JEE-Main-27-07-2021-Shift-1 (Memory Based)

PHYSICS

Question: Energy of an oscillating system is E. At a particular instant kinetic energy of system is 3E/4. Find

displacement of the oscillating particle from its mean position. Its amplitude of oscillation is A.

Options:

(A) $\frac{A}{2}$

(B) $\frac{A}{3}$

(C) $\frac{A}{4}$

(D) $\frac{A}{6}$

Answer: (A)

Solution:

$$\frac{1}{2}m\omega^2\left(A^2-x^2\right)=\frac{3}{4}\left(\frac{1}{2}m\omega^2 A^2\right)$$

$$\Rightarrow A^2-x^2=\frac{3}{4}A^2$$

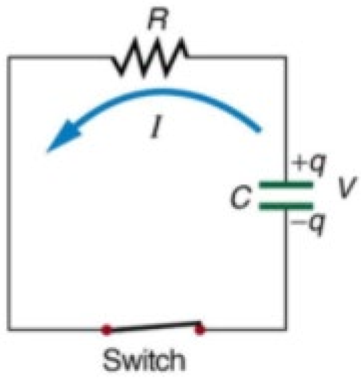
$$\Rightarrow x^2=\frac{A^2}{4}$$

$$\Rightarrow x=\frac{A}{2}$$

Question: A capacitor of capacitance $100\mu F$ discharges through a resistor R. At the same time a radioactive

substance decays with mean life 30 ms. If ratio of charge on capacitor to activity of substance does not change

with time, then find R.



Options:
(A) 300Ω
(B) 100Ω
(C) 200Ω
(D) 400Ω
Answer: (A)
Solution:

$$\frac{Q}{A}=\frac{Q_0 e^{-t/RC}}{A_o e^{-\lambda t}} \text{(ratio is time independent)}$$

$$\lambda=\frac{1}{RC}, \quad RC=\frac{1}{\lambda}=T_{\text{mean}} \text{ (mean life)}$$

$$R=\frac{T_{\text{mean}}}{C}=\frac{30\times10^{-3}}{100\times10^{-6}}=300\Omega$$

Question: Three balls A, B and C of masses m, 2 m and 2 m respectively are placed on a smooth horizontal
surface. A is initially moving with velocity 9 m / s, collides elastically with B (initially at rest), which in turn collides
inelastically with C (initially at rest). Assuming all collisions to be head on, final velocity of C is
Options:
(A) $\frac{9}{2}(m/s)$
(B) $9(m/s)$
(C) $6(m/s)$
(D) $3(m/s)$
Answer: (D)
Solution:

For collision between A and B

$$m_A=m, m_B=2m, u_A=9m/s, u_B=0, v_B=?$$

$$v_B = \frac{2m_A u_A}{m_A + m_B} - \left(\frac{m_A - m_B}{m_A + m_B}\right) u_B$$

$$= \frac{2 \times m \times 9}{m + 2m} - 0 = 6m/s$$

For collision between B and C

$$m_B = 2m, m_C = 2m, u_B = 6m/s, u_C = 0, v = ?$$

$$m_B u_B + m_C u_C = (m_B + m_C) v$$

$$2m \times 6 + 0 = 4mv$$

$$\Rightarrow v = \frac{12}{4} = 3m/s$$

Question: A particle of mass $9.1 \times 10^{-31} kg$ moving with a velocity $10^6 m/s$ has de Broglie wavelength λ_1. A

photon of momentum $10^{-27} kgm/s$ has wavelength λ_2 · Find $\frac{\lambda_2}{\lambda_1}$

Options:
(A) 910
(B) 667
(C) $\frac{1}{310}$
(D) 1
Answer: (A)
Solution: $\frac{\lambda_2}{\lambda_1} = \frac{h/p}{h/mv} = \frac{mv}{p} = \frac{9.1 \times 10^{-31} \times 10^6}{10^{-27}} = 910$

Question: Two disks having same surface mass density have radii r and R .

$I_1 \rightarrow$ Moment of inertia of 1st disk about an axis perpendicular to the plane and passing through centre.

$I_2 \rightarrow$ Moment of inertia of 2nd disk about one of its diameters. Find $\frac{I_1}{I_2}$ Memory based questions

Options:

(A) $\frac{r^4}{R^4}$

(B) $\frac{2r^4}{R^4}$

(C) $\frac{r^4}{2R^4}$

(D) $\frac{2r^2}{R^4}$

Answer: (B)

Solution:

Moment of inertia of a disc about it's center perpendicular to the plane of the disc is $\frac{MR^2}{2}$

& Moment of inertia of a disc about one of its diameter is $\frac{MR^2}{4}$

Now both dices have same surface mass density $(\sigma)(say)$

then mass of 1st discs with radius $r = (\pi r^2)\sigma$

$\therefore$ M.I of disc 1 about center, $\perp$ to the plane $= \frac{mr^2}{2} = (\pi r^2)\sigma \frac{r^2}{2}$

$= \frac{\pi \sigma r^4}{2} = I_1$

mass of 2nd disc with radius $R = (\pi R^2)\sigma$

$\therefore$ M.I of 2nd disc about one of it's diameters is $\frac{mR^2}{4}$

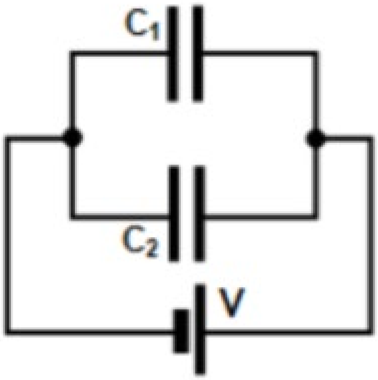
$= (\pi R^2)\sigma \frac{R^2}{4} = I_2 = \frac{\pi R^4 \sigma}{4}$

$$\frac{I_1}{I_2} = \frac{\left(\frac{\pi \sigma r^4}{2}\right)}{\frac{\pi \sigma R^4}{4}}$$

$= \frac{2r^4}{R^4}$

Question: Two capacitor C_1 with capacitor 2 C , and C_2 with capacitance C are connected in parallel. They are charged and then the battery is removed. If a material of dielectric constant K is inserted in

$C_{2'}$, find final potential across them



Options:
(A) KV
(B) $\frac{3V}{K+2}$
(C) $\frac{V}{K}$
(D) $\frac{3}{KV}$
Answer: (B)
Solution:
Both are in parallel, so p.d across them will be same.

$Q_1 = C_1V = (2C)V = 2CV$

$Q_2 = C_2V = CV$

Total charge = 3CV

When dielectric is inserted in it's capacitance becomes KC.

If V' is final common potential, then since total charge remains unchanged

$Q_{1f} + Q_{2f} = Q_1 + Q_2 = 3CV$

$2C\left(V'\right) + (KC)V' = 3CV$

$V' = \left(\frac{3V}{2+k}\right)$

Question: Pressure of a monatomic gas in a container is 2 atm. Average kinetic energy per molecule is $2\times10^{-9}J$.
Volume of gas is 1 litre. Find number of molecules of gas present in the container.
Options:
(A) $\frac{3}{2}\times10^{11}$
(B) $\frac{3}{2}\times10^{10}$

(C) $\frac{5}{2}\times10^{12}$

(D) $\frac{5}{2}\times10^{11}$

Answer: (A)

Solution:

No, of molecules = no of moles $\times N_A$

$= n\times N_A$

$n=\frac{PV}{RT}=\frac{PV}{N_A kT}$

Given avg. kinetic energy $=2\times10^{-9}J$

$\therefore \frac{3}{2}kT=2\times10^{-9}$

$\therefore kT=\frac{4\times10^{-9}}{3}$

Now given 2 atm pressure

$\therefore P=2\times1.013\times10^{5}N/m^{2}$

Volume $=1\text{litre}=10^{-3}\text{m}^{3}$

No of molecules $=\frac{PV}{N_A kT}\times N_A$

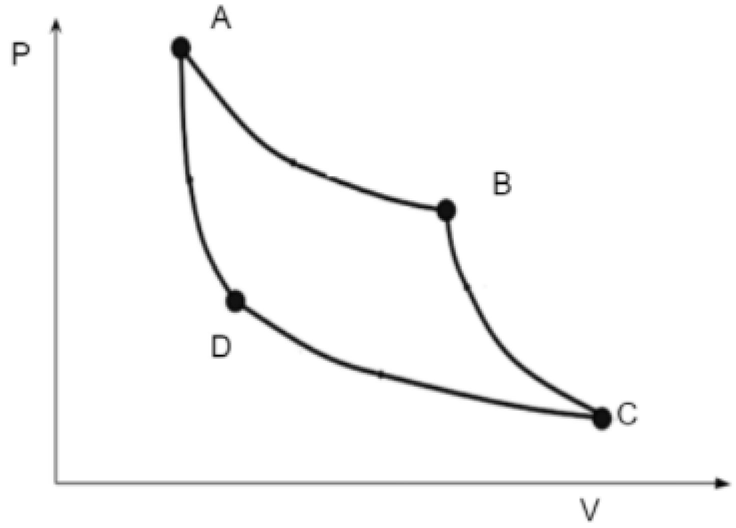
$=\frac{PV}{kT}$

$=\frac{2\times1.013\times10^{5}\times10^{-3}}{4/3\times10^{-9}}$

$\approx\frac{3}{2}\times10^{11}$

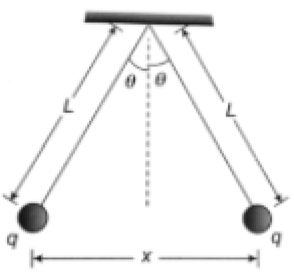
Question: If $V_A=V_0;V_B=3.5V,V_c=5.5V_0,V_0=1.5V_0,P_A=9P_0,P_D=P_0$ in the given indicator diagram, choose

correct option



Options:
(A) $W_{AB} < W_{CD}$
(B) $W_{AB} = W_{CD}$
(C) $W_{BC} + W_{DA} > 0$
(D) $W_{BC} = W_{AD}$
Answer: (D)
Solution:
$A \to B$ and $D \to C$ are isotherms
So. $T_A = T_B$ & $T_C = T_D$
Now, $B \to C$ and $D \to A$ are adiabatic
$\therefore |W_{BC}| = \frac{nR}{r-1}(T_B - T_C)$
$|W_{AD}| = \frac{nR}{r-1}(T_A - T_D) - \frac{nR}{r-1}(T_B - T_C)$
$\therefore W_{BC} = W_{AD}$

Question: Two point charges are suspended from a given point as shown in the figure. Find the equilibrium separation between them



Options:

(a) $q\sqrt{\dfrac{4\pi\varepsilon_0}{mg\tan\theta}}$

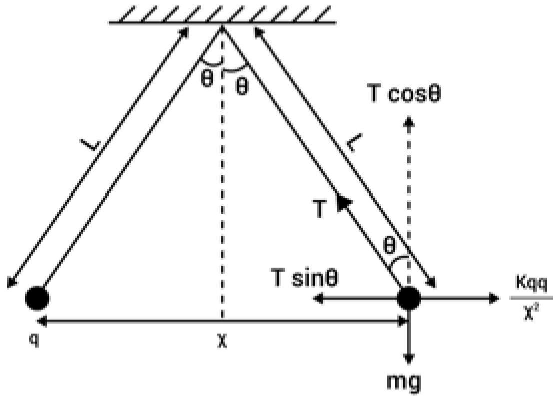
(b) $q\sqrt{\dfrac{\cot\theta}{4\pi\varepsilon_0 mg}}$

(c) $q\sqrt{\dfrac{\sin\theta}{mg4\pi\varepsilon_0}}$

(d) $q\sqrt{\dfrac{\cos\theta}{4\pi\varepsilon_0 mg}}$

Answer: (b)

Solution:



$T\sin\theta = \dfrac{Kq^2}{x^2}$ …(i)

$T\cos\theta = mg$ …(ii)

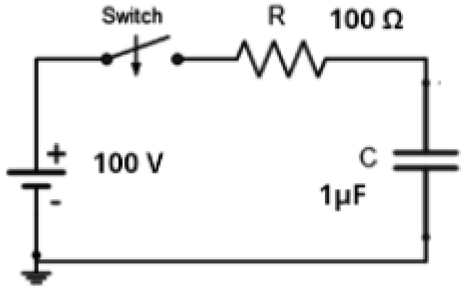
From eq (i) and (ii)

$\tan\theta = \dfrac{Kq^2}{x^2}\cdot\dfrac{1}{mg}$

$x = \sqrt{\dfrac{Kq^2}{mg\tan\theta}}$

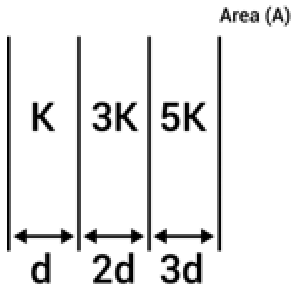
$x = q\sqrt{\dfrac{\cot\theta}{4\pi\varepsilon_0 mg}}$

Question: The switch is closed at t = 0. Find time after which voltage across capacitor becomes 50 volt. [take $\ell n2 = 0.6$]

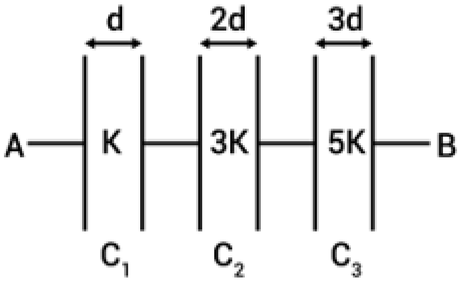


Options:
(a) $100\mu s$
(b) $60\mu s$
(c) $80\mu s$
(d) $70\mu s$
Answer: (b)
Solution:
$$V = V_0\left(1-e^{-\frac{t}{RC}}\right)$$
$$50 = 100\left(1-e^{-\frac{t}{RC}}\right)$$
$$\frac{1}{2} = 1-e^{-\frac{t}{RC}}$$
$$e^{-\frac{t}{RC}} = \frac{1}{2}$$
$$\frac{t}{RC} = \ell n 2$$
$$t = RC\ell n2$$
$$t = 100\times1\times10^{-6}\times0.6$$
$$t = 60\mu s$$

Question: Equivalent capacitance of following arrangement of identical parallel plates is



Options:
(a) $\frac{25k\varepsilon_0 A}{6d}$
(b) $\frac{6k\varepsilon_0 A}{25d}$
(c) $\frac{15k\varepsilon_0 A}{34d}$
(d) $\frac{2k\varepsilon_0 A}{15d}$
Answer: (c)
Solution:



$$\frac{1}{C_{eq}}=\frac{1}{C_1}+\frac{1}{C_2}+\frac{1}{C_3}$$

$$\frac{1}{C_{eq}}=\frac{d}{\varepsilon_0 AK}+\frac{2d}{\varepsilon_0 A3K}+\frac{3d}{\varepsilon_0 A5K}$$

$$\frac{1}{C_{eq}}=\frac{d}{\varepsilon_0 AK}\left\{1+\frac{2}{3}+\frac{3}{5}\right\}$$

$$\frac{1}{C_{eq}}=\frac{d}{\varepsilon_0 AK}\times\frac{34}{15}$$

$$C_{eq}=\frac{15\varepsilon_0 AK}{34d}$$

Question: Young's modulus of a string is $0.5\times10^9 Pa$, length of the wire without any force applied in 0.1 m and area is $0.04\times10^{-4} m^2$. If this wire is stretched by a length of 0.001 m. The energy stored in this string is transferred to a particle of mass 20 grams. Find speed of the partcile

Options:

(a) 1 m/s

(b) 0.5 m/s

(c) 2 m/s

(d) 0.25 m/s

Answer: (a)

Solution:

Young's modulus $Y=0.5\times10^3\, pa$

$\ell=0.1m$

$A=0.04\times10^{-4} m^2$

$\Delta\ell=0.001m$

$m=20g=20\times10^{-3}$

Energy stored in the string due to extension $=\frac{1}{2}\sigma\varepsilon\times\text{volume}$

$=\frac{1}{2}Y\varepsilon^2\cdot\text{volume}$

From energy conservation

$\frac{1}{2}mu^2=\frac{1}{2}Y\varepsilon^2\times\text{volume}$

$$20\times10^{-3}\times u^2 = 0.5\times10^9\times\left(\frac{0.001}{0.1}\right)^2\times0.04\times10^{-4}\times0.1$$

$$2\times10^{-2}u^2 = 2\times10^{-2}$$

$$u^2 = 1$$

$$u = 1m/s.$$

Question: In a YDSE setup, orange light is replaced by blue light. Then,

Options:

(a) Fringe width will increase

(b) Fringe width will decrease

(c) At center, instead of maxima, there would be a minima

(d) The intensity of central maxima will decrease

Answer: (b)

Solution:

Fringe width $\beta = \frac{\lambda D}{d}$

$$\lambda_{\text{orange}} > \lambda_{\text{blue}}$$

$$\beta_{\text{orange}} \propto \lambda_{\text{orange}} \quad \ldots\text{(i)}$$

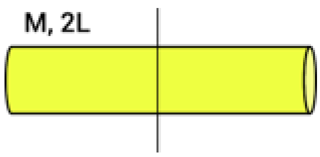
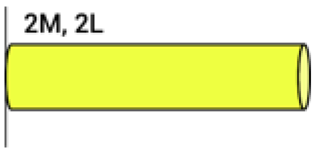
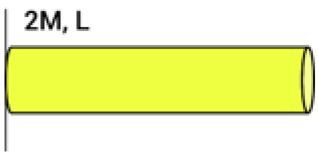
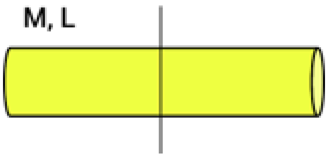
$$\beta_{\text{blue}} \propto \lambda_{\text{blue}} \quad \ldots\text{(ii)}$$

From (i) and (ii)

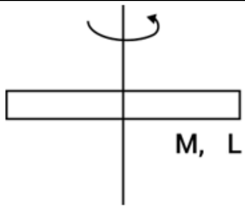
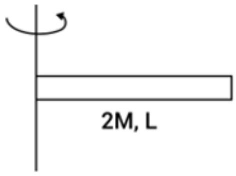
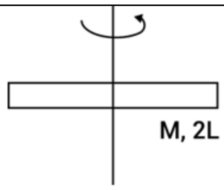
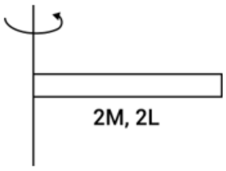
$$\frac{\beta_{\text{blue}}}{\beta_{\text{orange}}} = \frac{\lambda_{\text{blue}}}{\lambda_{\text{orange}}}$$

$$\beta_{\text{blue}} < \beta_{\text{orange}}.$$

Question: Match the moment of inertia of the rods with given mass and length in column A about the given axis.

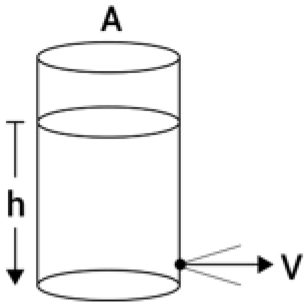
A		i	$\frac{8}{3}ML^2$
B		ii	$\frac{ML^2}{3}$
C		iii	$\frac{2ML^2}{3}$
D		iv	$\frac{ML^2}{12}$

Options:
(a) A(iv), B(iii), C(ii), D(i)
(b) A(ii), B(iii), C(iv), D(i)
(c) A(iii), B(ii), C(iv), D(i)
(d) A(ii), B(iv), C(i), D(iii)
Answer: (a)
Solution:

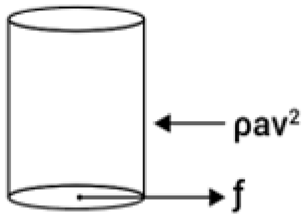
A	 $\rightarrow \frac{ML^2}{12}$
B	 $\rightarrow \frac{2ML^2}{3} = 2\frac{ML^2}{3}$
C	 $\rightarrow \frac{M(2L)^2}{12} = \frac{ML^2}{3}$
D	 $\rightarrow \frac{2M \cdot (2L)^2}{3} = \frac{8ML^2}{3}$

A(iv), B(iii), C(ii), D(i)

Question: A cylindrical massless container of cross-sectional area 'A' have a fluid filled upto height 'h' and have a small orifice of area 'a' in wall near its bottom. Find minimum coefficient between container and ground, so that container does not move.
Options:
(a) $\frac{2a}{A}$
(b) $\frac{a}{2A}$
(c) $\frac{A}{a}$
(d) None
Answer: (a)
Solution:



$v=\sqrt{2gh}$ as a << A



Thrust on container is $\rho a v^2$

where

$\rho \rightarrow$ density

$a \rightarrow$ area of cross section of orifice

$v \rightarrow$ speed of fluid

$f \rightarrow$ force of friction.

$f \le f_L$

$f \le \mu mg$

$f \le \mu \rho A h \cdot g$

For

$\rho a v^2 \le \rho A h g \times \mu$

Container does not move

$\rho a \times 2gh \le \rho A h g \times \mu$

$$\frac{2a}{A} \le \mu$$

$$\mu_{\min} = \frac{2a}{A}$$

Question: Two prisms P_1 and P_2 whose refractive index as a function of wavelength λ are μ_1 and μ_2 respectively where $\mu_1 = 1.2 + \frac{10.8\times10^{-14}}{\lambda^2}$ and $\mu_2 = 1.45 + \frac{1.8\times10^{-14}}{\lambda^2}$.

Find λ for which when P_1 and P_2 are put together, no deviation of light happened in the contact surface.

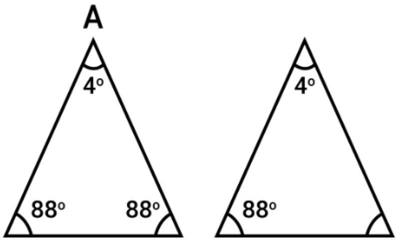
Assume both the prism are thin, and both having refracting angle A=4°

Options:

(a) 900 nm

(b) 600 nm

(c) 800 nm
(d) 700 nm
Answer: (b)
Solution:



$$\mu_1 = 1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2}$$

$$\mu_2 = 1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}$$

$\delta_1 + \delta_2 = 0$ for the system.

$$\delta_1 = (\mu_1 - 1) A_1$$

$$\delta_2 = (\mu_2 - 1) A_2$$

$$\delta_1 + \delta_2 = \mu_1 A_1 - A_1 + \mu_2 A_2 - A_2 = 0$$

$$\left|\frac{(\mu_1 - 1)}{(\mu_2 - 1)}\right| = \frac{A_2}{A_1}$$

$$\frac{1.2 + \frac{10.8 \times 10^{-14}}{\lambda^2} - 1}{1.45 + \frac{1.8 \times 10^{-14}}{\lambda^2} - 1} = +\frac{A_2}{A_1}$$

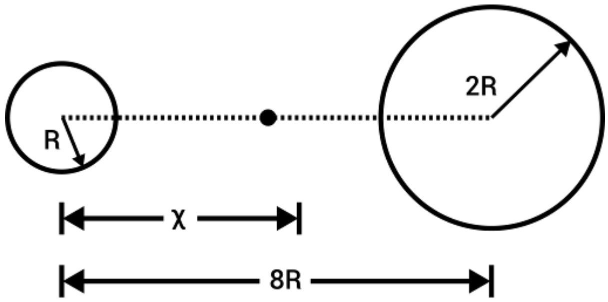
$$\frac{0.2 + \frac{10.8 \times 10^{-14}}{\lambda^2}}{0.45 + \frac{1.8 \times 10^{-14}}{\lambda^2}} = +\frac{A_2}{A_1}$$

on solving

$$\lambda = 600\ nm$$

Question: Two planets A and B of masses M and 9 M with radii R and 2 R respectively are present 8 R distance away from each other. Minimum velocity with which a particle is projected from surface of A such that it reaches plane B is given by $\sqrt{\frac{aGM}{7R}}$. Value of ‘a’ is

Answer: (4)
Solution:



We have to find the point where the gravitational field must be zero.This is because after that point particle itself pulled by greater mass , no need to give any amount of kinetic energy.

$$E_G = 0$$

$$\frac{GM}{x^2} = \frac{GM \times q}{(8R - x)^2}$$

$$\frac{(8R - x)^2}{x^2} = 9$$

$$\frac{8R - x}{x} = 3 \Rightarrow 8R - x = 3x$$

$$\boxed{x = 2R}$$

Potential at A

$$V_A = -\frac{GM}{R} - \frac{G9M}{7R} = -\frac{16GM}{7R}$$

Potential at point x distance apart from A

$$V_x = -\frac{GM}{2R} - \frac{G9M}{6R} = -\frac{12GM}{6R}$$

$$= -\frac{2GM}{R}$$

Potential difference is

$$\Delta V = V_x - V_A = -\frac{2GM}{R} + \frac{16GM}{7R} = \frac{2GM}{7R}$$

Applying conservation of energy principle

$$\Delta KE = \Delta U$$

$$\frac{1}{2} mV^2 = m\Delta V$$

$$\frac{1}{2} mV^2 = \frac{2GMm}{7R}$$

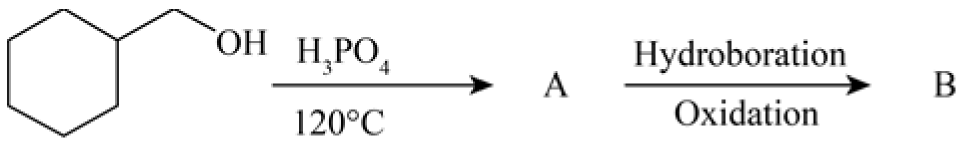
$$V = \sqrt{\frac{4GM}{7R}}$$

$$a = 4$$

JEE-Main-27-07-2021-Shift-1 (Memory Based)

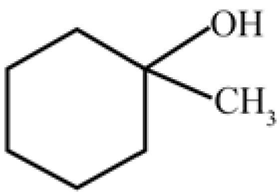
CHEMISTRY

Question: In the following reaction find ‘B’

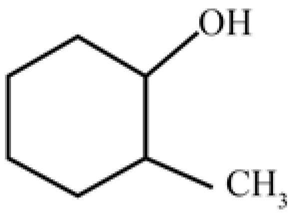


Options:

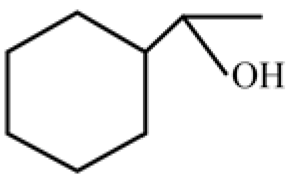
(a)



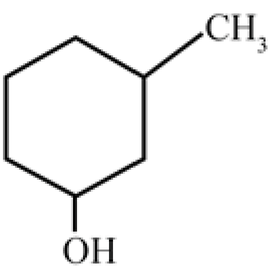
(b)



(c)

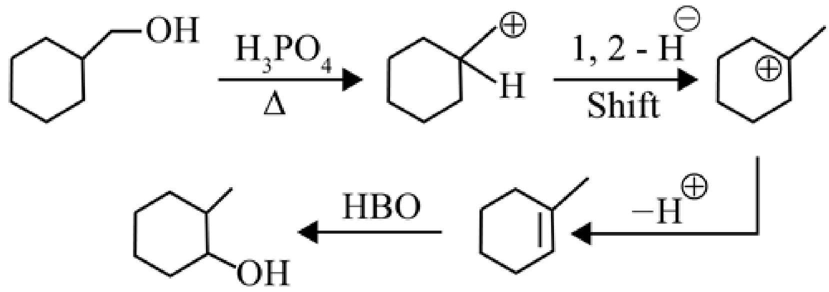


(d)



Answer: (b)

Solution:



(B) (A)

Question: If density of aqueous NaOH solution is 1.2 g/cm^3, then find its molality.

[Given that: density of water = 1 gm/cm^3 and molar mass of NaOH = 40 gm]

Options:

(a) 5 M

(b) 10 M

(c) 15 M

(d) 20 M

Answer: (a)

Solution:

Let the volume of solution = 1000 cm^3

∴ Weight of solution = 1200 g

Now, weight of water in the solution = 1000 g

Thus weight of NaOH = 200 g

∴ Moles of NaOH = $\frac{200}{40} = 5$

Thus, molarity = $\frac{5}{1} = 5$ M

Question: Difference between bond order of CO and NO^+ is x/2. Find x.

Options:

(a) 1

(b) 0

(c) 2

(d) 0.5

Answer: (b)

Solution: Electron is removed from anti-bonding of NO (bond order = 2.5) hence bond order increases by 0.5 hence NO^+ has bond order 3 and so does CO. The difference is zero

Question: $CH_3\text{-}I + I_2 \rightleftharpoons CH_4 + HI$

Which reagent can stop backward reaction?

Options:

(a) Dilute HNO_2

(b) Conc. HIO_3

(c) HClO

(d) NH_3(aq)

Answer: (b)

Solution: $5HI$ + Conc. $HIO_3 \rightarrow 3I_2 + 3H_2O$

Question: Which of the following is incorrect about Ellingham diagram?

Options:

(a) Graph gives idea about rate of reaction

(b) Graph gives idea about reduction of metal oxide

(c) Graph gives idea about free energy change

(d) Graph gives idea about phase change

Answer: (a)

Solution: Ellingham diagram does not give any information about rate of reaction

Question: Oxidation state of Phosphorus in $H_4P_2O_7$; $H_4P_2O_6$ and $H_4P_2O_5$ is respectively:

Options:

(a) +5, +4 and +3 respectively

(b) +5, +5 and +3 respectively

(c) +3, +4 and +5 respectively

(d) +7, +4 and +4 respectively

Answer: (a)

Solution:

$H_4P_2O_7$: $4 + 2x - 14 = 0$

$\Rightarrow 2x - 10 = 0$

$\Rightarrow 2x = 10$

$\Rightarrow x = +5$

$H_4P_2O_6$: $\Rightarrow 4 + 2x - 12 = 0$

$\Rightarrow 2x - 8 = 0$

$\Rightarrow 2x = 8$

$\Rightarrow x = +4$

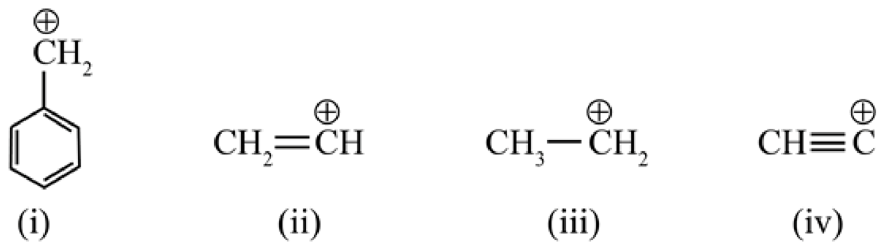
$H_4P_2O_5$: $4 + 2x - 10 = 0$

$\Rightarrow 2x - 6 = 0$

$\Rightarrow 2x = 6$

$\Rightarrow x = +3$

Question: Arrange the increasing order of stability order of the following:



Options:

(a) i > iii > ii > iv

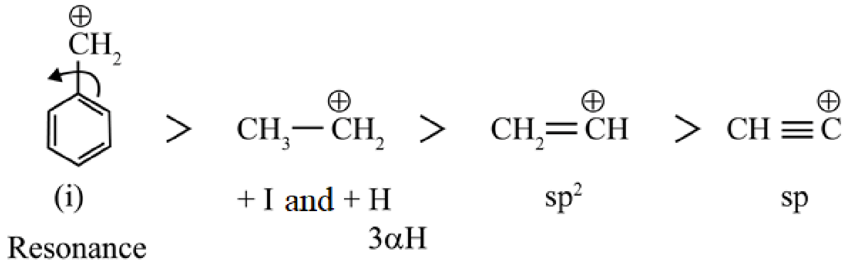
(b) ii > iii > i > iv

(c) iii > ii > i > iv

(d) iv > ii > iii > i

Answer: (a)

Solution:



Question: A: Aniline is less basic than acetamide.

R: Lone pair of N in aniline is involved in resonance.

Options:

(a) Both Assertion and Reason are correct.

(b) Assertion is correct and Reason is incorrect

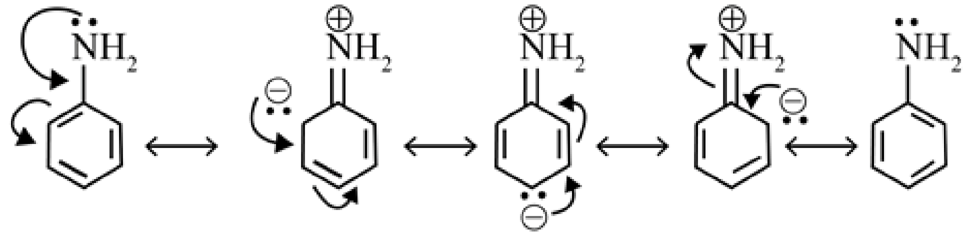
(c) Assertion is incorrect but Reason is correct.

(d) Both Assertion and Reason are incorrect.

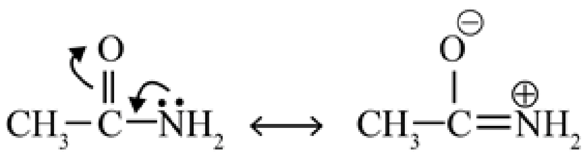
Answer: (c)

Solution: Acetamide is less basic then aniline because its lone pair of Nitrogen atoms involved in resonance with oxygen atom hence it is less available to donate

Aniline:



Amide:
(acetamide)



Question: S1: Halides of Lithium are generally covalent.

S2: Lithium has high polarisability.

Options:

(a) Both S1 and S2 are correct.

(b) S1 is correct but S2 is incorrect.

(c) S1 is incorrect but S2 is correct.

(d) Both S1 and S2 are incorrect.

Answer: (b)

Solution: Lithium halides are somewhat covalent. It is because of the high polarisation capability of lithium ion (The distortion of electron cloud of the anion by the cation is called polarisation). The Li^+ ion is very small in size and has high tendency to distort electron cloud around the negative halide ion. Since anion with large size can be easily distorted, among halides, lithium iodide is the most covalent in nature.

Polarisability is defined for anion

Question: S1: Rutherford's gold foil experiment didn't explain hydrogen spectrum.

S2: Bohr's model contradicted Heisenberg's uncertainty principle.

Options:

(a) Both S1 and S2 are correct.

(b) S1 is correct but S2 is incorrect.

(c) S1 is incorrect but S2 is correct.

(d) Both S1 and S2 are incorrect.

Answer: (a)

Solution: Both given statement are correct.

Question: Staggered and eclipsed form of ethane are:

Options:

(a) Rotamers

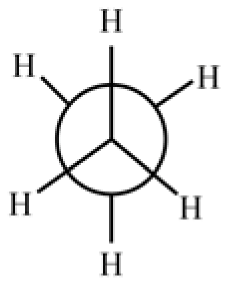
(b) Enantiomers

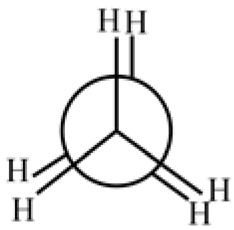
(c) Mirror images

(d) Polymers

Answer: (a)

Solution: Rotamers are any of a number of isomers of a molecule which can be inter converted by rotation of part of the molecule about a particular bond.





Staggered and Eclipsed form of ethane are inter convertible by rotation of the molecule about a bond. Hence these are rotamers.

Question: $PCl_5 \rightleftharpoons PCl_3 + Cl_2$, $K_C = 1.844$

If 3 moles of PCl_5 are taken in 1 L vessel. Find equilibrium concentration of PCl_5.

Options:

(a) 3.4 M

(b) 2.4 M

(c) 1.4 M

(d) 4.4 M

Answer: (c)

Solution:

$PCl_5 \rightleftharpoons PCl_3 + Cl_2$

3 – –

3-x x x

$$\frac{x^2}{3-x} = 1.844$$

$\Rightarrow x^2 = 5.532 - 1.844\,x$

$\Rightarrow x^2 + 1.844\,x - 5.532 = 0$

$\Rightarrow x = 1.6$ M

$\therefore$ Concentration of PCl_5 at equilibrium = 3 – x = 3 – 1.6 = 1.4 M

Question: Decomposition of N_2O_5 is a ________ order reaction

Options:

(a) Zero

(b) First

(c) Second

(d) Pseudo-first

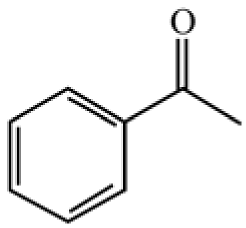
Answer: (b)

Solution: Decomposition of N_2O_5 is an example of first order reaction

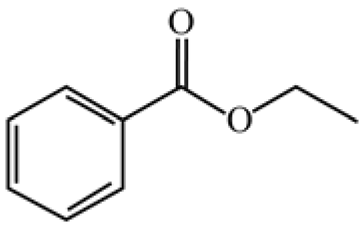
Question: Which gives orange colour with 2,4 DNP?

Options:

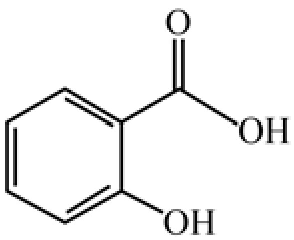
(a)



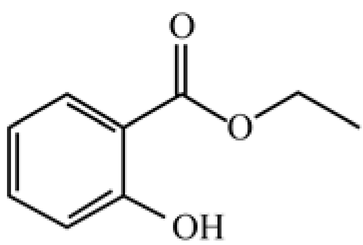
(b)



(c)

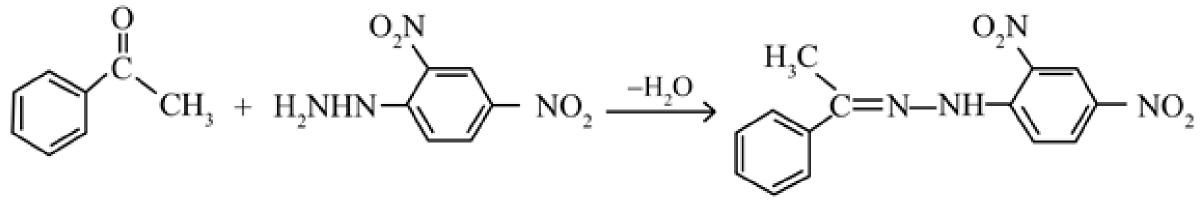


(d)



Answer: (a)

Solution: 2,4-dinitrophenyl hydrazine solution is used to detect ketones and aldehydes. A positive test is confirmed by the formation of a yellow, orange or red precipitate.



Question: Electrolysis of sulphate compound solution will give:

Options:

(a) HO_3SOOSO_3H

(b) HO_2SOOSO_2H

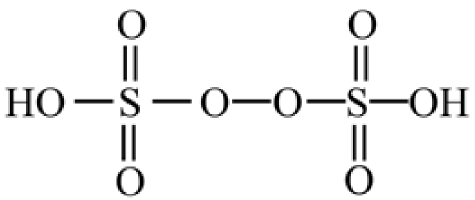
(c) HO_2SOSO_2H

(d) HO_3SOSO_3H

Answer: (a)

Solution: $2SO_4^{2-} \xrightarrow{\text{electrolysis}} S_2O_8^{2-} + 2e^-$

$H_2S_2O_8$:



Question: Match the column.

Column I	Column II
1. Arsphenamine	**(A)** Antibiotic
2. Valium	**(B)** Tranquilizer
3. Furacine	**(C)** Antiseptic
	(D) Synthetic antihistamine

Options:

(a) 1 → A; 2 → B; 3 → C

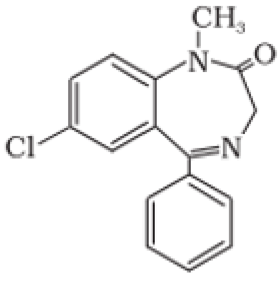
(b) 1 → B; 2 → D; 3 → C

(c) 1 → D; 2 → B; 3 → A

(d) 1 → C; 2 → B; 3 → A

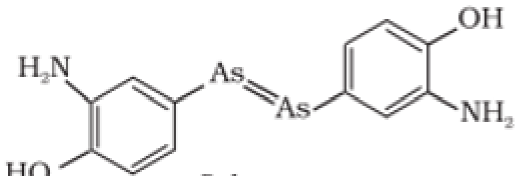
Answer: (a)

Solution:



Valium

Tranquilizer



Salvarsan

Arsphenamine, known as a salvarsan used as Antibiotic

Antiseptics are applied to the living tissues such as wounds, cuts, ulcers and diseased skin surfaces. Examples are furacine, soframicine.

Question: Identify the incorrect statement?

Options:

(a) Eutrophication pollutes water

(b) Eutrophication increases oxygen level in water

(c) Oxygen conc. is below 6 ppm, it is harmful for fish

(d) None of these

Answer: (b)

Solution: Eutrophication is the process in which a water body becomes overly enriched with nutrients, leading to plentiful growth of simple plant life. The excessive growth (or bloom) of algae and plankton in a water body are indicators of this process.

The excessive growth of algae in entrap, water is accompanied by the generation of a lame biomass of dead algae. These dead algae sink to the bottom of the water body where they are broken down by bacteria, which consume oxygen in the process.

Question: In crystal system, α = β = 90°, γ = 120º and a = 1.5, b = 2.5 and c = 3. Find the crystal?

Options:

(a) Monoclinic

(b) Orthorhombic

(c) Triclinic

(d) Hexagonal

Answer: (a)

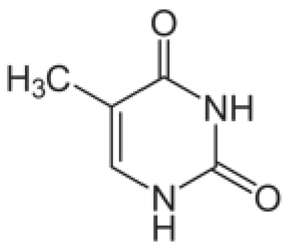
Solution: Given

α = β = 90°, $\gamma \neq$ 120º

a ≠ b ≠ c

Thus, crystal system is Monoclinic

Question: Which of the following is complementary base of the given structure in RNA?



Options:

(a) Uracil

(b) Cytosine

(c) Adenine

(d) Guanine

Answer: (a)

Solution: DNA contains four base adenine(A), guanine(G), cytosine (C) and thymine (T). RNA also contains four bases, the first bases are same as in DNA but the fourth one is uracil (U).

Question: Match the column.

Column I	**Column II**

(1) $Be(OH)_2$	**(A)** Acidic
(2) $B(OH)_3$	**(B)** Basic
(3) NaOH	**(C)** Amphoteric
(4) $Ca(OH)_2$	
(5) $Al(OH)_3$	

Options:

(a) 1 → C; 2 → A; 3 → B; 4 → B; 5 → C

(b) 1 → A; 2 → C; 3 → B; 4 → A; 5 → C

(c) 1 → B; 2 → A; 3 → C; 4 → A; 5 → C

(d) 1 → A; 2 → C; 3 → B; 4 → B; 5 → C

Answer: (a)

Solution:

$Be(OH)_2 \Rightarrow$ Amphoteric

$B(OH)_3 \Rightarrow$ Acidic

NaOH $\Rightarrow$ Basic

$Ca(OH)_2 \Rightarrow$ Basic

$Al(OH)_3 \Rightarrow$ Amphoteric

Question: How to differentiate between monosaccharides and disaccharides

Options:

(a) Iodine test

(b) Seliwanoff's test

(c) Barfoed test

(d) Tollen's test

Answer: (c)

Solution: Barfoed's test distinguishes monosaccharides from disaccharides. In this test, copper acetate in dilute acid is reduced in 30 seconds by monosaccharides whereas disaccharides take several minutes.

Question: If number of geometrical isomers of $[Co(NH_3)_3(NO_2)_3$ is a and that of $(Cr(C_2O_4)_3$ is b, then find a + b ?

Answer: 3.00

Solution:

$[Co(NH_3)_3(NO_2)_3 \Rightarrow Ma_3b_3$

Total geometric isomerism = 2

$Cr(C_2O_4)_3 \Rightarrow M(AA)_3$

Total geometric isomerism = 1

Thus, total = 3

* Please note that $Cr(C_2O_4)_3$ can exist in one form only and examiner might not consider it as geometric isomerism

Question: According to $x/m = kp^{1/n}$, when pressure is increased 2 times, the concentration becomes 64 times. Find the value of n [in terms of 10^{-2}] (Round off to nearest integer)

Answer: 17.00

Solution:

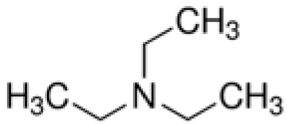
$$\frac{(x/m)_1}{(x/m)_2} = \left(\frac{p_1}{p_2}\right)^{1/n}$$

$$\Rightarrow \frac{1}{64} = \left(\frac{1}{2}\right)^{1/n}$$

$$\Rightarrow \frac{1}{n} = 6$$

$$\Rightarrow n = \frac{1}{6} = 0.167 = 16.7 \times 10^{-2} = 17$$

Question: The bond angle of C-N-C in $N(Et)_3$ is (Round of the answer to the nearest integer.)



Answer: 111.00

Solution: Triethylamine is a base, like ammonia. Also like ammonia, it has a trigonal pyramidal structure. The C-N-C bond angle is 110.9°, compared with 107.2° in NH_3, presumably due to greater repulsion between the ethyl groups

JEE-Main-27-07-2021-Shift-1 (Memory Based)

MATHEMATICS

Question: $\lim_{x \to 2} \frac{x^2 f(2) - 4f(x)}{(x-2)} =$

Where, $f(2) = 4 f'(2) = 1$

Options:

(a) 12

(b) 16

(c) 8

(d) 10

Answer: (a)

Solution:

$$\lim_{x \to 2} \frac{x^2 f(2) - 4f(x)}{x-2}$$

$$= \lim_{x \to 2} \frac{2xf(x) - 4f'(x)}{1}$$

$$= 2 \times 2 f(2) - 4f'(2)$$

$$= 2 \times 2 \times 4 - 4 \times 1$$

$$= 16 - 4 = 12$$

Question: If Coefficient of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ and x^{-7} in $\left[x - \frac{1}{bx^2}\right]^{11}$, $b = 0$, are equal then find b.

Options:

(a) 1

(b) -1

(c) 2

(d) -2

Answer: (a)

Solution:

Coeff. of x^7 in $\left(x^2 + \frac{1}{bx}\right)^{11}$ = coeff. of x^{-7} in $\left(x - \frac{1}{bx^2}\right)^{11}$

$$^{11}C_6\frac{1}{b^5}={}^{11}C_5\frac{1}{b^6}\Rightarrow b=1$$

Question: $\int_{\frac{-\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}$

Options:

(a) $\frac{\pi}{2\left(\sqrt{2}\right)}$

(b) $\frac{\pi}{4\left(\sqrt{2}\right)}$

(c) $\frac{\pi}{8\left(\sqrt{2}\right)}$

(d) $\frac{\pi}{\sqrt{2}}$

Answer: (a)

Solution:

$$I=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{dx}{\left(1+e^{x\cos x}\right)\left(\sin^4 x+\cos^4 x\right)}=\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}}\frac{e^{x\cos x}dx}{\left(\sin^4 x+\cos^4 x\right)\left(1+e^{x\cos x}\right)}$$

$$\therefore I=\int_0^{\frac{\pi}{4}}\frac{dx}{\sin^4 x+\cos^4 x}=\int_0^{\frac{\pi}{4}}\frac{\left(\tan^2 x+1\right)\sec^2 xdx}{\left(\tan^4 x+1\right)}$$

$$I=\int_0^1\frac{t^2+1}{t^4+1}dt=\int_0^1\frac{\left(1+\frac{1}{t^2}\right)}{\left(t-\frac{1}{t}\right)^2+2}dt$$

$$I=\int_{-\infty}^0\frac{du}{u^2+2}=\left[\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right)\right]_{-\infty}^0=\frac{\pi}{2\sqrt{2}}$$

Question: If α and β are the roots of equation $x^2+20^{\frac{1}{4}}x+5^{\frac{1}{2}}=0$ the find $\left(\alpha^8+\beta^8\right)$

Options:

(a) 100
(b) 50
(c) 200
(d) 300
Answer: (b)
Solution:

$$x^2+(20)^{\frac{1}{4}}x+(5)^{\frac{1}{2}}=0$$

$$\alpha+\beta=-(20)^{\frac{1}{4}},\alpha\beta=(5)^{\frac{1}{2}}\Rightarrow\alpha^2+\beta^2=(20)^{\frac{1}{2}}-2(5)^{\frac{1}{2}}=0$$

$$\alpha^4+\beta^4=-2\times5=-10$$
$$\Rightarrow\alpha^8+\beta^8=100-2\times25=50$$

Question: If $f(x)=\begin{cases}(1+|\sin x|)^{\frac{3a}{|\sin x|}}, & \left(\frac{-\pi}{4},0\right)\\ b, & x=0\\ e^{\cot\frac{4x}{\cot 2x}}, & x>0\end{cases}$. Find $6a+b^2$.

Options:
(a) $1+e$
(b) $1-e$
(c) e
(d) $e-1$
Answer: (a)
Solution:

$$f(0^-)=f(0)+f(0^+)\Rightarrow e^{3a}=b=e^{\frac{1}{2}}$$

$$\therefore 3a=\frac{1}{2}\text{ and } b=e^{\frac{1}{2}}\Rightarrow 6a+b^2=1+e$$

Question: If $\sin\theta+\cos\theta=\frac{1}{2}$. then find $16[\sin2\theta+\cos4\theta+\sin6\theta]$

Options:
(a) -27
(b) -23

(c) 27
(d) 23
Answer: (b)
Solution:

$$\sin\theta+\cos\theta=\frac{1}{2}$$

$$\Rightarrow \sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta=\frac{1}{4}$$

$$\Rightarrow 1+\sin 2\theta=\frac{1}{4}$$

$$\Rightarrow \sin 2\theta=\frac{-3}{4}$$

$$\cos 4\theta=1-2\sin^2 2\theta=1-2\left(\frac{-3}{4}\right)^2$$

$$=1-2\left(\frac{9}{16}\right)=1-\frac{9}{8}=-\frac{1}{8}$$

$$\sin 6\theta=3\sin 2\theta-4\sin^3 2\theta$$

$$=3\times\left(\frac{-3}{4}\right)-4\left(\frac{-3}{4}\right)^3$$

$$=\frac{-9}{4}+\frac{27}{16}=\frac{-36+27}{16}=\frac{-9}{16}$$

$$16\left(\sin 2\theta+\cos 4\theta+\sin 6\theta\right)$$

$$=16\left[\frac{-3}{4}+\left(\frac{-1}{8}\right)+\left(\frac{-9}{16}\right)\right]$$

$$=-12-2-9=-23$$

Question: The probability that a randomly selected 2-digit number belongs to the set $\left\{n\in N:\left(2^n-2\right)\text{ is a multiple of }3\right\}$ is equal to

Options:

(a) $\frac{2}{3}$

(b) $\frac{1}{3}$

(c) $\frac{1}{2}$

(d) $\frac{1}{6}$

Answer: (c)

Solution:

Total case = $\{10,11,12,....99\}=90$

$\left(2^n-2\right)=3k \Rightarrow n=\{11,13,15,....99\}=45$

$\therefore$ Required probability = $\frac{45}{90}=\frac{1}{2}$

Question: From point $(-1,1)$ two tangents drawn to $x^2+y^2-2x-6y+6=0$ that meet the circle A and B. A point D on the circle such that $AD=AB.$ Find are of $\Delta\, ABD.$

Options:

(a) 2

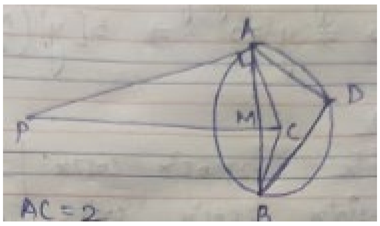
(b) 4

(c) $2+\sqrt{2}$

(d) 1

Answer: (b)

Solution:



$AP=2$

$CP=2\sqrt{2}$

$\angle CAP=90°,\ AC=2$

$\because\ AC=AP\ \Rightarrow \angle ACP=\angle APC=45°$

$\Rightarrow \angle BAC=\angle ABC=45°\ \Rightarrow \angle BAD=90°$

Now, $\sin\angle APM=\frac{1}{\sqrt{2}}=\frac{AM}{AP}\ \Rightarrow AM=\frac{2}{\sqrt{2}}=\sqrt{2}$

$\therefore\ AB=AD=2AM=2\sqrt{2}$

$\therefore$ Area of $\Delta\, ABD=\frac{1}{2}\times AB\times AD=4$

Question: Circle with centre (2, 3) passing through origin. P, Q are two points on the circle such that OC is perpendicular to both CP & CQ. Find points P & Q.

Options:

(a) (4, 0), (0, 6)

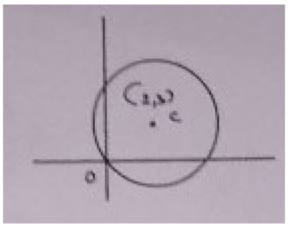
(b) (-1, 5), (5, 1)

(c)

(d)

Answer: (b)

Solution:



$$(x-2)^2+(y-3)^2=2^2+3^2$$

$$(x-2)^2+(y-3)^2=13$$

$$m_{oc}=\frac{3}{2} \quad m_{cp}=\frac{-2}{3}$$

Equation of $cp = y-3=\frac{-2}{3}(x-2)$

$$\Rightarrow 3y-9=-2x+4$$

$$\Rightarrow 3y+2x-13=0$$

$$\Rightarrow x=\frac{13-3y}{2}$$

Substituting in circle we get

$$\left(\frac{13-3y}{2}-2\right)^2+(y-3)^2=13$$

$$\left(\frac{9-3y}{2}\right)^2+(y-3)^2=13$$

$$81+9y^2-54y+4\left(y^2-6y+9\right)=52$$

$$81+9y^2-54y+4y^2-24y+36=52$$

$$13y^2-78y+117=52$$

$$13y^2-xy+65=0$$

$y^2 - 6y + 5 = 0$

$y = 1, 5$

$x = \frac{13 - 3y}{2} = 5, -1$

$(x, y) \equiv (5, 1)$ and $(-1, 5)$

Question: For the data 6, 10, 7, 13, a, 12, b, 12. If mean is 9, variance is $\frac{37}{4}$ find $(a - b)^2$.

Answer: 16.00

Solution:

$\overline{x} = \frac{60 + a + b}{8} = 9 \Rightarrow a + b = 12$

$\sigma = \frac{642 + a^2 + b^2}{8} - 81 = \frac{37}{4} \Rightarrow a^2 + b^2 = 80$

$\therefore (a + b)^2 = a^2 + b^2 + 2ab \Rightarrow ab = 32$

$(a - b)^2 = a^2 + b^2 - 2ab = 16$

Question: $\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{2j - 1 + 8n}{2j - 1 + 4n}$

Answer: $1 + 2\log\left(\frac{3}{2}\right)$

Solution:

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \frac{2\left(\frac{j}{n}\right) + 8 - \left(\frac{1}{n}\right)}{2\left(\frac{j}{n}\right) + 4 - \left(\frac{1}{n}\right)} \qquad \left\{\because \lim_{n\to\infty} \frac{1}{n} = 0\right\}$$

$$\Rightarrow \int_0^1 \frac{2x + 8}{2x + 4} dx = 1 + 4\int_0^1 \frac{dx}{2x + 4} = \left[1 + 2\log(2x + 4)\right]_0^1$$

$$= 1 + 2\log\left(\frac{3}{2}\right)$$

Question: If $\log_3 2, \log_3\left(2^x-5\right), \log_3\left(2x-\frac{7}{3}\right)$ are in an arithmetic progression then the value of x is equal to _____.

Answer: 3.00

Solution:

$$\log_3 2, \log_3 2^x-5, \log_3 2^x-\frac{7}{2} \Rightarrow \text{AP}$$

$$\Rightarrow 2\log_3 2^x-5=\log_3 2+\log_3 2^x-\frac{7}{2}$$

$$\Rightarrow \left(2^x-5\right)^2=2\left(2^x-\frac{7}{2}\right)$$

Let $2^x=t$

$$\Rightarrow t^2-10t+25=2t-7$$
$$\Rightarrow t^2-12t+32=0$$
$$\Rightarrow (t-8)(t-4)=0$$
$$\Rightarrow t=8,4$$

But $2^x=5$

$$\Rightarrow 2^x=8$$
$$\Rightarrow x=3$$

Question: Let $f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}; x\in(0,\pi]$. Then the maximum value of $f(x)$ is ?

Answer: 6.00

Solution:

$$f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2+\sin^2 x & \cos^2 x & \cos 2x \\ \sin^2 x & \cos^2 x & 1+\cos 2x \end{vmatrix}$$

$R_2 \to R_2-R_1;\ R_3\to R_3-R_1$

$$f(x)=\begin{vmatrix} \sin^2 x & -2+\cos^2 x & \cos 2x \\ 2 & 2 & 0 \\ 0 & 2 & 1 \end{vmatrix}$$

$f(x) = 2\sin^2 x + 4 - 2\cos^2 x + 4\cos 2x = 4 + 2\cos^2 x$

$f(x)_{\max} = 6$

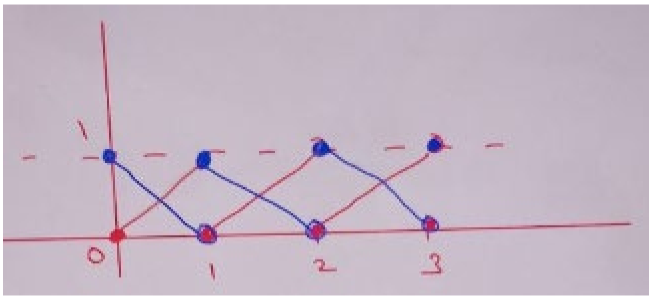
Question: $f(x) = \text{minimum } \{x - [x], 1 + [x] - x\} [0, 3] \to R,\ p$ is the number of points where it is discontinuous, q is the number of points where it is not differentiable. Find $p + q$.

Answer: 5.00

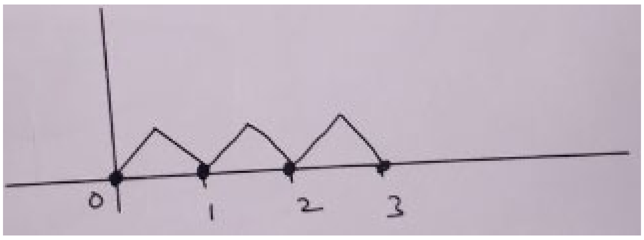
Solution:

$f(x) = \min\{x - [x], 1 + [x] - x\}\ \ x \in [0, 3]$

$= \min\{\{x\}, 1 - \{x\}\}$



$f(x) \to$



$p = 0,\ q = 5$

$p + q = 5$

Question: If $A = \begin{bmatrix} 1 & 2 \\ -1 & 4 \end{bmatrix}$ $A^{-1} = \alpha I + \beta A$, find $5(\alpha - \beta) = ?$

Answer: $\frac{9}{2}$

Solution:

$$A=\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}; A^2=\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}\begin{bmatrix}1 & 2\\-1 & 4\end{bmatrix}=\begin{bmatrix}-1 & 10\\-5 & 14\end{bmatrix}$$

$$\therefore\ I=\alpha A+\beta A^2 \Rightarrow \begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}=\begin{bmatrix}\alpha-\beta & 2\alpha+10\beta\\-\alpha-5\beta & 4\alpha+14\beta\end{bmatrix}$$

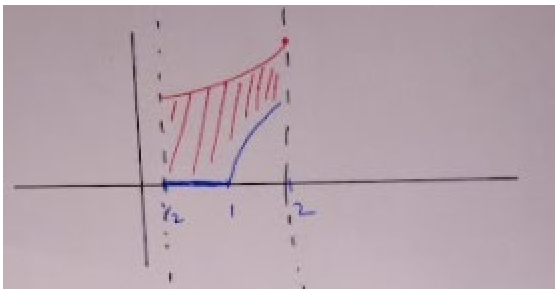
$$\therefore\ \alpha-\beta=1,\ \alpha+5\beta=0$$

$$\Rightarrow \beta=\frac{-1}{6},\ \alpha=\frac{5}{6}\Rightarrow 5\alpha-\beta=\frac{27}{6}=\frac{9}{2}$$

Question: Area enclosed by the figure: maximum $\{0, \log_e(x)\}\le y\le 2^x$ and $\frac{1}{2}\le x\le 2$.

Answer: ()

Solution:



Area = $=\int_{\frac{1}{2}}^{1} 2^x\,dx+\int_1^2\left(2^x-\ln^x\right)dx$

$$=2^x\ln 2\Big|_{\frac{1}{2}}^{1}+2^x\ln 2\Big|_1^2-\left(x\ln^x-x\right)\Big|_1^2$$

$$=\left(2\ln 2-2^{\frac{1}{2}}\ln 2\right)-\left(2^2\ln 2-2^1\ln 2\right)-\left[\left(2\ln^2-2\right)-\left(1\ln 1-1\right)\right]$$

$$=2\ln 2-\sqrt{2}\ln 2+4\ln 2-2\ln 2-2\ln 2+2=1$$

$$=2\ln 2-\sqrt{2}\ln 2+3$$

Question: $\overline{a}=\hat{i}+\hat{j}+2\hat{k}$, $\overline{b}=-\hat{i}+2\hat{j}+3\hat{k}$

$\left(\overline{a}+\overline{b}\right)\times\left(\left(\overline{a}\times\left(\overline{a}-\overline{b}\right)\times\overline{b}\right)\right)$

Answer:

Solution:

$$\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}\times\left(\left(\bar{a}-\bar{b}\right)\times b\right)\right)$$

$$=\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}\times\left(\bar{a}\times\bar{b}-\bar{b}\times\bar{b}\right)\right)$$

$$=\left(\bar{a}+\bar{b}\right)\times\left(\bar{a}\times\left(\bar{a}\times\bar{b}\right)\right)$$

$$=\left(\bar{a}+\bar{b}\right)\times\left(\left(\bar{a}\cdot\bar{b}\right)\bar{a}-\left(\bar{a}\cdot\bar{a}\right)\bar{b}\right)$$

$$=\left(-\left(\bar{a}\cdot\bar{a}\right)\left(\bar{a}\times\bar{b}\right)+\left(\bar{a}\cdot\bar{b}\right)\left(\bar{b}\times\bar{a}\right)\right)$$

$$=\left(-6\left(\bar{a}\times\bar{b}\right)+(7)\left(\bar{b}\times\bar{a}\right)\right)$$

$$=\left(6\left(\bar{b}\times\bar{a}\right)+7\left(\bar{b}\times\bar{a}\right)\right)$$

$$=13\left(\bar{b}\times\bar{a}\right)=13\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ -1 & 2 & 3\\ 1 & 1 & 2\end{vmatrix}$$

$$=13\left[\hat{i}\left(4-3\right)-\hat{j}\left(-2-3\right)+\hat{k}\left(-1-2\right)\right]$$

$$=13\left[\hat{i}+3\hat{j}-3\hat{k}\right]$$

Question: $\ln\left(\frac{dy}{dx}\right)=3x+4y$ and $y(0)=0$. Find $y\left(\frac{-2}{3}\ln 2\right)$

Answer: ()

Solution:

$$\ln\left(\frac{dy}{dx}\right)=3x+4y$$

$$\frac{dy}{dx}=e^{3x+4y}$$

$$\frac{dy}{dx}=e^{3x+4y}$$

$$\frac{e^{-4y}}{-4}=\frac{e^{3x}}{3}+c$$

$$y(0)=0$$

$$\Rightarrow\frac{-1}{4}=\frac{1}{3}+c$$

$$\Rightarrow c = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$$\Rightarrow 3e^{-4y} = -4e^{3x} + 7$$

$$x = \frac{-2}{3}\ln 2$$

$$\Rightarrow 3e^{-4y} = -4e^{-2\ln 2} + 7$$

$$\Rightarrow \frac{-1}{4} = \frac{1}{3} + c$$

$$\Rightarrow c = \frac{-1}{4} - \frac{1}{3} = \frac{-7}{12}$$

$$\frac{e^{-4y}}{-4} = \frac{e^{3x}}{3} - \frac{7}{12}$$

$$\Rightarrow 3e^{-4y} = -4e^{3x} + 7$$

$$x = \frac{-2}{3}\ln 2$$

$$\Rightarrow 3e^{-4y} = -4e^{-2\ln 2} + 7$$

$$\Rightarrow 3e^{-4y} = -4 \times \frac{1}{4} + 7$$

$$\Rightarrow 3e^{-4y} = 6$$

$$\Rightarrow e^{-4y} = 2$$

$$\Rightarrow -4y = \ln 2$$

$$\Rightarrow y = -\frac{1}{4}\ln 2$$

Question: Let a plane p pass through the point $(3, 7, -9)$ and contain the line,

$\frac{x-2}{-3} = \frac{y-3}{2} = \frac{x-2}{1}$ or distance or the plane p from the origin d_1 then d_2 is equal to

_____.

Answer: ()

Solution:

Let plane be $ax+by+cz=d$

It passes through A(3, 7, -9)

It contains $\frac{x-2}{-3}=\frac{y-3}{2}=\frac{z+2}{1}$

$\Rightarrow$ Plane passes through B(2, 3, -2)

$\overrightarrow{AB}=-\hat{i}-4\hat{j}+7\hat{k}$

Normal to plane is $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -1 & -4 & 7 \\ -3 & 2 & 1 \end{vmatrix}$

$=\hat{i}\left(-4-14\right)-\hat{j}\left(-1+21\right)+\hat{k}\left(-2-12\right)$

$=-18\hat{i}-20\hat{j}-14\hat{k}$

$\vec{n}=9\hat{i}+19\hat{j}+7\hat{k}$

Equation of plane is $9x+10y+7z=d$

It passes through (3, 7, -9)

$\Rightarrow 9\times 3+10\times 7+7\times\left(-9\right)=d$

$\Rightarrow 27+70-63=d$

$\Rightarrow 97-63=d$

$\Rightarrow 34=d$

Plane is $9x+10y+7z=34$

$d^2=\frac{34^2}{9^2+10^2+7^2}=\frac{34\times 34}{230}=\frac{578}{115}$

Question: If $\sec x\left(\frac{dy}{dx}\right)-\sin\left(x+y\right)-\sin\left(x-y\right)=0$ and $y\left(0\right)=0$. Find $5\left[y'\left(\frac{\pi}{2}\right)\right]$.

Answer: 0.00

Solution:

$\sec x\left(\frac{dy}{dx}\right)-\sin\left(x+y\right)-\sin\left(x-y\right)=2\sin x\cos y$

$\int \sec y\,dy=\int \sin 2x.dx \Rightarrow \log\left(\sec y+\tan y\right)=\frac{-\cos 2x}{2}+c$

At $x=0,\ y=0$

$$\therefore \sec y + \tan y = e^{\frac{1-\cos 2x}{2}} = e^{\sin^2 x}$$

$$\Rightarrow \left(\sec y \tan y + \sec^2 y\right) y' = e^{\sin^2 x} \sin 2x = 0$$

$$\Rightarrow y' = 0 \Rightarrow 5\left[y'\left(\frac{\pi}{2}\right)\right] = 0$$