FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

1. A torque meter is calibrated to reference standards of mass, length and time each with 5% accuracy. After calibration, the measured torque with this torque meter will have net accuracy of :

(A) 15% (B) 25% (C) 75% (D) 5%

Official Ans. by NTA (B)

Sol. Dimensional formula for Torque

$[\tau] = [ML^2T^{-2}]$

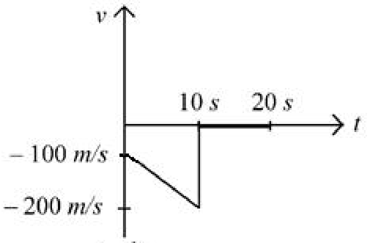
Now

Percentage error in torque = % τ = % M + 2 % L 2 % T

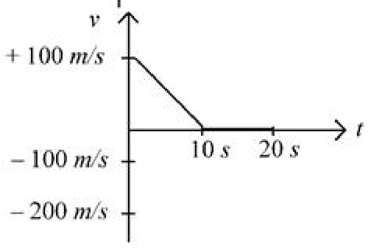
% τ = 25%

2. A bullet is shot vertically downwards with an initial velocity of 100 m/s from a certain height. Within 10 s, the bullet reaches the ground and instantaneously comes to rest due to the perfectly inelastic collision. The velocity-time curve for total time t = 20 s will be : (Take g = 10 m/s^2)

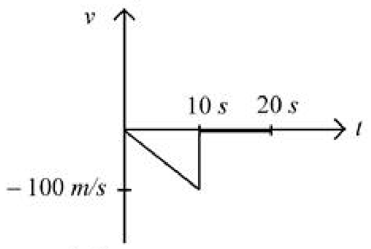
(A)



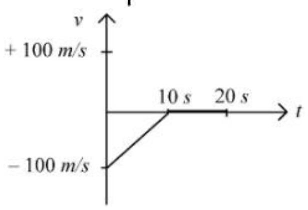
(B)



(C)



(D)



Official Ans. by NTA (A)

Sol. V = –100 –10t

3. Sand is being dropped from a stationary dropper at a rate of 0.5 kgs^{-1} on a conveyor belt moving with a velocity of 5 ms^{-1}. The power needed to keep belt moving with the same velocity will be :

(A) 1.25 W (B) 2.5 W

(C) 6.25 W (D) 12.5 W

Official Ans. by NTA (D)

Sol. Thrust = λV_{rel}

= 2.5 N

Now, Power =F × V = 12.5 W

4. A bag is gently dropped on a conveyor belt moving at a speed of 2 m/s. The coefficient of friction between the conveyor belt and bag is 0.4 Initially, the bag slips on the belt before it stops due to friction. The distance travelled by the bag on the belt during slipping motion is : [Take g = 10 m/s^{-2}]

(A) 2 m (B) 0.5 m

(C) 3.2 m (D) 0.8 ms

Official Ans. by NTA (B)

Sol. In frame of belt

$a = \mu g = 4$ m/s^2 , v = 2m/s , u = 0

$v^2 = u^2 + 2as$

$\Rightarrow$ s = 0.5 m

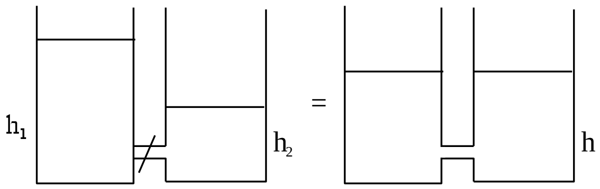
5. Two cylindrical vessels of equal cross-sectional area 16 cm^2 contain water upto heights 100 cm and 150 cm respectively. The vessels are interconnected so that the water levels in them become equal. The work done by the force of gravity during the process, is [Take density of water = 10^3 kg/m^3 and g = 10 ms^{-2}]

(A) 0.25 J (B) 1 J

(C) 8 J (D) 12 J

Official Ans. by NTA (B)

Sol.



$$h = \frac{h_1 + h_2}{2}$$

Now,

$W = U_i - U_f$

$$W = (\rho A h_1)\ g\ \frac{h_1}{2} + (\rho\ A h_2) g \frac{h_2}{2} - \rho\ A(h_1+h_2)g$$

$$\left(\frac{h_1 + h_2}{4}\right)$$

$$W = \frac{\rho A g}{2}\left[h_1^2 + h_2^2 - \frac{(h_1+h_2)^2}{2}\right]$$

$W = 1\ J$

6. Two satellites A and B having masses in the ratio 4:3 are revolving in circular orbits of radii 3r and 4 r respectively around the earth. The ratio of total mechanical energy of A to B is :

(A) 9 : 16 (B) 16 : 9

(C) 1 : 1 (D) 4 : 3

Official Ans. by NTA (B)

Sol. Given that $\frac{m_1}{m_2} = \frac{4}{3}$, $\frac{r_1}{r_2} = \frac{3}{4}$

$$\text{Now TE} = \frac{1}{2}\ mv^2 + \left(\frac{-GMm}{r}\right)$$

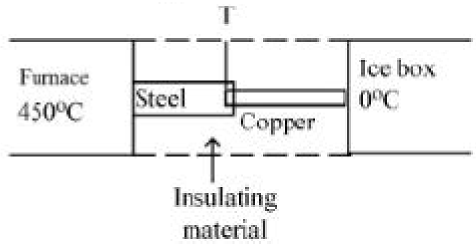
$$\text{but } \frac{mv^2}{r} = \frac{GMm}{r^2} \Rightarrow mv^2 = \frac{GMm}{r}$$

$$\Rightarrow TE = -\frac{GMm}{2r} \propto \frac{m}{r}$$

$$\frac{TE_1}{TE_2} = \frac{m_1}{m_2} \cdot \frac{r_2}{r_1} = \frac{4}{3} \times \frac{4}{3} = \frac{16}{9}$$

7. If K_1 and K_2 are the thermal conductivities L_1 and L_2 are the lengths and A_1 and A_2 are the cross sectional areas of steel and copper rods respectively such that $\frac{K_2}{K_1} = 9, \frac{A_1}{A_2} = 2, \frac{L_1}{L_2} = 2$.

Then, for the arrangement as shown in the figure. The value of temperature T of the steel – copper junction in the steady state will be :

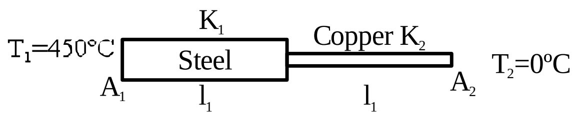


(A) 18ºC (B) 14 ºC

(C) 45 ºC (D) 150 ºC

Official Ans. by NTA (C)

Sol.



$$\frac{d\theta}{dt} = \frac{K_1 A_1}{l_1}(T_1 - T) = \frac{K_2 A_2}{l_2}(T - T_2)$$

$$\Rightarrow \frac{450 - T}{T - 0} = \frac{K_2 A_2 l_1}{K_1 A_1 l_2} = 9 \times \frac{1}{2} \times 2$$

$\Rightarrow 450 - T = 9T \Rightarrow T = 45ºC$

8. Read the following statements :

A. When small temperature difference between a liquid and its surrounding is doubled the rate of loss of heat of the liquid becomes twice.

B. Two bodies P and Q having equal surface areas are maintained at temperature 10ºC and 20 ºC. The thermal radiation emitted in a given time by P and Q are in the ratio 1 : 1.15

C. A carnot Engine working between 100 K and 400 K has an efficiency of 75%

D. When small temperature difference between a liquid and its surrounding is quadrupled, the rate of loss of heat of the liquid becomes twice.

Choose the correct answer from the options given below :

(A) A, B, C only (B) A, B only

(C) A, C only (D) B, C, D only

Official Ans. by NTA (A)

Sol. Heat Transfer

A. by Newton's low of colling $\frac{d\theta}{dt} = \propto \Delta T$

B. $H = \frac{d\theta}{dt} = \sigma e A T^4 \Rightarrow \frac{H_P}{H_Q} = \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{283}{293}\right)^4$

$H_P : H_Q = 1\ (1.03)^4 = 1 : (1.03)^4 = 1 : 1.15$

$\Rightarrow$ B is correct

C. $\eta = 1 - \frac{100}{400} = \frac{3}{4} = 75\%$

D. is wrong as $\frac{d\theta}{dt} \propto \Delta T$

9. Same gas is filled in two vessels of the same volume at the same temperature. If the ratio of the number of molecules is 1:4, then

A. The r.m.s. velocity of gas molecules in two vessels will be the same.

B. The ratio of pressure in these vessels will be 1 : 4

C. The ratio of pressure will be 1 : 1

D. The r.m.s. velocity of gas molecules in two vessels will be in the ratio of 1 : 4

(A) A and C only (B) B and D only

(C) A and B only (D) C and D only

Official Ans. by NTA (C)

Sol. **KTG**

A. $V_{Rms} = \sqrt{\frac{3RT}{M_w}} \Rightarrow V_{Rms}$ is same

B. $\frac{P_1}{P_2} = \frac{N_1}{N_2} \Rightarrow$ B is correct

Ans [A & B only are correct]

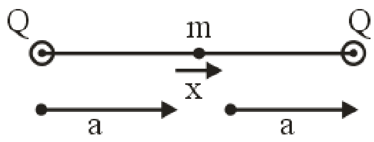
10. Two identical positive charges Q each are fixed at a distance of '2a' apart from each other. Another point charge q_0 with mass 'm' is placed at midpoint between two fixed charges. For a small displacement along the line joining the fixed charges, the charge q_0 executes SHM. The time period of oscillation of charge q_0 will be :

(A) $\sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{q_0 Q}}$ (B) $\sqrt{\frac{q_0 Q}{4\pi^3\varepsilon_0 ma^3}}$

(C) $\sqrt{\frac{2\pi^2\varepsilon_0 ma^3}{q_0 Q}}$ (D) $\sqrt{\frac{8\pi^3\varepsilon_0 ma^3}{q_0 Q}}$

Official Ans. by NTA (A)

Sol. **Electrostatics**



$F = m\,acc^n = \frac{KQq_0}{(a-x)^2} - \frac{KQq_0}{(a+x)^2}$

$m\,acc^n = \frac{KQq_0[2a][2x]}{(a^2 - x^2)^2}$

$\Rightarrow acc^n \approx \left(\frac{4kQq_0}{ma^3}\right)x$

$T = 2\pi\sqrt{\frac{\pi\varepsilon_0 ma^3}{Qq_0}}$

$T = \sqrt{\frac{4\pi^3\varepsilon_0 ma^3}{Qq_0}}$

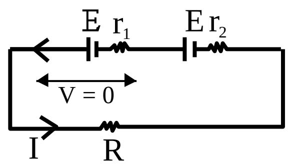
11. Two sources of equal emfs are connected in series. This combination is connected to an external resistance R. The internal resistances of the two sources are r_1 and r_2 ($r_1 > r_2$). If the potential difference across the source of internal resistance r_1 is zero then the value of R will be

(A) $r_1 - r_2$ (B) $\frac{r_1 r_2}{r_1 + r_2}$

(C) $\frac{r_1 + r_2}{2}$ (D) $r_2 - r_1$

Official Ans. by NTA (A)

Sol.



$I = \frac{2E}{r_1 + r_2 + R}$

$IR = E - Ir_2$

$I\,(R + r_2) = E$

$I = \frac{E}{R + r_2}$

$\frac{2E}{r_1 + r_2 + R} = \frac{E}{R + r_2}$

$2R + 2r_2 = r_1 + r_2 + R$

$R = r_1 - r_2$

12. Two bar magnets oscillate in a horizontal plane in earth's magnetic field with time periods of 3 s and 4 s respectively. If their moments of inertia are in the ratio of 3 : 2 then the ratio of their magnetic moments will e :

(A) 2 : 1 (B) 8 : 3

(C) 1 : 3 (D) 27 : 16

Official Ans. by NTA (B)

Sol. $T = 2\pi\sqrt{\dfrac{I}{MB_H}}$

$$\frac{T_1}{T_2} = \frac{2\pi\sqrt{\dfrac{I_1}{M_1B_H}}}{2\pi\sqrt{\dfrac{I_2}{M_2B_H}}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}\times\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{I_1}{I_2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\sqrt{\frac{3}{2}}\times\sqrt{\frac{M_2}{M_1}} = \frac{3}{4}$$

$$\frac{3}{2}\times\frac{M_2}{M_1} = \frac{9}{16}$$

$$\frac{M_1}{M_2} = \frac{8}{3}$$

13. A magnet hung at 45° with magnetic meridian makes an angle of 60° with the horizontal. The actual value of the angle of dip is

(A) $\tan^{-1}\left(\sqrt{\frac{3}{2}}\right)$ (B) $\tan^{-1}\left(\sqrt{6}\right)$

(C) $\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)$ (D) $\tan^{-1}\left(\sqrt{\frac{1}{2}}\right)$

Official Ans. by NTA (A)

Sol. $\tan\theta' = \dfrac{\tan\theta}{\cos\alpha}$

$\theta' = 60°$

$\alpha = 45°$

$$\sqrt{3} = \frac{\tan\theta}{\dfrac{1}{\sqrt{2}}}$$

$$\tan\theta = \sqrt{\frac{3}{2}}$$

$$\theta = \tan^{-1}\sqrt{\frac{3}{2}}$$

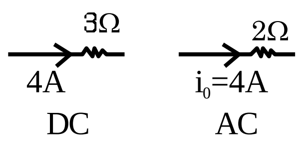
14. A direct current of 4 A and an alternating current of peak value 4 A flow through resistance of 3 Ω and 2 Ω respectively. The ratio of heat produced in the two resistances in same interval of time will be :

(A) 3 : 2 (B) 3 : 1

(C) 3 : 4 (D) 4 : 3

Official Ans. by NTA (B)

Sol.



$H_1 = i^2R_1t$ $\quad H_2 = i_{rms}^2R_2t\left\{i_{rms} = \dfrac{i_0}{\sqrt{2}}\right\}$

$H_1 = 16(3)t$ $\quad H_2 = \dfrac{i_0^2}{2}R_2t$

$H_2 = 16t$

$H_1 : H_2 = 3 : 1$

15. A beam of light travelling along X-axis is described by the electric field $E_y = 900\sin\omega(t - x/c)$. The ratio of electric force to magnetic force on a charge q moving along Y-axis with a speed of $3\times10^7\ ms^{-1}$ will be :

[Given speed of light = $3\times10^8\ ms^{-1}$]

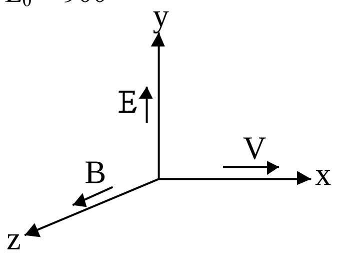
(A) 1 : 1 (B) 1 : 10

(C) 10 : 1 (D) 1 : 2

Official Ans. by NTA (C)

Sol. $E_y = 900\sin\left(\omega t - \frac{\omega x}{c}\right)$

$E_0 = 900$



$F_E = qE_0$

$F_B = qvB_0$

$$\frac{F_E}{F_B} = \frac{E_0}{vB_0} = \frac{c}{v} = \frac{3\times10^8}{3\times10^7} = 10:1$$

16. A microscope was initially placed in air (refractive index 1). It is then immersed in oil (refractive index 2). For a light whose wavelength in air is λ, calculate the change of microscope's resolving power due to oil and choose the correct option

(A) Resolving power will be $\frac{1}{4}$ in the oil than it was in the air

(B) Resolving power will be twice in the oil than it was in the air.

(C) Resolving power will be four times in the oil than it was in the air.

(D) Resolving power will be $\frac{1}{2}$ in the oil than it was in the air.

Official Ans. by NTA (C)

Sol. $(R.P)_{air} = \frac{2\sin\theta}{1.22\lambda}$

$$(R.P)_{oil} = \frac{2\sin\theta}{1.22\lambda_{oil}} = \frac{2\sin\theta\times\mu}{1.22\lambda}$$

$(R.P)_{oil} = (R.P)_{air} \times 2$

17. An electron (mass m) with an initial velocity $\vec{v} = v_0\hat{i}(v_0 > 0)$ is moving in an electric field $\vec{E} = -E_0\hat{i}(E_0 > 0)$ where E_0 is constant. If at t = 0 de Broglie wavelength is $\lambda_0 = \frac{h}{mv_0}$, then its de Broglie wavelength after time t is given by

(A) λ_0

(B) $\lambda_0\left(1+\frac{eE_0t}{mv_0}\right)$

(C) $\lambda_0 t$

(D) $\frac{\lambda_0}{\left(1+\frac{eE_0t}{mv_0}\right)}$

Official Ans. by NTA (D)

Sol. $e \longrightarrow V_0$

$E = -E_0\hat{i}$

$$\lambda_0 = \frac{h}{mv_0}$$

$$v = v_0 + \frac{eE_0t}{m}$$

$$\lambda = \frac{h}{mv} = \frac{h}{m\left(v_0+\frac{eE_0}{m}t\right)}$$

$$\lambda' = \frac{h}{mv_0\left(1+\frac{eE_0}{mv_0}t\right)}$$

$$\lambda' = \frac{\lambda_0}{\left(1+\frac{eE_0}{mv_0}t\right)}$$

18. What is the half-life period of a radioactive material if its activity drops to 1/16th of its initial value of 30 years ?

(A) 9.5 years

(B) 8.5 years

(C) 7.5 years

(D) 10.5 years

Official Ans. by NTA (C)

Sol. $A = A_0e^{-\lambda t}$

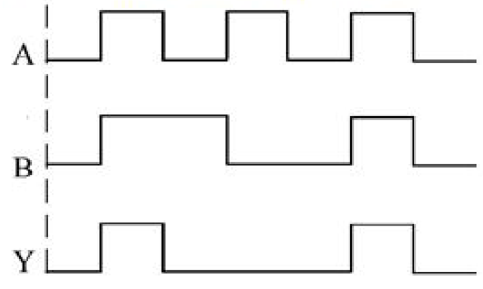
$$\Rightarrow -\lambda t = \ell n\left(\frac{A}{A_0}\right)$$

$$\Rightarrow -\frac{\ell n2}{t_{1/2}}\times30 = \ell n\left(\frac{1}{16}\right)$$

$$\Rightarrow -\frac{\ell n2}{t_{1/2}}\times30 = -4\ell n2$$

$$\Rightarrow t_{1/2} = \frac{30}{4} = 7.5 \text{ yrs}$$

19. A logic gate circuit has two inputs A and B and output Y. The voltage waveforms of A, B and Y are shown below



The logic gate circuit is

(A) AND gate (B) OR gate

(C) NOR gate (D) NAND gate

Official Ans. by NTA (A)

Sol. By making Truth table

A	B	Output
0	0	0
1	1	1
0	1	0
1	0	0

Comparing with output of AND gate

A	B	AND
0	0	0
0	1	0
1	0	0
1	1	1

$\Rightarrow$ logic gate present is AND gate

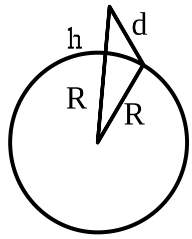
20. At a particular station, the TV transmission tower has a height of 100 m. To triple its coverage range, height of the tower should be increased to

(A) 200 m (B) 300 m

(C) 600 m (D) 900 m

Official Ans. by NTA (D)

Sol.



Let d be range

$d^2 = (h+R)^2 - R^2$

$= h^2 + R^2 + 2RH - R^2$

$d^2 = h^2 + 2Rh$

as R >>>> h then

$d \approx \sqrt{2Rh}$ .... (1)

Now, if coverage is to be increased 3 times

$3d = \sqrt{2Rh'}$ .... (2)

Divide 2 and 1 $\frac{3d}{d} = \sqrt{\frac{2Rh'}{2Rh}}$

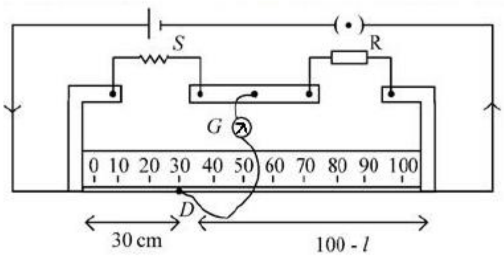
$9 = \frac{h'}{h}$

9h = h'

If h = 100 m then tower of height 900 m is required

SECTION-B

1. In meter bridge experiment for measuring unknown resistance 'S', the null point is obtained at a distance 30 cm from the left side as shown at point D. If R is 5.6 kΩ, then the value of unknown resistance 'S' will be ________ Ω.



Official Ans. by NTA (2400)

Sol. $\frac{S}{30} = \frac{5.6\times10^3}{70}$

$S = \frac{3}{7}\times5.6\times10^3 = 2400$

2. The one division of main scale of vernier callipers reads 1 mm and 10 divisions of Vernier scale is equal to the 9 divisions on main scale. When the two jaws of the instrument touch each other the zero of the Vernier lies to the right of zero of the main scale and its fourth division coincides with a main scale division. When a spherical bob is tightly placed between the two jaws, the zero of the Vernier scale lies in between 4.1 cm and 4.2 cm and 6th Vernier division coincides with a main scale division. The diameter of the bob will be_____ 10^{-2} cm

Official Ans. by NTA (412)

Sol. 10 VSD = 9 MSD

1 VST = .9 MSD

L.C. = .1 mm = .01 cm

+ve zero error = .4mm

= 0.04 cm

Negative zero error = 4.1 cm + 6 × .01

= 4.12 cm

= 412 × 10^{-2} cm

3. Two beams of light having intensities I and 4I interfere to produce a fringe pattern on a screen. The phase difference between the two beams are $\pi/2$ and $\pi/3$ at points A and B respectively. The difference between the resultant intensities at the two points is xI. The value of x will be _____ .

Official Ans. by NTA (2)

Sol. $\phi_A = \frac{\pi}{2}$

$\phi_B = \frac{\pi}{3}$

$I_A = I + 4I + 2\sqrt{I}\ \sqrt{4I}\ \cos\left(\frac{\pi}{2}\right)$

$= 5I + 4I\ (0) = 5I$

$I_B = I + 4I + 2\sqrt{I}\ \sqrt{4I}\ \cos(60^o)$

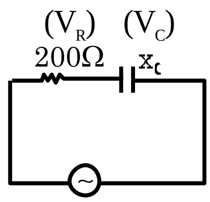
$= 5I + 4I \times \frac{1}{2} = 7I$

$I_B - I_A = 7I - 5I = 2I$, (x = 2)

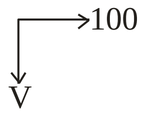
4. To light, a 50 W, 100 V lamp is connected, in series with a capacitor of capacitance $\frac{50}{\pi\sqrt{x}}\mu F$, with 200 V, 50Hz AC source. The value of x will be ____ .

Official Ans. by NTA (3)

Sol. $P = \frac{V^2}{R} \Rightarrow R = \frac{V^2}{P}$



200 V , 50 Hz



$R = \frac{100 \times 10^2}{50} = R = 200\Omega$

$V_R^2 + V_C^2 = V^2$

$(100)^2 + V_C^2 = (200)^2$

$i = \frac{100}{200} = \frac{1}{2}$; $V^2 = 40000$

$V = I \times X_C$; $V_C^2 = 30000$

$V_C = 100\sqrt{3}$

$X_C = 200\sqrt{3}$

$200\sqrt{3} = \frac{1}{\omega C}$

$C = \frac{1}{20 \times 50 \times 20\sqrt{3}} = \frac{50 \times 10^{-6}}{\sqrt{x}}$

$\sqrt{x} = 50 \times 10^{-6} \times 100 \times 200\sqrt{3}$

X = 3

5. A 1 m long copper wire carries a current of 1 A. If the cross section of the wire is 2.0 mm^2 and the resistivity of copper is 1.7×10^{-8} Ω m. the force experienced by moving electron in the wire is ______ $\times$ 10^{-23} N. (charge on electron $= 1.6 \times 10^{-19}$ C)

Official Ans. by NTA (136)

Sol. $l = 1$m

$i = 1A$

Area $= 2 \times 10^{-6}$

$\rho = 1.7 \times 10^{-8}$

$$R = \frac{\rho \ell}{A} = \frac{1.7 \times 10^{-8} \times 1}{2 \times 10^{-5}} = \frac{1.7}{2} \times 10^{-2}$$

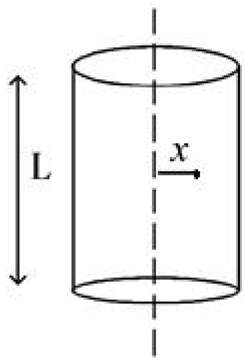
$$v = \frac{1.7}{2} \times 10^{-2}$$

$$F = 1.6 \times 10^{-19} \times \frac{1.7}{2} \times 10^{-2}$$

$= 1.36 \times 10^{-21}$

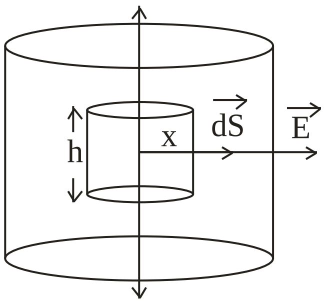
$= 136 \times 10^{-23}$

6. A long cylindrical volume contains a uniformly distributed charge of density ρ Cm^{-3}. The electric field inside the cylindrical volume at a distance $x = \frac{2\varepsilon_0}{\rho}$ m from its axis is ______ Vm^{-1}



Official Ans. by NTA (1)

Sol.



$$\int E ds \cos 0 = \frac{q}{\varepsilon_0}$$

$$\Rightarrow E.2\pi xh = \frac{\rho \times \pi x^2 h}{\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho x}{2\varepsilon_0}$$

$$\Rightarrow E = \frac{\rho}{2\varepsilon_0} \times \frac{2\varepsilon_0}{\rho} = 1$$

7. A mass 0.9 kg, attached to a horizontal spring, executes SHM with an amplitude A_1. When this mass passes through its mean position, then a smaller mass of 124 g is placed over it and both masses move together with amplitude A_2. If the ratio $\frac{A_1}{A_2}$ is $\frac{\alpha}{\alpha - 1}$, then the value of α will be ____ .

Official Ans. by NTA (16)

Sol. $$\frac{1}{2} kA^2 = \frac{p^2}{2m}$$

$$\Rightarrow \left(\frac{A_1}{A_2}\right)^2 = \frac{m_2}{m_1} = \frac{1024}{900}$$

$$\Rightarrow \frac{A_1}{A_2} = \frac{32}{30} = \frac{16}{15} = \frac{16}{16-1}$$

$$\therefore \alpha = 16$$

8. A square aluminium (shear modulus is 25×10^9 Nm^{-2}) slab of side 60 cm and thickness 15 cm is subjected to a shearing force (on its narrow face) of 18.0×10^4 N. The lower edge is riveted to the floor. The displacement of the upper edge is ____ μ m.

Official Ans. by NTA (48)

Sol. $\frac{F}{A} = \eta \frac{x}{\ell} \Rightarrow \frac{F\ell}{A\eta} = x$

$\Rightarrow x = \frac{18 \times 10^4 \times 60 \times 10^{-2}}{60 \times 10^{-2} \times 15 \times 10^{-2} \times 25 \times 10^9}$

$= 48 \times 10^{-6}$ m $= 48$ μ m

9. A pulley of radius 1.5 m is rotated about its axis by a force $F = (12t - 3t^2)$ N applied tangentially (while t is measured in seconds). If moment of inertia of the pulley about its axis of rotation is 4.5 kg m^2, the number of rotations made by the pulley before its direction of motion is reversed, will be $\frac{K}{\pi}$. The value of K is ____ .

Official Ans. by NTA (18)

Sol. $\tau = I\,\alpha \Rightarrow (12t - 3t^2)1.5 = 4.5\,\alpha$

$\Rightarrow \alpha = 4t - t^2$

$\Rightarrow \frac{d\omega}{dt} = 4t - t^2 \Rightarrow \omega = \int_0^t (4t - t^2)dt$

$\Rightarrow \omega = 2t^2 - t^3/3$

For $\omega = 0 = 2t^2 - \frac{t^3}{3} \Rightarrow t^2\left(2 - \frac{t}{3}\right) = 0$

$\Rightarrow t = 0, 6.$

$\frac{d\theta}{dt} = 2t^2 - \frac{t^3}{3} \Rightarrow \theta = \int_0^6 (2t^2 - \frac{t^3}{3})dt$

$= \left[\frac{2t^3}{3} - \frac{t^4}{12}\right]_0^6$

$= 6^3\left(\frac{2}{3} - \frac{6}{12}\right) = 6^3\left(\frac{8-6}{12}\right)$

$= \frac{6^3}{6} = 36$

No. of revolutions $= \frac{36}{2\pi} = \frac{18}{\pi}$

$\therefore K = 18$

10. A ball of mass m is thrown vertically upward. Another ball of mass 2 m is thrown an angle θ with the vertical. Both the balls stay in air for the same period of time. The ratio of the heights attained by the two balls respectively is $\frac{1}{x}$. The value of x is _____ .

Official Ans. by NTA (1)

Sol. Time of flight is same

$\Rightarrow$ vertical component of velocity is same

$\Rightarrow H_{max}$ is same

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. 250 g solution of D-glucose in water contains 10.8% of carbon by weight. The molality of the solution is nearest to

(Given: Atomic Weights are H, 1u ; C, 12u ; O, 16u)

(A) 1.03 (B) 2.06

(C) 3.09 (D) 5.40

Official Ans. by NTA (B)

Sol. $C_6H_{12}O_6 \rightarrow$ Glucose

We know: $\dfrac{\text{mass of C}}{\text{mass of glucose}} = \dfrac{72}{180}$

Given: $\%C = 10.8 = \dfrac{\text{mass of C}}{\text{mass of solution}} \times 100$

$\dfrac{10.8 \times 250}{100} = \text{mass of C} \Rightarrow \text{Mass of C} = 27\text{ gm}$

$\therefore$ mass of glucose = 67.5 gm

$\therefore$ moles of glucose = 0.375 moles

Mass of solvent = 250 – 67.5 gm = 182.5 gm

$\therefore \text{Molality} = \dfrac{0.375}{0.1825} = 2.055 \approx 2.06$

2. Given below are two statements.

Statement I : O_2 , Cu^{2+} and Fe^{3+} are weakly attracted by magnetic field and are magnetized in the same direction as magnetic field.

Statement II : NaCl and H_2O are weakly magnetized in opposite direction to magnetic field.

In the light of the above statements, choose the ***most appropriate*** answer form the options given below :

(A) Both Statement I and Statement II are correct.

(B) Both Statement I and Statement II are incorrect.

(C) Statement I is correct but Statement II is incorrect.

(D) Statement I is incorrect but Statement II is correct.

Official Ans. by NTA (A)

Sol. O_2, Cu^{2+} and Fe^{3+} are paramagnetic,

$\therefore$ Weakly attracted by magnetic field.

NaCl and H_2O are diamagnetic,

$\therefore$ Weakly repelled by magnetic field.

3. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Energy of 2s orbital of hydrogen atom is greater than that of 2s orbital of lithium.

Reason R : Energies of the orbitals in the same subshell decrease with increase in the atomic number.

In the light of the above statements, choose the ***correct*** answer from the options given below.

(A) Both **A** and **R** are true and **R** is the correct explanation of **A.**

(B) Both **A** and **R** are true but **R** is NOT the correct explanation of **A.**

(C) **A** is true but **R** is false.

(D) **A** is false but **R** is true.

Official Ans. by NTA (A)

Sol. Energy of orbitals decreases on increasing the atomic number.

4. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : Activated charcoal adsorbs SO_2 more efficiently than CH_4.

Reason R : Gases with lower critical temperatures are readily adsorbed by activated charcoal.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A.**

(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A.**

(C) **A** is correct but **R** is not correct.

(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (C)

Sol. SO_2 is absorbed to a greater extent than CH_4 on activated charcoal under same conditions.

Gases with **higher critical temperature** are readily absorbed by activated charcoal.

5. Boiling point of a 2% aqueous solution of a non-volatile solute A is equal to the boiling point of 8% aqueous solution of a non-volatile solute B. The relation between molecular weights of A and B is.

(A) $M_A = 4M_B$ (B) $M_B = 4M_A$

(C) $M_A = 8M_B$ (D) $M_B = 8M_A$

Official Ans. by NTA (B)

Sol. **For A :** 100 gm solution $\rightarrow$ 2 gm solute A

$$\therefore \text{Molality} = \frac{2/M_A}{0.098}$$

For B : 100 gm solution $\rightarrow$ 8 gm solute B

$$\therefore \text{Molality} = \frac{8/M_B}{0.092}$$

$$\because (\Delta T_B)_A = (\Delta T_B)_B$$

$\therefore$ Molality of A = Molality of B

$$\therefore \frac{2}{0.098 M_A} = \frac{8}{0.092 M_B}$$

$$\frac{2}{98} \times \frac{92}{8} = \frac{M_A}{M_B}$$

$$\frac{1}{4.261} = \frac{M_A}{M_B}$$

$$\therefore M_B = 4.261 \times M_A$$

6. The **incorrect** statement is

(A) The first ionization enthalpy of K is less than that of Na and Li

(B) Xe does not have the lowest first ionization enthalpy in its group

(C) The first ionization enthalpy of element with atomic number 37 is lower than that of the element with atomic number 38.

(D) The first ionization enthalpy of Ga is higher than that of the d-block element with atomic number 30.

Official Ans. by NTA (D)

Sol. Ionization enthalpy order :

Li > Na > K

He > Ne > Ar > Kr > Xe > Rn

Sr > Rb

Zn > Ga

7. Which of the following methods are not used to refine any metal?

(A) Liquation (B) Calcination

(C) Electrolysis (D) Leaching

(E) Distillation

Choose the **correct** answer from the options given below:

(A) B and D only (B) A, B, D and E only

(C) B, D and E only (D) A, C and E only

Official Ans. by NTA (A)

Sol. Calcination and leaching are the methods of concentration of ore and not that of refining.

8. Given below are two statements:

Statement I : Hydrogen peroxide can act as an oxidizing agent in both acidic and basic conditions.

Statement II: Density of hydrogen peroxide at 298 K is lower than that of D_2O.

In the light of the above statements. Choose the ***correct*** answer from the options.

(A) Both statement I and Statement II are ture

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (C)

Sol. Depending on the nature of reducing agent H_2O_2 can act as an oxidising agent in both acidic as well as basic medium.

Density of D_2O = 1.1 g/cc

Density of H_2O_2 = 1.45 g/cc

9. Given below are two statements:

Statement I : The chlorides of Be and Al have Cl-bridged structure. Both are soluble in organic solvents and act as Lewis bases.

Statement II: Hydroxides of Be and Al dissolve in excess alkali to give beryllate and aluminate ions.

In the light of the above statements. Choose the correct answer from the options given below.

(A) Both statement I and Statement II are true

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. Be_2Cl_4 is lewis acid and Al_2Cl_6 has complete octet. Be and Al are amphoteric metals therefore dissolve in acid as well as alkaline solution and form beryllate and aluminate ions in excess alkali.

10. Which oxoacid of phosphorous has the highest number of oxygen atoms present in its chemical formula?

(A) Pyrophosphorous acid

(B) Hypophosphoric acid

(C) Phosphoric acid

(D) Pyrophosphoric acid

Official Ans. by NTA (D)

Sol. Pyrophosphorous acid $\rightarrow H_4P_2O_5$.

Hypophosphoric acid $\rightarrow H_4P_2O_6$.

Phosphoric acid $\rightarrow H_3PO_4$.

Pyrophosphoric acid $\rightarrow H_4P_2O_7$.

11. Given below are two statements:

Statement I : Iron (III) catalyst, acidified $K_2Cr_2O_7$ and neutral $KMnO_4$ have the ability to oxidise I^- to I_2 independently.

Statement II: Manganate ion is paramagnetic in nature and involves $p\pi - p\pi$ bonding.

In the light of the above statements, choose the ***correct*** answer from the options.

(A) Both statement I and Statement II are ture

(B) Both statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (B)

Sol. Neutral $KMnO_4$ oxidises I^- to IO_3^-

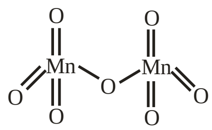
Manganate ion has $d\pi$-$p\pi$ bonding.

12. The total number of Mn = O bonds in Mn_2O_7 is ____

(A) 4 (B) 5

(C) 6 (D) 3

Official Ans. by NTA (C)

Sol. 

13. Match List I with List II

List I Pollutant	List II Disease /sickness
A. Sulphate (>500 ppm)	I. Methemoglobinemia
B. Nitrate (>50 ppm)	II. Brown mottling of teeth
C. Lead (> 50 ppb)	III. Laxative effect
D. Fluoride (>2 ppm)	IV. Kidney damage

Choose the correct answer from the options given below:

(A) A-IV, B –I, C-II, D-III

(B) A-III, B –I, C-IV, D-II

(C) A-II, B –IV, C-I, D-III

(D) A-II, B –IV, C-III, D-I

Official Ans. by NTA (B)

Sol. A. Sulphate (>500 ppm) - Causes Laxative effect that leads to dehydration

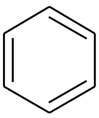
B. Nitrate (>50 ppm) - Causes Methemoglobinemia, skin appears blue

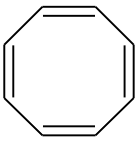
C. Lead (> 50 ppb) – It damage kidney and RBC

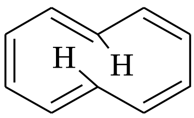
D. Fluoride (>2 ppm) – It Causes Brown mottling of teeth

14. Given below are two statements. One is labelled as **Assertion A** and the other is labelled as **Reason R.**

Assertion A : [6] Annulene. [8] Annulene and cis –[10] Annulene, are respectively aromatic, not-aromatic and aromatic.

[6] Annulene 

[8] Annulene 

Cis-[10] Annulene 

Reason R : Planarity is one of the requirements of aromatic systems.

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A.**

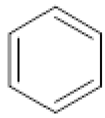
(B) Both **A** and **R** are correct but **R** is NOT the correct explanation of **A.**

(C) **A** is correct but **R** is not correct.

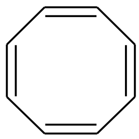
(D) **A** is not correct but **R** is correct.

Official Ans. by NTA (A)

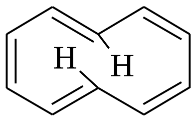
Sol. Assertion A : Not correct , Reason R : correct

 Arom atic

[6] – annulene

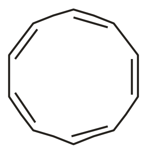
 - Not aromatic

[8] – annulene

 - Not aromatic

[10] – annulene

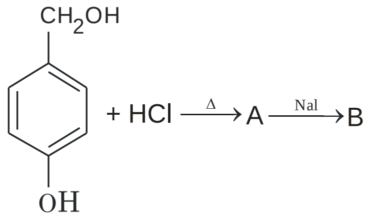
In [10] –Annulene – the hydrogen atoms in the 1 and 6 position interfere with each other and force the molecule out of planarity



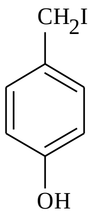
all -cis(10)annulene

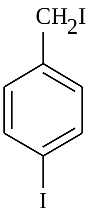
If this annulene with five cis double bonds were planar, each internal angle would be 144°. Since a normal double bond has bond angle of 120°, this would be from ideal. This compound can be made but it does not adopt a planar conformation and therefore is not aromatic even though it has ten π electrons.

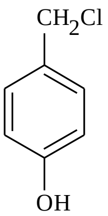
15.

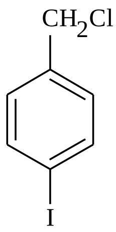


In the above reaction product B is:

(A) 

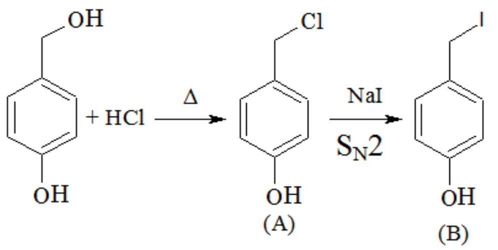
(B) 

(C) 

(D) 

Official Ans. by NTA (A)

Sol.



16. Match **List I** with **List II**

List I **Polymers**	**List II** **Commenrcial names**
A. Phenol-formaldehyde resin	I. Glyptal
B. Copolymer of 1,3-butadiene and styrene	II. Novolac
C. Polyester of glycol and phthalic acid	III. Buna-S
D. Polyester of glycol and terephthalic acid	IV. Dacron

Choose the correct answer from the options given below:

(A) A-II, B –III, C-IV, D-I

(B) A-II, B –III, C-I, D-IV

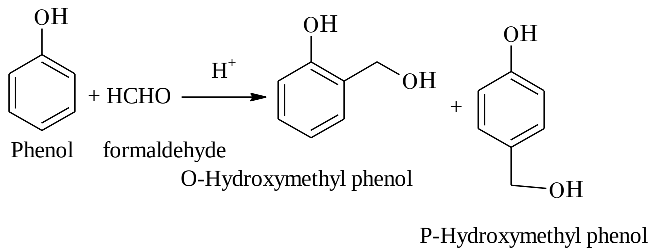
(C) A-II, B –I, C-III, D-IV

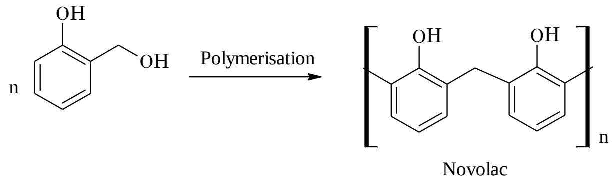
(D) A-III, B –II, C-IV, D-I

Official Ans. by NTA (B)

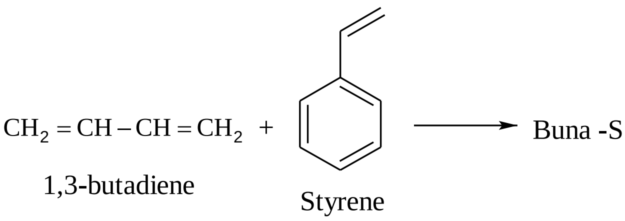
Sol.

(A)

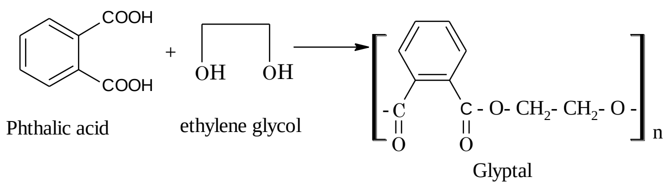




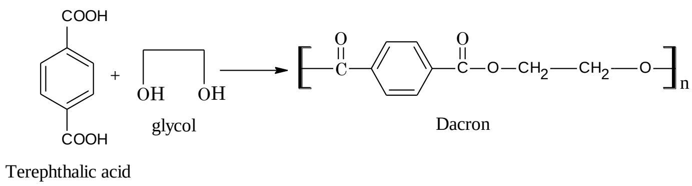
(B)



(C)



(D)



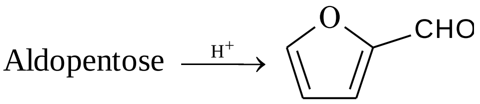
17. A sugar 'X' dehydrates very slowly under acidic condition to give furfural which on further reaction with resorcinol gives the coloured product after sometime. Sugar 'X' is

(A) Aldopentose (B) Aldotetrose

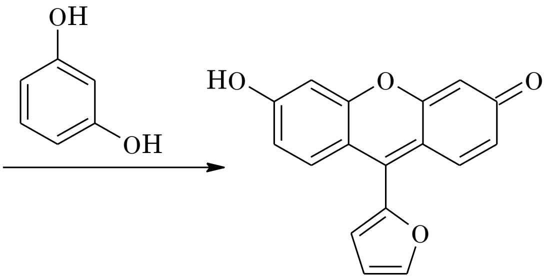
(C) Oxalic acid (D) Ketotetrose

Official Ans. by NTA (A)

Sol.

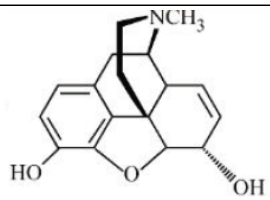
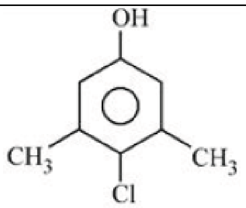
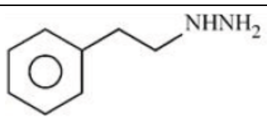
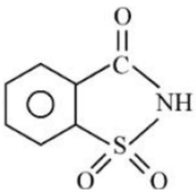


Furfural



Cherry red product
(seliwanoff's test)

18. Match **List I** with **List II**

List I	**List II**
A. 	I. Anti-depressant
B. 	II. 550 times sweeter than cane sugar
C. 	III. Narcotic analgesic
D. 	IV. Antiseptic

Choose the correct answer from the options given below:

(A) A-IV, B –III, C-II, D-I

(B) A-III, B –I, C-II, D-IV

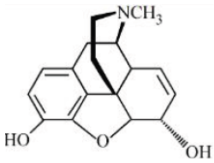
(C) A-III, B –IV, C-I, D-II

(D) A-III, B –I, C-IV, D-II

Official Ans. by NTA (C)

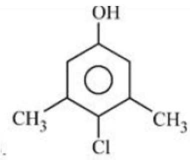
Sol.

(A)



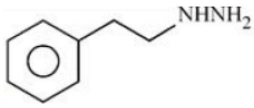
It is morphine use for relief for pain, known for narcotic analgesic

(B)



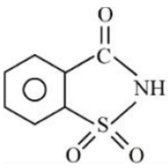
Chloroxylenol used as an antiseptic

(C)



Phenelzine (Nardil) use as Antidepressant

(D)



Saccharin   550 times sweeter than cane sugar

19. In Carius method of estimation of halogen. 0.45 g of an organic compound gave 0.36 g of AgBr. Find out the percentage of bromine in the compound.

(Molar masses : AgBr = 188 g mol^{-1}: Br = 80 g mol^{-1})

(A) 34.04% (B) 40.04%

(C) 36.03% (D) 38.04%

Official Ans. by NTA (A)

Sol. Mass of organic compound = 0.45 gm

Mass of AgBr obtained = 0.36 gm

$\therefore$ Moles of AgBr = $\frac{0.36}{188}$

$\therefore$ Mass of Bromine = $\frac{0.36}{188}\times 80 = 0.1532\text{gm}$

$\therefore$ % Br in compound = $\frac{0.1532}{0.45}\times 100 = 34.04\%$

20. Match **List I** with **List II**

List I	**List II**
A. Benzenesulphonyl chloride	I. Test for primary amines
B. Hoffmann bromamide reaction	II. Anti Saytzeff
C. Carbylamine reaction	III. Hinsberg reagent
D. Hoffmann orientation	IV. Known reaction of Isocyanates.

Choose the correct answer from the options given below:

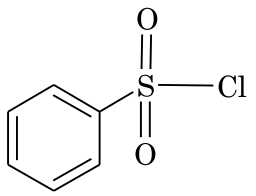
(A) A-IV, B –III, C-II, D-I

(B) A-IV, B –II, C-I, D-III

(C) A-III, B –IV, C-I, D-II

(D) A-IV, B –III, C-I, D-II

Official Ans. by NTA (C)

Sol. (A)  $\rightarrow$ Hinsberg reagent

Benzen sulphonyl chloride

(B) Hoffmann bromamide reaction $\rightarrow$ known reaction of isocynates

$R-CO-NH_2 + X_2 + 4\,NaOH \rightarrow R\text{-}NH_2 + 2NaX + Na_2CO_3 + 2H_2O$

Intermediate : R- N = C = O (isocyanate)

(C) Carbylamine reaction $\rightarrow$ Test for primary amine

$R\text{-}NH_2$ or $Ar-NH_2 + CHCl_3 + 3KOH \rightarrow RNC$ or $Ar\text{-}NC + 3KCl + 3H_2O$

(D) Hoffmann orientation $\rightarrow$ Anti saytzeff (Formation of less substituted alkene as major product)

SECTION-B

1. 20 mL of 0.02 M $K_2Cr_2O_7$ solution is used for the titration of 10 mL of Fe^{2+} solution in the acidic medium.

The molarity of Fe^{2+} solution is ______ $\times 10^{-2}$ M. (Nearest Integer)

Official Ans. by NTA (24)

Sol. Eq. of $K_2Cr_2O_7$ = Eq. of Fe^{2+}

$\Rightarrow$ (Molarity $\times$ volume $\times$ n.f) of $K_2Cr_2O_7$ = (molarity $\times$ volume $\times$ n.f) of Fe^{2+}

$\Rightarrow 0.02 \times 20 \times 6 = M \times 10 \times 1$

$\Rightarrow M = 0.24$

$\Rightarrow$ Molarity $= 24 \times 10^{-2}$

2. $2NO + 2H_2 \rightarrow N_2 + 2H_2O$

The above reaction has been studied at 800°C. The related data are given in the table below

Reaction serial number	Initial pressure of H_2 / kPa	Initial Pressure of NO/ kPa	Initial rate $\left(\frac{-dp}{dt}\right)/(kPa/s)$
1	65.6	40.0	0.135
2	65.6	20.1	0.033
3	38.6	65.6	0.214
4	19.2	65.6	0.106

The order of the reaction with respect to NO is____

Official Ans. by NTA (2)

Sol. On decreasing pressure of NO by a factor of '2' the rate of reaction decreases by a factor of '4'.

$\therefore$ Order of reaction w.r.t. 'NO' = 2

3. Amongst the following the number of oxide(s) which are paramagnetic in nature is

Na_2O, KO_2, NO_2, N_2O, ClO_2, NO, SO_2, Cl_2O

Official Ans. by NTA (4)

Sol. KO_2, NO_2, ClO_2, NO are paramagnetic.

4. The molar heat capacity for an ideal gas at constant pressure is 20.785 J $K^{-1}mol^{-1}$. The change in internal energy is 5000 J upon heating it from 300K to 500K. The number of moles of the gas at constant volume is ___ [Nearest integer]

(Given: R = 8.314 J K^{-1} mol^{-1})

Official Ans. by NTA (2)

Sol. $C_{p,m} = C_{v,m} + R$

$\Rightarrow C_{v,m} = 20.785 - 8.314 = 12.471$ J k^{-1} ml^{-1}

$\Delta U = nC_{v,m}\Delta T$

$$\Rightarrow n = \frac{5000}{12.471\times 200} = \frac{25}{12.471} \approx 2$$

5. According to MO theory, number of species/ions from the following having identical bond order is_____:

$CN^-, NO^+, O_2, O_2^+, O_2^{2+}$

Official Ans. by NTA (3)

Sol. CN^-, NO^+, O_2^{2+} have bond order = 3

6. At 310 K, the solubility of CaF_2 in water is 2.34×10^{-3}g /100 mL. The solubility product of CaF_2 is ___ $\times$ 10^{-8} $(mol/L)^3$. (Given molar mass : CaF_2 = 78 g mol^{-1})

Official Ans. by NTA (0)

Sol. Solubility of CaF_2 = S mole/L

$$S = \frac{2.34\times 10^{-3}}{0.1\times 78} = \frac{2.34}{78}\times 10^{-2} = 3 \times 10^{-4} \text{ mol/L}$$

K_{sp} $(CaF_2) = 4S^3 = 4(3 \times 10^{-4})^3$

$= 108 \times 10^{-12}$

$= 0.0108 \times 10^{-8}$ $(mol/L)^3$

7. The conductivity of a solution of complex with formula $CoCl_3(NH_3)_4$ corresponds to 1 : 1 electrolyte, then the primary valency of central metal ion is______

Official Ans. by NTA (1)

Sol. $[Co(NH_3)_4Cl_2]Cl$

Primary valency = oxidation no. = +3

8. In the titration of $KMnO_4$ and oxalic acid in acidic medium, the change in oxidation number of carbon at the end point is_____

Official Ans. by NTA (1)

Sol. Oxidation state of carbon changes from +3 to +4.

$$2KMnO_4 + 5H_2C_2O_4 + 3H_2SO_4(\text{dil.}) \rightarrow$$
$$K_2SO_4 + 2MnSO_4 + 10CO_2 + 8H_2O$$

9. Optical activity of an enantiomeric mixture is +12.6° and the specific rotation of (+) isomer is +30°. The optical purity is____%

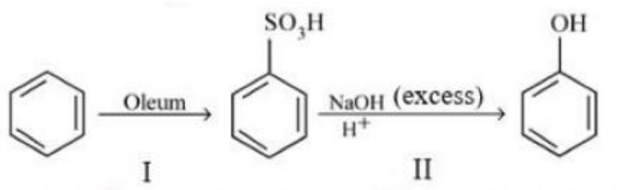
Official Ans. by NTA (42)

Sol.

$$\% \text{ optical purity} = \frac{\text{observed rotation of mixture}\times 100}{\text{rotation of pure enantiomer}}$$

$$= \frac{+12.6^\circ}{+30^\circ}\times 100 = 42$$

10. In the following reaction

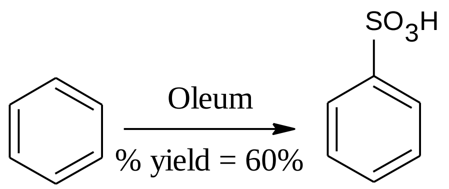


The % yield for reaction I is 60% and that of reaction II is 50%. The overall yield of the complete reaction is ___% [nearest integer]

Official Ans. by NTA (30)

Sol.

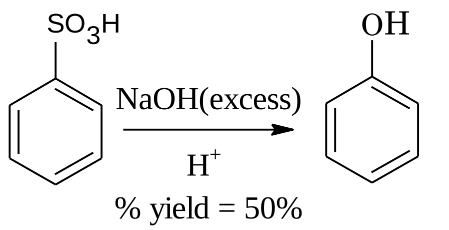
(I)



Let initial moles of reactant taken = n

Total moles obtained for benzene sulphonic acid (with % yield = 60%) = 0.6n

(II)



Moles of benzene sulphonic acid before reaction II = 0.6n

Moles obtained for phenol (with % yield = 50%) = $0.6\times0.5n = 0.3n$

So over all % yield of complete reaction = $\frac{0.3n}{n}\times100=30$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. Let R_1 and R_2 be two relations defined on $\mathbb{R}$ by $a\,R_1\,b \Leftrightarrow ab \ge 0$ and $a\ R_2 b \Leftrightarrow a \ge b$, then

(A) R_1 is an equivalence relation but not R_2

(B) R_2 is an equivalence relation but not R_1

(C) both R_1 and R_2 are equivalence relations

(D) neither R_1 nor R_2 is an equivalence relation

Official Ans. by NTA (D)

Sol. $R_1 = \{xy \ge 0, x, y \in R\}$

For reflexive $x \times x \ge 0$ which is true.

For symmetric

If $xy \ge 0 \Rightarrow yx \ge 0$

If x = 2, y = 0 and z = -2

Then $x.y \ge 0$ & $y.z \ge 0$ but x.z ≥ 0 is not true

$\Rightarrow$ not transitive relation.

$\Rightarrow R_1$ is not equivalence

R_2 if $a \ge b$ it does not implies $b \ge a$

$\Rightarrow R_2$ is not equivalence relation

$\Rightarrow D$

2. Let $f, g : \mathbb{N} - \{1\} \to \mathbb{N}$ be functions defined by $f(a) = \alpha$, where α is the maximum of the powers of those primes p such that p^{α} divides a, and $g(a) = a + 1$, for all $a \in \mathbb{N} - \{1\}$. Then, the function $f + g$ is

(A) one-one but not onto

(B) onto but not one-one

(C) both one-one and onto

(D) neither one-one nor onto

Official Ans. by NTA (D)

Sol. $f : N - \{1\} \to N$ $\quad f(a) = \alpha$

Where α is max of powers of prime P such that p^{α} divides a. Also $g(a) = a + 1$

$\therefore$ f(2) = 1 $\quad$ g(2) = 3

f (3) = 1 $\quad$ g(3) = 4

f(4) = 2 $\quad$ g(4) = 5

f(5) = 1 $\quad$ g(5) = 6

$\Rightarrow$ f (2) + g (2) = 4

$(f(3) + g(3)) = 5$

f(4) + g (4) = 7

f (5) + g (5) = 7

$\therefore$ Many one f(x) + g(x) does not cotain 1

$\Rightarrow$ into function

$\therefore$ Ans. (D) [neither one-one nor onto]

3. Let the minimum value v_0 of $v = |z|^2 + |z-3|^2 + |z-6i|^2$, $z \in \mathbb{C}$ is attained at $z = z_0$. Then $|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$ is equal to

(A) 1000 $\quad$ (B) 1024

(C) 1105 $\quad$ (D) 1196

Official Ans. by NTA (A)

Sol. $z_0 = \left(\frac{0+3+0}{3}, \frac{0+6+0}{3}\right) = (1, 2)$

$v_0 = |1+2i|^2 + |1+2i-3|^2 + |1+2i-6i|^2 = 30$

Then $|2z_0^2 - \bar{z}_0^3 + 3|^2 + v_0^2$

$= |2(1+2i)^2 - (1-2i)^3 + 3|^2 + 900$

$= |2(1-4+4i) - (1-4-4i)(1-2i) + 3|^2 + 900$

$= |8+6i|^2 + 900 = 100 + 900 = 1000$

4. Let $A=\begin{pmatrix}1 & 2\\ -2 & -5\end{pmatrix}$. Let $\alpha,\beta\in\mathbb{R}$ be such that $\alpha A^2+\beta A=2I$. Then $\alpha+\beta$ is equal to -

(A) -10 (B) -6

(C) 6 (D) 10

Official Ans. by NTA (D)

Sol. Characteristic equation of matric A

$|A-\lambda I|=0$

$$\begin{vmatrix}1-\lambda & 2\\ -2 & -5-\lambda\end{vmatrix}=0$$

$\Rightarrow \lambda^2+4\lambda=1$

$\Rightarrow A^2+4A=I$

$\Rightarrow 2A^2+8A=2I$ .... (1)

Given that $\alpha A^2+\beta A=2I$ .... (2)

Comparing equation (1) & (2) we get

$\alpha=2,\quad \beta=8$

$\therefore \alpha+\beta=10$

Ans. (D) (10)

5. The remainder when $(2021)^{2022}+(2022)^{2021}$ is divided by 7 is

(A) 0 (B) 1

(C) 2 (D) 6

Official Ans. by NTA (A)

Sol. $(2021)^{2022}+(2022)^{2021}$

$=(2023-2)^{2022}+(2023-1)^{2021}$

$=7n_1+2^{2022}+7n_2-1$

$=7(n_1+n_2)+8^{674}-1$

$=7(n_1+n_2)+(7-1)^{674}-1$

$=7(n_1+n_2)+7n_3+1-1$

$=7(n_1+n_2+n_3)$

$\therefore$ Given number is divisible by 7 hence remainder is zero

6. Suppose $a_1, a_2, \ldots, a_n, \ldots$ be an arithmetic progression of natural numbers. If the ratio of the sum of the first five terms of the sum of first nine terms of the progression is 5 : 17 and $110 < a_{15} < 120$, then the sum of the first ten terms of the progression is equal to -

(A) 290 (B) 380

(C) 460 (D) 510

Official Ans. by NTA (B)

Sol. $$\frac{S_5}{S_9}=\frac{5}{17}\Rightarrow\frac{\frac{5}{2}(2a+4d)}{\frac{9}{2}(2a+8d)}=\frac{5}{17}$$

$\Rightarrow d=4a$

$a_{15}=a+14d=57a$

Now, $110<a_{15}<120$

$\Rightarrow 110<57a<120$

$\Rightarrow a=2 \therefore d=8$

$S_{10}=\frac{10}{2}(2\times2+9\times8)=380$

7. Let $f:\mathbb{R}\to\mathbb{R}$ be a function defined as

$f(x)=a\sin\left(\frac{\pi[x]}{2}\right)+[2-x]$, $a\in\mathbb{R}$, where $[t]$ is the greatest integer less than or equal to t. If $\lim\limits_{x\to-1}f(x)$ exists, then the value of $\int_0^4 f(x)\,dx$ is equal to :

(A) -1 (B) -2

(C) 1 (D) 2

Official Ans. by NTA (B)

Sol. $\lim_{x\to -1^+} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=-a+2$

$\lim_{x\to -1^-} a\sin\left(\pi\frac{[x]}{2}\right)+[2-x]=0+3=3$

$\lim_{x\to -1} f(x)$ exist when $a=-1$

Now,

$\int_0^4 f(x)dx=\int_0^1 f(x)dx+\int_1^2 f(x)dx+\int_2^3 f(x)dx+\int_3^4 f(x)dx$

$=\int_0^1(0+1)dx+\int_1^2(-1+0)dx+\int_2^3(0-1)dx+\int_3^4(1-2)dx$

$=1-1-1-1=-2$

8. $I=\int_{\pi/4}^{\pi/3}\left(\frac{8\sin x-\sin 2x}{x}\right)dx$. Then

(A) $\frac{\pi}{2}<I<\frac{3\pi}{4}$

(B) $\frac{\pi}{5}<I<\frac{5\pi}{12}$

(C) $\frac{5\pi}{12}<I<\frac{\sqrt{2}}{3}\pi$

(D) $\frac{3\pi}{4}<I<\pi$

Official Ans. by NTA (C)

Sol. Consider

f(x) = 8sinx – sin2x

f'(x) = 8sinx – 2cos2x

f''(x) = –8sinx + 4 sin2x

= –8sinx (1 – cosx)

$\therefore$ f''(x) < 0 $x\in\left(\frac{\pi}{4},\frac{\pi}{3}\right)$

$\therefore$ f'(x) is ↓ function

$f'\left(\frac{\pi}{3}\right)<f'(x)<f'\left(\frac{\pi}{4}\right)$

$5<f'(x)<\frac{8}{\sqrt{2}}$

$5<f'(x)<4\sqrt{2}$

$5x<f(x)<4\sqrt{2}x$

$5<\frac{f(x)}{x}<4\sqrt{2}$

$\int_{\pi/4}^{\pi/3}5<\int\frac{f(x)}{x}<\int_{\pi/4}^{\pi/3}4\sqrt{2}$

$\int_{\pi/4}^{\pi/3}5<\int\frac{8\sin x-\sin 2x}{x}<\int_{\pi/4}^{\pi/3}4\sqrt{2}$

$\frac{5\pi}{12}<I<\frac{\sqrt{2}\pi}{3}$

9. The area of the smaller region enclosed by the curves $y^2=8x+4$ and $x^2+y^2+4\sqrt{3}x-4=0$ is equal to

(A) $\frac{1}{3}\left(2-12\sqrt{3}+8\pi\right)$

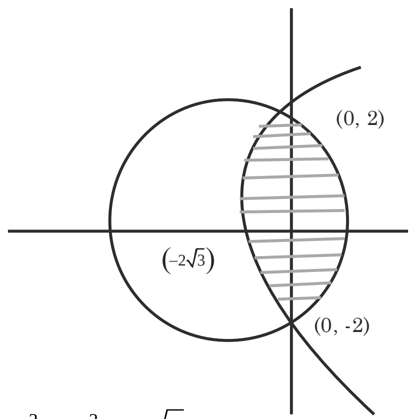
(B) $\frac{1}{3}\left(2-12\sqrt{3}+6\pi\right)$

(C) $\frac{1}{3}\left(4-12\sqrt{3}+8\pi\right)$

(D) $\frac{1}{3}\left(4-12\sqrt{3}+6\pi\right)$

Official Ans. by NTA (C)

Sol.



$x^2+y^2+4\sqrt{3}x-4=0$

$y^2=8x+4$

Point of intersections are (0, 2) & (0, -2)

Both are symmetric about x-axis

Area $=2\int_0^2\left(\sqrt{16-y^2}-2\sqrt{3}\right)-\left(\frac{y^2-4}{8}\right)dy$

On solving $Area=\frac{1}{3}\left[8\pi+4-12\sqrt{3}\right]$

10. Let y = y_1 (x) and y = y_2(x) be two distinct solutions of the differential equation $\frac{dy}{dx} = x + y$, with y_1 (0) = 0 and y_2 (0) = 1 respectively. Then, the number of points of intersection of y = y_1 (x) and y = y_2(x) is

(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (A)

Sol. $\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

1f = e^{-x}

$\therefore$ solution is $ye^{-x} = \int xe^{-x}dx$

$\Rightarrow ye^{-x} = -xe^{-x} - e^{-x} + c$

$\Rightarrow y = -x - 1 + ce^{x}$

$y_1(0) = 0 \Rightarrow c = 1$

$\therefore y_1 = -x - 1 + e^{x}$ ....(1)

y_2 (0) = 1 $\Rightarrow$ c = 2

$\therefore y_2 = -x - 1 + 2e^{x}$ .....(2)

Now $y_2 - y_1 = e^{x} > 0 \therefore y_2 \neq y_1$

$\therefore$ Number of points of intersection of y_1 & y_2 is zero.

11. Let P (*a*, b) be a point on the parabola y^2 = 8x such that the tangent at P passes through the centre of the circle $x^2 + y^2 - 10x - 14y + 65 = 0$. Let A be the product of all possible values of *a* and *B* be the product of all possible values of *b*. Then the value of A + B is equal to :

(A) 0 (B) 25

(C) 40 (D) 65

Official Ans. by NTA (D)

Sol. P(a, b) is point on y^2 = 8x, such that tangent at P pass through centre of $x^2 + y^2 - 10x - 14y + 65 = 0$ i.e. (5, 7)

Tangent at P(at^2, 2at) is ty = x + at^2

A = 2 & it pass through (5, 7)

7t = 5 + $2t^2$

$\Rightarrow t = 1,\ t = \frac{5}{2}$

$\therefore P\left(at^2, 2at\right) \Rightarrow (2,4)$ when t = 1

& $\left(\frac{25}{2}, 10\right)$ when $t = \frac{5}{2}$

$\therefore A = 2 \times \frac{25}{2} = 25$

B = 4 × 10 = 40 $\therefore A + B = 65$

12. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}$ and $\vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$ be two vectors, such that $\vec{a} \times \vec{b} = -\hat{i} + 9\hat{i} + 12k$. Then the projection of $\vec{b} - 2\vec{a}$ on $\vec{b} + \vec{a}$ is equal to

(A) 2 (B) $\frac{39}{5}$

(C) 9 (D) $\frac{46}{5}$

Official Ans. by NTA (D)

Sol. Let $\vec{a} = \alpha\hat{i} + \hat{j} + \beta\hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\vec{a} \times \vec{b} = -\hat{i} + 9\hat{j} + 12\hat{k}$

$$\Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \alpha & 1 & \beta \\ 3 & -5 & 4 \end{vmatrix}$$

$\Rightarrow (4 + 5\beta)\hat{i} + (3\beta - 4\alpha)\hat{j} + (-5\alpha - 3)\hat{k}$

$= -\hat{i} + 9\hat{j} + 12\hat{k}$

$\therefore 4 + 5\beta = -1,\ 3\beta - 4\alpha = 9,\ -5\alpha - 3 = 12$

$\beta = -1,\ \ \alpha = -3$

$\therefore \vec{a} = -3\hat{i} + \hat{j} - \hat{k}, \vec{b} = 3\hat{i} - 5\hat{j} + 4\hat{k}$

$\therefore \vec{a} + \vec{b} = -4\hat{j} + 3\hat{k}$

$|\vec{a}|^2 = 11,\ |\vec{b}|^2 = 50$

$\vec{a}.\vec{b} = -9 + (-5) - 4 = -18$

$\therefore$ Projectile of $\left(\vec{b}-2\vec{a}\right)$ on $\vec{a}+\vec{b}$ is

$$\frac{\left(\vec{b}-2\vec{a}\right).\left(\vec{a}+\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}$$

$$=\frac{\left|\vec{b}\right|^2-2\left|\vec{a}\right|^2-\left(\vec{a}.\vec{b}\right)}{\left|\vec{a}+\vec{b}\right|}=\frac{50-22-(-18)}{5}=\frac{46}{5}$$

Ans. $\left(\frac{46}{5}\right)$

13. Let $\vec{a}=2\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$. If $\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right).\hat{k}=\frac{23}{2}$, then $\left|\vec{b}\times 2\hat{j}\right|$ is equal to

(A) 4 (B) 5

(C) $\sqrt{21}$ (D) $\sqrt{17}$

Official Ans. by NTA (B)

Sol. $\vec{a}=2\hat{i}-\hat{j}+5\hat{k},\ \vec{b}=\alpha\hat{i}+\beta\hat{j}+2\hat{k}$

$\left(\left(\vec{a}\times\vec{b}\right)\times\hat{i}\right).\hat{k}=\frac{23}{2}$, then $\left|\vec{b}\times 2\hat{j}\right|$ is

$$\left(\left(\vec{a}.\hat{i}\right)\vec{b}-\left(\vec{b}.\hat{i}\right)\vec{a}\right).\hat{k}=\frac{23}{2}$$

$$\left(\vec{a}.\hat{i}\right)\left(\vec{b}.\hat{i}\right)-\left(\vec{b}.\hat{i}\right)\left(\vec{a}.\hat{k}\right)=\frac{23}{2}$$

$$2\times2-\alpha\times5=\frac{23}{2}\Rightarrow 5\alpha=4-\frac{23}{2}\Rightarrow\alpha=\frac{-3}{2}$$

$$\vec{b}\times2\hat{j}=\begin{vmatrix}\hat{i}&\hat{j}&\hat{k}\\ \alpha&\beta&2\\ 0&2&0\end{vmatrix}=-4\hat{i}+2\alpha\hat{k}$$

$$\therefore\left|\vec{b}\times2\hat{j}\right|=\sqrt{16+4\alpha^2}=\sqrt{16+4\times\frac{9}{4}}=5$$

14. Let S be the sample space of all five digit numbers. If p is the probability that a randomly selected number from S, is a multiple of 7 but not divisible by 5, then $9p$ is equal to

(A) 1.0146 (B) 1.2085

(C) 1.0285 (D) 1.1521

Official Ans. by NTA (C)

Sol. n(S) = all 5 digit nos $= 9\times10^4$

A : no is multiple of 7 but not divisible by 5

Smallest 5 digit divisible by 7 is 10003

Largest 5 digit divisible by 7 is 99995

$\therefore$ 99995 = 10003 + (n – 1) 7 n = 12857

Numbers divisible by 35

99995 = 10010 + (P-1) 35 $\Rightarrow$ P = 2572

$\therefore$ Numbers divisible by 7 but not by 35 are

12857 – 2572 = 10285

$\therefore P=\frac{10285}{90000}$ $\therefore$ 9P = 1.0285

Ans. (C) [1.0285]

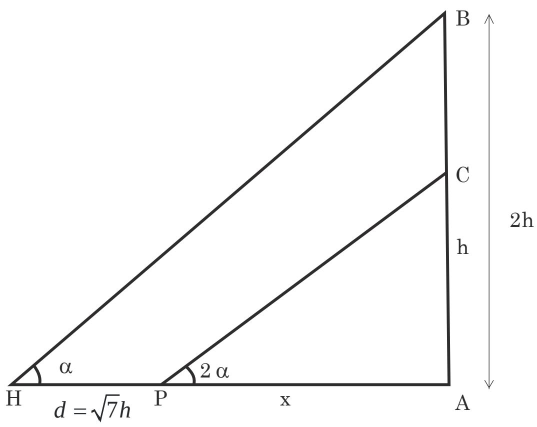
15. Let a vertical tower AB of height $2h$ stands on a horizontal ground. Let from a point P on the ground a man can see upto height h of the tower with an angle of elevation 2α. When from P, he moves a distance d in the direction of $\overrightarrow{AP}$, he can see the top B of the tower with an angle of elevation α. If $d=\sqrt{7}h$, then tan α is equal to

(A) $\sqrt{5}-2$ (B) $\sqrt{3}-1$

(C) $\sqrt{7}-2$ (D) $\sqrt{7}-\sqrt{3}$

Official Ans. by NTA (C)

Sol.



$$\tan 2\alpha=\frac{h}{x}$$

and $\tan\alpha = \dfrac{2h}{x+\sqrt{7}h}$

$$\tan\alpha = \frac{2h}{h\cot 2\alpha + \sqrt{7}h}$$

$$\tan\alpha = \frac{2}{\frac{(1-\tan^2\alpha)}{2\tan\alpha}+\sqrt{7}}$$

Put $\tan\alpha = t$ & simplify

$\Rightarrow \tan\alpha = \sqrt{7}-2$

16. $(p \wedge r) \Leftrightarrow (p \wedge (\sim q))$ is equivalent to $(\sim p)$ when r is

(A) p (B) $\sim p$

(C) q (D) $\sim q$

Official Ans. by NTA (C)

Sol. Given $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

Taking r = q

p	q	~p	~q	p^q	P^~q	$(p\wedge r)\Leftrightarrow(p\wedge(\sim q))$
T	T	F	F	T	F	F
T	F	F	T	F	T	F
F	T	T	F	F	F	T
F	F	T	T	F	F	T

So, clear $(p \wedge r) \Leftrightarrow (p \wedge (\sim q)) \equiv (\sim p)$

17. If the plane P passes through the intersection of two mutually perpendicular planes 2x + ky – 5z = 1 and 3kx – ky + z = 5, k < 3 and intercepts a unit length on positive x-axis, then the intercept made by the plane P on the y-axis is

(A) $\dfrac{1}{11}$ (B) $\dfrac{5}{11}$

(C) 6 (D) 7

Official Ans. by NTA (D)

Sol. Two given planes mutually perpendicular

2(3k) + k (-k) + (-5) 1 = 0

k = 1, 5

but k < 3 So k = 1

Plane passing through these planes is

$2x + y - 5z - 1 + \lambda(3x - y + z - 5) = 0$

$$\frac{x}{\frac{5\lambda+1}{2+3\lambda}} + \frac{y}{\frac{5\lambda+1}{1-\lambda}} + \frac{z}{\frac{5\lambda+1}{\lambda-5}} = 1$$

Given $\dfrac{5\lambda+1}{2+3\lambda} = 1 \Rightarrow \lambda = \dfrac{1}{2}$

So intercept on y - axis = $\dfrac{5\lambda+1}{1-\lambda} = 7$

18. Let A(1, 1), B(-4, 3) C(-2, -5) be vertices of a triangle ABC, P be a point on side BC, and Δ_1 and Δ_2 be the areas of triangle APB and ABC. Respectively.

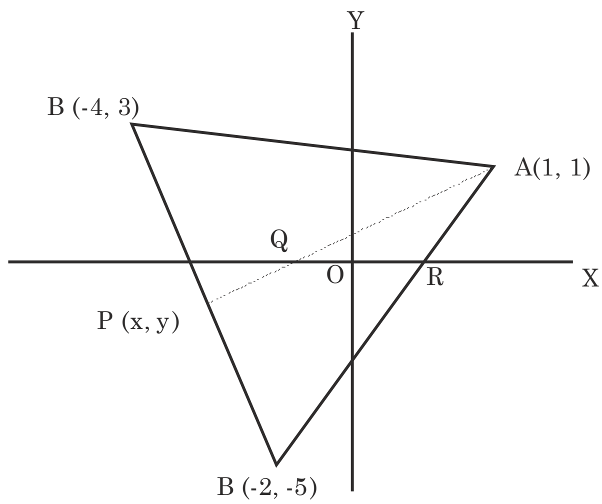
If $\Delta_1 : \Delta_2 = 4:7$, then the area enclosed by the lines AP, AC and the x-axis is

(A) $\dfrac{1}{4}$ (B) $\dfrac{3}{4}$

(C) $\dfrac{1}{2}$ (D) 1

Official Ans. by NTA (C)

Sol.



Given $\Delta_1 = \frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 1 & 1 \\ -4 & 3 & 1 \end{vmatrix}$

& $\Delta_2 = \frac{1}{2}\begin{vmatrix} 1 & 1 & 1 \\ -4 & 3 & 1 \\ -2 & -5 & 1 \end{vmatrix}$

Given $\frac{\Delta_1}{\Delta_2} = \frac{4}{7} \Rightarrow \frac{-2x-5y+7}{36} = \frac{4}{7}$

$\Rightarrow 14x + 35y = -95$ ….(1)

Equation of BC is 4x + y = -13 ….(2)

Solve equation (1) & (2)

Point $P\left(\frac{-20}{7}, \frac{-11}{7}\right)$

Here point $Q\left(\frac{-1}{2}, 0\right)$ & $R\left(\frac{1}{2}, 0\right)$

So Area of triangle AQR $= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

19. If the circle $x^2 + y^2 - 2gx + 6y - 19c = 0$, $g, c \in \mathbb{R}$ passes through the point (6, 1) and its centre lies on the line x – 2cy = 8, then the length of intercept made by the circle on x-axis is

(A) $\sqrt{11}$ (B) 4

(C) 3 (D) $2\sqrt{23}$

Official Ans. by NTA (D)

Sol. Given circle $x^2 + y^2 - 2gx + 6y - 19c = 0$

Passes through (6, 1)

12g + 19 c = 43 ….(1)

Centre (g, -3) lies on given line

So, g + 6c = 8 ….(2)

Solve equation (1) & (2)

c = 1 & g = 2

equation of circle $x^2 + y^2 - 4x + 6y - 19 = 0$

Length of intercept on x-axis

$= 2\sqrt{g^2 - c} = 2\sqrt{23}$

20. Let a function $f : \mathbb{R} \to \mathbb{R}$ be defined as :

$$f(x) = \begin{cases} \int_0^x (5 - |t-3|)\,dt, & x > 4 \\ x^2 + bx, & x \le 4 \end{cases}$$

where $b \in \mathbb{R}$. If f is continuous at x = 4, then which of the following statements is NOT true ?

(A) f is not differentiable at x = 4

(B) $f'(3) + f'(5) = \frac{35}{4}$

(C) f is increasing in $\left(-\infty, \frac{1}{8}\right) \cup (8, \infty)$

(D) f has a local minima at $x = \frac{1}{8}$

Official Ans. by NTA (C)

Sol. Given $f(x) \begin{cases} \int_0^x (5 - |t-3|)\,dt, & x > 4 \\ x^2 + bx, & x \le 4 \end{cases}$

f(x) is continuous at x = 4

So $\lim_{x \to 4^-} f(x) = \lim_{x \to 4^+} f(x) = f(4)$

So 16 + 4b $= \int_0^3 (2-t)\,dt + \int_3^4 (8-t)dt$

$\Rightarrow 16 + 4b = 15$

So $b = \frac{-1}{4}$

At x = 4

LHD = 2x + b = $\frac{31}{4}$

RHD = 5 - |x – 3| = 4

LHD $\neq$ RHD

Option (A) is true

and f'(3) + f'(5) = $\frac{23}{4} + 3 = \frac{35}{4}$

Option (B) is true

$\because f(x) = x^2 - \frac{x}{4}$ at $x \le 4$

$$f'(x) = 2x - \frac{1}{4}$$

This function is not increasing.

In the interval in $x \in \left(-\infty, \frac{1}{8}\right)$

Option (C) is NOT TRUE.

This function f(x) is also local minima at $x = \frac{1}{8}$

SECTION-B

1. For $k \in \mathbb{R}$, let the solutions of the equation $\cos\left(\sin^{-1}\left(x\cot\left(\tan^{-1}\left(\cos\left(\sin^{-1}x\right)\right)\right)\right)\right) = k, 0 < |x| < \frac{1}{\sqrt{2}}$ be α and β, where the inverse trigonometric functions take only principal values. If the solutions of the equation x^2 – bx – 5 = 0 are $\frac{1}{\alpha^2} + \frac{1}{\beta^2}$ and $\frac{\alpha}{\beta}$, then $\frac{b}{k^2}$ is equal to ______.

Official Ans. by NTA (12)

Sol. $\cos\left(\sin^{-1}x\right) = \cos\left(\cos^{-1}\sqrt{1-x^2}\right) = \sqrt{1-x^2}$

$$\cot\left(\tan^{-1}\sqrt{1-x^2}\right) = \cot\ \cot^{-1}\left(\sqrt{\frac{1}{\sqrt{1-x^2}}}\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \cos\left(\sin^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right) = \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{\sqrt{1-2x^2}}{\sqrt{1-x^2}} = k$$

$$\Rightarrow 1-2x^2 = k^2\left(1-x^2\right)$$

$$\Rightarrow \left(k^2-2\right)x^2 = k^2-1$$

$$x^2 = \frac{k^2-1}{k^2-2}$$

$$\alpha = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \alpha^2 = \frac{k^2-1}{k^2-2}$$

$$\beta = \sqrt{\frac{k^2-1}{k^2-2}} \Rightarrow \beta^2 = \frac{k^2-1}{k^2-2}$$

$$\frac{1}{\alpha^2} + \frac{1}{\beta^2} = 2\left(\frac{k^2-2}{k^2-1}\right) \ \& \ \frac{\alpha}{\beta} = -1$$

Sum of roots = $\frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{\alpha}{\beta} = b$

$$\Rightarrow \frac{2\left(k^2-2\right)}{k^2-1} - 1 = b \ .....(1)$$

Product of roots $= \left(\frac{1}{\alpha^2} + \frac{1}{\beta^2}\right)\frac{\alpha}{\beta} = -5$

$$\Rightarrow \frac{2\left(k^2-2\right)}{k^2-1}(-1) = -5$$

$$\Rightarrow 2k^2 - 4 = 5k^2 - 5$$

$$\Rightarrow 3k^2 = 1 \Rightarrow k^2 = \frac{1}{3} \ \text{.... Put in (1)}$$

$$\Rightarrow b = \frac{2\left(k^2-2\right)}{k^2-1} - 1 = 5 - 1 = 4$$

$$\frac{b}{k^2} = \frac{4}{\frac{1}{3}} = 12$$

2. The mean and variance of 10 observations were calculated as 15 and 15 respectively by a student who took by mistake 25 instead of 15 for one observation. Then, the correct standard deviation is __________.

Official Ans. by NTA (2)

Sol. n = 10, $\overline{x} = \frac{\sum x_i}{10} = 15$

$$6^2 = \frac{\sum x_i^2}{10} - \left(\overline{x}\right)^2 = 15$$

$$\Rightarrow \sum_{i=1}^{10} x_i = 150$$

$\Rightarrow \sum_{i=1}^{9} x_i + 25 = 150$

$\Rightarrow \sum_{i=1}^{9} x_i = 125$

$\Rightarrow \sum_{i=1}^{9} x_i + 15 = 140$

Actual mean $= \frac{140}{10} = 14 = \overline{x}_{new}$

$\sum_{i=1}^{9} \frac{x_i^2 + 25^2 - 15^2}{10} = 15$

$\Rightarrow \sum_{i=1}^{9} x_i^2 + 625 = 2400$

$\sum_{i=1}^{9} x_i^2 = 1775$

$\sum_{i=1}^{9} x_i^2 + 15^2 = 2000 = \left(\sum x_i^2\right)_{actual}$

$6^2_{actual} = \frac{\left(\sum x_i^2\right)_{actual} - \left(\overline{x}_{new}\right)^2}{10}$

$= \frac{2000}{10} - 14^2$

= 200-196 = 4

$(S.D)_{actual} = 6 = 2$

3. Let the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4}$ intersect the plane containing the lines $\frac{x-4}{1} = \frac{y+1}{-2} = \frac{z}{1}$ and $4ax - y + 5z - 7a = 0$ = 2x – 5y – z – 3, $a \in \mathbb{R}$ at the point $P(\alpha, \beta, \gamma)$. Then the value of $\alpha + \beta + \gamma$ equals ________.

Official Ans. by NTA (12)

Sol. Equation of plane

$4ax - y + 5z - 7a + \lambda$ (2x – 5y – z – 3)= 0

this satisfy (4, -1, 0)

$16a + 1 - 7a + \lambda(8 + 5 - 3) = 0$

$9a + 1 + 10\lambda = 0$ .....(1)

Normal vector of the plane A is $(4a + 2\lambda, -1 - 5\lambda, 5 - \lambda)$ vector along the line which contained the plane A is

i – 2j + k

$\therefore 4a + 2\lambda + 2 + 10\lambda + 5 - \lambda = 0$

$11\lambda + 4a + 7 = 0$ ......(2)

Solve (1) and (2) to get a = 1, $\lambda = -1$

Now equation of plane

$x + 2y + 3z - 2 = 0$

Let the point in the line $\frac{x-3}{7} = \frac{y-2}{-1} = \frac{z-3}{-4} = t$

is $(7t + 3, -t + 2, -4t + 3)$ satisfy the equation of plane A

7t + 3 – 2t + 4 + 9 -12t – 2 = 0

t = 2

So $\alpha + \beta + \gamma = 2t + 8 = 12$

4. An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$. Let the major and minor axes of the ellipse E coincide with the transverse and conjugate axes of the hyperbola H. Let the product of the eccentricities of E and H be $\frac{1}{2}$. If l is the length of the latus rectum of the ellipse E, then the value of $113l$ *is* equal to ________.

Official Ans. by NTA (1552)

Sol. Hyp : $\frac{y^2}{64} - \frac{x^2}{49} = 1$

An ellipse $E: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ passes through the vertices of the hyperbola $H: \frac{x^2}{49} - \frac{y^2}{64} = -1$.

So $b^2 = 64$

$$e_H = \sqrt{1+\frac{a^2}{b^2}} = \sqrt{1+\frac{49}{64}}$$

Ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$$e_E = \sqrt{1-\frac{a^2}{b^2}} = \sqrt{1-\frac{a^2}{64}}$$

$$b=8, \sqrt{\frac{1-a^2}{64}} \times \frac{\sqrt{113}}{8} = \frac{1}{2} \Rightarrow \sqrt{64-a^2} \times \sqrt{113} = 32$$

$$\left(64-a^2\right) = \frac{32^2}{113}$$

$$\Rightarrow a^2 = 64 - \frac{32^2}{113}$$

$$l = \frac{2a^2}{b} = \frac{2}{8}\left(64-\frac{32^2}{113}\right) = \frac{1552}{113}$$

$$113l = 1552$$

5. Let y = y(x) be the solution curve of the differential equation

$$\sin\left(2x^2\right)\log_e\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx = 0,$$

$0 < x < \sqrt{\frac{\pi}{2}}$, which passes through the point $\left(\sqrt{\frac{\pi}{6}},1\right)$. Then $\left|y\left(\sqrt{\frac{\pi}{3}}\right)\right|$ is equal to _______.

Official Ans. by NTA (1)

Sol. $$\sin\left(2x^2\right)\ln\left(\tan x^2\right)dy + \left(4xy - 4\sqrt{2}x\sin\left(x^2-\frac{\pi}{4}\right)\right)dx = 0$$

$$\ln\left(\tan x^2\right)dy + \frac{4xydx}{\sin\left(2x^2\right)} - \frac{4\sqrt{2}\,x\sin\left(x^2-\frac{\pi}{4}\right)}{\sin\left(2x^2\right)}dx = 0$$

$$d\left(y.\ln\left(\tan x^2\right)\right) - 4\sqrt{2}x\frac{\left(\sin x^2 - \cos x^2\right)}{\sqrt{2}-2\sin x^2\cos x^2}dx = 0$$

$$d\left(y\ln\left(\tan x^2\right)\right) - \frac{4x\left(\sin x^2 - \cos x^2\right)}{\left(\sin x^2 + \cos^2\right)-1}dx = 0$$

$$\Rightarrow \int d\left(y\ln\left(\tan x^2\right)\right) + 2\int\frac{dt}{t^2-1} = \int 0$$

$$\Rightarrow y\ln\left(\tan x^2\right) + 2.\frac{1}{2}\ln\left|\frac{t-1}{t+1}\right| = c$$

$$y\ln\left(\tan x^2\right) + \ln\left(\frac{\sin x^2 + \cos x^2 - 1}{\sin x^2 + \cos x^2 + 1}\right) = c$$

Put y = 1 and $x = \sqrt{\frac{\pi}{6}}$

$$1\ln\left(\frac{1}{\sqrt{3}}\right) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)} = c$$

Now

$$x = \sqrt{\frac{\pi}{3}} \Rightarrow y\left(\ln\sqrt{3}\right) + \ln\frac{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}-1\right)}{\left(\frac{1}{2}+\frac{\sqrt{3}}{2}+1\right)} = \ln\left(\frac{1}{\sqrt{3}}\right) + \ln\left(\frac{\sqrt{3}-1}{\sqrt{3}+3}\right)$$

$$y\left(\ln\sqrt{3}\right) = \ln\left(\frac{1}{\sqrt{3}}\right)$$

$\Rightarrow$ y = -1

$$|y| = 1$$

6. Let *M* and *N* be the number of points on the curve $y^5 - 9xy + 2x = 0$, where the tangents to the curve are parallel to x-axis and y-axis, respectively. Then the value of M + N equals ________.

Official Ans. by NTA (2)

Sol. $y^5 - 9xy + 2x = 0$

$$5y^4\frac{dy}{x} - 9x\frac{dy}{dx} - 9y + 2 = 0$$

$$\frac{dy}{dx}\left(5y^4 - 9x\right) = 9y - 2$$

$$\frac{dy}{dx} = \frac{9y-2}{5y^4-9x} = 0 \text{ (for horizontal tangent)}$$

$y = \frac{2}{9} \Rightarrow$ Which does not satisfy the original equation $\Rightarrow$ M = 0.

Now $5y^4 - 9x = 0$ (for vertical tangent)

$5y^4(9y-2) - 9y^5 = 0$

$y^4[45y - 10 - 9y] = 0$

$y = 0$ (Or) $36y = 10$

$$y = \frac{5}{18}$$

$$y = 0 \Rightarrow x = 0 \ \& \ y = \frac{5}{18} \Rightarrow x =$$

$$(0,0) \qquad \left(x, \frac{5}{18}\right)$$

$N = 2$

$M + N = 0 + 2 = 2$

7. Let $f(x) = 2x^2 - x - 1$ and $S = \{n \in \mathbb{Z} : |f(n)| \le 800\}$. Then, the value of $\sum_{n \in S} f(n)$ is equal to ________.

Official Ans. by NTA (10620)

Sol. $f(x) = 2x^2 - x - 1$

$|f(x)| \le 800$

$2n^2 - n - 801 \le 0$

$$n^2 - \frac{1}{2}n - \frac{801}{2} \le 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{801}{2} - \frac{1}{16} \le 0$$

$$\left(n - \frac{1}{4}\right)^2 - \frac{6409}{16} \le 0$$

$$\left(n - \frac{1}{4} - \frac{\sqrt{6409}}{4}\right)\left(n - \frac{1}{4} + \frac{\sqrt{6409}}{16}\right) \le 0$$

$$\frac{1 - \sqrt{6409}}{4} \le n \le \frac{1 + \sqrt{6409}}{4}$$

$$n = \{-19, -18 - 17, ...0, 1, 2, ..., 20\}$$

$$\sum_{n \in S} f(x) = \sum\left(2x^2 - x - 1\right)$$

$$= 2\left[19^2 + 18^2 + ... + 1^2 + 1^2 + 2^2 + ... + 19^2 + 20^2\right]$$

$$= 4\left[1^2 + 2^2 + ... + 19^2\right] + 2\left[20^2\right] - 20 - 40$$

$$= \frac{4 \times 19 \times 20 \times (2 \times 19 + 1)}{6} + 2 \times 400 - 60$$

$$= \frac{4 \times 19 \times 20 \times 39}{6} + 800 - 60 - 9880 + 800 - 60$$

$= 10620$

8. Let S be the set containing all 3×3 matrices with entries from {-1, 0, 1}. The total number of matrices A $\in S$ such that the sum of all the diagonal elements of $A^T A$ is 6 is ________.

Official Ans. by NTA (5376)

Sol. $Tr\left(AA^T\right) = 6$

$$AA^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & i \end{bmatrix}\begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}$$

Now given $a^2 + d^2 + g^2 + b^2 + e^2 + h^2 + c^2 + f^2 + i^2 = 6$

$$= {}^9C_3 \times 2^6$$

$= 5376$

9. If the length of the latus rectum of the ellipse $x^2 + 4y^2 + 2x + 8y - \lambda = 0$ is 4, and l is the length of its major axis, then $\lambda + l$ is equal to _______.

Official Ans. by NTA (75)

Sol. $\lambda + \ell = 75$

$$x^2 + 4y^2 + 2x + 8y - \lambda = 0$$

$$\frac{(x+1)^2}{\lambda+5} + \frac{(y+1)^2}{\frac{\lambda+5}{4}} = 1$$

$$\because \frac{2b^2}{a} = 4$$

$$\frac{2(\lambda+5)}{4} = 4\left(\sqrt{\lambda+5}\right)$$

$$\Rightarrow \lambda = 59$$

$$\lambda \neq -5$$

$$l = 2a = 2\sqrt{\lambda+5} = 2\sqrt{65} = 16$$

$$\Rightarrow \lambda + \ell = 59 + 16 = 75$$

10. Let $S = \{z \in \mathbb{C} : z^2 + \bar{z} = 0\}$. Then $\sum_{z \in S} (\operatorname{Re}(z) + \operatorname{Im}(z))$ is equal to _______.

Official Ans. by NTA (0)

Sol. $S = \{z \in C : z^2 + \bar{z} = 0\}$

Let $z = x + iy$

$z^2 = x^2 - y^2 + 2ixy$

$\bar{z} = x - iy$

$$z^2 + \bar{z} = x^2 - y^2 + x + i(2xy - y) = 0$$

$$\Rightarrow x^2 + x - y^2 = 0 \ \& \ 2xy - y = 0$$

$$y = 0 \text{ or } x = \frac{1}{2}$$

If $y = 0$; $x = 0, -1$

If $x = \frac{1}{2}; y = \frac{\sqrt{3}}{2}, \frac{-\sqrt{3}}{2}$

$$\sum_{z \in S}\left(\operatorname{Re}(z) + \operatorname{Im}(z) = \left(0 - 1 + \frac{1}{2} + \frac{1}{2}\right) + 0 + 0 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right)$$