FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. An expression for a dimensionless quantity P is given by $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$; where α and β are constants, x is distance ; k is Boltzmann constant and t is the temperature. Then the dimensions of α will be :

(A) $[M^0L^{-1}T^0]$ (B) $[ML^0T^{-2}]$

(C) $[MLT^{-2}]$ (D) $[ML^2T^{-2}]$

Official Ans. by NTA (C)

Sol. $P = \frac{\alpha}{\beta}\log_e\left(\frac{kt}{\beta x}\right)$

$\frac{kt}{\beta x} = 1 \Rightarrow \beta = \frac{kt}{x} = \frac{ML^2T^{-2}}{L}$

$\left(\because E = \frac{1}{2}kt\right)$

As P is dimensionless

$\Rightarrow [\alpha] = [\beta] = [MLT^{-2}]$

2. A person is standing in an elevator. In which situation, he experiences weight loss ?

(A) When the elevator moves upward with constant acceleration

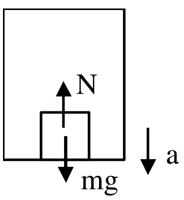
(B) When the elevator moves downward with constant acceleration

(C) When the elevator moves upward with uniform velocity

(D) When the elevator moves downward with uniform velocity

Official Ans. by NTA (B)

Sol.



$mg - N = ma$

$\Rightarrow N = m(g - a)$

∴ Person experiences weightloss, when acceleration of lift is downward.

3. An object is thrown vertically upwards. At its maximum height, which of the following quantity becomes zero ?

(A) Momentum (B) Potential energy

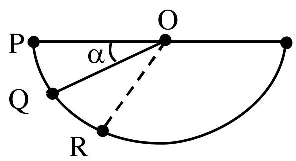
(C) Acceleration (D) Force

Official Ans. by NTA (A)

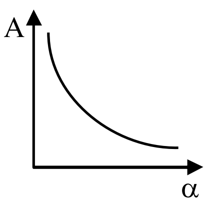
Sol. At maximum height, V = 0

∴ Momentum of object is zero.

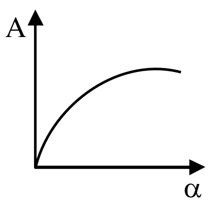
4. A ball is released from rest from point P of a smooth semi-spherical vessel as shown in figure. The ratio of the centripetal force and normal reaction on the ball at point Q is A while angular position of point Q is α with respect to point P. Which of the following graphs represent the correct relation between A and α when ball goes from Q to R ?



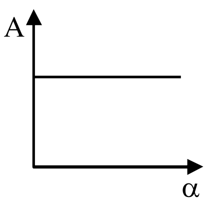
(A)



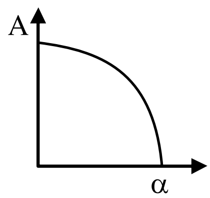
(B)



(C)



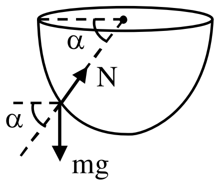
(D)



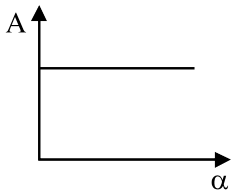
Official Ans. by NTA (C)

Sol. $V = \sqrt{2gR\sin\alpha}$

$N - mg\sin\alpha = \frac{mv^2}{R} = 2mg\sin\alpha$



$\frac{N}{2mg\sin\alpha} = \frac{1}{2} + 1 = \frac{3}{2}$



$\Rightarrow$ A = constant

5. A thin circular ring of mass M and radius R is rotating with a constant angular velocity 2 rads^{-1} in a horizontal plane about an axis vertical to its plane and passing through the center of the ring. If two objects each of mass m be attached gently to the opposite ends of a diameter of ring, the ring will then rotate with an angular velocity (in rads^{-1}).

(A) $\frac{M}{(M+m)}$ (B) $\frac{(M+2m)}{2M}$

(C) $\frac{2M}{(M+2m)}$ (D) $\frac{2(M+2m)}{M}$

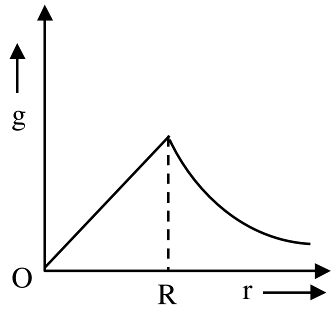
Official Ans. by NTA (C)

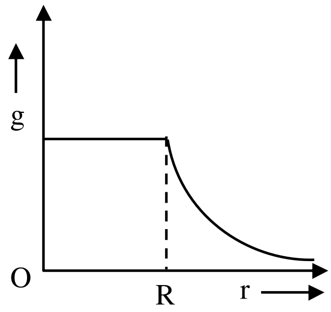
Sol. Applying conservation of angular momentum

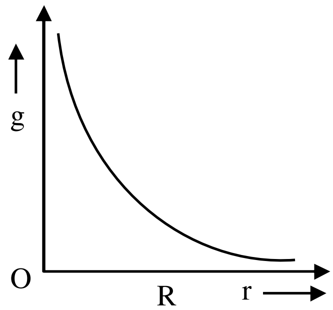
$MR^2\omega = (MR^2 + 2mR^2)\omega'$

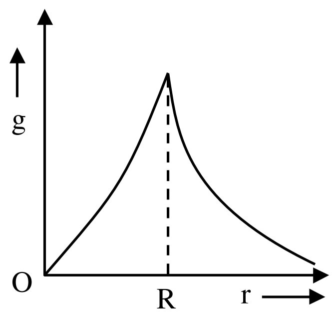
$\omega' = \frac{2M}{M+2m}$

6. The variation of acceleration due to gravity (g) with distance (r) from the center of the earth is correctly represented by : (Given R = radius of earth)

(A) 

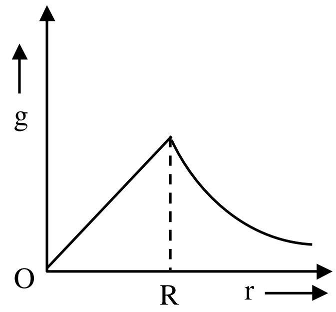
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol. $g = \begin{cases} \frac{GMr}{R^3}, r \le R \\ \frac{GM}{r^2}, r \ge R \end{cases}$



7. The efficiency of a Carnot's engine, working between steam point and ice point, will be :

(A) 26.81% (B) 37.81%

(C) 47.81% (D) 57.81%

Official Ans. by NTA (A)

Sol. $\eta = \left[1 - \frac{T_L}{T_n}\right] \times 100\%$

$T_L = 0°C = 273K, T_n = 373 K$

$\therefore \eta = 26.809\%$

8. Time period of a simple pendulum in a stationary lift is 'T'. If the lift accelerates with $\frac{g}{6}$ vertically upwards then the time period will be :

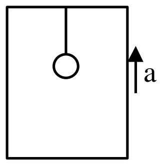
(where g = acceleration due to gravity)

(A) $\sqrt{\frac{6}{5}}T$ (B) $\sqrt{\frac{5}{6}}T$

(C) $\sqrt{\frac{6}{7}}T$ (D) $\sqrt{\frac{7}{6}}T$

Official Ans. by NTA (C)

Sol. $T = 2\pi\sqrt{\frac{\ell}{g_{eff}}}$



(a) when a = 0, $T = 2\pi\sqrt{\frac{\ell}{g}}$

(b) when $a = \frac{g}{6}$, $T' = 2\pi\sqrt{\frac{\ell}{g + \frac{g}{6}}}$

$\therefore T' = \sqrt{\frac{6}{7}}T$

9. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats 1.4. Vessel is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surrounding and vessel temperature of the gas increases by : (R = universal gas constant)

(A) $\frac{Mv^2}{7R}$

(B) $\frac{Mv^2}{5R}$

(C) $2\frac{Mv^2}{7R}$

(D) $7\frac{Mv^2}{5R}$

Official Ans. by NTA (B)

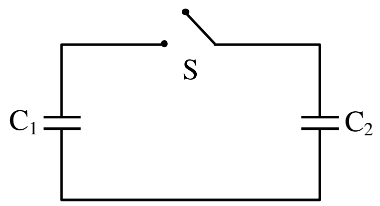
Sol. $\frac{C_P}{C_v} = 1 + \frac{2}{F} = 1.4 \Rightarrow F = 5$

By conservation of energy

$\frac{F}{2}nR\Delta T = \frac{1}{2}[nm]v^2$

$\Delta T = \frac{mv^2}{FR} = \frac{Mv^2}{5R}$

10. Two capacitors having capacitance C_1 and C_2 respectively are connected as shown in figure. Initially, capacitor C_1 is charged to a potential difference V volt by a battery. The battery is then removed and the charged capacitor C_1 is now connected to uncharged capacitor C_2 by closing the switch S. The amount of charge on the capacitor C_2, after equilibrium is :



(A) $\frac{C_1C_2}{(C_1+C_2)}V$ (B) $\frac{(C_1+C_2)}{C_1C_2}V$

(C) $(C_1 + C_2)V$ (D) $(C_1 - C_2)V$

Official Ans. by NTA (A)

Sol. Charge on capacitor C_2

$$= \frac{C_2 \times Q_{total}}{C_{total}} = \frac{C_2[C_1V]}{C_1+C_2} = \frac{C_1C_2V}{C_1+C_2}$$

11. **Assertion (A) :** Non-polar amterials do not have my permanent dipole moment.

Reason (R) : When an non-polar material is placed in a electric field. the centre of the positive charge distribution of it's individual atom or molecule coinsides with the centre of the negative charge distribution.

In the light of above statements, choose the most appropriate answer from the options given below.

(A) Both (A) and (R) are correct and (R) is the correct explanation of (A).

(B) Both (A) and (R) are correct and (R) is not the correct explanation of (A).

(C) (A) is correct but (R) is not correct.

(D) (A) is not correct but (R) is correct.

Official Ans. by NTA (C)

Sol. **S1 :** In nonpolar molecules, centre of +ve charge coincides with centre of –ve charge, hence net dipole moment is comes to zero.

S2 : When non polar material is placed in external field, centre of charges does not coincide, hence give non zero moment in field

12. The magnetic flux through a coil perpendicular to its plane is varying according to the relation $\phi = (5t^3 + 4t + 2t - 5)$ Weber. If the resistant of the coil is 5 ohm, then the induced current through the coil at t = 2 sec will be:

(A) 15.6 A (B) 16.6 A

(C) 17.6 A (D) 18.6 A

Official Ans. by NTA (A)

Sol. $\phi = 5t^3 + 4t^2 + 2t - 5$

$$|e| = \frac{d\phi}{dt} = 15t^2 + 8t + 2$$

At t = 2, $|e| = 15 \times 2^2 + 8 \times 2 + 2$

$$\Rightarrow e = 78V \Rightarrow I = \frac{e}{R} = \frac{78}{5} = 15.60$$

13. An aluminium wire is stretched to make its length, 04% larger. Then percentage change in resistance is:

(A) 0.4 % (B) 0.2 %

(C) 0.8 % (D) 0.6 %

Official Ans. by NTA (C)

Sol. $R = \frac{\rho\ell}{A}$

$$\frac{\Delta R}{R} = \frac{\Delta\ell}{\ell} - \frac{\Delta A}{A}$$

$\ell A = k$

$$\frac{\Delta\ell}{\ell} + \frac{\Delta A}{A} = 0$$

$$\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell}$$

$$\frac{\Delta R}{R} = 2 \times 0.4 = 0.8\%$$

14. A proton and an alpha particle of the same enter in a uniform magnetic field which is acting perpendicular to their direction of motion. The ratio of the circular paths described by the alpha particle and proton is:

(A) 1 : 4 (B) 4 : 1

(C) 2 : 1 (D) 1 : 2

Official Ans. by NTA (C)

Sol. $\frac{R_\alpha}{R_P} = \frac{M_\alpha}{M_P} \times \frac{q_P}{q_\alpha}$

$\frac{R_\alpha}{R_P} = \frac{4}{1} \times \frac{1}{2} = 2$

15. If electric field intensity of a uniform plane electro magnetic wave is given as

E = –301.6 sin(kz – ωt) $\hat{a}_x$ + 452.4 sin(kz – ωt) $\hat{a}_y \frac{V}{m}$

Then, magnetic intensity H of this wave in Am^{-1} will be:'

[Given: Speed of light in vacuum c = 3 × 10^8 ms^{-1}, permeability of vacuum $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}]

(A) $+0.8\sin(kz-\omega t)\hat{a}_y + 0.8\sin(kz-\omega t)\hat{a}_x$

(B) $+1.0\times10^{-6}\sin(kz-\omega t)\hat{a}_y + 1.5\times10^{-6}(kz-\omega t)\hat{a}_x$

(C) $-0.8\sin(kz-\omega t)\hat{a}_y - 1.2\sin(kz-\omega t)\hat{a}_x$

(D) $-1.0\times10^{-6}\sin(kz-\omega t)\hat{a}_y - 1.5\times10^{-6}\sin(kz-\omega t)\hat{a}_x$

Official Ans. by NTA (C)

Sol. $\vec{E} = 301.6\sin(kz-\omega t)(-\hat{a}_x) + 452.4\sin(kz-\omega t)\hat{a}_y$

$\vec{B} = \frac{301.6}{C}\sin(kz-\omega t)(-\hat{a}_y)$

$+ \frac{452.4}{C}\sin(kz-\omega t)(-\hat{a}_x)$

$\vec{H} = \frac{\vec{B}}{\mu_0} = \frac{301.6}{\mu C}\sin(kz-\omega t)(-\hat{a}_y)$

$+\frac{452.4}{\mu C}\sin(kz-\omega t)(-\hat{a}_x)$

$\vec{H} = -0.8\sin(kz-\omega t)\hat{a}_y - 1.2\sin(kz-\omega t)\hat{a}_x$

For direction

$\vec{E}\times\vec{B}$ is direction of $\vec{C}$

For first part $\hat{E} = -\hat{i}, \hat{B} = ?$

$\hat{E}\times\hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{j}$

Similarly for second

$\hat{E} = \hat{j}, \hat{B} = ?$

$\hat{E}\times\hat{B} = \hat{k} \Rightarrow \hat{B} = -\hat{i}$

16. In free space, an electromagnetic wave of 3 GHz of 3 GHz frequency strikes over the edge of an object of size $\frac{\lambda}{100}$, where λ is the wavelength of the wave in free space. The phenomenon, which happens there will be:

(A) Reflection (B) Refraction

(C) Diffraction (D) Scattering

Official Ans. by NTA (D)

Sol. $\frac{a}{\lambda} = \frac{1}{100}$

For reflection size of obstacle must be much larger than wavelength, for diffraction size should be order of wavelength.

Since the object is of size $\frac{\lambda}{100}$, much smaller than wavelength, so scattering will occur.

17. An electron with speed v and a photon with speed c have the same de-Broglie wavelength. If the kinetic energy and momentum of electron are E_e and p_e and that of photon are E_{ph} and p_{ph} respectively. Which of the following is correct?

(A) $\frac{E_e}{E_{ph}} = \frac{2c}{v}$ (B) $\frac{E_e}{E_{ph}} = \frac{v}{2c}$

(C) $\frac{p_e}{p_{ph}} = \frac{2c}{v}$ (D) $\frac{p_e}{p_{ph}} = \frac{v}{2c}$

Official Ans. by NTA (B)

Sol. $\lambda_e = \lambda_{photon}$

$$\frac{h}{mv} = \frac{h}{P_{photon}} \Rightarrow P_{photon} = mv$$

$$\frac{E_e}{E_{ph}} = \frac{\frac{1}{2}mv^2}{\frac{hc}{\lambda}} = \frac{1}{2}\frac{mv}{P_{Ph}C} \times v = \frac{v}{2C}$$

18. How many alpha and beta particles are emitted when Uranium ${}_{92}U^{238}$ decays to lead ${}_{82}Pb^{206}$?

(A) 3 alpha particles and 5 beta particles

(B) 6 alpha particles and 4 beta particles

(C) 4 alpha particles and 5 beta particles

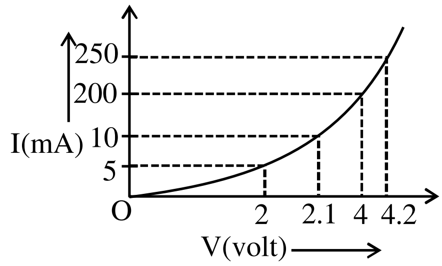
(D) 8 alpha particles and 6 beta particles

Official Ans. by NTA (D)

Sol. ${}^{238}_{92}U \rightarrow \boxed{8}\,{}^{4}_{2}He + \boxed{6}\,{}^{0}_{-1}e + {}^{206}_{82}Pb$

8α particles and 6β particles are emitted.

19. The I-V characteristics of a p-n junction diode in forward bias is shown in the figure. The ratio of dynamic resistance, corresponding to forward bias voltages of 2V and 4V respectively, is :



(A) 1 : 2 (B) 5 : 1

(C) 1 : 40 (D) 20 : 1

Official Ans. by NTA (B)

Sol. $R = \frac{\Delta V}{\Delta i}$

$$\frac{R_1}{R_2} = \frac{\Delta v_1}{\Delta v_2}\frac{\Delta i_2}{\Delta i_1} = \frac{0.1}{0.2} \times \frac{50}{5} = 5$$

20. Choose the correct statement for amplitude modulation:

(A) Amplitude of modulating is varied in accordance with the information signal.

(B) Amplitude of modulated is varied in accordance with the information signal.

(C) Amplitude of carrier signal is varied in accordance with the information signal.

(D) Amplitude of modulated is varied in accordance with the modulating signal.

Official Ans. by NTA (C)

Sol. In amplitude modulation the amplitude of high frequency carrier wave is varied in accordance with message signal

SECTION-B

1. A fighter jet is flying horizontally at a certain altitude with a speed of 200 ms^{-1}. When it passes directly overhead an anti-aircraft gun, bullet is fired from the gun, at an angle θ with the horizontal, to hit the jet. If the bullet speed is 400 m/s, the value of θ will be .... °.

Official Ans. by NTA (60)

Sol. Both should have same horizontal component of velocity

$200 = 400 \cos\theta$

$\theta = 60°$

2. A ball of mass 0.5 kg is dropped from the height of 10m. The height, at which the magnitude of velocity becomes equal to the magnitude of acceleration due to gravity, is .... m. (Use g = 10 m/s^2).

Official Ans. by NTA (5)

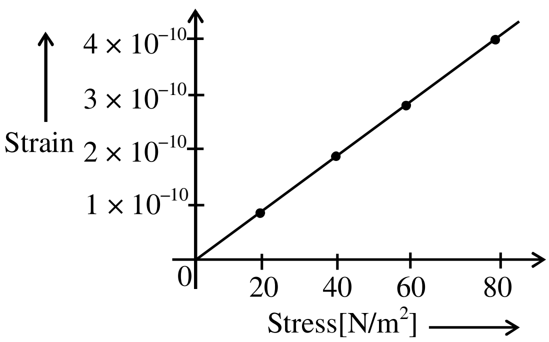
Sol. $v^2 = u^2 + 2as$

$100 = 0 + 2(10)s$

S = 5m

Height from ground = 10 – 5 = 5m

3. The elastic behaviour of material for linear streass and linear strain, is shown in the figure. The energy density for a linear strain of 5 × 10^{-4} is .... kJ/m^3. Assume that material is elastic upto the linear strain of 5 × 10^{-4}.



Official Ans. by NTA (25)

Sol. $y = \dfrac{\text{stress}}{\text{strain}} = 2.0\times10^{10}$

Energy density = $\dfrac{1}{2}\text{stress}\times\text{strain}$

$= \dfrac{1}{2}(\text{strain})^2\, y \;= \dfrac{1}{2}\left(5\times10^{-4}\right)^2 \times 20\times10^{10}$

$= 25 \times 10^2 \times 10 = 25\dfrac{\text{kJ}}{\text{m}^3}$

Ans. 25

4. The elongation of a wire on the surface of the earth is 10^{-4} m. The same wire of same dimensions is elongated by 6 × 10^{-5}m on another planet. The acceleration due to gravity on the planet will be .... ms^{-2}. (Take acceleration due to gravity on the surface of earth = 10 m/s^{-2})

Official Ans. by NTA (6)

Sol. $\Delta\ell \propto g$

$\dfrac{\Delta\ell_{\text{earth}}}{\Delta\ell_{\text{planet}}} = \dfrac{g_{\text{earth}}}{g_{\text{planet}}} = \dfrac{10^{-4}}{6\times10^{-5}}$

$g_{\text{planet}} = 6\ m/s^2$

Ans. 6.00

5. A 10Ω, 20 mH coil carrying constant current is connected to a battery of 20 V through a switch is opened current becomes zero in 100μs. The average emf induced in the coil is .... V.

Official Ans. by NTA (400)

Sol. $<\varepsilon>= \dfrac{\int \varepsilon dt}{\int dt} = \dfrac{\int (L di/dt)dt}{\int dt} = \dfrac{L\int di}{\int dt}$

$<\varepsilon>= \dfrac{L\Delta i}{\Delta i}$

$i_0 = \dfrac{V}{R} = \dfrac{20}{10} = 2A$, if $i = 0A$

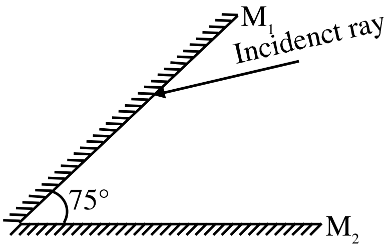
T = 100 μs, L = 20 mH

$<\varepsilon>= \dfrac{20\times10^{-3}\times(2-0)}{100\times10^{-6}}$

$= \dfrac{2\times10^3}{5}$

$<\varepsilon>= 400V$

6. A light ray is incident, at an incident angle θ_1, on the system of two plane mirrors M_1 and M_2 having an inclination angle 75° between them (as shown in figure). After reflecting from mirror M_1 it gets reflected back by the mirror M_2 with an angle of reflection 30°. The total deviation of the ray will be .... degree.

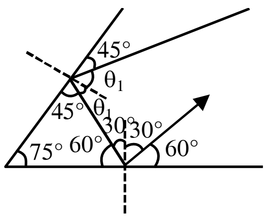


Official Ans. by NTA (210)

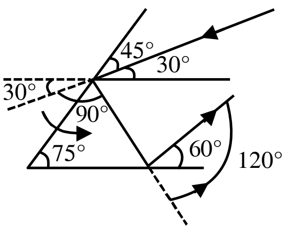
Sol. $\delta_{total} = 360° - 2\theta$

$= 360° - 2 \times 75°$

$\delta_{totakl} = 210°$



$\theta_1 = 45°$



$\delta = 120° + 90° = 210°$

7. In a vernier callipers, each cm on the main scale is divided into 20 equal parts. If tenth vernier scale division coincides with nineth main scale division. Then the value of vernier constant will be .... $\times 10^{-2}$ mm.

Official Ans. by NTA (5)

Sol. 20 MSD = 1cm

$1\text{MSD} = \frac{1}{20}\text{cm}$

10 VSD = 9MSD

$1\text{VSD} = \frac{9}{10}\text{MSD}$

$= \frac{9}{10} \times \frac{1}{20}\text{cm}$

$1\text{VSD} = \frac{9}{200}\text{cm}$

VC = 1 MSD – 1 VSD

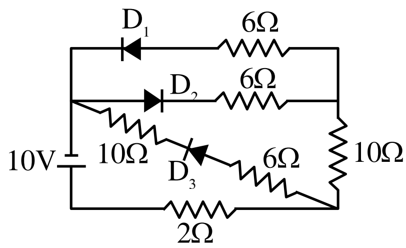
$= \frac{1}{20}\text{cm} - \frac{9}{200}\text{cm}$

$= \frac{1}{200} \times 10\text{mm}$

$\text{VC} = 5 \times 10^{-2}$ mm

Ans. 5

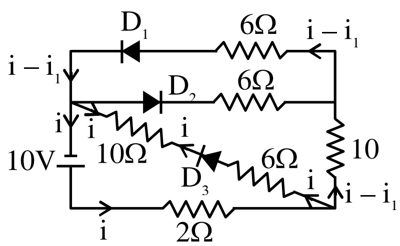
8. As per the given circuit, the value of current through the battery will be .... A.

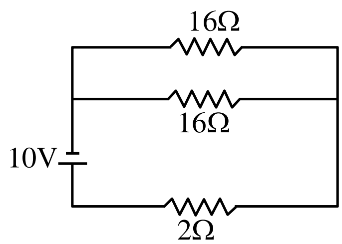


Official Ans. by NTA (1)

)

Sol.





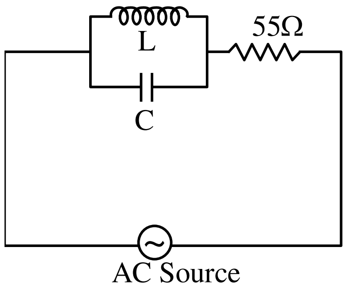
$V = IR_{net}$

$10 = I \times 10$

$I = 1A$

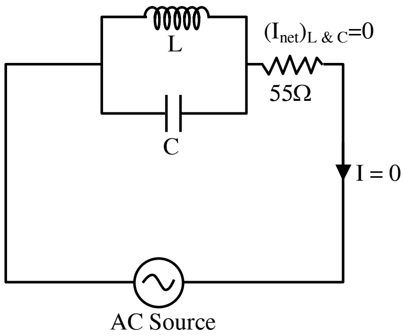
Ans. 1

9. A 110 V , 50 Hz, AC source is connected in the circuit (as shown in figure). The current through the resistance 55 Ω, at resonance in the circuit, will be .... A.



Official Ans. by NTA (0)

Sol. At resonance $I_L = I_C$



Alternatively,

$$\frac{1}{Z} = \sqrt{\left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

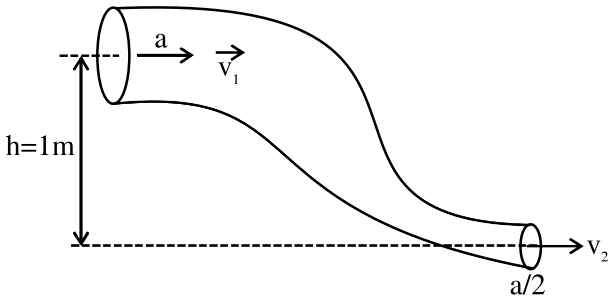
At resonance, $X_L = X_C$ & $Z \to \infty$

$\therefore Z_{\text{total circuit}} \to \infty$ i.e, $I = 0$

Ans. 0

10. An ideal fluid of density 800 kgm^{-3}, flows smoothly through a bent pipe (as shown in figure) that tapers in cross-sectional area from a to $\frac{a}{2}$. The pressure difference between the wide and narrow sections of pipe is 4100 Pa. At wider section, the velocity of fluid is $\frac{\sqrt{x}}{6} ms^{-1}$ for x = ...

(Given g = 10 m^{-2})



Official Ans. by NTA (363)

Sol. From continuity equation

$$av_1 = \frac{a}{2}v_2$$

$v_2 = 2v_1$

From Bernoulli's theorem,

$$P_1 + \rho g h_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$$

$$P_1 - P_2 = \rho\left[\left(\frac{v_2^2 - v_1^2}{2}\right) + g(h_2 - h_1)\right]$$

$$4100 = 800\left[\left(\frac{4v_1^2 - v_1^2}{2}\right) + 10 \times (0-1)\right]$$

$$\frac{41}{8} + 10 = \frac{3v_1^2}{2}$$

$$\frac{121}{8} \times \frac{2}{3} = v_1^2$$

$$v_1 = \sqrt{\frac{121}{4 \times 3} \times \frac{3}{3}}$$

$$v_1 = \frac{\sqrt{363}}{6} m/s$$

X = 363.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. A commercially sold conc. HCl is 35% HCl by mass. If the density of this commercial acid is 1.46 g/mL, the molarity of this solution is :
(Atomic mass : Cl = 35.5 amu, H = 1 amu)

(A) 10.2 M (B) 12.5 M
(C) 14.0 M (D) 18.2 M

Official Ans. by NTA (C)

Sol. Let total volume = 1000 mL = 1L

total mass of solution = 1460 g

mass of HCl = $\frac{35}{100} \times 1460$

moles of HCl = $\frac{35 \times 1460}{100 \times 36.5}$

So molarity = $\frac{35 \times 1460}{100 \times 36.5} = 14M$

2. An evacuated glass vessel weighs 40.0 g when empty, 135.0 g when filled with a liquid of density 0.95 g mL^{-1} and 40.5 g when filled with an ideal gas at 0.82 atm at 250 K. The molar mass of the gas in g mol^{-1} is :
(Given : R = 0.082 L atm K^{-1} mol^{-1})

(A) 35 (B) 50
(C) 75 (D) 125

Official Ans. by NTA (D)

Sol. Mass of liquid = 135 – 40 = 95 g

Volume of liquid = $\frac{\text{mass}}{\text{density}} = \frac{95}{.95}$ mL

= 100 mL = 0.1 L

mass of ideal gas = 40.5 – 40 g = 0.5 g

PV = nRT

$0.82 \times 0.1 = \left(\frac{0.5}{M}\right) \times 0.082 \times 250$

M = 125

3. If the radius of the 3^{rd} Bohr's orbit of hydrogen atom is r_3 and the radius of 4^{th} Bohr's orbit is r_4. Then :

(A) $r_4 = \frac{9}{16} r_3$ (B) $r_4 = \frac{16}{9} r_3$

(C) $r_4 = \frac{3}{4} r_3$ (D) $r_4 = \frac{4}{3} r_3$

Official Ans. by NTA (B)

Sol. $r = 0.529 \times \frac{n^2}{z}$ Å

$r_3 = 0.529 \times \frac{3^2}{1}$

$r_4 = 0.529 \times \frac{4^2}{1}$

$\frac{r_4}{r_3} = \frac{4^2}{3^2} = \frac{16}{9}$

$r_4 = \frac{16 r_3}{9}$

4. Consider the ions/molecule

O_2^+, O_2, O_2^-, O_2^{2-}

For increasing bond order the correct option is :

(A) $O_2^{2-} < O_2^- < O_2 < O_2^+$

(B) $O_2^- < O_2^{2-} < O_2 < O_2^+$

(C) $O_2^- < O_2^{2-} < O_2^+ < O_2$

(D) $O_2^- < O_2^+ < O_2^{2-} < O_2$

Official Ans. by NTA (A)

Sol.

ion/molecule	Number of e^- in BMO	Number of e^- in ABMO	Bond order
O_2^+	10	5	2.5
O_2	10	6	2
O_2^-	10	7	1.5
O_2^{2-}	10	8	1

Bond order $O_2^{2-} < O_2^- < O_2 < O_2^+$

5. The $\left(\frac{\partial E}{\partial T}\right)_P$ of different types of half cells are as follows :

A	B	C	D
1×10^{-4}	2×10^{-4}	0.1×10^{-4}	0.2×10^{-4}

(Where E is the electromotive force)

Which of the above half cells would be preferred to be used as reference electrode ?

(A) A (B) B

(C) C (D) D

Official Ans. by NTA (C)

Sol. A cell with less variation in EMF with temperature is preferred as reference electrode because it can be used for wider range of temperature without much derivation from standard value so a cell with less $\left(\frac{\partial E}{\partial T}\right)_P$ is preferred.

6. Choose the correct stability order of group 13 elements in their +1 oxidation state.

(A) Al < Ga < In < Tl (B) Tl < In < Ga < Al

(C) Al < Ga < Tl < In (D) Al < Tl < Ga < In

Official Ans. by NTA (A)

Sol. Moving down the group stability of lower oxidation state increases

Al < Ga < In < Tl

7. Given below are two statements :

Statement I : According to the Ellingham diagram, any metal oxide with higher $\Delta G°$ is more stable than the one with lower $\Delta G°$.

Statement II : The metal involved in the formation of oxide placed lower in the Ellingham diagram can reduce the oxide of a metal placed higher in the diagram.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (D)

Sol. Metal oxide with lower $\Delta G°$ is more stable

Statement II is correct

8. Consider the following reaction :

$$2HSO_4^-(aq) \xrightarrow[\text{(2) Hydrolysis}]{\text{(1) Electrolysis}} 2HSO_4^- + 2H^+ + \mathbf{A}$$

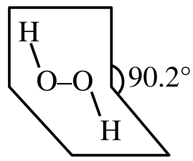
The dihedral angle in product **A** in its solid phase at 110 K is :

(A) 104° (B) 111.5°

(C) 90.2° (D) 111.0°

Official Ans. by NTA (C)

Sol. $2HSO_4^-(aq.) \xrightarrow[\text{(2) Hydrolysis}]{\text{(1) Electrolysis}} 2HSO_4^- + 2H^+ + \underset{(A)}{H_2O_2}$



Solid phase.

9. The correct order of melting point is :

(A) Be > Mg > Ca > Sr (B) Sr > Ca > Mg > Be

(C) Be > Ca > Mg > Sr (D) Be > Ca > Sr > Mg

Official Ans. by NTA (D)

Sol.

	M.P
Be	1560 K
Mg	924 K
Ca	1124 K
Sr	1062 K

10. The correct order of melting points of hydrides of group 16 elements is :

(A) $H_2S < H_2Se < H_2Te < H_2O$

(B) $H_2O < H_2S < H_2Se < H_2Te$

(C) $H_2S < H_2Te < H_2Se < H_2O$

(D) $H_2Se < H_2S < H_2Te < H_2O$

Official Ans. by NTA (A)

Sol.

	M.P
H_2O	273 K
H_2S	188 K
H2Se	208 K
H_2Te	222 K

11. Consider the following reaction :

A + alkali $\rightarrow$ B (Major Product)

If B is an oxoacid of phosphorus with no P–H bond, then A is :

(A) White P_4 (B) Red P_4

(C) P_2O_3 (D) H_3PO_3

Official Ans. by NTA (B)

Red P_4 + Alkali $\rightarrow$ $H_4P_2O_6$ (No P–H bond)

12. Polar stratospheric clouds facilitate the formation of :

(A) $ClONO_2$ (B) HOCl

(C) ClO (D) CH_4

Official Ans. by NTA (B)

Sol. Polar stratospheric clouds provide surface on which hydrolysis of $ClONO_2$ takes place to form HOCl (Hypochlorous acid)

$ClONO_2(g) + H_2O(g) \rightarrow HOCl(g) + HNO_3(g)$

13. Given below are two statements :

Statement I : In 'Lassaigne's Test, when both nitrogen and sulphur are present in an organic compound, sodium thiocyanate is formed.

Statement II : If both nitrogen and sulphur are present in an organic compound, then the excess of sodium used in sodium fusion will decompose the sodium thiocyanate formed to give NaCN and Na_2S.

In the light of the above statements, choose the **most appropriate** answer from the options given below :

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (A)

Sol. Both statement I & statement II are correct.

14. $(C_7H_5O_2)_2 \xrightarrow{h\nu} [X] + 2\dot{C}_6H_5 + 2CO_2$

Consider the above reaction and identify the intermediate 'X'

(A) $C_6H_5-\overset{O}{\overset{\|}{C}}{}^{\oplus}$

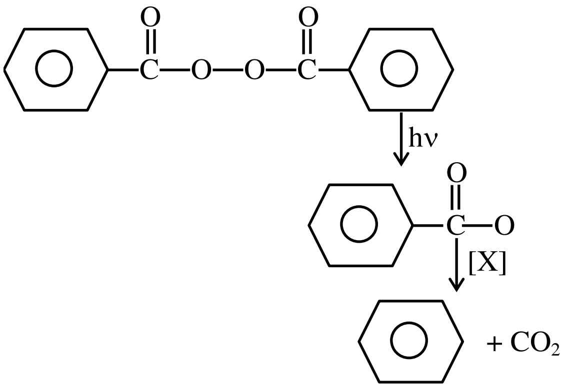
(B) $C_6H_5-\overset{O}{\overset{\|}{C}}-O^{\ominus}$

(C) $C_6H_5-\overset{O}{\overset{\|}{C}}-\ddot{\underset{\cdot\cdot}{O}}$

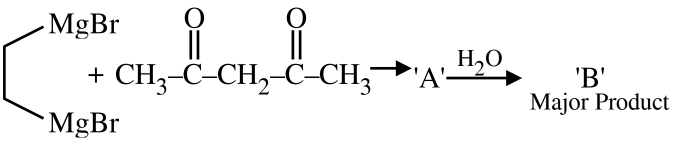
(D) $C_6H_5-\overset{O}{\overset{\|}{C}}-\ddot{\underset{\cdot\cdot}{O}}\cdot$

Official Ans. by NTA (D)

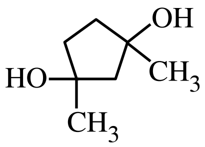
Sol.

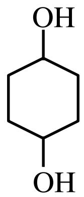


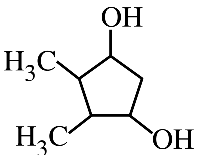
15.

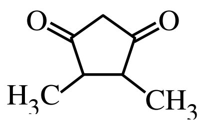


Consider the above reaction sequence and identify the product **B.**

(A) 

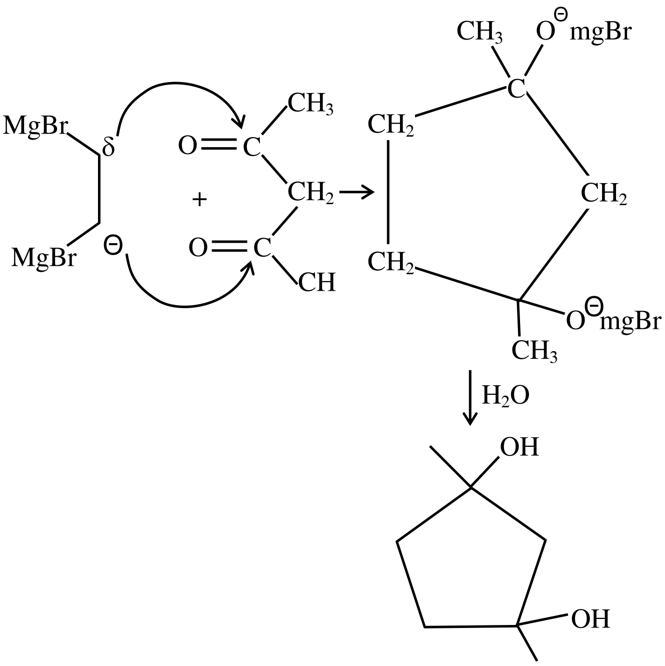
(B) 

(C) 

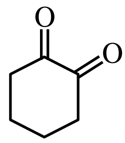
(D) 

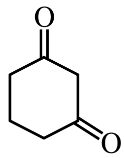
Official Ans. by NTA (A)

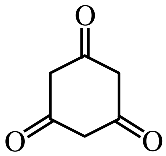
Sol. Although Acetyl Acetone predominantly gives Acid base reaction with G.R due to Active methylene group but according to given option ans should be based on nucleophilic addition reaction (NAR).

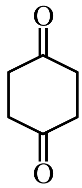


16. Which will have the highest enol content ?

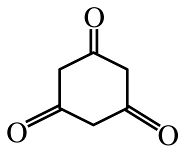
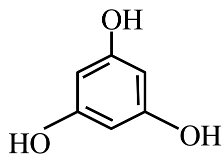
(A) 

(B) 

(C) 

(D) 

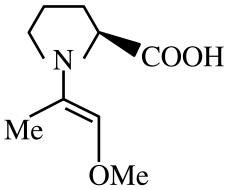
Official Ans. by NTA (C)

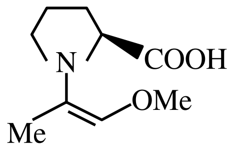
Sol.  is tautomer of 

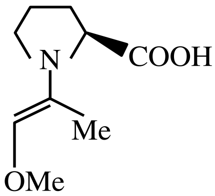
, Which is aromatic in nature.

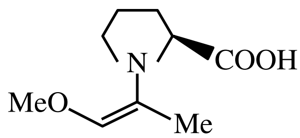
17. Among the following structures, which will show the most stable enamine formation ?

(Where Me is $-CH_3$)

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (C)

Sol. All these enamines are interconvertible through their resonating structures. So most stable form is 'C' due to steric factor.

18. Which of the following sets are **correct** regarding polymer ?

(A) Copolymer : Buna–S

(B) Condensation polymer : Nylon–6,6

(C) Fibre : Nylon–6,6

(D) Thermosetting polymer : Terylene

(E) Homopolymer : Buna–N

Choose the **correct** answer from given options below:

(A) (A), (B) and (C) are correct

(B) (B), (C) and (D) are correct

(C) (A), (C) and (E) are correct

(D) (A), (B) and (D) are correct

Official Ans. by NTA (A)

Sol. Which of the following set are correct regarding polymer.

Bona - 5 is copolymer of butadiene + styrene

Nylon 6.6 is condensation polymer of adipic Acid and hexanediamine.

Nylon 6.6 is fiber

Terylene is fiber not themosetting polymer

Buna-N is copolymer nol Homopolymer

19. A chemical which stimulates the secretion of pepsin is :

(A) Anti histamine (B) Cimetidine

(C) Histamine (D) Zantac

Official Ans. by NTA (C)

Sol. Histamine (It is use for secretion of pepsin & HCl in stomach)

20. Which statement is **not** true with respect to nitrate ion test ?

(A) A dark brown ring is formed at the junction of two solutions.

(B) Ring is formed due to nitroferrous sulphate complex.

(C) The brown complex is $[Fe(H_2O)_5(NO)]SO_4$.

(D) Heating the nitrate salt with conc. H_2SO_4, light brown fumes are evolved.

Official Ans. by NTA (B)

Sol. Ring is formed due to formation of nitrosoferrous sulphate

SECTION–B

1. For complete combustion of methanol

$$CH_3OH(l)+\frac{3}{2}O_2(g)\rightarrow CO_2(g)+2H_2O(l)$$

the amount of heat produced as measured by bomb calorimeter is 726 kJ mol^{-1} at 27°C. The enthalpy of combustion for the reaction is –x kJ mol^{-1}, where x is ______. (Nearest integer)

(Given : R = 8.3 JK^{-1} mol^{-1})

Official Ans. by NTA (727)

Sol. $\Delta U = -726$ KJ/mol

$\Delta ng = 1-3/2 = \frac{-1}{2}$

$\Delta H = \Delta U + \Delta ngRT$

$= -726 - \frac{1}{2}\times\frac{8.3\times300}{1000}$

$= -727.245$

2. A 0.5 percent solution of potassium chloride was found to freeze at –0.24°C. The percentage dissociation of potassium chloride is ________. (Nearest integer)

(Molal depression constant for water is 1.80 K kg mol^{-1} and molar mass of KCl is 74.6 g mol^{-1})

Official Ans. by NTA (98)

Sol. 0.5% solution of KCl

So $m = \frac{0.5}{74.6} \times \frac{1}{0.1}$

$\Delta T_f = i \times m \times K_f$

$0.24 = i \times \frac{0.5}{74.6} \times \frac{1.80}{0.1}$

$i = \frac{0.24 \times 74.6}{0.5 \times 1.80} \times 0.1$

= 1.989

1.989 = 1 + α (n–1)

1.989 = 1 + α

α = .989

% α = 98.9%

Ans 99%

If mass of H_2O = 99.5

$m = \frac{0.5}{74.5} \times \frac{1}{.0995}$

$i = \frac{0.24 \times 74.6 \times .0995}{.5 \times 1.80}$

= 1.979

1.979 = 1 + α (n–1)

1.979 = 1 + α

α = .979

% α = 97.9 %

Ans 98%

3. 50 mL of 0.1 M CH_3COOH is being titrated against 0.1 M NaOH. When 25 mL of NaOH has been added, the pH of the solution will be ________ $\times 10^{-2}$. (Nearest integer)

(Given : pK_a (CH_3COOH) = 4.76)

log 2 = 0.30

log 3 = 0.48

log 5 = 0.69

log 7 = 0.84

log 11 = 1.04

Official Ans. by NTA (476)

Sol. Moles of CH_3COOH = 5 m mole

moles of NaOH = 2.5 m mole

$NaOH + CH_3COOH \longrightarrow CH_3COONa + H_2O$

2.5 m mole 2.5 m mole

0 2.5 m mole 2.5 m mole

so buffer is formed

$pH = pKa + \log\left(\frac{2.5/75}{2.5/75}\right) = pKa$

pH = 4.76

$= 476 \times 10^{-2}$

4. A flask is filled with equal moles of A and B. The half lives of A and B are 100 s and 50 s respectively and are independent of the initial concentration. The time required for the concentration of A to be four times that of B is ________s.

(Given : ln 2 = 0.693)

Official Ans. by NTA (200)

Sol. $k_A = \frac{\ln 2}{100}$; $k_B = \frac{\ln 2}{50}$

$A_t = A_0 \times e^{-k_A t}$

$A_t = A_0 \times e^{\left(\frac{-\ln 2}{100} \times t\right)}$

$B_t = B_0 \times e^{\left(\frac{-\ln 2}{50} \times t\right)}$

$A_0 = B_0$

& $A_t = 4B_t$

$e^{-\frac{\ln 2}{100} \times t} = 4 \times e^{-\frac{\ln 2}{50} \times t}$

$$e^{\frac{\ln 2}{100}\times t} = 4$$

$$e^{\frac{\ln 2}{100}\times t} = 4$$

$$\frac{\ln 2}{100}\times t = \ln 4 = 2\ln 2$$

t = 200 sec

5. 2.0 g of H_2 gas is adsorbed on 2.5 g of platinum powder at 300 K and 1 bar pressure. The volume of the gas adsorbed per gram of the adsorbent is ________ mL.

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (9960)

Sol. Volume of $H_2 = \frac{nRT}{p} = \frac{2}{2}\times\frac{0.083\times 300}{1}$

= 24.92 L

= 24900 mL

So 1 g platinum adsorb $= \frac{24900}{2.5}\,mLH_2$

= 9960

6. The spin–only magnetic moment value of the most basic oxide of vanadium among V_2O_3, V_2O_4 and V_2O_5 is ________ B.M. (Nearest Integer)

Official Ans. by NTA (3)

Sol. Most basic oxide is V_2O_3

$V^{+3} \rightarrow [A_r]\ 3d^2$

$\mu = \sqrt{2(2+2)} = 2.84\ BM \approx 3$

7. The spin–only magnetic moment value of an octahedral complex among $CoCl_3.4NH_3$, $NiCl_2.6H_2O$ and $PtCl_4.2HCl$, which upon reaction with excess of $AgNO_3$ gives 2 moles of AgCl is ________ B.M. (Nearest Integer)

Official Ans. by NTA (3)

Sol. $CoCl_3.\ 4NH_3 \rightarrow [Co(NH_3)_4Cl_2]Cl$

$NiCl_2.6H_2O \rightarrow [Ni(H_2O)_6]Cl_2$

$PtCl_4\ .\ 2HCl \rightarrow H_2[PtCl_6]$

$[Ni(H_2O)_6]Cl_2 \xrightarrow{2AgNO_3} 2AgCl\downarrow + [Ni(H_2O)_6](NO_3)_2$

1	1

⇅	⇅	⇅

$\mu = \sqrt{2(2+2)}$ B.M = 2.84 BM ≈ 3

8. On complete combustion 0.30 g of an organic compound gave 0.20 g of carbon dioxide and 0.10 g of water. The percentage of carbon in the given organic compound is ________ (Nearest Integer)

Official Ans. by NTA (18)

Sol. $C_xHyOz + \left(x+\frac{y}{4}-\frac{z}{2}\right)O_2 \rightarrow xCO_2 + \frac{y}{2}\ H_2O$

0.3g   0.2g   .1g

$$\frac{n_{CO_2}}{n_{H_2O}} = \frac{x}{y/2} = \frac{0.2/44}{.1/18}$$

$$\frac{2x}{y} = \frac{36}{44} = \frac{9}{11}$$

$$x = \frac{9y}{22}$$

$$\frac{n_{C_xH_yO_z}}{n_{CO_2}} = \frac{1}{x}$$

$$\frac{0.3}{12x+y+16z}\times\frac{44}{0.2} = \frac{1}{x}$$

66x = 12 x + y + 16 z

54x = y + 16 z

$$\frac{54\times 9y}{22} - y = 16\ z$$

$$\frac{464y}{22} = 16z$$

$z = \frac{29y}{22}$

$C_xH_yO_z = C_xH_yO_z$

$C_{\frac{9y}{22}}H_yO_{\frac{29y}{22}}$

$C_9\,H_{22}\,O_{29}$

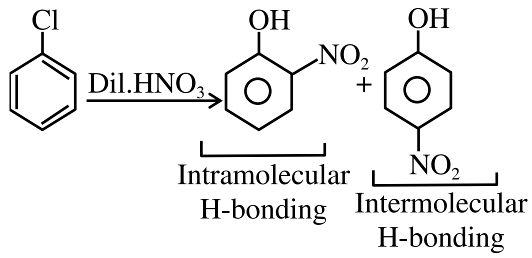
$$\% \text{ of } C = \frac{12\times 9}{(12\times 9+22+29\times 16)}\times 100 = \frac{108}{594}\times 100$$

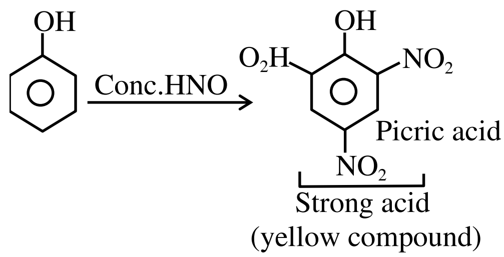
18.18%

9. Compound 'P' on nitration with dil. HNO_3 yields two isomers (A) and (B). These isomers can be separated by steam distillation. Isomers (A) and (B) show the intramolecular and intermolecular hydrogen bonding respectively. Compound (P) on reaction with conc. HNO_3 yields a yellow compound 'C', a strong acid. The number of oxygen atoms is present in compound 'C' ________.

Official Ans. by NTA (7)

Sol.

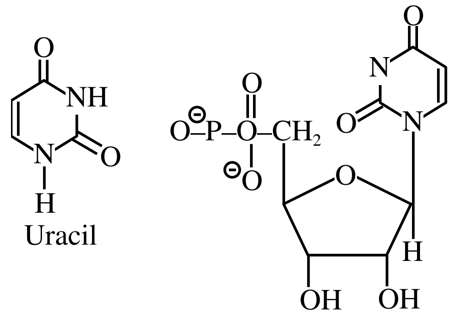




10. The number of oxygens present in a nucleotide formed from a base, that is present only in RNA is ________.

Official Ans. by NTA (9)

Sol. Uracil is the base which only present is RNA.



Structure of nucleotides number of 0-9.

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Sunday 26th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Let $f(x)=\frac{x-1}{x+1}, x \in R-\{0,-1,1\}$. If $f^{n+1}(x)=f(f^{n}(x))$ for all n ∈ N, then $f^6(6)+f^7(7)$ is equal to:

(A) $\frac{7}{6}$ (B) $-\frac{3}{2}$ (C) $\frac{7}{12}$ (D) $-\frac{11}{12}$

Official Ans. by NTA (B)

Sol. $f(x)=\frac{x-1}{x+1}$

$$\Rightarrow f^2(x)=f(f(x))=\frac{\frac{x-1}{x+1}-1}{\frac{x-1}{x+1}+1}=-\frac{1}{x}$$

$$f^3(x)=f(f^2(x))=f\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^4(x)=f\left(\frac{x+1}{1-x}\right)=-\frac{1}{x}$$

$$\Rightarrow f^6(x)=-\frac{1}{x}\Rightarrow f^6(6)=-\frac{1}{8}$$

$$f^7(x)=\left(-\frac{1}{x}\right)=\frac{x+1}{1-x}$$

$$\Rightarrow f^7(7)=\frac{8}{-6}=-\frac{4}{3}$$

$$\therefore -\frac{1}{6}+-\frac{4}{3}=-\frac{3}{2}$$

2. Let $A=\left\{z\in C:\left|\frac{z+1}{z-1}\right|<1\right\}$

and $B=\left\{z\in C:\arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}\right\}$.

Then $A\cap B$ is :

(A) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second and third quadrants only

(B) a portion of a circle centred at $\left(0,-\frac{1}{\sqrt{3}}\right)$ that lies in the second quadrant only

(C) an empty set

(D) a portion of a circle of radius $\frac{2}{\sqrt{3}}$ that lies in the third quadrant only

Official Ans. by NTA (B)

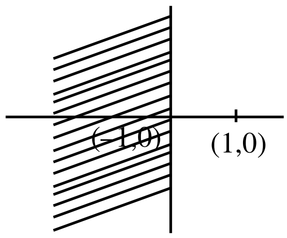
Sol. Set A

$$\Rightarrow \left|\frac{z+1}{z-1}\right|<1$$

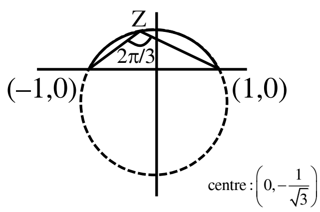
$$\Rightarrow |z+1|<|z-1|$$

$$\Rightarrow (x+1)^2+y^2<(x-1)^2+y^2$$

$$\Rightarrow x<0$$



Set B



$$\Rightarrow \arg\left(\frac{z-1}{z+1}\right)=\frac{2\pi}{3}$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x-1}\right)-\tan^{-1}\left(\frac{y}{x+1}\right)=\frac{2\pi}{3}$$

$$\Rightarrow x^2+y^2+\frac{2y}{\sqrt{3}}-1=0$$

$A\cap B$

$$\Rightarrow \text{Centre}\left(0,-\frac{1}{\sqrt{3}}\right)$$

3. Let A be a 3 × 3 invertible matrix. If |adj (24A)| = adj(3adj(2A))|, then $|A|^2$ is equal to :

(A) 6^6 (B) 2^{12} (C) 2^6 (D) 1

Official Ans. by NTA (C)

Sol. |adj (24A)| = |adj 3(adj 2A)|

$\Rightarrow |24a|^2 = (3\ \text{adj}(2A))^2$

$\Rightarrow (24^3\ |A|)^2 = (3^3|\text{adj}\ (2A)|)^2$

$= 3^6(|2A|^2)^2$

$$\Rightarrow 24^6\,|A|^2 = \left(24^3\,|A|\right)^2 = 3^6 \times 2^{12}\,|A|^4$$

$$\Rightarrow |A|^2 = \frac{24^6}{3^6 \times 2^{12}} = 64$$

4. The ordered pair (a, b), for which the system of linear equations

$3x - 2y + z = b$

$5x - 8y + 9z = 3$

$2x + y + az = -1$

has no solution, is :

(A) $\left(3, \frac{1}{3}\right)$ (B) $\left(-3, \frac{1}{3}\right)$

(C) $\left(-3, -\frac{1}{3}\right)$ (D) $\left(3, -\frac{1}{3}\right)$

Official Ans. by NTA (C)

Sol. $$\begin{vmatrix} 3 & -2 & 1 \\ 5 & -8 & 9 \\ 2 & 1 & a \end{vmatrix} = 0$$

$3(-8a - 9) + 2(5a - 18) + 1(21) = 0$

$\Rightarrow a = -3$

$$\text{Also } \Delta_2 = \begin{vmatrix} 3 & -2 & b \\ 5 & 8 & 3 \\ 2 & 1 & -1 \end{vmatrix}^{\frac{1}{3}}$$

If $b = \frac{1}{3}$

$\Delta_2 = 0$

So b must be equal to

$-\frac{1}{3}$

5. The remainder when $(2021)^{2023}$ is divided by 7 is :

(A) 1 (B) 2 (C) 5 (D) 6

Official Ans. by NTA (C)

Sol. $(2021)^{2023} = (7\lambda - 2)^{2023}$

$= {}^{2023}C_0(7A)^{2023} - \ldots\ldots\ {}^{2023}C_{2023}2^{2023}$

$= 7t - 2^{2023}$

$\therefore\ -2^{2023} = -2 \times 2^{2022}$

$= -2 \times \left(2^3\right)^{674}$

$= -2\left(1 + 7\mu\right)^{674}$

$= -(7\alpha + 2)$

$\Rightarrow$ remainder $= -2$ or + 5

6. $\lim\limits_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$ is equal to :

(A) $\sqrt{2}$ (B) $-\sqrt{2}$

(C) $\frac{1}{\sqrt{2}}$ (D) $-\frac{1}{\sqrt{2}}$

Official Ans. by NTA (D)

Sol. $$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin(\cos^{-1} x) - x}{1 - \tan(\cos^{-1} x)}$$

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sin\left(\sin^{-1}\sqrt{1 - x^2}\right) - x}{1 - \tan\left(\tan^{-1}\left(\frac{\sqrt{1 - x^2}}{x}\right)\right)}$$

$$\lim_{x \to \frac{1}{\sqrt{2}}} \frac{\sqrt{1 - x^2} - x}{1 - \left(\frac{\sqrt{1 - x^2}}{x}\right)}$$

$$\lim_{x\to\frac{1}{\sqrt{2}}}(-x)=-\frac{1}{\sqrt{2}}$$

7. Let f, g : R → R be two real valued functions defined as $f(x)=\begin{cases}-|x+3| & , x<0\\ e^x & , x\geq 0\end{cases}$ and

$g(x)=\begin{cases}x^2+k_1x & , x<0\\ 4x+k_2 & , x\geq 0\end{cases}$, where k_1 and k_2 are real constants. If (gof) is differentiable at x = 0, then (gof) (–4) + (gof) (4) is equal to :

(A) $4(e^4 + 1)$ (B) $2(2e^4 + 1)$

(C) $4e^4$ (D) $2(2e^4 – 1)$

Official Ans. by NTA (D)

Sol. $f(x)=\begin{cases}x+3 & ; x<-3\\ -(x+3) & ; -3\leq x<0\\ e^x & ; x\geq 0\end{cases}$

$g(x)=\begin{cases}x^2+k_1x & ; x<0\\ 4x+k_2 & ; x\geq 0\end{cases}$

$g(f(x))=\begin{cases}f(x)^2+k_1f(x) & ; f(x)<0\\ 4f(x)+k_2 & ; f(x)\geq 0\end{cases}$

$g(f(x))=\begin{cases}(x+3)^2+k_1(x+3) & ; x<-3\\ (x+3)^2-k_1(x+3) & ; -3\leq x<0\\ 4e^x+k_2 & ; x>0\end{cases}$

check continuity at x = 0

$gof(0)=g(f(0^-))=g(f(0^+))$

$4+k_2=9-3k_1=4+k_2$

$3k_1+k_2=5$ ...(a)

differentiate

$(g(f(x)))'=\begin{cases}2(x+3)+k_1 & ; x<-3\\ 2(x+3)-k_1 & ; -3\leq x<0\\ 4e^x & ; x\geq 0\end{cases}$

$6-k_1=4$

$k_1=2$ ...(b)

$\therefore k_1=2, k_2=-1$

$gof(x)=\begin{cases}(x+3)^2+2(x+3) & ; x<-3\\ (x+3)^2-2(x+3) & ; -3\leq x<0\\ 4e^x-1 & ; x\geq 0\end{cases}$

gof(–4) + gof(4) = $4e^4-2$

$\Rightarrow 2(2e^4-1)$

8. The sum of the absolute minimum and the absolute maximum values of the function $f(x)=|3x-x^2+2|-x$ in the interval [–1, 2] is :

(A) $\frac{\sqrt{17}+3}{2}$ (B) $\frac{\sqrt{17}+5}{2}$

(C) 5 (D) $\frac{9-\sqrt{17}}{2}$

Official Ans. by NTA (A)

Sol. $f(x)=\begin{cases}x^2-4x-2, & \forall x\in\left(-1,\frac{3-\sqrt{17}}{2}\right)\\ -x^2+2x+2, & \forall x\in\left(\frac{3-\sqrt{17}}{2},2\right)\end{cases}$

$f'(x)$ when $x\in\left(-1,\frac{3-\sqrt{17}}{2}\right)$

$f'(x) = 2x – 4 = 0 \Rightarrow x = 2$

$f'(x) = 2(x – 2)$ $\Rightarrow f'(x)$ is always ↓

$f(2) = 2$

$f(–1) = 3$

$f\left(\frac{3-\sqrt{17}}{2}\right)=\frac{\sqrt{17}-3}{2}$

$f'(x)$ when $x\in\left(\frac{3-\sqrt{17}}{2},2\right)$

$f'(x) = –2x + 2$

$f'(x) = –2(x – 1)$

$f'(x) = 0$ when $x = 1$

$f(1) = 3$

absolute minimum value = $\frac{\sqrt{17}-3}{2}$

absolute maximum value = 3

Sum = $\frac{\sqrt{17}-3}{2}+3=\frac{\sqrt{17}+3}{2}$

9. Let S be the set of all the natural numbers, for which the line $\frac{x}{a}+\frac{y}{b}=2$ is a tangent to the curve $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$ at the point (a, b), ab ≠ 0. Then:

(A) $S=\phi$ (B) $n(S)=1$

(C) $S=\{2k : k\in N\}$ (D) $S=N$

Official Ans. by NTA (D)

Sol. $\left(\frac{x}{a}\right)^n+\left(\frac{y}{b}\right)^n=2$

Slope of tangent at (a, b)

$$n.\left(\frac{x}{a}\right)^{n-1}.\frac{1}{a}+n\left(\frac{x}{b}\right)^{n-1}.\frac{1}{b}\frac{dy}{dx}=0$$

$$\left.\frac{dy}{dx}\right|_{(a,b)}=-\frac{b}{a}$$

∴ Equation of tangent

$$y-b=-\frac{b}{a}(x-a)$$

$$\frac{x}{a}+\frac{y}{b}=2\ \forall\ n\in N$$

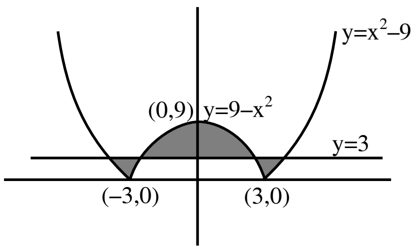
10. The area bounded by the curve $y=|x^2-9|$ and the line y = 3 is :

(A) $4\left(2\sqrt{3}+\sqrt{6}-4\right)$ (B) $4\left(4\sqrt{3}+\sqrt{6}-4\right)$

(C) $8\left(4\sqrt{3}+3\sqrt{6}-9\right)$ (D) $8\left(4\sqrt{3}+\sqrt{6}-9\right)$

Official Ans. by NTA (DROP)

Sol.



Area of shaded region

$$=2\int_0^3\left(\sqrt{9+y}-\sqrt{9-y}\right)dy+2\int_3^9\left(\sqrt{9-y}\right)dy$$

$$=2\left[\int_0^3(9+y)^{1/2}dy-\int_0^3(9-y)^{1/2}dy+\int_3^9(9-y)^{1/2}dy\right]$$

$$=2\left[\frac{2}{3}\left[(9+y)^{3/2}\right]_0^3+\frac{2}{3}\left[(9-y)^{3/2}\right]_0^3-\frac{2}{3}\left[(9-y)^{3/2}\right]_3^9\right]$$

$$=\frac{4}{3}\left[12\sqrt{12}-27+6\sqrt{6}-27-\left(0-6\sqrt{6}\right)\right]$$

$$=\frac{4}{3}\left[24\sqrt{3}+12\sqrt{6}-54\right]$$

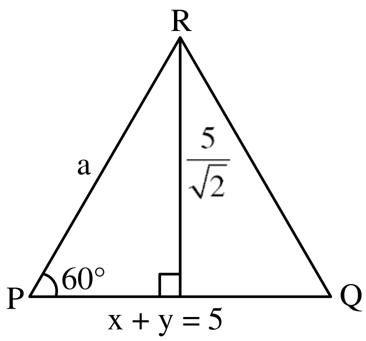
$$=8\left(4\sqrt{3}+2\sqrt{6}-9\right)$$

11. Let R be the point (3, 7) and let P and Q be two points on the line x + y = 5 such that PQR is an equilateral triangle. Then the area of ΔPQR is :

(A) $\frac{25}{4\sqrt{3}}$ (B) $\frac{25\sqrt{3}}{2}$ (C) $\frac{25}{\sqrt{3}}$ (D) $\frac{25}{2\sqrt{3}}$

Official Ans. by NTA (D)

Sol.



$$\sin 60°=\frac{5/\sqrt{2}}{a}$$

$$a=\frac{5\sqrt{2}}{3}$$

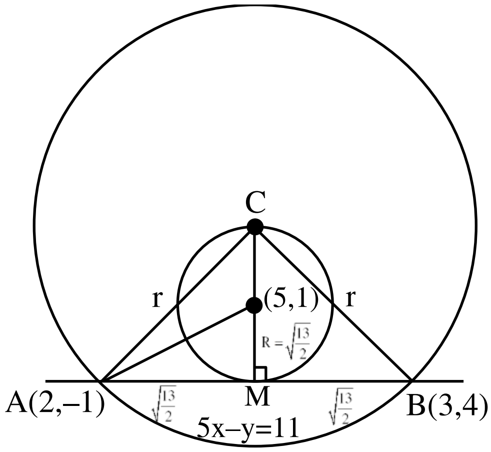
Area of ΔPQR = $\frac{\sqrt{3}}{4}a^2=\frac{25}{2\sqrt{3}}$

12. Let C be a circle passing through the points A(2, –1) and B(3, 4). The line segment AB is not a diameter of C. If r is the radius of C and its centre lies on the circle $(x-5)^2+(y-1)^2=\frac{13}{2}$, then r^2 is equal to :

(A) 32 (B) $\frac{65}{2}$ (C) $\frac{61}{2}$ (D) 30

Official Ans. by NTA (B)

Sol.



AB = $\sqrt{26}$

$r^2 = CM^2 + AM^2$

$= \left(2\times\sqrt{\frac{13}{2}}\right)^2 + \left(\sqrt{\frac{13}{2}}\right)^2$

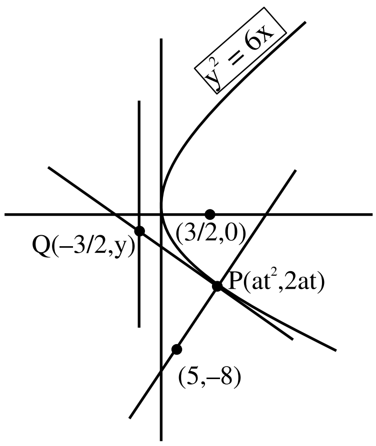
$r^2 = \frac{65}{2}$

13. Let the normal at the point P on the parabola $y^2 = 6x$ pass through the point (5, –8). If the tangent at P to the parabola intersects its directrix at the point Q, then the ordinate of the point Q is :

(A) –3 (B) $-\frac{9}{4}$ (C) $-\frac{5}{2}$ (D) –2

Official Ans. by NTA (B)

Sol.



Equation of normal : $y = -tx + 2at + at^3$ $\left(a = \frac{3}{2}\right)$

since passing through (5, –8), we get $t = -2$

Co-ordinate of Q : (6, –6)

Equation of tangent at Q : x + 2y + 6 = 0

Put $x = \frac{-3}{2}$ to get $R\left(\frac{-3}{2}, \frac{-9}{4}\right)$

14. If the two lines $l_1 : \frac{x-2}{3} = \frac{y+1}{-2}$, z = 2 and $l_2 : \frac{x-1}{1} = \frac{2y+3}{\alpha} = \frac{z+5}{2}$ perpendicular, then an angle between the lines l_2 and $l_3 : \frac{1-x}{3} = \frac{2y-1}{-4} = \frac{z}{4}$ is :

(A) $\cos^{-1}\left(\frac{29}{4}\right)$ (B) $\sec^{-1}\left(\frac{29}{4}\right)$

(C) $\cos^{-1}\left(\frac{2}{29}\right)$ (D) $\cos^{-1}\left(\frac{2}{\sqrt{29}}\right)$

Official Ans. by NTA (B)

Sol. $l_1 : \frac{x-2}{3} = \frac{y+1}{-2} = \frac{z-2}{0}$

$l_2 : \frac{x-1}{1} = \frac{y+3/2}{\alpha/2} = \frac{z+5}{2}$

$l_3 : \frac{x-1}{-3} = \frac{y-1/2}{-2} = \frac{z-0}{4}$

$l_1 \perp l_2 \Rightarrow \frac{|3-\alpha+0|}{\sqrt{13}\sqrt{1+\frac{\alpha^2}{4}+4}} = 0 \Rightarrow \alpha = 3$

angle between l_2 & l_3

$$\cos\theta = \frac{|1\times(-3)+(-2)(\alpha/2)+2\times4|}{\sqrt{1+4+\frac{\alpha^2}{4}}\sqrt{9+16+4}}$$

$$\cos\theta = \frac{|-3-\alpha+8|}{\sqrt{5+\frac{\alpha^2}{4}}\sqrt{29}}$$

put $\alpha = 3$

$$\cos\theta = \frac{2}{\sqrt{\frac{29}{4}}\sqrt{29}} = \frac{4}{29}$$

$$\theta = \cos^{-1}\left(\frac{4}{29}\right) \Rightarrow \theta = \sec^{-1}\left(\frac{29}{4}\right)$$

15. Let the plane 2x + 3y + z + 20 = 0 be rotated through a right angle about its line of intersection with the plane x – 3y + 5z = 8. If the mirror image of the point $\left(2,-\frac{1}{2},2\right)$ in the rotated plane is B(a, b, c), then :

(A) $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$ (B) $\frac{a}{4}=\frac{b}{5}=\frac{c}{-2}$

(C) $\frac{a}{8}=\frac{b}{-5}=\frac{c}{4}$ (D) $\frac{a}{4}=\frac{b}{5}=\frac{c}{2}$

Official Ans. by NTA (A)

Sol. Let equation of rotated plane be :

$(2x + 3y + z + 20) + \lambda(x - 3y + 5z - 8) = 0$

$(2 + \lambda)x + (3 - 3\lambda)y + (1 + 5\lambda)z + 20 - 8\lambda = 0$

Above plane is perpendicular to $2x + 3y + z + 20 = 0$

So, $(2 + \lambda).2 + (3 - 3\lambda).3 + (1 + 5\lambda).1 = 0 \Rightarrow \lambda = 7$

$\Rightarrow$ Equation of rotated plane : $x - 2y + 4z - 4 = 0$

Mirror image of $A\left(2,\frac{-1}{2},2\right)$ in rotated plane is B(a, b, c)

Equation of AB : $\frac{x-2}{1}=\frac{y+1/2}{-2}=\frac{z-2}{4}=k$

Let coordinate of B be $\left(2+k, \frac{-1}{2}-2k, 2+4k\right)$

midpoint of AB is $\left(2+\frac{k}{2},\frac{-1}{2}-k,2+2k\right)$ which will lie on the plane $x - 2y + 4z - 4 = 0$

Hence $k=\frac{-2}{3}$

Therefore B is $\left(\frac{4}{3},\frac{5}{6},\frac{-2}{3}\right)\equiv\left(\frac{8}{6},\frac{5}{6},\frac{-4}{6}\right)$

So, $\frac{a}{8}=\frac{b}{5}=\frac{c}{-4}$

16. If $\vec{a}\cdot\vec{b}=1$, $\vec{b}\cdot\vec{c}=2$ and $\vec{c}\cdot\vec{a}=3$, then the value of $\left[\vec{a}\times\left(\vec{b}\times\vec{c}\right), \vec{b}\times\left(\vec{c}\times\vec{a}\right), \vec{c}\times\left(\vec{b}\times\vec{a}\right)\right]$ is :

(A) 0 (B) $-6\vec{a}\cdot\left(\vec{b}\times\vec{c}\right)$

(C) $12\vec{c}\cdot\left(\vec{a}\times\vec{b}\right)$ (D) $-12\vec{b}\cdot\left(\vec{c}\times\vec{a}\right)$

Official Ans. by NTA (A)

Sol. $\vec{a}\times\left(\vec{b}\times\vec{c}\right)=(\vec{a}.\vec{c})\vec{b}-(\vec{a}.\vec{b})\vec{c}=3\vec{b}-\vec{c}$

$\vec{b}\times\left(\vec{c}\times\vec{a}\right)=(\vec{b}.\vec{a})\vec{c}-(\vec{b}.\vec{c})\vec{a}=\vec{c}-2\vec{a}$

$\vec{c}\times\left(\vec{b}\times\vec{a}\right)=(\vec{c}.\vec{a})\vec{b}-(\vec{c}.\vec{b})\vec{a}=3\vec{b}-2\vec{a}$

$\left[3\vec{b}-\vec{c},\vec{c}-2\vec{a}, 3\vec{b}-2\vec{a}\right]$

$(3\vec{b}-\vec{c}).\left[(\vec{c}-2\vec{a})\times(3\vec{b}-2\vec{a})\right]$

$(3\vec{b}-\vec{c}).\left[3(\vec{c}\times\vec{b})-2(\vec{c}\times\vec{a})-6(\vec{a}\times\vec{b})\right]$

$-6\left[\vec{b}\ \vec{c}\ \vec{a}\right]+6\left[\vec{c}\ \vec{a}\ \vec{b}\right]$

17. Let a biased coin be tossed 5 times. If the probability of getting 4 heads is equal to the probability of getting 5 heads, then the probability of getting atmost two heads is:

(A) $\frac{275}{6^5}$ (B) $\frac{36}{5^4}$ (C) $\frac{181}{5^5}$ (D) $\frac{46}{6^4}$

Official Ans. by NTA (D)

Sol. P(H) = x, P(T) = 1 – x

P(4H, 1T) = P(5H)

${}^5C_1(x)^4\,(1-x)^1 = {}^5C_5\,x^5$

$5(1 - x) = x$

$6x = 5 = 0 \quad x=\frac{5}{6}$

P(atmost 2H)

= P(OH, 5T) + P(1H, 4T) + P(2H, 3T)

$={}^5C_0\left(\frac{1}{6}\right)^5+{}^5C_1\frac{5}{6}.\left(\frac{1}{6}\right)^4+{}^5C_2\left(\frac{5}{6}\right)^3\left(\frac{1}{6}\right)^3$

$=\frac{1}{6^5}(1+25+250)=\frac{276}{6^5}$

$= \frac{46}{6^4}$

18. The mean of the numbers a, b, 8, 5, 10 is 6 and their variance is 6.8. If M is the mean deviation of the numbers about the mean, then 25 M is equal to:

(A) 60 (B) 55 (C) 50 (D) 45

Official Ans. by NTA (A)

Sol. $\sigma^2 = \frac{\sum_{i=1}^{5}(x_i - \bar{x})^2}{n}$

Mean = 6

$\frac{a+b+8+5+10}{5} = 6$

$a + b = 7$

$b = 7 - a$

$6.8 = \frac{(a-6)^2+(b-6)^2+(8-6)^2+(5-6)^2+(10-6)^2}{5}$

$34 = (a-6)^2 + (7-a-6)^2 + 4 + 1 + 18$

$a^2 - 7a + 12 = 0 \Rightarrow a = 4$ or $a = 3$

a = 4 a = 3

b = 3 b = 4

$M = \frac{\sum_{i=1}^{5}|x_i - x|}{n}$

$M = \frac{|a-6|+|b-6|+|8-6|+|5-6|+|10-6|}{5}$

when a = 3, b = 4 when a = 4, b = 3

$M = \frac{3+2+2+1+4}{5}$ $M = \frac{2+3+2+1+7}{5}$

$M = \frac{12}{5}$ $M = \frac{12}{5}$

$25M = 25 \times \frac{12}{5} = 60$

19. Let $f(x) = 2\cos^{-1}x + 4\cot^{-1}x - 3x^2 - 2x + 10$, $x \in [-1, 1]$. If [a, b] is the range of the function then 4a – b is equal to:

(A) 11 (B) $11 - \pi$ (C) $11 + \pi$ (D) $15 - \pi$

Official Ans. by NTA (B)

Sol. $f'(x) = \frac{-2}{\sqrt{1-x^2}} - \frac{4}{1+x^2} - 6x - 2$

$= -2\left[\frac{1}{\sqrt{1-x^2}} + \frac{2}{1+x^2} + 3x + 1\right]$

$f'(x) < 0 \Rightarrow f(x)$ is a dec. function

$f(1) = \pi + 5$

$f(-1) = 5\pi + 5$

Range : $[a, b] \equiv [\pi + 5, 5\pi + 5]$

$a = \pi + 5, b = 5\pi + 5 \Rightarrow 4a - b = 11 - \pi$.

20. Let $\Delta, \nabla \in \{\wedge, \vee\}$ be such that $p \nabla q \Rightarrow ((p \Delta q) \nabla r)$ is a tautology. Then $(p \nabla q) \Delta r$ is logically equivalent to :

(A) $(p \Delta r) \vee q$ (B) $(p \Delta r) \wedge q$

(C) $(p \wedge r) \Delta q$ (D) $(p \nabla r) \wedge q$

Official Ans. by NTA (A)

Sol. **Case-I** If $\Delta \equiv \nabla \equiv \wedge$

$(p \wedge q) \to ((p \wedge q) \wedge r)$

it can be false if r is false,

so not a tautology

Case-II If $\Delta \equiv \nabla \equiv \vee$

$(p \vee q) \to ((p \vee q) \vee r) \equiv$ tautology

then $(p \vee q) \vee r \equiv (p \Delta r) \vee q$

Case-III if $\Delta = \vee, \nabla = \wedge$

then $(p \wedge q) \to \{(p \vee q) \wedge r\}$

Not a tautology

(Check $p \to T, q \to T, r \to F$)

Case-IV if $\Delta = \wedge, \nabla = \vee$

$(p\wedge q)\rightarrow\{(p\wedge q)\vee r\}$

Not a tautology

SECTION-B

1. The sum of the cubes of all the roots of the equation $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$ is ______.

Official Ans. by NTA (36)

Sol. $x^4 - 3x^3 - 2x^2 + 3x + 1 = 10$

x = 0 is not the root of this equation so divide it by x^2

$$x^2 - 3x - 2 + \frac{3}{x} + \frac{1}{x^2} = 0$$

$$x^2 + \frac{1}{x^2} - 2 + 2 - 3\left(x - \frac{1}{x}\right) - 2 = 0$$

$$\left(x - \frac{1}{x}\right)^2 - 3\left(x - \frac{1}{x}\right) = 0$$

$x - \frac{1}{x} = 0$, $\quad x - \frac{1}{x} = 3$

$x^2 - 1 = 0$ $\quad x^2 - 3x - 1 = 0$

$x = \pm 1$ $\quad \gamma + \delta = 3$

$\alpha = 1, \beta = -1$ $\quad \gamma\delta = -1$

$\alpha^3 + \beta^3 + \gamma^3 + \delta^3$

$1 - 1 + (\gamma + \delta)((\gamma + \delta)^2 - 3\gamma\delta)$

$0 + 3(9 - 3(-1))$

$+ 3(12) = 36$

2. There are ten boys $B_1, B_2, ..., B_{10}$ and five girls $G_1, G_2, ..., G_5$ in a class. Then the number of ways of forming a group consisting of three boys and three girls, if both B_1 and B_2 together should not be the members of a group, is__________.

Official Ans. by NTA (1120)

Sol. n(B) = 10

n(a) = 5

The number of ways of forming a group of 3 girls of 3 boys.

$= {}^{10}C_3 \times {}^5C_3$

$$= \frac{10\times9\times8}{3\times2}\times\frac{5\times4}{2} = 1200$$

The number of ways when two particular boys B_1 of B_2 be the member of group together

$= {}^8C_1 \times {}^5C_3 = 8 \times 10 = 80$

Number of ways when boys B_1 of B_2 hot in the same group together

$= 1200 \times 80 = 1120$

3. Let the common tangents to the curves $4(x^2 + y^2) = 9$ and $y^2 = 4x$ intersect at the point Q. Let an ellipse, centered at the origin O, has lengths of semi-minor and semi-major axes equal to OQ and 6, respectively. If e and l respectively denote the eccentricity and the length of the latus rectum of this ellipse, then $\frac{l}{e^2}$ is equal to__________.

Official Ans. by NTA (4)

Sol. $x^2 + y^2 = \frac{9}{4}$ $\qquad$ y = 4x

Equation tangent in slope form

$$y = mx \pm \frac{3}{2}\sqrt{(1+m^2)} \quad ...(1)$$

$$y = mx + \frac{1}{m} \quad ...(2)$$

compare (1) & (2)

$$\pm\frac{3}{2}\sqrt{(1+m^2)} = \frac{1}{m^2}$$

$9m^2(1 + m^2) = 4$

$9m^4 + 9m^2 - 4 = 0$

$9m^4 + 12m^2 - 3m^2 - 4 = 0$

$3m^2(3m^2 + 4) - (3m^2 + 4) = 0$

$m^2 = -\frac{4}{3}$ (Rejected)

$m^2 = \frac{1}{3} \Rightarrow m = \pm\frac{1}{\sqrt{3}}$

Equation of common tangent

$$y = \frac{1}{\sqrt{3}}x + \sqrt{3}$$

on X axis y = 0

OQ = –3

b = |OQ| = 3

a = 6

$b^2 = a^2(1 - e^2) \Rightarrow e^2 = 1 - \frac{9}{36} = \frac{3}{4}$

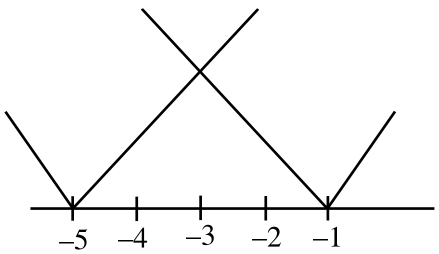
$$e = \frac{2b^2}{a} = \frac{2\times 9}{6} = 3$$

$$\frac{e}{e^2} = \frac{3}{3/4} = 4$$

4. Let f(x) = max{|x + 1|, |x + 2|, ..., |x + 5|}. Then $\int_{-6}^{0} f(x)\,dx$ is equal to ______________.

4. Official Ans. by NTA (21)

Sol. f(x) = max{|x+1|, |x+2|, |x + 3|, |x + 4|, |x + 5|}



$$\int_{-6}^{0} f(x)\,dx = \int_{-6}^{-3} |x+1|\,dx + \int_{-3}^{0} |x+5|\,dx$$

$$= -\int_{-6}^{-3} (x+1)\,dx + \int_{-3}^{0} (x+5)\,dx$$

$$= -\left[\frac{x^2}{2} + x\right]_{-6}^{-3} + \left[\frac{x^2}{2} + 5x\right]_{-3}^{0}$$

$$= -\left[\left(\frac{9}{2} - 3\right) - (18-6)\right] + \left[0 - \left(\frac{9}{2} - 15\right)\right]$$

$$= -\left[\frac{3}{2} - 12\right] + \frac{21}{2} = \frac{21}{2} + \frac{21}{2} = 21$$

5. Let the solution curve y = y(x) of the differential equation $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx = 0$ pass through the origin. Then y(2) is equal to________.

Official Ans. by NTA (12)

Sol. $(4 + x^2)dy - 2x(x^2 + 3y + 4)dx$

$$(x^2 + 4)\frac{dy}{dx} = 2x^3 + 6xy + 8x$$

$$(x^2 + 4)\frac{dy}{dx} - 6xy = 2x^3 + 8x$$

$$\frac{dy}{dx} - \frac{6x}{x^2+4}y = \frac{2x^3+8x}{x^2+y}$$

L.I. $\frac{dy}{dx} + py = \phi$

$$p = \frac{-6x}{x^2+4} \qquad \phi = \frac{2x^3+8x}{x^2+4}$$

$$\text{I.F.} = e^{-\int\frac{6x}{x^2+4}dx} = e^{-3\log_e(x^2+4)}$$

$$= e^{\log_e(x^2+4)^{-3}} = \frac{1}{(x^2+4)^3}$$

Sol.

$$y.\frac{1}{(x^2+4)^3} = \int \frac{2x^3+8x}{(x^2+4)^3(x^2+4)}dx$$

$$\frac{y}{(x^2+4)^3} = \int \frac{2x(x^2+4)}{(x^2+4)^3(x^2+4)}dx$$

$x^2 + 4 = t$

2xdx = dt

$$\frac{y}{(x^2+4)^3} = \int \frac{dt}{t^3}$$

$$\frac{y}{(x^2+4)^3} = \frac{-1}{2(x^2+4)^2} + C$$

passes through origin (0, 0)

$$0 = \frac{-1}{2\times 16} + C$$

$$\frac{y}{(x^2+4)^3} = \frac{-1}{2(x^2+4)^2} + \frac{1}{32}$$

$$y = \frac{-(x^2+4)}{2} + \frac{(x^2+4)^3}{32}$$

$$y(2) = -\frac{8}{2} + \frac{8\times 8\times 8}{32} = 12$$

6. If $\sin^2(10°)\sin(20°)\sin(40°)\sin(50°)\sin(70°) = \alpha - \frac{1}{16}\sin(10°)$, then $16 + \alpha^{-1}$ is equal to ___________.

Official Ans. by NTA (80)

Sol. $\sin 10°\left(\frac{1}{2}.2\sin 20°\sin 40°\right).\sin 10°\sin(60°-10°)\sin(60°+10°)$

$\sin 10°\frac{1}{2}(\cos 20°-\cos 60°).\frac{1}{4}\sin 30°$

$\frac{1}{2}.\frac{1}{4}.\frac{1}{2}.\sin 10°\left(\cos 20°-\frac{1}{2}\right)$

$=\frac{1}{32}(2\sin 10°\cos 20°-\sin 10°)$

$=\frac{1}{32}(\sin 30°-\sin 10°-\sin 10°)$

$=\frac{1}{32}\left(\frac{1}{2}-2\sin 10°\right)$

$=\frac{1}{64}(1-4\sin 10°)$

$=\frac{1}{64}-\frac{1}{16}\sin 10°$

Hence $\alpha=\frac{1}{64}$

$16+\alpha^{-1}=80$

7. Let A = {n ∈ N : H.C.F. (n, 45) = 1} and

Let B = {2k : k ∈ {1, 2, ..., 100}}. Then the sum of all the elements of A ∩ B is __________.

Official Ans. by NTA (5264)

Sol. Sum of elements in A ∩ B

$=\underbrace{(2+4+6+...+200)}_{\text{Multiple of 2}}-\underbrace{(6+12+...+198)}_{\text{Multiple of 2 \& 3 i.e. 6}}$

$-\underbrace{(10+20+...+200)}_{\text{Multiple of 5 \& 2 i.e. 10}}+\underbrace{(30+60+...+180)}_{\text{Multiple of 2, 5 \& 3 i.e. 30}}$

= 5264

8. The value of the integral $\frac{48}{\pi^4}\int_0^{\pi}\left(\frac{3\pi x^2}{2}-x^3\right)\frac{\sin x}{1+\cos^2 x}dx$ is equal to _______.

Official Ans. by NTA (6)

Sol. $I=\frac{48}{\pi^4}\int_0^{\pi}x^2\left(\frac{3\pi}{2}-x\right)\frac{\sin x}{1+\cos^2 x}dx$ ...(1)

Apply king property

$I=\frac{48}{\pi^4}\int_0^{\pi}(\pi-x)^2\left(\frac{\pi}{2}+x\right)\frac{\sin x}{1+\cos^2 x}dx$ ...(2)

(1) + (2)

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+(\pi-2).x.(\pi-2x)\right]dx$ ...(3)

Apply king again

$I=\frac{12}{\pi^3}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[\pi^2+(\pi-2)(\pi-x)(2x-\pi)\right]dx$ ...(4)

(3) + (4)

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(\pi-2x)\right]dx$ ...(5)

Apply king

$I=\frac{6}{\pi^2}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}\left[2\pi+(\pi-2)(2x-\pi)\right]dx$ ...(6)

(5) + (6)

$I=\frac{12}{\pi}\int_0^{\pi}\frac{\sin x}{1+\cos^2 x}dx$

Let cosx = t ⇒ sinxdx = –dt

$I=\frac{12}{\pi}\int_1^{-1}\frac{-dt}{1+t^2}=6$

9. Let $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$ and $B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$. Then A + B is equal to _________.

Official Ans. by NTA (1100)

Sol. $A=\sum_{i=1}^{10}\sum_{j=1}^{10}\min\{i,j\}$

$B=\sum_{i=1}^{10}\sum_{j=1}^{10}\max\{i,j\}$

$A=\sum_{j=1}^{10}\min(i,1)+\min(j,2)+...\min(i,10)$

$=\underbrace{(1+1+1+...+1)}_{\text{19 times}}+\underbrace{(2+2+2...+2)}_{\text{17 times}}+\underbrace{(3+3+3...+3)}_{\text{15 times}}$

+ ... (1) 1 times

$$B = \sum_{j=1}^{10} \max(i,1) + \max(j,2) + \ldots \max(i,10)$$

$$= \underbrace{(10+10+\ldots+10)}_{19\text{ times}} + \underbrace{(9+9+\ldots+9)}_{17\text{ times}} + \ldots + 1 \text{ 1 times}$$

A + B = 20(1 + 2 + 3 + ... +10)

$$= 20 \times \frac{10\times 11}{2} = 10 \times 110 = 1100$$

10. Let $S = (0,2\pi) - \left\{\frac{\pi}{2}, \frac{3\pi}{4}, \frac{3\pi}{2}, \frac{7\pi}{4}\right\}$. Let y = y(x), x ∈ S, be the solution curve of the differential equation $\frac{dy}{dx} = \frac{1}{1+\sin 2x}$, $y\left(\frac{\pi}{4}\right) = \frac{1}{2}$. if the sum of abscissas of all the points of intersection of the curve y = y(x) with the curve $y = \sqrt{2}\sin x$ is $\frac{k\pi}{12}$, then k is equal to __________.

Official Ans. by NTA (42)

Sol. $$\frac{dy}{dx} = \frac{1}{1+\sin 2x}$$

$$\int dy = \int \frac{dx}{(\sin x + \cos x)^2}$$

$$\int dy = \int \frac{\sec^2 x}{(1+\tan x)^2}$$

$$y(x) = -\frac{1}{1+\tan x} + C$$

$$y\left(\frac{\pi}{4}\right) = \frac{1}{2} = -\frac{1}{2} + C$$

C = 1

$$y(x) = \frac{-1}{1+\tan x} + 1$$

$$y(x) = \frac{-1+1+\tan x}{1+\tan x}$$

$$y(x) = \frac{\tan x}{1+\tan x}$$

Solving with $y = \sqrt{2}\sin x$

$$\frac{\tan x}{1+\tan x} = \sqrt{2}\sin x$$

$\sin x = 0$, $\quad \frac{1}{\sqrt{2}} = \sin x + \cos x$

$x = \pi \quad \frac{1}{2} = \sin\left(x + \frac{\pi}{4}\right)$

$$\sin\frac{\pi}{6} = \sin\left(x + \frac{\pi}{4}\right)$$

$$x + \frac{\pi}{4} = \pi - \frac{\pi}{6},\ 2\pi + \frac{\pi}{6}$$

$$x = \frac{5\pi}{6} - \frac{\pi}{4},\quad x = \frac{13\pi}{6} - \frac{\pi}{4}$$

$$x = \frac{7\pi}{12},\ x = \frac{23\pi}{12}$$

sum of sol.

$$= \pi + \frac{7\pi}{12} + \frac{23\pi}{12}$$

$$= \frac{12\pi + 7\pi + 23}{12} = \frac{42\pi}{12} = \frac{k\pi}{12}$$

$\Rightarrow k = 42$