. by NTA (30)

Sol. Here 1[st] digit is 1 or 2 only

Case-I

If first digit is 1

Then last two digits can be 24, 32, 36, 52, 56, 64

1			

$1 \times 3 \times 6 = 18$ ways

Case – II

If first digit is 2 then last two digit can be 16, 36, 56, 64

2			

$1 \times 3 \times 4 = 12$ ways

Total ways = 12 + 18 = 30 ways

4. If $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$, where m and n are co-prime, then m + n is equal to

Official Ans. by NTA (166)

Sol. $\sum_{k=1}^{10} \frac{k}{k^4 + k^2 + 1} = \frac{m}{n}$

$$\Rightarrow \frac{1}{2}\sum_{k=1}^{10} \frac{(k^2 + k + 1) - (k^2 - k + 1)}{(k^2 + k + 1)(k^2 - k + 1)}$$

$$\Rightarrow \frac{1}{2}\left(\sum_{k=1}^{10}\left(\frac{1}{(k^2 - k + 1)} - \frac{1}{k^2 + k + 1}\right)\right)$$

$$\Rightarrow \frac{55}{111} = \frac{m}{n}$$

m + n = 166

5. If the sum of solutions of the system of equations $2\sin^2\theta - \cos 2\theta = 0$ and $2\cos^2\theta + 3\sin\theta = 0$ in the interval $[0, 2\pi]$ is $k\pi$, then k is equal to _______.

Official Ans. by NTA (3)

Sol. $2\sin^2\theta - \cos 2\theta = 0$

$2\sin^2\theta - (1 - 2\sin^2\theta) = 0$

$$\Rightarrow \sin^2\theta = \left(\frac{1}{2}\right)^2$$

$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$

$2\cos^2\theta + 3\sin\theta = 0$

$\Rightarrow 2\sin^2\theta - 3\sin\theta - 2 = 0$

$\therefore \sin\theta = -\frac{1}{2}$

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

So, the common solution is

$\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$

Sum = $\frac{7\pi + 11\pi}{6} = 3\pi = k\pi$

K = 3

6. The mean and standard deviation of 40 observations are 30 and 5 respectively. It was noticed that two of these observations 12 and 10 were wrongly recorded. If σ is the standard deviation of the data after omitting the two wrong observations from the data, then $38\sigma^2$ is equal to___________.

Official Ans. by NTA (238)

Sol. Wrong mean = $\mu_1 = 30$

Wrong S.D = $\sigma_1 = 5$

$\frac{\sum x_i}{40} = 30$

$\Rightarrow \sum x_i = 1200$

$\sigma_1^2 = 25$

$\Rightarrow \frac{\sum x_i^2}{40} - 30^2 = 25$

$\Rightarrow \sum x_i^2 = 925 \times 40 = 37000$

New sum = $\sum x_i' = 1200 - 10 - 12 = 1178$

New mean = $\mu_1' = \frac{1178}{38} = 31$

New $\sum x_i^2 = 37000 - (10)^2 - (12)^2 = 36756$

New S.D, $\sigma_1' = \sqrt{\frac{36756}{38} - (31)^2} = \sigma$

$36756 - (31)^2 \times 38 = 38\sigma^2$

$\Rightarrow 38\sigma^2 = 238$

7. The plane passing through the line L: $\ell x - y + 3(1-\ell) z = 1$, $x + 2y - z = 2$ and perpendicular to the plane $3x + 2y + z = 6$ is $3x - 8y + 7z = 4$. If θ is the acute angle between the line L and the y-axis, then 415 $\cos^2\theta$ is equal to_____.

Official Ans. by NTA (125)

Sol. $\vec{n}_1 = \ell\hat{i} - \hat{j} + 3(1-\ell)\hat{k}$

$\vec{n}_2 = \hat{i} + 2\hat{j} - \hat{k}$

Direction ratio of line = $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \ell & -1 & 3(1-\ell) \\ 1 & 2 & -1 \end{vmatrix}$

$= (6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

$3x - 8y + 7z = 4$ will contain the line $(6\ell - 5)\hat{i} + (3 - 2\ell)\hat{j} + (2\ell + 1)\hat{k}$

Normal of $3x - 8y + 7z = 4$ will be perpendicular to the line

$= 3(6\ell - 5) + (3 - 2\ell)(-8) + 7(2\ell + 1) = 0$

$\Rightarrow \ell = \frac{2}{3}$

$\therefore$ direction ratio of line $\left(-1, \frac{5}{3}, \frac{7}{3}\right)$

Angle with y axis

$\cos\theta = \frac{5/3}{\sqrt{1 + \frac{25}{9} + \frac{49}{9}}}$

$\cos\theta = \frac{5}{\sqrt{83}}$

$\therefore 415\cos^2\theta = \frac{25}{83} \times 415 = 125$

8. Suppose y = y(x) be the solution curve to the differential equation $\frac{dy}{dx} - y = 2 - e^{-x}$ such that $\lim_{x\to\infty} y(x)$ is finite. If a and b are respectively the x- and y- intercepts of the tangent to the curve at x=0, then the value of a-4b is equal to __________.

Official Ans. by NTA (3)

Sol. $\frac{dy}{dx} - y = 2 - e^{-x}$

I.F. = $e^{-\int dx} = e^{-x}$

$\therefore$ solution of D.E

$y \cdot e^{-x} = \int \left(2e^{-x} - e^{-2x}\right)dx$

$\Rightarrow y = -2 + \frac{e^{-x}}{2} + C.e^{x}$

$\because \lim_{x\to\infty} y$ is finite

$\therefore \lim_{x\to\infty}\left(-2 + \frac{e^{-x}}{2} + C.e^{x}\right) \to$ finite

This is possible only when C = 0

$\therefore y = y(x) = -2 + \frac{e^{-x}}{2}$

$\frac{dy}{dx} = -\frac{1}{2}e^{-x}$

$\left.\frac{dy}{dx}\right|_{x=0} = -\frac{1}{2} = m$, $y(0) = -2 + \frac{1}{2} = \frac{-3}{2}$

$\therefore$ equation of tangent

$y + \frac{3}{2} = -\frac{1}{2}(x - 0)$

$\Rightarrow x + 2y = -3$

$a = -3,\ b = \frac{-3}{2}$

$a - 4b = -3 + 6 = 3$

9. Different A.P.'s are constructed with the first term 100, the last term 199, And integral common differences. The sum of the common differences of all such, A.P's having at least 3 terms and at most 33 terms is.

Official Ans. by NTA (53)

Sol. 1^{st} term = 100 = a

Last term = 199 = ℓ

If 3 term

a, a + d, a + 2d

$a_n = \ell = a + (n-1)d$

$d_i = \frac{\ell - a}{n-1}$

n $\to$ number of terms

$n=3,\ d_1 = \frac{199-100}{2}$

$= \frac{99}{2} \notin I$

$n = 4,\ d_2 = \frac{99}{3} = 33 \in I$

$n = 10,\ d_3 = \frac{99}{9} = 11 \in I$

$n = 12,\ d_4 = \frac{99}{11} = 9 \in I$

$\therefore \sum d_i = 33 + 11 + 9 = 53$

10. The number of matrices $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, where $a,b,c,d \in \{-1,0,1,2,3,\ldots\ldots,10\}$, such that $A = A^{-1}$, is_______.

Official Ans. by NTA (50)

Sol. $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

Given $A = A^{-1}$

$\therefore A^2 = A \cdot A^{-1} = I$

$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} a^2 + bc & ab + bd \\ ac + cd & bc + d^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\therefore a^2 + bc = 1$ ....(1)

$ab + bd = 0$ .....(2)

$ac + cd = 0$ ...(3)

$bc + d^2 = 1$ ....(4)

(1) – (4) gives

$a^2 - d^2 = 0$

$\Rightarrow (a + d) = 0$ or $a - d = 0$

Case – I

$a + d = 0 \Rightarrow (a, d) = (-1, 1), (0, 0), (1, -1)$

(a) $(a, d) = (-1, 1)$

$\therefore$ from equation (1)

$1 + bc = 1 \Rightarrow bc = 0$

$b = 0$ $C = 12$ possibilities

$c = 0$ $b = 12$ possibilities

but (0, 0) is repeated

$\therefore 2 \times 12 = 24$

24 – 1 (repeated) = 23 pairs

(b) $(a, d) = (1, -1) \Rightarrow bc = 0 \to 23$ pairs

(c) $(a, d) = (0, 0) \Rightarrow bc = 1$

$\Rightarrow (b, c) = (1, 1)$ & $(-1, -1)$, 2 pairs

Case – II

$a = d$

from (2) and (3)

$a \neq 0$ then $b = c = 0$

$a^2 = 1$

$a = \pm 1 = d$

$(a, d) = (1, 1), (-1, -1) \to 2$ pairs

$\therefore$ Total = 23 + 23 + 2 + 2

= 50 pairs