FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS	TEST PAPER WITH SOLUTION

SECTION-A

1. An expression of energy density is given by $u = \frac{\alpha}{\beta}\sin\left(\frac{\alpha x}{kt}\right)$, where α, β are constants, x is displacement, k is Boltzmann constant and t is the temperature. The dimensions of β will be :

(A) $[ML^2T^{-2}\theta^{-1}]$ (B) $[M^0L^2T^{-2}]$

(C) $[M^0L^0T^0]$ (D) $[M^0L^2T^0]$

Official Ans. by NTA (D)

Sol. $\frac{\alpha[L]}{[ML^2T^{-2}]} = [M^0L^0T^0]$

$\alpha = [ML^1T^{-2}]$

$\frac{\alpha}{\beta} = \frac{[ML^2T^{-2}]}{[L^3]} \Rightarrow \beta = \frac{[ML^1T^{-2}][L^3]}{ML^2T^{-2}}$

2. A body of mass 10 kg is projected at an angle of 45° with the horizontal. The trajectory of the body is observed to pass through a point (20, 10). If T is the time of flight, then its momentum vector, at time $t = \frac{T}{\sqrt{2}}$, is ________

[Take g = 10 m/s^2]

(A) $100\hat{i} + (100\sqrt{2} - 200)\hat{j}$

(B) $100\sqrt{2}\,\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(C) $100\,\hat{i} + (100 - 200\sqrt{2})\hat{j}$

(D) $100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\hat{j}$

Official Ans. by NTA (D)

Sol. $y = x - \frac{10x^2}{2u^2\left(\frac{1}{2}\right)} \Rightarrow 10 = 20 - \frac{(10)(100)}{u^2}$

$u = 20$

$T = \frac{(2)(20)}{\sqrt{2}(10)} = 2\sqrt{2}$

$\vec{v} = 10\sqrt{2}\,\hat{i} + (10\sqrt{2} - 10(2))\hat{j}$

Momentum $\vec{p} = M\vec{v} = 100\sqrt{2}\,\hat{i} + (100\sqrt{2} - 200)\hat{j}$

3. A block of mass M slides down on a rough inclined plane with constant velocity. The angle made by the incline plane with horizontal is θ. The magnitude of the contact force will be :

(A) Mg

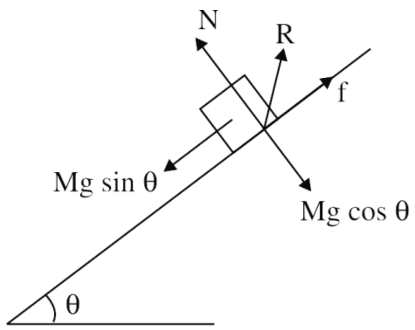
(B) Mg cos θ

(C) $\sqrt{Mg\sin\theta + Mg\cos\theta}$

(D) $Mg\sin\theta\sqrt{1+\mu}$

Official Ans. by NTA (A)

Sol.



$N = Mg\cos\theta$

$f = Mg\sin\theta$

$R = \sqrt{N^2 + f^2}$

$R = Mg$

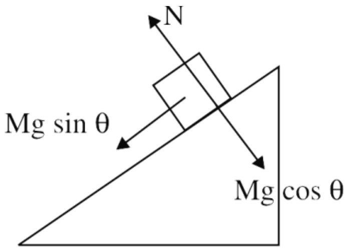
4. A block 'A' takes 2 s to slide down a frictionless incline of 30° and length 'l', kept inside a lift going up with uniform velocity 'v'. If the incline is changed to 45°, the time taken by the block, to slide down the incline, will be approximately:

(A) 2.66 s (B) 0.83 s

(C) 1.68 s (D) 0.70 s

Official Ans. by NTA (C)

Sol.



$a = g \sin \theta$

$$\ell = \frac{1}{2} g \sin 30° (2)^2 \quad \text{....(1)}$$

$$\ell = \frac{1}{2} g \sin 45° \, t^2 \quad \text{....(2)}$$

$$\left(\frac{1}{2}\right)(4) = \frac{1}{\sqrt{2}} t^2 \Rightarrow t = \sqrt{2\sqrt{2}} \simeq 1.68$$

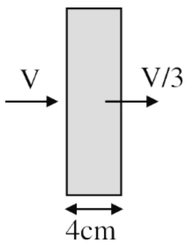
5. The velocity of the bullet becomes one third after it penetrates 4 cm in a wooden block. Assuming that bullet is facing a constant resistance during its motion in the block. The bullet stops completely after travelling at (4 + x) cm inside the block. The value of x is:

(A) 2.0 (B) 1.0

(C) 0. 5 (D) 1.5

Official Ans. by NTA (C)

Sol.



$$\left(\frac{V}{3}\right)^2 = V^2 - 2a(4) \Rightarrow a = \frac{8V^2}{9(8)} = \frac{V^2}{9}$$

$$0 = V^2 - 2a(4 + x)$$

$$\Rightarrow \quad V^2 = 2\left(\frac{V^2}{9}\right)(4 + x)$$

$$4.5 = 4 + x$$

$$x = 0.5$$

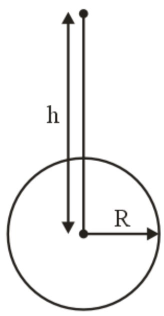
6. A body of mass m is projected with velocity λv_e in vertically upward direction from the surface of the earth into space. It is given that v_e is escape velocity and $\lambda < 1$. If air resistance is considered to the negligible, then the maximum height from the centre of earth, to which the body can go, will be (R : radius of earth)

(A) $\frac{R}{1+\lambda^2}$ (B) $\frac{R}{1-\lambda^2}$

(C) $\frac{R}{1-\lambda}$ (D) $\frac{\lambda^2 R}{1-\lambda^2}$

Official Ans. by NTA (B)

Sol.



$$-\frac{GMm}{R} + \frac{1}{2} m\lambda^2 V_e^2 = -\frac{GMm}{h}$$

$$-\frac{GMm}{R} + \frac{1}{2}\lambda^2 \frac{2GMm}{R} = -\frac{GMm}{h}$$

$$\frac{\lambda^2}{R} - \frac{1}{R} = \frac{-1}{h}$$

$$\frac{1}{h} = \frac{1-\lambda^2}{R}$$

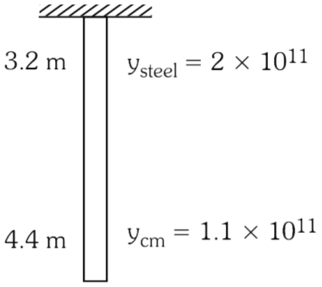
$$h = \frac{R}{1-\lambda^2}$$

7. A steel wire of length 3.2 m ($Y_S = 2.0 \times 10^{11}$ Nm^{-2}) and a copper wire of length 4.4 M ($Y_C = 1.1 \times 10^{11}$ Nm^{-2}), both of radius 1.4 mm are connected end to end. When stretched by a load, the net elongation is found to be 1.4 mm. The load applied, in Newton, will be: (Given $\pi = \frac{22}{7}$)

(A) 360 (B) 180 (C) 1080 (D) 154

Official Ans. by NTA (D)

Sol.

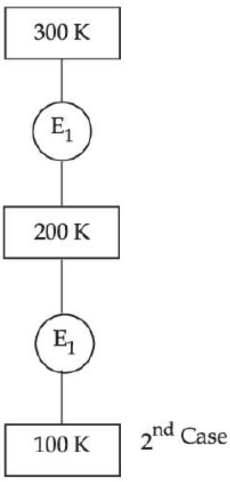


$$\Delta\ell_1 + \Delta\ell_2 = \Delta\ell$$

$$\frac{F\ell_1}{A_1 y_1} + \frac{F\ell_2}{A_2 y_2} = \Delta\ell$$

$$F = \frac{\Delta\ell}{\frac{\ell_1}{A_1 y_1} + \frac{\ell_2}{A_2 y_2}} = 1.54 \times 10^2 = 154$$

8. In 1st case, Carnot engine operates between temperatures 300 K and 100 K. In 2nd case, as shown in the figure, a combination of two engines is used. The efficiency of this combination (in 2nd case) will be :



(A) same as the 1st case

(B) always greater than the 1st case

(C) always less than the 1st case

(D) may increase or decrease with respect to the 1st case

Official Ans. by NTA (C)

Sol. First case : $\eta = 1 - \frac{100}{300} = \frac{2}{3}$

Second case : $\eta_{net} = \eta_1 + \eta_2 - \eta_1\eta_2$

$$\eta_1 = 1 - \frac{200}{300} = \frac{1}{3}$$

$$\eta_2 = 1 - \frac{100}{200} = \frac{1}{2}$$

$$\eta_{net} = \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$$

η (first case) = η (second case)

9. Which statements are correct about degrees of freedom?

A. A molecule with n degrees of freedom has n^2 different ways of storing energy.

B. Each degree of freedom is associated with $\frac{1}{2}RT$ average energy per mole.

C. A monoatomic gas molecule has 1 rotational degree of freedom where as diatomic molecule has 2 rotational degrees of freedom

D. CH_4 has a total to 6 degrees of freedom

Choose the correct answer from the option given below:

(A) B and C only (B) B and D only

(C) A and B only (D) C and D only

Official Ans. by NTA (B)

Sol. Methane molecule is tetrahedron

Degree of freedom due to rotation = 3

Degree of freedom due to translation = 3

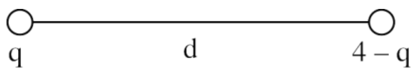
10. A charge of 4 μC is to be divided into two. The distance between the two divided charges is constant. The magnitude of the divided charges so that the force between them is maximum, will be:

(A) 1 μC and 3 μC (B) 2 μC and 2 μC

(C) 0 and 4 μC (D) 1.5 μC and 2.5 μC

Official Ans. by NTA (B)

Sol.



$$F = \frac{Kq(4-q)}{d^2}$$

$$\frac{dF}{dq} = \frac{K}{d^2}[4 - 2q] = 0$$

$$q = 2$$

11. A. The drift velocity of electrons decreases with the increase in the temperature of conductor.
B. The drift velocity is inversely proportional to the area of cross-section of given conductor.
C. The drift velocity does not depend on the applied potential difference to the conductor.
D. The drift velocity of electron is inversely proportional to the length of the conductor.
E. The drift velocity increases with the increase in the temperature of conductor.

Choose the correct answer from the options given below:

(A) A and B only (B) A and D only
(C) B and E only (D) B and C only

Official Ans. by NTA (B)

Sol. Drift velocity = $\left(\frac{e\tau}{m}\right)E$

$$v_d = \left(\frac{e\tau}{m}\right)\left(\frac{\Delta V}{\ell}\right)$$

ΔV = Potential difference applied across the wire

As temperature increases, relaxation time decreases, hence V_d decreases.

As per formula, $V_d \propto \frac{1}{\ell}$

$v_d = \frac{I}{neA}$, as it is not mentioned that current is at steady state neither it is mentioned that n is constant for given conductor. So it can't be said that v_d is inversely proportional to A.

$$I = neAv_d = \frac{V}{R} = \frac{V}{\rho\ell}A$$

$$v_d = \frac{V}{\rho\ell ne} \quad \left(E = \frac{V}{\ell}\right)$$

$$v_d = \frac{eE\tau}{m}$$

τ decrease with temperature increase.

First and fourth statements are correct.

12. A compass needle of oscillation magnetometer oscillates 20 times per minute at a place P of dip 30°. The number of oscillations per minute become 10 at another place Q of 60° dip. The ratio of the total magnetic field at the two places ($B_Q : B_P$) is:

(A) $\sqrt{3}:4$ (B) $4:\sqrt{3}$
(C) $\sqrt{3}:2$ (D) $2:\sqrt{3}$

Official Ans. by NTA (A)

Sol. $T = 2\pi\sqrt{\frac{I}{B_H M}}$

$$T_1 = 3\,\text{sec} = 2\pi\sqrt{\frac{I}{(B_P\cos 30°)M}}$$

$$T_2 = 6\,\text{sec} = 2\pi\sqrt{\frac{I}{(B_Q\cos 60°)M}}$$

$$\frac{3}{6} = \sqrt{\frac{1}{\left(B_P\frac{\sqrt{3}}{2}\right)} \times \left(B_Q/2\right)}$$

$$\frac{3}{6} = \sqrt{\left(\frac{B_Q}{\sqrt{3}B_P}\right)}$$

$$\frac{\sqrt{3}}{4} = \frac{B_Q}{B_P}$$

$$B_Q : B_P = \sqrt{3}:4$$

13. A cyclotron is used to accelerate protons. If the operating magnetic field is 1.0 T and the radius of the cyclotron 'dees' is 60 cm, the kinetic energy of the accelerated protons in MeV will be :

[use m_p = 1.6 × 10^{-27} kg, e = 1.6 × 10^{-19} C]

(A) 12 (B) 18
(C) 16 (D) 32

Official Ans. by NTA (B)

Sol. Kinetic energy of electron in cyclotron

$$= \left[\frac{q^2B^2r_0^2}{2m}\right]$$

= 18 MeV

14. A series LCR circuit has L = 0.01 H, R = 10 Ω and C = 1 μF and it is connected to ac voltage of amplitude (V_m) 50 V. At frequency 60% lower than resonant frequency, the amplitude of current will be approximately :

(A) 466 mA (B) 312 mA
(C) 238 mA (D) 196 mA

Official Ans. by NTA (C)

Sol. Resonant frequency, $\omega_0 = \frac{1}{\sqrt{LC}} = 10^4$ rad/sec

$\omega' = .4 \times 10^4 = 4000$ rad/sec

$i_0 = \frac{V_0}{\sqrt{R^2 + (X'_C - X'_L)^2}} = 238$ mA

15. Identify the correct statements from the following descriptions of various properties of electromagnetic waves.

A. In a plane electromagnetic wave electric field and magnetic field must be perpendicular to each other and direction of propagation of wave should be along electric field or magnetic field.

B. The energy in electromagnetic wave is divided equally between electric and magnetic fields.

C. Both electric field and magnetic field are parallel to each other and perpendicular to the direction of propagation of wave.

D. The electric field, magnetic field and direction of propagation of wave must be perpendicular to each other.

E. The ratio of amplitude of magnetic field to the amplitude of electric field is equal to speed of light.

Choose the most appropriate answer from the options given below:

(A) D only
(B) B and D only
(C) B, C and E only
(D) A, B and E only

Official Ans. by NTA (B)

Sol. Second and fourth statements are correct.

16. Two coherent sources of light interfere. The intensity ratio of two sources is 1 : 4. For this interference pattern if the value of $\frac{I_{max} + I_{min}}{I_{max} - I_{min}}$ is equal to $\frac{2\alpha + 1}{\beta + 3}$, then $\frac{\alpha}{\beta}$ will be :

(A) 1.5 (B) 2 (C) 0.5 (D) 1

Official Ans. by NTA (B)

Sol. $\frac{I_1}{I_2} = \frac{1}{4}$

$I_2 = 4I_1$

$I_{max} = I_1 + 4I_1 + 2\sqrt{I_1 4I_1} = 9I_1$

$I_{min} = I_1 + 4I_1 - 2\sqrt{I_1 4I_1} = I_1$

$\therefore \frac{9I_1 + I_1}{9I_1 - I_1} = \frac{10}{8} = \frac{5}{4} = \frac{2\alpha + 1}{\beta + 1}$

$\alpha = 2 \quad \beta = 1$

$\therefore \frac{\alpha}{\beta} = \frac{2}{1} = 2$

17. With reference to the observations in photo-electric effect, identify the correct statements from below:

A. The square of maximum velocity of photoelectrons varies linearly with frequency of incident light.

B. The value of saturation current increases on moving the source of light away from the metal surface.

C. The maximum kinetic energy of photo-electrons decreases on decreasing the power of LED (light emitting diode) source of light.

D. The immediate emission of photo-electrons out of metal surface can not be explained by particle nature of light/electromagnetic waves.

E. Existence of threshold wavelength can not be explained by wave nature of light/electromagnetic waves.

Choose the correct answer from the options given below:

(A) A and B only (B) A and E only
(C) C and E only (D) D and E only

Official Ans. by NTA (B)

Sol. $\frac{1}{2}mV_{max}^2 = hf - \phi$

Photoelectric effect can be explained by particle nature of light. Threshold λ is max wavelength at which emission takes place.

18. The activity of a radioactive material is 6.4×10^{-4} curie. Its half life is 5 days. The activity will become 5×10^{-6} curie after :

(A) 7 days (B) 15 days

(C) 25 days (D) 35 days

Official Ans. by NTA (D)

Sol. $A_0 = 6.4 \times 10^{-4}\,\text{Curie}$

$T_{1/2} = 5\text{ days} = \dfrac{\ln 2}{\lambda}$

$A = A_0 e^{-\lambda t}$

$5 \times 10^{-6} = 6.4 \times 10^{-4} e^{-\lambda t}$

$\dfrac{5}{6.4} \times 10^{-2} = e^{-\lambda t}$

$7.8 \times 10^{-3} = e^{-\lambda t}$

$\log(7.8 \times 10^{-3}) = -\lambda t \ln e$

$\ln(7.8 \times 10^{-3}) = -\dfrac{\lambda n 2}{5} \cdot t$

$\therefore \dfrac{5 \times 4.853}{0.693} = t = 35\text{ days}$

19. For a constant collector-emitter voltage of 8V, the collector current of a transistor reached to the value of 6 mA from 4 mA, whereas base current changed from 20 μA to 25 μA value. If transistor is in active state, small signal current gain (current amplification factor) will be :

(A) 240 (B) 400

(C) 0.0025 (D) 200

Official Ans. by NTA (B)

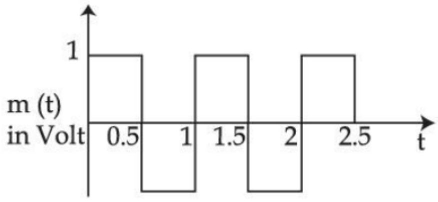
Sol. $V_{CE} = 8\text{ V} = I_C = 6\text{ mA from }4\text{ mA},$

$I_B = 20\ \mu A \text{ to } 25\ \mu A$

Current gain $\beta_{av} = \dfrac{I_C' - I_C}{I_B' - I_B} = \dfrac{2mA}{5\mu A}$

$\beta_{av} = \dfrac{2}{5} \times 10^3 = \dfrac{2000}{5} = 400$

20. A square wave of the modulating signal is shown in the figure. The carrier wave is given by $C(t) = 5\sin(8\pi t)$ Volt. The modulation index is :



(A) 0.2 (B) 0.1

(C) 0.3 (D) 0.4

Official Ans. by NTA (A)

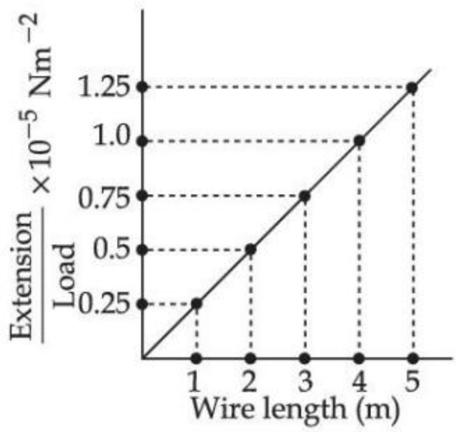
Sol. Modulation Index $\mu = \dfrac{A_m}{A_C} = \dfrac{1}{5} = 0.2$

A_m = amp. of modulating signal

A_C = amp. of carrier wave

SECTION-B

1. In an experiment to determine the Young's modulus, steel wires of five different lengths (1, 2, 3, 4 and 5 m) but of same cross section (2 mm^2) were taken and curves between extension and load were obtained. The slope (extension/load) of the curves were plotted with the wire length and the following graph is obtained. If the Young's modulus of given steel wires is $x \times 10^{11}\text{ Nm}^{-2}$, then the value of x is ________ .



Official Ans. by NTA (2)

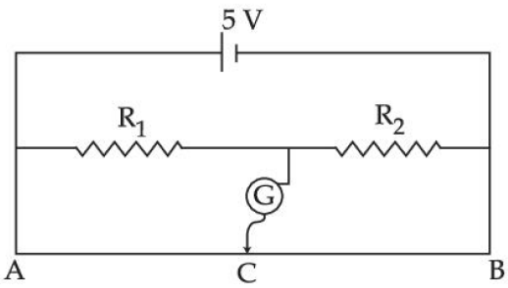
Sol. $\text{Slope} = \dfrac{\Delta l / w}{L} = \dfrac{\Delta l / L}{w} = \dfrac{1}{YA}$

$\Rightarrow Y = \dfrac{1}{(\text{slope})A}$

$Y = \dfrac{1}{2 \times 10^{-6}(0.25 \times 10^{-5})}$

$Y = 2 \times 10^{11} \text{N/m}^2$

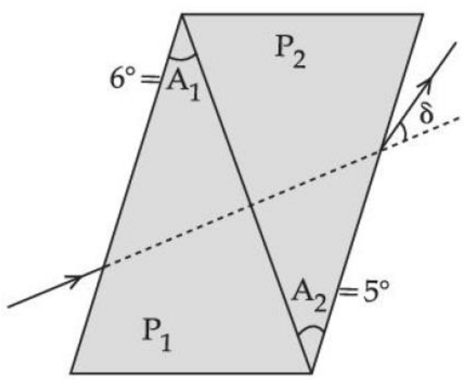
2. In the given figure of meter bridge experiment, the balancing length AC corresponding to null deflection of the galvanometer is 40 cm. The balancing length, if the radius of the wire AB is doubled, will be……….cm.



Official Ans. by NTA (40)

Sol. Independent of area in case of uniform wire.

3. A thin prism of angle 6° and refractive index for yellow light (n_Y)1.5 is combined with another prism of angle 5° and n_Y = 1.55. The combination produces no dispersion. The net average deviation (δ) produced by the combination is $\left(\frac{1}{x}\right)^{\circ}$. The value of x is…….



Official Ans. by NTA (4)

Sol. $\delta = A(\mu_y - 1) - A'(\mu_y' - 1)$

$= 6(1.5 - 1) - 5(1.55 - 1)$

$= \dfrac{1}{4}$

4. A conducting circular loop is placed in X – Y plane in presence of magnetic field $\vec{B} = \left(3t^3\hat{j} + 3t^2\hat{k}\right)$ in SI unit. If the radius of the loop is 1m, the induced emf in the loop, at time, t = 2s is $n\pi$V. The value of n is……..

Official Ans. by NTA (12)

Sol. $\phi = \vec{B} \cdot \vec{A}$

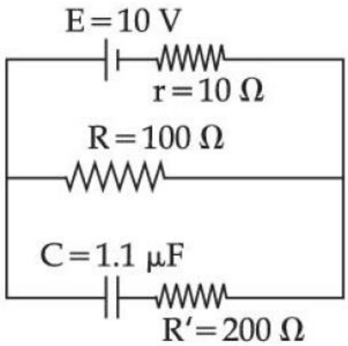
$= (3t^3\hat{j} + 3t^2\hat{k}) \cdot (\pi(1)^2\hat{k})$

$\phi = 3t^2\pi$

$\varepsilon_{IND} = \left|\dfrac{d\phi}{dt}\right| = 6t\pi$

at $t = 2,\ \varepsilon_{IND} = 12$

5. As show in the figure, in steady state, the charge stored in the capacitor is……. × 10^{-6}C.



Official Ans. by NTA (10)

Sol. $q = C\,V_{100\Omega}$

$= (1.1 \times 10^{-6})\left(\dfrac{10}{R + r}R\right)$

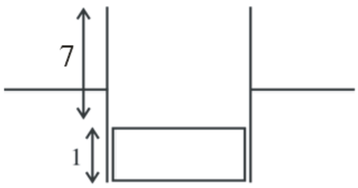
$= 1.1 \times 10^{-6}\left(\dfrac{10}{110} \times 100\right)$

$= 10\ \mu C$

6. A parallel plate capacitor with width 4 cm, length 8 cm and separation between the plates of 4mm is connected to a battery of 20 V. A dielectric slab of dielectric constant 5 having length 1cm, width 4 cm and thickness 4 mm is inserted between the plates of parallel plate capacitor. The electrostatic energy of this system will be……… ϵ_0 J. (Where ϵ_0 is the permittivity of free space)

Official Ans. by NTA (240)

Sol.



$$C_{eff} = \left[\frac{\varepsilon_0(7\times4)}{4/10} + \frac{5\varepsilon_0(1\times4)}{4/10}\right]\times10^{-2}$$

$$C_{eff} = 1.2\varepsilon_0$$

$$\text{Energy} = \frac{1}{2}C_{eff}V^2$$

$$= \frac{1}{2}(1.2)\varepsilon_0(20)(20) = 240\varepsilon_0$$

7. A wire of length 30 cm, stretched between rigid supports, has it's n^{th} and $(n + 1)^{th}$ harmonics at 400 Hz and 450 Hz, respectively. If tension in the string is 2700 N, it's linear mass density is……..kg/m.

Official Ans. by NTA (3)

Sol. $\frac{nv}{0.6} = 400$ & $\frac{(n+1)v}{0.6} = 450$

$$\Rightarrow \left[\frac{0.6\times400}{v} + 1\right]\frac{v}{0.6} = 450$$

$$\Rightarrow = v = 30$$

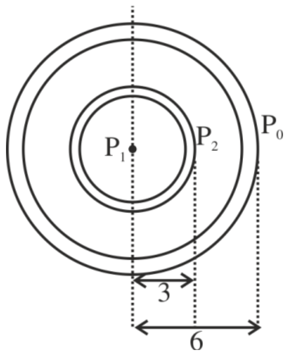
$$\Rightarrow \sqrt{\frac{T}{\mu}} = 30$$

$$\Rightarrow \frac{2700}{\mu} = 900 = \mu = 3$$

8. A spherical soap bubble of radius 3 cm is formed inside another spherical soap bubble of radius 6 cm. If the internal pressure of the smaller bubble of radius 3 cm in the above system is equal to the internal pressure of the another single soap bubble of radius r cm. The value of r is…….

Official Ans. by NTA (2)

Sol.

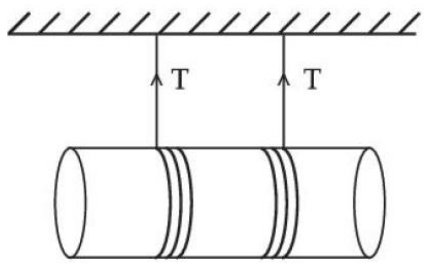


$$P_2 - P_0 = \frac{4T}{6} \;\&\; P_1 - P_2 = \frac{4T}{3}$$

$$\Rightarrow P_1 - P_0 = \frac{4T}{2} \quad = 2$$

9. A solid cylinder length is suspended symmetrically through two massless strings, as shown in the figure. The distance from the initial rest position, the cylinder should by unbinding the strings to achieve a speed of 4 ms^{-1}, is…….cm.

(take g = 10 ms^{-2})



Official Ans. by NTA (120)

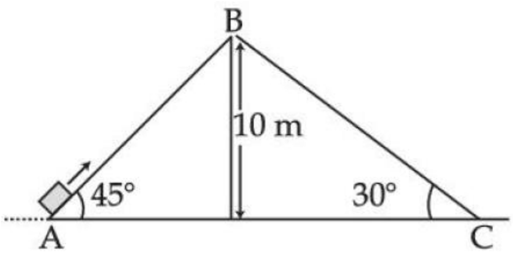
Sol. From energy conservation

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\frac{mR^2}{2}\omega^2$$

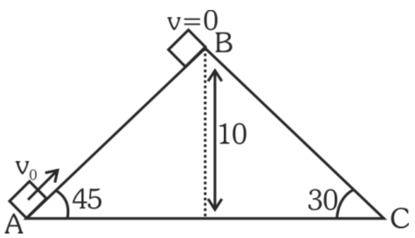
$$10h = \frac{16}{2} + \frac{16}{4} \Rightarrow h = 1.2m = 120cm$$

10. Two inclined planes are placed as shown in figure. A block is projected from the Point A of inclined plane AB along its surface with a velocity just sufficient to carry it to the top Point B at a height 10 m. After reaching the Point B the block slides down on inclined plane BC. Time it takes to reach to the point C from point A is $t(\sqrt{2}+1)$s. The value of t is……..(use g = 10 m/s^2)



Official Ans. by NTA (2)

Sol.



From E.C. = $\frac{1}{2}mv_0^2 = mgh$

$$v_0 = 10\sqrt{2}$$

For A $\rightarrow$ B

at B, $v = 0$

$$a = -g\sin 45° = \frac{-10}{\sqrt{2}}$$

$$v = u + at_1 \Rightarrow 0 = 10\sqrt{2} - \frac{10}{\sqrt{2}}t_1 \Rightarrow t_1 = 2\sec$$

For B $\rightarrow$ C

$$s = ut_2 + \frac{1}{2}at_2^2$$

$$\frac{10}{\sin 30°} = \frac{1}{2}(10\sin 30°)t_2^2$$

$$t_2 = 2\sqrt{2}$$

So total time

$$T = t_1 + t_2$$

$$= 2\sqrt{2} + 2$$

$$= 2(\sqrt{2}+1)\sec$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. The correct decreasing order of energy, for the orbitals having, following set of quantum numbers:

(A) n = 3, l = 0, m = 0

(B) n = 4, l = 0, m = 0

(C) n = 3, l = 1, m = 0

(D) n = 3, l = 2, m = 1

(A) (D) > (B) > (C) > (A)

(B) (B) > (D) > (C) > (A)

(C) (C) > (B) > (D) > (A)

(D) (B) > (C) > (D) > (A)

Official Ans. by NTA (A)

Sol. (A) $n + \ell = 3 + 0 = 3$

(B) $n + \ell = 4 + 0 = 4$

(C) $n + \ell = 3 + 1 = 4$

(D) $n + \ell = 3 + 2 = 5$

Higher $n + \ell$ value, higher the energy & if same $n + \ell$ value, then higher n value, higher the energy.

Thus : D > B > C > A.

2. Match List-I with List-II

List-I	List-II
(A) $\Psi_{MO} = \Psi_A - \Psi_B$	(I) Dipole moment
(B) $\mu = Q \times r$	(II) Bonding molecular orbital
(C) $\frac{N_b - N_a}{2}$	(III) Anti-bonding molecualr orbital
(D) $\Psi_{MO} = \Psi_A + \Psi_B$	(IV) Bond order

(A) (A)-(II), (B)-(I), (C)-(IV), (D)-(III)

(B) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

Official Ans. by NTA (C)

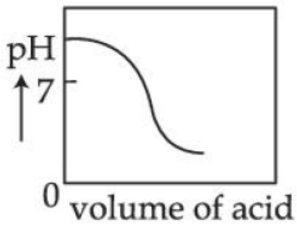
Sol. (A) $\psi_{MO} = \psi_A - \psi_B$ (III) ABMO

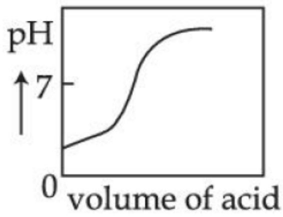
(B) $\mu = Q \times r$ (I) Dipole moment

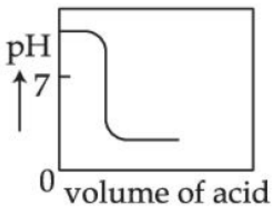
(C) $\frac{N_b - N_a}{2}$ (IV) Bond order

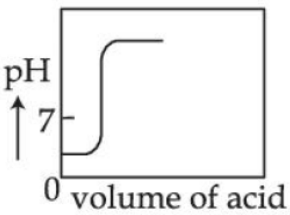
(D) $\psi_{MO} = \psi_A + \psi_B$ (II) BMO

3. The Plot of pH-metric titration of weak base NH_4OH vs strong acid HCl looks like:

(A)


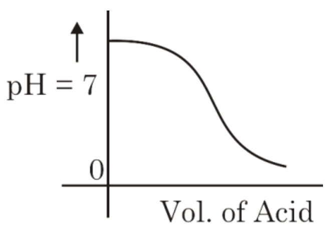
(B)


(C)


(D)


Official Ans. by NTA (A)

Sol. Titration curve of NH_4OH vs HCl (WB + SA).



4. Given below are two statements:

Statement I: For KI, molar conductivity increases steeply with dilution.

Statement II: For carbonic acid, molar conductivity increases slowly with dilution.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both **Statement I** and **Statement II** are true

(B) Both **Statement I** and **Statement II** are false

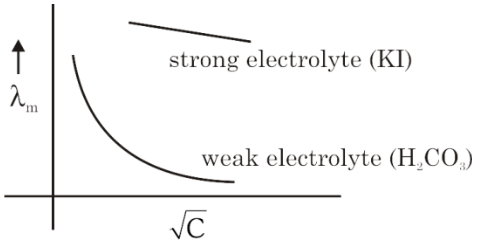
(C) **Statement I** is true but **Statement II** is false

(D) **Statement I** is false but **Statement II** is true

Official Ans. by NTA (B)

Sol. **Statement I:** KI is strong electrolyte thus almost constant on dilution.

Statement II: In weak electrolyte it increases, sharply.



5. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Dissolved substances can be removed from a colloidal solution by diffusion through a parchment paper.

Reason (R) : Particles in a true solution cannot pass through parchment paper but the collodial particles can pass through the parchment paper.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both **(A)** and **(R)** are correct and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are correct but **(R)** is not the correct explanation of **(A)**

(C) **(A)** is correct but **(R)** is not correct

(D) **(A)** is not correct but **(R)** is correct

Official Ans. by NTA (C)

Sol. **Assertion (A):** Correct.

Reason(R): Incorrect.

Particles of true solution pass through parchment paper thus answer is (C).

6. Outermost electronic configurations of four elements A, B, C, D are given below:

(A) $3s^2$

(B) $3s^2 3p^1$

(C) $3s^2 3p^3$

(D) $3s^2 3p^4$

The **correct** order of first ionization enthalpy for them is:

(A) (A) < (B) < (C) < (D)

(B) (B) < (A) < (D) < (C)

(C) (B) < (D) < (A) < (C)

(D) (B) < (A) < (C) < (D)

Official Ans. by NTA (B)

Sol. (A) $3s^2 \rightarrow Mg$

(B) $3s^2 3p^1 \rightarrow Al$

(C) $3s^2 3p^3 \rightarrow P$

(D) $3s^2 3p^4 \rightarrow S$

$\underbrace{P > S}_{\text{Half filled stability}} > \underbrace{Mg > Al}_{\text{Penetrating power of s>p.}}$

C > D > A > B.

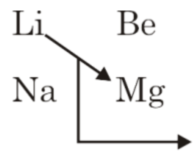
7. An element A of group 1 shows similarity to an element B belonging to group 2. If A has maximum hydration enthalpy in group 1 then B is:

(A) Mg (B) Be

(C) Ca (D) Sr

Official Ans. by NTA (A)

Sol.



Diagonal relationship

$Li^+ \rightarrow$ Maximum hydration enthalpy in group 1 due to small size.

So 'B' is Mg.

8. Given below are two statements: one is labelled as **Assertion (A)** and the other is labelled as **Reason (R)**

Assertion (A) : Boron is unable to form BF_6^{3-}

Reason (R) : Size of B is very small.

In the light of the above statements, choose the **correct** answer from the options given below:

(A) Both **(A)** and **(R)** are true and **(R)** is the correct explanation of **(A)**

(B) Both **(A)** and **(R)** are true but **(R)** is **not** the correct explanation of **(A)**

(C) **(A)** is true but **(R)** is false

(D) **(A)** is false but **(R)** is true

Official Ans. by NTA (B)

Sol. **Assertion (A):** True

Reason (R): True but not correct explanation.

Correct explanation: Expansion of octet not possible for 'B'.

9. In neutral or alkaline solution, MnO_4^- oxidises thiosulphate to:

(A) $S_2O_7^{2-}$ (B) $S_2O_8^{2-}$

(C) SO_3^{2-} (D) SO_4^{2-}

Official Ans. by NTA (D)

Sol. $8MnO_4^- + 3S_2O_3^{2-} + H_2O \xrightarrow[\text{alk. solution}]{\text{neutral or}} 8MnO_2 + 6SO_4^{2-} + 2OH^-$

10. Low oxidation state of metals in their complexes are common when ligands:

(A) have good π -accepting character

(B) have good σ -donor character

(C) are havind good π -donating ability

(D) are havind poor σ -donating ability

Official Ans. by NTA (A)

Sol. When metal is in low oxidation state then it forms complexes when ligands have good π -accepting character.

11. Given below are two statements:

Statement I : The non bio-degradable fly ash and slag from steel industry can be used by cement industry.

Statement II : The fuel obtained from plastic waste is lead free.

In the light of the above statements, choose the **most appropriate** answer from the options given below:

(A) Both **Statement I** and **Statement II** are correct

(B) Both **Statement I** and **Statement II** are incorrect

(C) **Statement I** is correct but **Statement II** is incorrect

(D) **Statement I** is incorrect but **Statement II** is correct

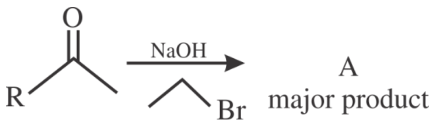
Official Ans. by NTA (A)

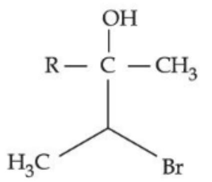
Sol. (I) Fly ash and slag from steel industry are utilised by cement industry.

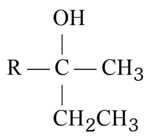
(II) Fuel obtained from plastic waste has high octane rating. It contains no lead and it is known as green fuel.

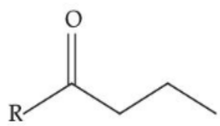
Both statement (I) & (II) are correct.

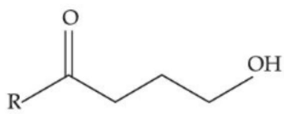
12. The structure of A in the given reaction is:



(A)


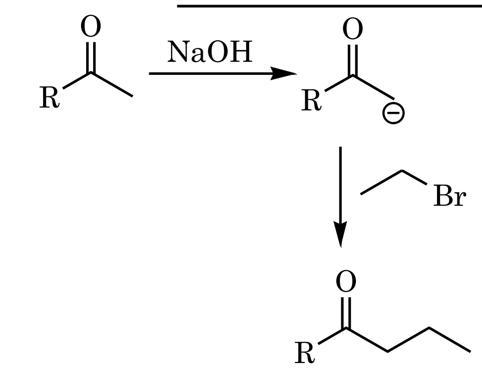
(B)


(C)


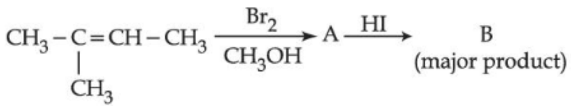
(D)


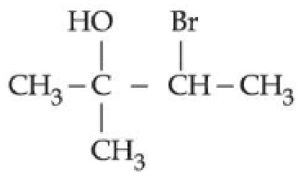
Official Ans. by NTA (C)

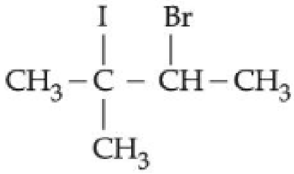
Sol.

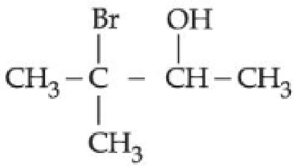


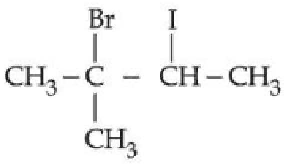
13. Major product 'B' of the following reaction sequence is:



(A) 

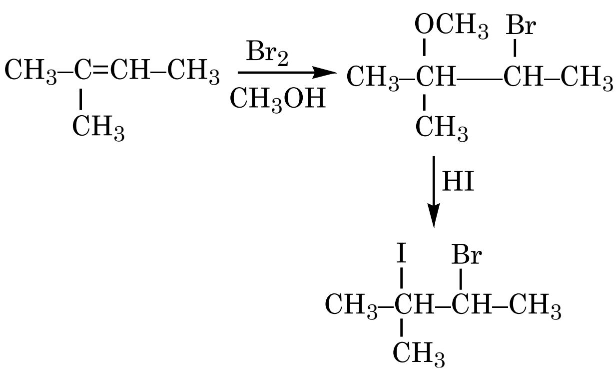
(B) 

(C) 

(D) 

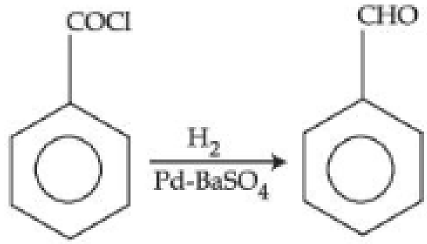
Official Ans. by NTA (B)

Sol

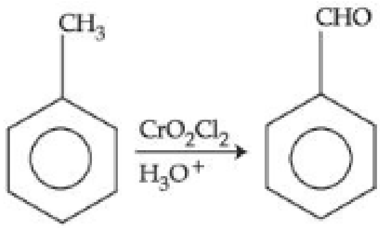


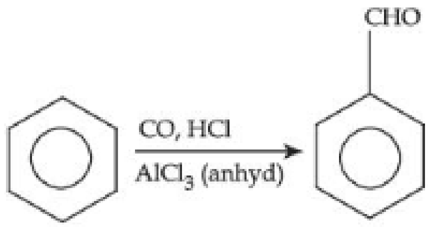
14. Match List-I with List-II.

List-I

(A) 

(B) $CH_3-CN \xrightarrow[H_3O^+]{SnCl_2/HCl} CH_3-CHO$

(C) 

(D) 

Lits-II

(I) Gatterman Koch reaction

(II) Etard reaction

(III) Stephen reaction

(IV) Rosenmund reaction

Choose the **correct** answer from the options given below:

(A) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

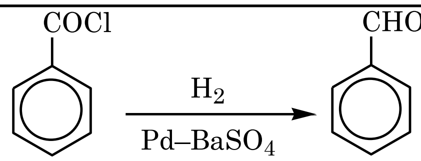
(B) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

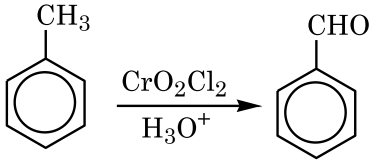
Official Ans. by NTA (A)

Sol.

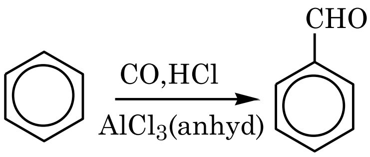
(A) 

(Rosenmund reaction)

(B) $CH_3-C\equiv N \xrightarrow[H_3O^+]{SnCl_2-HCl} CH_3-CHO$
(Stephen reaction)

(C) 

(Etard reaction)

(D) 

(Gattermann Koch reaction)

15. Match List-I with List-II.

List-I	**List-II**
(Polymer)	**(Monomer)**
(A) Neoprene	(I) Acrylonitrile
(B) Teflon	(II) Chloroprene
(C) Acrilan	(III) Tetrafluoroethene
(D) Natural rubber	(IV) Isoprene

Choose the correct answer from the option given below:

(A) (A)-(II), (B)-(III), (C)-(I), (D-(IV)

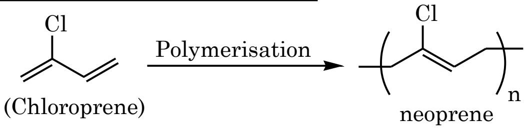
(B) (A)-(II), (B)-(I), (C)-(III), (D-(IV)

(C) (A)-(II), (B)-(I), (C)-(IV), (D-(III)

(D) (A)-(I), (B)-(II), (C)-(III), (D-(IV)

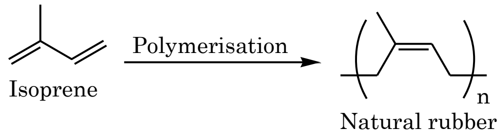
Official Ans. by NTA (A)

Sol.

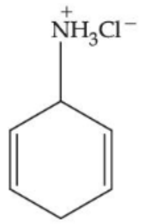
(A) 

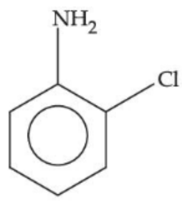
(B) $F_2C{=}CF_2 \xrightarrow{\text{Polymerisation}} {-\!\!(CF_2-CF_2)\!\!-}_n$
Tetrafluoro ethene — Teflon

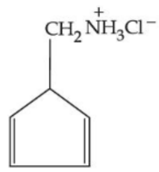
(C) $H_2C{=}CH(CN) \xrightarrow{\text{Polymerisation}} {-\!\!(CH_2-CH(CN))\!\!-}_n$
Acrylonitrile — Acrilan

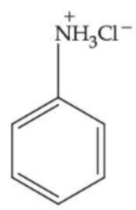
(D) 

16. An organic compound 'A' contains nitrogen and chlorine. It dissolves readily in water to give a solution that turns litmus red. Titration of compound 'A' with standard base indicates that the molecular weight of 'A' is 131 ± 2. When a sample of 'A' is treated with aq. NaOH, a liquid separates which contains N but not Cl. Treatment of the obtained liquid with nitrous acid followed by phenol gives orange precipitate. The compound 'A' is :

(A) 

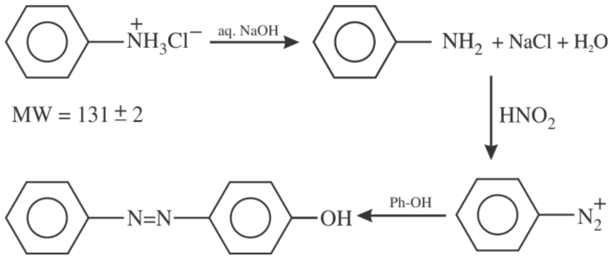
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol.



p-hydroxy azobenzene
(Orange dye)

17. Match List-I with List-II

List-I

(A) Glucose + HI

(B) Glucose + Br_2 water

(C) Glucose + acetic anhydride

(D) Glucose + HNO_3

List-II

(I) Gluconic acid

(II) Glucose pentacetate

(III) Saccharic acid

(IV) Hexane

Choose the correct answer from the options given below:

(A) (A)-(IV), (B)-(I), (C)-(II), (D)-(III)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(D) (A)-(I), (B)-(III), (C)-(IV), (D)-(II)

Official Ans. by NTA (A)

Sol.

(A) $\text{Glucose} \xrightarrow{HI} \text{n-hexane}$

(B) $\text{Glucose} \xrightarrow[H_2O]{Br_2} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ CH_2OH \end{array}$
Gluconic acid

(C) $\text{Glucose} \xrightarrow[\text{anhydride}]{\text{5 acetic}} \text{Glucose pentacetate}$

(D) $\text{Glucose} \xrightarrow{HNO_3} \begin{array}{c} COOH \\ | \\ (CHOH)_4 \\ | \\ COOH \end{array}$
Saccharic acid

18. Which of the following enhances the lathering property of soap?

(A) Sodium stearate

(B) Sodium carbonate

(C) Sodium rosinate

(D) Trisodium phosphate

Official Ans. by NTA (C)

Sol. Rosin is added to soaps which forms sodium rosinate which lathers well.

19. Match List-I with List-II

List-I (Mixture)

(A) Chloroform & Aniline

(B) Benzoic acid & Napthalene

(C) Water & Aniline

(D) Napthalene & Sodium chloride

List-II (Purification Process)

(I) Steam distillation

(II) Sublimation

(III) Distillation

(IV) Crystallisation

(A) (A)-(IV), (B)-(III), (C)-(I), (D)-(II)

(B) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(C) (A)-(III), (B)-(IV), (C)-(II), (D)-(I)

(D) (A)-(III), (B)-(IV), (C)-(I), (D)-(II)

Official Ans. by NTA (D)

Sol. (A) Chloroform + Aniline → (III) Distillation

(B) Benzoic acid + Napthalene → (IV) Crystallisation

(C) Water + Aniline → (I) Steam distillation

(D) Napthalene + Sodium chloride →(II) Sublimation

20. Fe^{3+} cation gives a prussian blue precipitate on addition of potassium ferrocyanide solution due to the formation of:

(A) $[Fe(H_2O)_6]_2\,[Fe(CN)_6]$

(B) $Fe_2[Fe(CN)_6]_2$

(C) $Fe_3[Fe(OH)_2(CN)_4]_2$

(D) $Fe_4[Fe(CN)_6]_3$

Official Ans. by NTA (D)

Sol. $4Fe^{3+} + 3[Fe(CN)_6]^{-4} \longrightarrow \underset{\text{Prussian Blue}}{Fe_4[Fe(CN)_6]_3}$

SECTION-B

1. The normality of H_2SO_4 in the solution obtained on mixing 100 mL of 0.1 M H_2SO_4 with 50 mL of 0.1 M NaOH is_______$\times 10^{-1}$ N. (Nearest Integer)

Official Ans. by NTA (1)

Sol. No. of equivalents of $H_2SO_4 = 100 \times 0.1 \times 2 = 20$

No. of equivalents of NaOH = $50 \times 0.1 = 5$

No. of equivalents of H_2SO_4 left = $20 - 5 = 15$

$\Rightarrow\ 150 \times x = 15$

$$x = \frac{1}{10} = 0.1\text{N} = 1 \times 10^{-1}\ \text{N}$$

2. for a real gas at 25°C temperature and high pressure (99 bar) the value of compressibility factor is 2, so the value of Vander Waal's constant 'b' should be_________$\times 10^{-2}$ L mol^{-1} (Nearest integer) (Given R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by N

Sol. For real gas under high pressure

$$Z = 1 + \frac{Pb}{RT} \qquad \Rightarrow b = \frac{RT}{P}$$

$$= \frac{0.083 \times 298}{99}$$

$$= 0.25 \times 10^{-2}\ \text{L mol}^{-1}$$

3. A gas (Molar mass = 280 g mol^{-1}) was burnt in excess O_2 in a constant volume calorimeter and during combustion the temperature of calorimeter increased from 298.0 K to 298.45 K. If the heat capacity of calorimeter is 2.5 kJ K^{-1} and enthalpy of combustion of gas is 9 kJ mol^{-1} then amount of gas burnt is_______g. (Nearest Integer)

Official Ans. by NTA (35)

Sol. Let x g is burnt

$$\text{moles} = \frac{x}{280}$$

$$\text{heat released by } \frac{x}{280} \text{ mole} = 2.5 \times 0.45 \text{ kJ}$$

$$\text{heat released by 1 mole} = \frac{2.5 \times 0.45 \times 280}{x} \text{kJ}$$

$$\Delta H = \Delta U + \Delta ngRT$$

$$\Delta H \simeq \Delta U$$

$$9 = \frac{2.5 \times 280 \times 0.45}{x}$$

x = **35 g**

4. When a certain amount of solid A is dissolved in 100 g of water at 25°C to make a dilute solution, the vapour pressure of the solution is reduced to one-half of that of pure water. The vapour pressure of pure water is 23.76 mmHg. The number of moles of solute A added is________. (Nearest Integer)

Official Ans. by NTA (3)

Sol. $\because$ Diliute solution given:

$$\frac{P^0 - P_S}{P^0} \sim \frac{{}^{n}\text{solute}}{{}^{n}\text{solvent}}$$

$$\frac{P^0 - P^0/2}{P^0} = \frac{{}^{n}\text{solute}}{{}^{n}\text{solvent}}$$

$$^{n}\text{solute} \sim \frac{{}^{n}\text{solvent}}{2} = \frac{100}{18 \times 2} = 2.78\,\text{mol}$$

More accurate approach:

$$\frac{P^0 - P_S}{P_S} = \frac{{}^{n}\text{solute}}{{}^{n}\text{solvent}}$$

$$\frac{P^0 - P^0/2}{P^0/2} = \frac{{}^{n}\text{solute}}{{}^{n}\text{solvent}}$$

$$^{n}\text{solute} = {}^{n}\text{solvent} = \frac{100}{18} = 5.55\,\text{mol}$$

5. [A] $\rightarrow$ [B]

Reactant Product

If formation of compound [B] follows the first order of kinetics and after 70 minutes the concentration of [A] was found to be half of its initial concentration. Then the rate constant of the reaction is x × 10^{-6} s^{-1}. The value of x is______. (Nearest Integer)

Official Ans. by NTA (165)

Sol. $K = \dfrac{0.693}{t_{1/2}} = \dfrac{0.693}{70 \times 60}$

$= \dfrac{6930}{7 \times 6} \times 10^{-6}$

$= 165 \times 10^{-6}\ s^{-1}$

6. Among the following ores Bauxite, Siderite, Cuprite, Calamine, Haematite, Kaolinite, Malachite, Magnetite, Sphalerite, Limonite, Cryolite, the number of principal ores if (of) iron is_______.

Official Ans. by NTA (4)

Sol.

Bauxite	—	$AlO_X(OH)_{3-2x}$(where $0 < x < 1$)
✓Siderite	—	$FeCO_3$
Cuprite	—	Cu_2O
Calamine	—	$ZnCO_3$
✓Haematite	—	Fe_2O_3
Kaolinite	—	$Al_2(OH)_4Si_2O_5$
Malachite	—	$CuCO_3 . Cu(OH)_2$
✓Magnetite	—	Fe_3O_4
Sphalerite	—	ZnS
✓Limonite	—	$Fe_2O_3.3H_2O$
Cryolite	—	Na_3AlF_6

7. The oxidation state of manganese in the product obtained in a reaction of potassium permanganate and hydrogen peroxide in basic medium is______.

Official Ans. by NTA (4)

Sol. $2KMnO_4 + 3H_2O_2 \xrightarrow{\text{basic medium}} 2\overset{+4}{Mn}O_2 + 3O_2 + 2H_2O + 2KOH$

8. The number of molecule(s) or ion(s) from the following having non-planar structure is______.

$NO_3^-, H_2O_2, BF_3, PCl_3, XeF_4,$
$SF_4, XeO_3, PH_4^+, SO_3, [Al(OH)_4]^-$

Official Ans. by NTA (6)

Sol.

SO_3	—	sp^2	Planar
BF_3	—	sp^2	Planar
NO_3^-	—	sp^2	Planar
SF_4	—	sp^3d	Non-planar
H_2O_2	—	sp^3	Non-planar
PCl_3	—	sp^3	Non-planar
$[Al(OH)_4]^-$	—	sp^3	Non-planar
XeF_4	—	sp^3d^2	Planar
XeO_3	—	sp^3	Non-planar
PH_4^+	—	sp^3	Non-planar

9. The spin only magnetic moment of the complex present in Fehling's reagent is______ B.M. (Nearest integer).

Official Ans. by NTA (2)

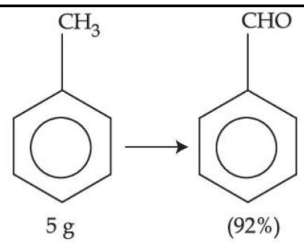
Sol. Fehling solution is a complex of Cu^{++}

$Cu^{++} = 3d^9$

No. of unpaired $e^- = 1$

$M.M = \sqrt{1(1+2)} = \sqrt{3} = 1.73$ BM

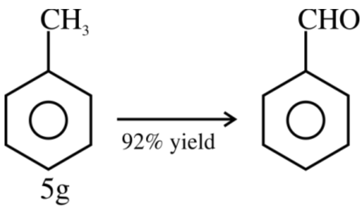
10.



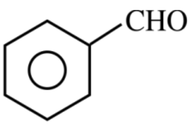
In the above reaction, 5 g of toluene is converted into benzaldehyde with 92% yield. The amount of benzaldehyde produced is______$\times 10^{-2}$ g. (Nearest integer)

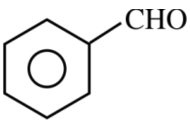
Official Ans. by NTA (530)

Sol.



moles $= \frac{5}{92}$

moles of  $= \frac{5}{92} \times \frac{92}{100} = 5 \times 10^{-2}$

mass of  $= 106 \times 5 \times 10^{-2} = 5.3$ g

$= 530 \times 10^{-2}$ g

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Wednesday 27th July, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS	TEST PAPER WITH SOLUTION

SECTION-A

1. The domain of the function

$$f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right),$$

where [t] is the greatest integer function, is :

(A) $\left(-\sqrt{\frac{5}{2}},\frac{5-\sqrt{5}}{2}\right)$ (B) $\left(\frac{5-\sqrt{5}}{2},\frac{5+\sqrt{5}}{2}\right)$

(C) $\left(1,\frac{5-\sqrt{5}}{2}\right)$ (D) $\left[1,\frac{5+\sqrt{5}}{2}\right)$

Official Ans. by NTA (C)

Sol. $f(x)=\sin^{-1}[2x^2-3]+\log_2\left(\log_{\frac{1}{2}}\left(x^2-5x+5\right)\right)$

$P_1 : -1\le\left[2x^2-3\right]<1$

$\Rightarrow -1\le 2x^2-3<2$

$\Rightarrow 2<2x^2<5$

$\Rightarrow 1<x^2<\frac{5}{2}$

$\Rightarrow P_1 :\ x\in\left(-\sqrt{\frac{5}{2}},-1\right)\cup\left(1,\sqrt{\frac{5}{2}}\right)$

$P_2 : x^2-5x+5>0$

$\Rightarrow\left(x-\left(\frac{5-\sqrt{5}}{2}\right)\right)\left(x-\left(\frac{5+\sqrt{5}}{2}\right)\right)>0$

$P_3 :\ \log_{\frac{1}{2}}\left(x^2-5x+5\right)>0$

$\Rightarrow x^2-5x-5<1$

$\Rightarrow x^2-5x+4<0$

$\Rightarrow P_3 :\ x\in(1,4)$

So, $P_1\cap P_2\cap P_3=\left(1,\frac{5-\sqrt{5}}{2}\right)$

2. Let S be the set of all (α,β), $\pi<\alpha,\beta<2\pi$, for which the complex number $\frac{1-i\sin\alpha}{1+2i\sin\alpha}$ is purely imaginary and $\frac{1+i\cos\beta}{1-2i\cos\beta}$ is purely real. Let $Z_{\alpha\beta}=\sin 2\alpha+i\cos 2\beta,(\alpha,\beta)\in S$.

Then $\sum_{(\alpha,\beta)\in S}\left(iZ_{\alpha\beta}+\frac{1}{i\bar{Z}_{\alpha\beta}}\right)$ is equal to :

(A) 3 (B) 3 i

(C) 1 (D) 2 – i

Official Ans. by NTA (C)

Sol. $\pi<\alpha,\beta<2\pi$

$\frac{1-i\sin\alpha}{1+i(2\sin\alpha)}$ = Purely imaginary

$\Rightarrow\frac{(1-i\sin\alpha)\,(1-i(2\sin\alpha))}{1+4\sin^2\alpha}$ = Purely imaginary

$\Rightarrow\frac{1-2\sin^2\alpha}{1+4\sin^2\alpha}=0$

$\Rightarrow\sin^2\alpha=\frac{1}{2}$

$\Rightarrow\alpha=\left\{\frac{5\pi}{4},\frac{7\pi}{4}\right\}$

& $\frac{1+i\cos\beta}{1+i(-2\cos\beta)}$ = Purely real

$\Rightarrow\frac{(1+i\cos\beta)\,(1+2i\cos\beta)}{1+4\cos^2\beta}$ = Purely real

$\Rightarrow 3\cos\beta=0$

$\Rightarrow\boxed{\beta=\frac{3\pi}{2}}$

$\Rightarrow Z_{\alpha\beta}=\sin\frac{5\pi}{2}+i\cos 3\pi=1-i$

or

$Z_{\alpha\beta}=\sin\frac{7\pi}{2}+i\cos 3\pi=-1-i$

Required value $=\left[i(1-i)+\frac{1}{i(1+i)}\right]+\left[i(-1-i)+\frac{1}{i(-1+i)}\right]$

$=i(-2i)+\frac{1}{i}\frac{2i}{(-2)}\Rightarrow 2-1=1$

3. If α, β are the roots of the equation

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log_5 3)^{\frac{2}{3}}}-1\right)=0$$

then the equation, whose roots are

$\alpha+\frac{1}{\beta}$ and $\beta+\frac{1}{\alpha}$,

(A) $3x^2 - 20x - 12 = 0$

(B) $3x^2 - 10x - 4 = 0$

(C) $3x^2 - 10x + 2 = 0$

(D) $3x^2 - 20x + 16 = 0$

Official Ans. by NTA (B)

Sol. Bonus because 'x' is missing the correct will be,

$$x^2-\left(5+3^{\sqrt{\log_3 5}}-5^{\sqrt{\log_5 3}}\right)x+3\left(3^{(\log_3 5)^{\frac{1}{3}}}-5^{(\log_5 3)^{\frac{2}{3}}}-1\right)=0$$

$$3^{\sqrt{\log_3 5}}=3^{\sqrt{\log_3 5}.\sqrt{\log_3 5}.\sqrt{\log_5 3}}=3^{\log_3 5.\sqrt{\log_5 3}}$$

$$=\left(3^{\log_3 5}\right)^{\sqrt{\log_5 3}}=5^{\sqrt{\log_5 3}}$$

$$3^{\sqrt[3]{\log_3 5}}=3^{\log_3 5.\sqrt[3]{(\log_5 3)^2}}=\left(3^{\log_3 5}\right)^{(\log_5 3)^{2/3}}$$

$$=5^{(\log_5 3)^{2/3}}$$

So, equation is $x^2 - 5x - 3 = 0$ and roots are α & β

$\{\alpha + \beta = 5;\ \alpha\beta = -3\}$

New roots are $\alpha+\frac{1}{\beta}$ & $\beta+\frac{1}{\alpha}$

i.e., $\frac{\alpha\beta+1}{\beta}$ & $\frac{\alpha\beta+1}{\alpha}$ i.e., $\frac{-2}{\beta}$ & $\frac{-2}{\alpha}$

Let $\frac{-2}{\alpha}=t \Rightarrow \alpha=\frac{-2}{t}$

As $\alpha^2 - 5\alpha - 3 = 0$

$$\Rightarrow \left(\frac{-2}{t}\right)^2-5\left(\frac{-2}{t}\right)-3=0$$

$$\Rightarrow \frac{4}{t^2}+\frac{10}{t}-3=0$$

$\Rightarrow 4 + 10t - 3t^2 = 0$

$\Rightarrow 3t^2 - 10t - 4 = 0$

i.e., $3x^2 - 10x - 4 = 0$

4. Let $A=\begin{pmatrix}4 & -2\\ \alpha & \beta\end{pmatrix}$

If $A^2 + \gamma A + 18I = O$, then det (A) is equal to ____________.

(A) –18 (B) 18 (C) –50 (D) 50

Official Ans. by NTA (B)

Sol. The characteristic equation for A is $|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix}4-\lambda & -2\\ \alpha & \beta-\lambda\end{vmatrix}=0$$

$\Rightarrow (4-\lambda)(\beta-\lambda)+2\alpha=0$

$\Rightarrow \lambda^2-(\beta+4)\lambda+4\beta+2\alpha=0$

Put λ = A

$A^2-(\beta+4)A+(4\beta+2\alpha)\,I=0$

On comparison

$-9(\beta+4)=\gamma$ & $4\beta+2\alpha=18$

and $|A| = 4\beta + 2\alpha = 18$

5. If for $p\neq q\neq 0$, then function

$$f(x)=\frac{\sqrt[7]{p(729+x)}-3}{\sqrt[3]{729+qx}-9}$$ is continuous at x = 0, then:

(A) 7pq f(0) – 1 = 0 (B) 63q f(0) – p^2 = 0

(C) 21q f(0) – p^2 = 0 (D) 7pq f(0) – 9 = 0

Official Ans. by NTA (B)

Sol. $f(0)=\lim\limits_{x\to 0} f(x)$

Limit should be $\frac{0}{0}$ form

So, $\sqrt[7]{p.729}-3=0 \Rightarrow p.3^6=3^7 \Rightarrow p=3$

$$\text{Now, } f(0)=\lim_{x\to 0}\frac{\sqrt[7]{3(3^6+x)}-3}{\sqrt[3]{3^6+qx}-9}$$

$$=\lim_{x\to 0}\frac{3\left[\left(1+\frac{x}{3^6}\right)^{1/7}-1\right]}{9\left[\left(1+\frac{qx}{3^6}\right)^{1/3}-1\right]}=\frac{3}{9}\times\frac{\frac{1}{7.3^6}}{\frac{q}{3.3^6}}$$

$$\Rightarrow f(0)=\frac{1}{3}\times\frac{3}{7q}=\frac{1}{7q}$$

$\Rightarrow 7qf(0) - 1 = 0$

$\Rightarrow 7.p^2.qf(0) - p^2 = 0$ (for option)

$\Rightarrow 63qf(0) - p^2 = 0$

6. Let $f(x) = 2 + |x| - |x - 1| + |x + 1|$, $x \in \mathbf{R}$. Consider

$$(S1): f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=2$$

$$(S2): \int_{-2}^{2} f(x)dx = 12$$

Then,

(A) both (S1) and (S2) are correct

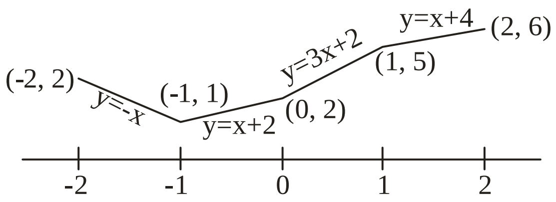
(B) both (S1) and (S2) are wrong

(C) only (S1) is correct

(D) only (S2) is correct

Official Ans. by NTA (D)

Sol.



$$(S1): f'\left(-\frac{3}{2}\right)+f'\left(-\frac{1}{2}\right)+f'\left(\frac{1}{2}\right)+f'\left(\frac{3}{2}\right)=4$$

$$(S2): \int_{-2}^{2} f(x)dx = 12$$

$\therefore$ (D)

7. Let the sum of an infinite G.P., whose first term is a and the common ratio is r, be 5. Let the sum of its first five terms be $\frac{98}{25}$. Then the sum of the first 21 terms of an AP, whose first term is 10ar, n^{th} term is a_n and the common difference is $10ar^2$, is equal to :

(A) $21\ a_{11}$ (B) $22\ a_{11}$

(C) $15\ a_{16}$ (D) $14\ a_{16}$

Official Ans. by NTA (A)

Sol. $S_{21} = \frac{21}{2}[20\,ar + 20.10\,ar^2]$

$= 21\ [10\ ar + 100\ ar^2]$

$= 21.\ a_{11}$

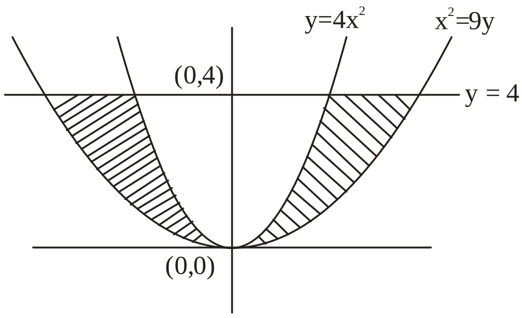
8. The area of the region enclosed by

$y \le 4x^2, x^2 \le 9y$ and $y \le 4$, is equal to :

(A) $\frac{40}{3}$ (B) $\frac{56}{3}$ (C) $\frac{112}{3}$ (D) $\frac{80}{3}$

Official Ans. by NTA (D)

Sol.



$$\Delta = 2\cdot\int_0^4\left(3\sqrt{y}-\frac{\sqrt{y}}{2}\right)dy$$

$$= 2\cdot\int_0^4 \frac{5}{2}\sqrt{y}\,dy = \frac{80}{3}$$

9. $\int_0^2\left(\left|2x^2-3x\right|+\left[x-\frac{1}{2}\right]\right)dx$,

where [t] is the greatest integer function, is equal to:

(A) $\frac{7}{6}$ (B) $\frac{19}{12}$ (C) $\frac{31}{12}$ (D) $\frac{3}{2}$

Official Ans. by NTA (B)

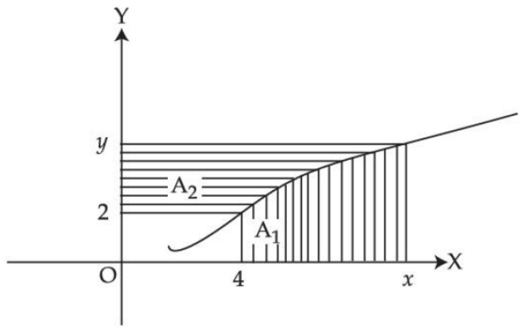
Sol. $\int_0^2\left|2x^2-3x\right|dx$

$$= \int_0^{\frac{3}{2}}(3x-2x^2)dx + \int_{\frac{3}{2}}^{2}(2x^2-3x)dx = \frac{19}{12}.$$

$$\int_0^2\left[x-\frac{1}{2}\right]dx = \int_{-\frac{1}{2}}^{\frac{3}{2}}[t]dt$$

$$= \int_{-\frac{1}{2}}^{0}(-1)dt + \int_0^1 0\cdot dt + \int_1^{\frac{3}{2}} 1\cdot dt = 0.$$

10. Consider a curve $y = y(x)$ in the first quadrant as shown in the figure. Let the area A_1 is twice the area A_2. Then the normal to the curve perpendicular to the line $2x - 12y = 15$ does **NOT** pass through the point.



(1) (6, 21) (2) (8, 9)

(3) (10, –4) (4) (12, –15)

Official Ans. by NTA (C)

Sol. Given that $A_1 = 2A_2$

from the graph $A_1 + A_2 = xy - 8$

$\Rightarrow \frac{3}{2}A_1 = xy - 8$

$\Rightarrow A_1 = \frac{2}{3}xy - \frac{16}{3}$

$\Rightarrow \int_4^x f(x)dx = \frac{2}{3}xy - \frac{16}{3}$

$\Rightarrow f(x) = \frac{2}{3}\left(x\frac{dy}{dx} + y\right)$

$\Rightarrow \frac{2}{3}x\frac{dy}{dx} = \frac{y}{3}$

$\Rightarrow 2\int \frac{dy}{y} = \int \frac{dx}{x}$

$\Rightarrow 2\ell ny = \ell nx + \ell nc$

$\Rightarrow y^2 = cx$

As $f(4) = 2 \Rightarrow c = 1$

so $y^2 = x$

slope of normal = –6

$y = -6(x) - \frac{1}{2}(-6) - \frac{1}{4}(-6)^3$

$\Rightarrow y = -6x + 3 + 54$

$\Rightarrow y + 6x = 57$

Now check options and (C) will not satisfy.

11. The equations of the sides AB, BC and CA of a triangle ABC are $2x + y = 0$, $x + py = 39$ and $x - y = 3$ respectively and P(2, 3) is its circumcentre. Then which of the following is NOT true :

(A) $(AC)^2 = 9p$

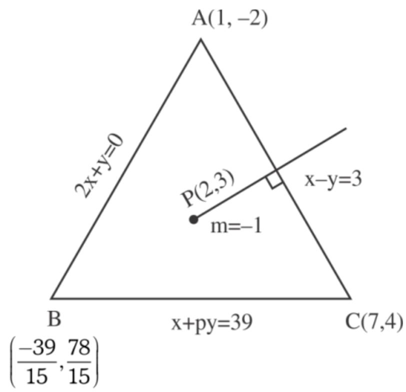
(B) $(AC)^2 + p^2 = 136$

(C) $32 <$ area $(\Delta ABC) < 36$

(D) $34 <$ area $(\Delta ABC) < 38$

Official Ans. by NTA (D)

Sol.



Perpendicular bisector of AB

$x + y = 5$

Take image of A

$\frac{x-1}{1} = \frac{y+2}{1} = \frac{-2(-6)}{2} = 6$

(7, 4)

$7 + 4p = 39$

$p = 8$

solving $x + 8y = 39$ and $y = -2x$

$x = \frac{-39}{15}$ $y = \frac{78}{15}$

$AC^2 = 72 = 9p$

$AC^2 + p^2 = 72 + 64 = 136$

$$\Delta ABC = \frac{1}{2}\begin{vmatrix} 1 & -2 & 1 \\ 7 & 4 & 1 \\ \frac{-39}{15} & \frac{78}{15} & 1 \end{vmatrix}$$

$$= \frac{1}{2}\left[4 - \frac{78}{15} + 2\left(7 + \frac{39}{15}\right) + 7\left(\frac{78}{15}\right) + \frac{4 \times 39}{15}\right]$$

$$= \frac{1}{2}\left[18 + 18 \times \frac{13}{5}\right]$$

$$= 9\left[\frac{18}{5}\right] = \frac{162}{5} = 32.4$$

Ans. (D)

12. A circle C_1 passes through the origin O and has diameter 4 on the positive x-axis. The line y = 2x gives a chord OA of a circle C_1. Let C_2 be the circle with OA as a diameter. If the tangent to C_2 at the point A meets the x-axis at P and y-axis at Q, then QA : AP is equal to :

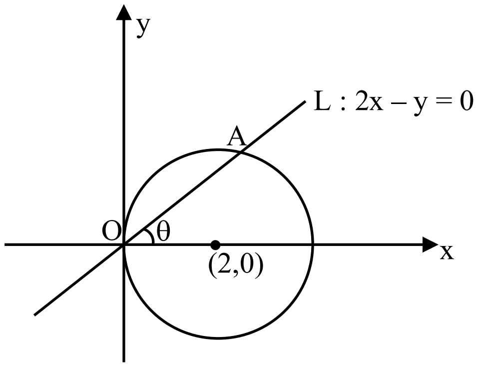
(A) 1 : 4 (B) 1 : 5

(C) 2 : 5 (D) 1 : 3

Official Ans. by NTA (A)

Sol. $C_1 : x \; + y \; - 4x = 0$

$\tan\theta = 2$



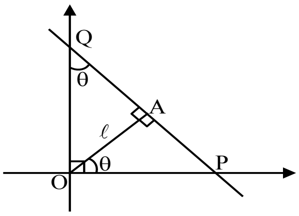
C_2 is a circle with OA as diameter.

So, tangent at A on C_2 is perpendicular to OR

Let OA = ℓ

$\therefore \dfrac{QA}{AP} = \dfrac{\ell\cot\theta}{\ell\tan\theta}$

$= \dfrac{1}{\tan^2\theta} = \dfrac{1}{4}$



13. If the length of the latus rectum of a parabola, whose focus is (a, a) and the tangent at its vertex is x + y = a, is 16, then |a| is equal to :

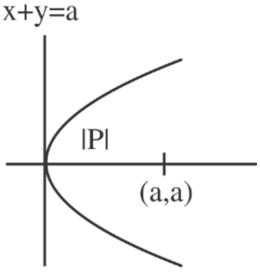
(A) $2\sqrt{2}$ (B) $2\sqrt{3}$

(C) $4\sqrt{2}$ (D) 4

Official Ans. by NTA (C)

Sol.

x+y=a



$$|P| = \left|\frac{a}{\sqrt{2}}\right| = \frac{16}{4} = 4$$

$$|a| = 4\sqrt{2}$$

Ans. (C)

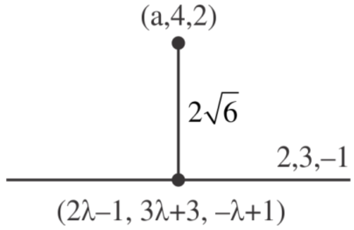
14. If the length of the perpendicular drawn from the point P(a, 4, 2), a > 0 on the line $\dfrac{x+1}{2} = \dfrac{y-3}{3} = \dfrac{z-1}{-1}$ is $2\sqrt{6}$ units and $Q(\alpha_1, \alpha_2, \alpha_3)$ is the image of the point P in this line, then $a + \sum_{i=1}^{3} \alpha_i$ is equal to :

(A) 7 (B) 8

(C) 12 (D) 14

Official Ans. by NTA (B)

Sol.



$(2\lambda - 1 - a)2 + (3\lambda - 1)3 + (-\lambda - 1)(-1) = 0$

$\Rightarrow 4\lambda - 2 - 2a + 9\lambda - 3 + \lambda + 1 = 0$

$\Rightarrow 14\lambda - 4 - 2a = 0$

$\Rightarrow 7\lambda - 2 - a = 0$

and,

$(2\lambda - 1 - a)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow (5\lambda - 1)^2 + (3\lambda - 1)^2 + (\lambda + 1)^2 = 24$

$\Rightarrow 35\lambda^2 - 14\lambda - 21 = 0$

$\Rightarrow (\lambda - 1)(35\lambda + 21) = 0$

For, $\lambda = 1 \quad \Rightarrow a = 5$

Let $(\alpha_1, \alpha_2, \alpha_3)$ be reflection of point P

$\alpha_1 + 5 = 2 \qquad \alpha_2 + 4 = 12 \qquad \alpha_3 + 2 = 0$

$\alpha_1 = -3 \qquad \alpha_2 = 8 \qquad \alpha_3 = -2$

$a + \alpha_1 + \alpha_2 + \alpha_3 = 8$

15. If the line of intersection of the planes $ax + by = 3$ and $ax + by + cz = 0$, a > 0 makes an angle 30° with the plane $y - z + 2 = 0$, then the direction cosines of the line are :

(A) $\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}, 0$ (B) $\dfrac{1}{\sqrt{2}}, \dfrac{-1}{\sqrt{2}}, 0$

(C) $\dfrac{1}{\sqrt{5}}, -\dfrac{2}{\sqrt{5}}, 0$ (D) $\dfrac{1}{2}, -\dfrac{\sqrt{3}}{2}, 0$

Official Ans. by NTA (B)

Sol. $\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a & b & 0 \\ a & b & c \end{vmatrix}$

$= bc\hat{i} - ac\hat{j}$

Direction ratios of line are **b, –a, 0**

Direction ratios of normal of the plane are **0, 1, – 1**

$$\cos 60^\circ = \left| \frac{-a}{\left|\sqrt{2}\right|\sqrt{b^2+a^2}} \right| = \frac{1}{2}$$

$$\Rightarrow \left| \frac{a}{\sqrt{a^2+b^2}} \right| = \frac{1}{\sqrt{2}}$$

$\Rightarrow b = \pm a$

So, D.R.'s can be (±a, –a, 0)

$\therefore$ D.C.'s can be $\pm\left(\frac{\pm 1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0\right)$

16. Let X have a binomial distribution B(n, p) such that the sum and the product of the mean and variance of X are 24 and 128 respectively. If $P(X > n-3) = \frac{k}{2^n}$, then k is equal to

(A) 528 (B) 529 (C) 629 (D) 630

Official Ans. by NTA (B)

Sol. Let $\alpha = $ Mean & $\beta = $ Variance $(\alpha > \beta)$

So, $\alpha + \beta = 24$, $\alpha\beta = 128$

$\Rightarrow \alpha = 16$ & $\beta = 8$

$\Rightarrow np = 16$ $npq = 8 \Rightarrow q = \frac{1}{2}$

$\therefore p = \frac{1}{2}, n = 32$

$$p(x > n-3) = \frac{1}{2^n}({}^nC_{n-2} + {}^nC_{n-1} + {}^nC_n)$$

$$\therefore k = {}^{32}C_{30} + {}^{32}C_{31} + {}^{32}C_{32} = \frac{32 \times 31}{2} + 32 + 1$$

$= 496 + 33 = 529$

17. A six faced die is biased such that 3 × P(a prime number) = 6 × P(a composite number) = 2 × P(1). Let X be a random variable that counts the number of times one gets a perfect square on some throws of this die. If the die is thrown twice, then the mean of X is :

(A) $\frac{3}{11}$ (B) $\frac{5}{11}$

(C) $\frac{7}{11}$ (D) $\frac{8}{11}$

Official Ans. by NTA (D)

Sol. Let $\frac{P(\text{a prime number})}{2} = \frac{P(\text{a composite})}{1} = \frac{P(1)}{3} = k$

So, P(a prime number) = 2k,

P(a composite number) = k,

& P(1) = 3k

& $3 \times 2k + 2 \times k + 3k = 1$

$\Rightarrow k = \frac{1}{11}$

$P(\text{success}) = P(1 \text{ or } 4) = 3k + k = \frac{4}{11}$

Number of trials, n = 2

$\therefore$ mean = np = $2 \times \frac{4}{11} = \frac{8}{11}$

18. The angle of elevation of the top P of a vertical tower PQ of height 10 from a point A on the horizontal ground is 45°. Let R be a point on AQ and from a point B, vertically above R, the angle of elevation of P is 60°. If $\angle BAQ = 30°$, AB = d and the area of the trapezium PQRB is α, then the ordered pair (d, α) is :

(A) $\left(10\left(\sqrt{3}-1\right), 25\right)$ (B) $\left(10\left(\sqrt{3}-1\right), \frac{25}{2}\right)$

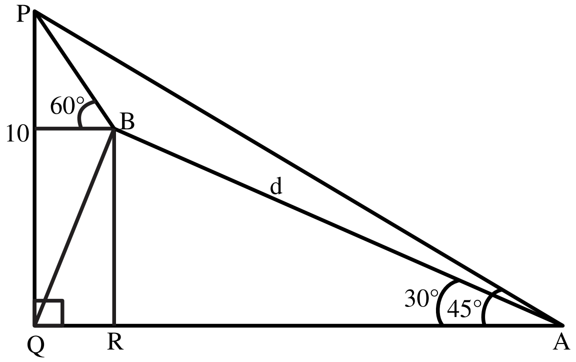
(C) $\left(10\left(\sqrt{3}+1\right), 25\right)$ (D) $\left(10\left(\sqrt{3}+1\right), \frac{25}{2}\right)$

Official Ans. by NTA (A)

Sol. $QA = 10 \quad RA = d\cos 30° = \frac{\sqrt{3}d}{2}$

$QR = 10 - \frac{\sqrt{3}d}{2}$

$BR = d\sin 30° = \frac{d}{2}$



$$\tan 60° = \frac{PQ - BR}{QR} = \frac{10 - \frac{d}{2}}{10 - \frac{\sqrt{3}d}{2}}$$

$$\Rightarrow \sqrt{3} = \frac{20 - d}{20 - \sqrt{3}d}$$

$$\Rightarrow 20\sqrt{3} - 3d = 20 - d$$

$$\Rightarrow 2d = 20(\sqrt{3} - 1)$$

$$\Rightarrow d = 10(\sqrt{3} - 1)$$

$$ar(PQRB) = \alpha = \frac{1}{2}(PQ + BR)\cdot QR$$

$$= \frac{1}{2}\left(10 + \frac{d}{2}\right)\cdot\left(10 - \frac{\sqrt{3}d}{2}\right)$$

$$= \frac{1}{2}\left(10 + 5\sqrt{3} - 5\right)\left(10 - 15 + 5\sqrt{3}\right)$$

$$= \frac{1}{2}\left(5\sqrt{3} + 5\right)\left(5\sqrt{3} - 5\right) = \frac{1}{2}(75 - 25) = 25$$

19. Let $S = \left\{\theta \in \left(0, \frac{\pi}{2}\right) : \sum_{m=1}^{9} \sec\left(\theta + (m-1)\frac{\pi}{6}\right)\sec\left(\theta + \frac{m\pi}{6}\right) = -\frac{8}{\sqrt{3}}\right\}$

Then

(A) $S = \left\{\frac{\pi}{12}\right\}$ (B) $S = \left\{\frac{2\pi}{3}\right\}$

(C) $\sum_{\theta \in S} \theta = \frac{\pi}{2}$ (D) $\sum_{\theta \in S} \theta = \frac{3\pi}{4}$

Official Ans. by NTA (C)

Sol. Let $\alpha = \theta + (m-1)\frac{\pi}{6}$

& $\beta = \theta + m\frac{\pi}{6}$

So, $\beta - \alpha = \frac{\pi}{6}$

Here, $\sum_{m=1}^{9} \sec\alpha \cdot \sec\beta = \sum_{m=1}^{9} \frac{1}{\cos\alpha \cdot \cos\beta}$

$$= 2\sum_{m=1}^{9} \frac{\sin(\beta - \alpha)}{\cos\alpha \cdot \cos\beta} = 2\sum_{m=1}^{9} (\tan\beta - \tan\alpha)$$

$$= 2\sum_{m=1}^{9}\left(\tan\left(\theta + m\frac{\pi}{6}\right) - \tan\left(\theta + (m-1)\frac{\pi}{6}\right)\right)$$

$$= 2\left(\tan\left(\theta + \frac{9\pi}{6}\right) - \tan\theta\right) = 2(-\cot\theta - \tan\theta) = -\frac{8}{\sqrt{3}}$$

(Given)

$$\therefore \quad \tan\theta + \cot\theta = \frac{4}{\sqrt{3}}$$

$$\Rightarrow \tan\theta = \frac{1}{\sqrt{3}} \text{ or } \sqrt{3}$$

So, $S = \left\{\frac{\pi}{6}, \frac{\pi}{3}\right\}$

$$\sum_{\theta \in S} \theta = \frac{\pi}{6} + \frac{\pi}{3} = \frac{\pi}{2}$$

20. If the truth value of the statement $(P \wedge (\sim R)) \rightarrow ((\sim R) \wedge Q)$ is F, then the truth value of which of the following is F ?

(A) $P \vee Q \rightarrow \sim R$ (B) $R \vee Q \rightarrow \sim P$

(C) $\sim(P \vee Q) \rightarrow \sim R$ (D) $\sim(R \vee Q) \rightarrow \sim P$

Official Ans. by NTA (D)

Sol. $X \Rightarrow Y$ is a false

when X is true and Y is false

So, $P \rightarrow T, Q \rightarrow F, R \rightarrow F$

(A) $P \vee Q \rightarrow \sim R$ is T

(B) $R \vee Q \rightarrow \sim P$ is T

(C) $\sim(P \vee Q) \rightarrow \sim R$ is T

(D) $\sim(R \vee Q) \rightarrow \sim P$ is F

SECTION-B

1. Consider a matrix $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$,

where α, β, γ are three distinct natural numbers.

If $\dfrac{\det\left(\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}A\right)\right)\right)\right)}{(\alpha-\beta)^{16}(\beta-\gamma)^{16}(\gamma-\alpha)^{16}} = 2^{32} \times 3^{16}$, then the number of such 3 – tuples (α, β, γ) is _______ .

Official Ans. by NTA (42)

Sol. $A = \begin{bmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ \beta+\gamma & \gamma+\alpha & \alpha+\beta \end{bmatrix}$

$R_3 \to R_3 + R_1$

$\Rightarrow |A| = |\alpha+\beta+\gamma| \begin{vmatrix} \alpha & \beta & \gamma \\ \alpha^2 & \beta^2 & \gamma^2 \\ 1 & 1 & 1 \end{vmatrix}$

$\Rightarrow |A| = (\alpha+\beta+\gamma)(\alpha-\beta)(\beta-\gamma)(\gamma-\alpha)$

$\because$ $|\operatorname{adj} A| = |A|^{n-1}$

$|\operatorname{adj}(\operatorname{adj} A)| = |A|^{(n-1)^2}$

$\left|\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}\left(\operatorname{adj}A\right)\right)\right)\right| = |A|^{(n-1)^4} = |A|^{2^4} = |A|^{16}$

$\therefore (\alpha + \beta + \gamma)^{16} = 2^{32} . 3^{16}$

$\Rightarrow (\alpha + \beta + \gamma)^{16} = (2^2 . 3)^{16} = (12)^{16}$

$\Rightarrow \alpha + \beta + \gamma = 12$

$\because \alpha, \beta, \gamma \in N$

$(\alpha - 1) + (\beta - 1) + (\gamma - 1) = 9$

number all tuples (α, β, γ) = ${}^{11}C_2 = 55$

1 case for α = β = γ

& 12 case when any two of these are equal

So, No. of distinct tuples (α, β, γ)

$= 55 - 13 = 42$

2. The number of functions f, from the set

$A = \{x \in N : x^2 - 10x + 9 \le 0\}$ to the set

$B = \{n^2 : n \in N\}$ such that $f(x) \le (x-3)^2 + 1$, for every $x \in A$, is __________ .

Official Ans. by NTA (1440)

Sol. $(x^2 - 10x + 9) \le 0 \Rightarrow (x - 1)(x - 9) \le 0$

$\Rightarrow x \in [1, 9] \Rightarrow A = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$f(x) \le (x - 3)^2 + 1$

$x = 1 : f(1) \le 5 \Rightarrow 1^2, 2^2$

$x = 2 : f(2) \le 2 \Rightarrow 1^2$

$x = 3 : f(3) \le 1 \Rightarrow 1^2$

$x = 4 : f(4) \le 2 \Rightarrow 1^2$

$x = 5 : f(5) \le 5 \Rightarrow 1^2, 2^2$

$x = 6 : f(6) \le 10 \Rightarrow 1^2, 2^2, 3^2$

$x = 7 : f(7) \le 17 \Rightarrow 1^2, 2^2, 3^2, 4^2$

$x = 8 : f(8) \le 26 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2$

$x = 9 : f(9) \le 37 \Rightarrow 1^2, 2^2, 3^2, 4^2, 5^2, 6^2$

Total number of such function

$= 2(6!) = 2(720) = 1440$

3. Let for the 9^{th} term in the binomial expansion of $(3 + 6x)^n$, in the increasing powers of 6x, to be the greatest for $x = \frac{3}{2}$, the least value of n is n_0. If k is the ratio of the coefficient of x^6 to the coefficient of x^3, then $k + n_0$ is equal to:

Official Ans. by NTA (24)

Sol. $(3 + 6x)^n = {}^nC_0 3^n + {}^nC_1 3^{n-1}(6x)^1 + \ldots$

$T_{r+1}\ {}^nC_r 3^{n-r}.(6x)r = {}^nC_r 3^{n-r}.6^r.\ x^r$

$= {}^nC_r 3^{n-r}.3^r.2^r.\left(\frac{3}{2}\right)^r = {}^nC_r 3^n.3^r \qquad [\text{for } x = \frac{3}{2}]$

T_9 is greatest of $x = \frac{3}{2}$

So, $T_9 > T_{10}$ and $T_9 > T_8$

(concept of numerically greatest term)

Here, $\frac{T_9}{T_{10}} > 1$ and $\frac{T_9}{T_8} > 1$

$\Rightarrow \frac{{}^nC_8 3^n.3^8}{{}^nC_9 3^n.3^9} > 1$ and $\frac{{}^nC_8 3^n.3^8}{{}^nC_7 3^n.3^7} > 1$

and $\frac{{}^nC_8}{{}^nC_7} > \frac{1}{3}$

and $\frac{n-7}{8} > \frac{1}{3}$

$\Rightarrow \frac{29}{3} < n < 11 \Rightarrow n = 10 = n_0$

So, in $(3 + 6x)^n$ for $n = n_0 = 10$

i.e., in $(3 + 6x)^{10}$, here $T_{r+1} = {}^{10}C_r 3^{10-r}6^r x^r$

$T_7 = {}^{10}C_6 3^4.6^6.x^6 = 210.3^{10}.2^6x^6$

$T_4 = {}^{10}C_3 3^7 6^3 x^3 = 120.3^{10}.2^3x^3$

Ratio of coefficient of x^6 and coefficient of $x^3 = k$

$\therefore k = \frac{210.3.^{10}2^6}{120.3^{10}.2^3} = \frac{7}{4} \times 2^3 = 14$

So, $k + n_0 = 14 + 10 = 24$

4. $\frac{2^3 - 1^3}{1 \times 7} + \frac{4^3 - 3^3 + 2^3 - 1^3}{2 \times 11} + \frac{6^3 - 5^3 + 4^3 - 3^3 + 2^3 - 1^3}{3 \times 15} + \ldots.. + \frac{30^3 - 29^3 + 28^3 - 27^3 + \ldots + 2^3 - 1^3}{15 \times 63}$

is equal to _____.

Official Ans. by NTA (120)

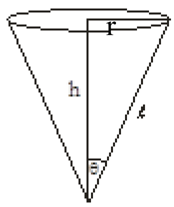
Sol. $T_n = \frac{2\sum_{r=1}^{n}(2r)^3 - (\sum_{r=1}^{2n} r^3)}{n(4n+3)}$

$\Rightarrow T_n = n$

So, $\sum_{n=1}^{15} T_n = 120$

5. A water tank has the shape of a right circular cone with axis vertical and vertex downwards. Its semi-vertical angle is $\tan^{-1}\frac{3}{4}$. Water is poured in it at a constant rate of 6 cubic meter per hour. The rate (in square meter per hour), at which the wet curved surface area of the tank is increasing, when the depth of water in the tank is 4 meters, is ________.

Official Ans. by NTA (5)



$\tan\theta = \frac{3}{4} = \frac{r}{h}$

$\frac{dV}{dt} = 6$

$V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi h^3 \tan^2\theta = \frac{9\pi}{48}h^3 = \frac{3\pi}{16}h^3$

$\Rightarrow \frac{dV}{dt} = \frac{3\pi}{16}.3h^2.\frac{dh}{dt} = 6 \Rightarrow \left(\frac{dh}{dt}\right)_{h=4} = \frac{2}{3\pi} m/hr$

Now, $S = \pi r\ell = \frac{15}{16}\pi h^2$

$\Rightarrow \frac{dS}{dt} = \frac{15\pi}{16}.2h\frac{dh}{dt}$

$\Rightarrow \left(\frac{dS}{dt}\right)_{h=4} = 5m^2/hr$

6. For the curve C : $(x^2 + y^2 - 3) + (x^2 - y^2 - 1)^5 = 0$, the value of $3y' - y^3y''$, at the point (α, α), $\alpha > 0$, on C, is equal to ________.

Official Ans. by NTA (16)

Sol. (α, α) lies on

$C:\ x^2 + y^2 - 3\ +\ x^2 - y^2 - 1\ ^5 = 0$

Put (α, α), $\ 2\alpha^2 - 3\ +\ -1\ ^5 = 0$

$\Rightarrow \alpha = \sqrt{2}$

Now, differentiate C

$2x + 2y\cdot y' + 5\left(x^2 - y^2 - 1\right)^4\left(2x - 2yy'\right) = 0 \ \ldots (1)$

At $\left(\sqrt{2}, \sqrt{2}\right)$

$$\sqrt{2}+\sqrt{2}y'+5(-1)^4\left(\sqrt{2}-\sqrt{2}y'\right)=0$$

$$\Rightarrow \quad y'=\frac{3}{2} \qquad \ldots.(2)$$

Diff. (1) w.r.t. x

Again, Diff. (1) w.r.t. x

$$1+(y')^2+yy''+20\left(x^2-y^2-1\right)^3(x-yy')^2.2$$

$$+5\left(x^2-y^2-1\right)^4\left(1-(y')^2-yy''\right)=0$$

At $\left(\sqrt{2},\sqrt{2}\right)$ and $y'=\frac{3}{2}$

We have,

$$\left(1+\frac{9}{4}\right)+\sqrt{2}y''-40\left(\sqrt{2}-\sqrt{2}\cdot\frac{3}{2}\right)^2$$

$$+5(1)\left(1-\frac{9}{4}-\sqrt{2}y''\right)=0$$

$$\Rightarrow \quad 4\sqrt{2}y''=-23$$

$$\therefore \quad 3y'-y^3y''=\frac{9}{2}+\frac{23}{2}=16$$

7. Let f(x) = min{[x – 1], [x – 2], …, [x – 10]} where [t] denotes the greatest integer ≤ t. Then $\int_0^{10} f(x)dx+\int_0^{10}(f(x))^2dx+\int_0^{10}|f(x)|dx$ is equal to ___.

Official Ans. by NTA (385)

Sol. f(x) = [x] – 10

$$\int_0^{10} f(x)\cdot dx=-10-9-8-.....-1$$

$$=-\frac{10\cdot 11}{2}=-55$$

$$\int_0^{10}\left(f(x)\right)^2dx=10^2+9^2+8^2+...+1^2$$

$$=\frac{10\cdot 11\cdot 21}{6}=385$$

$$\int_0^{10}\left|f(x)\right|=10+9+8+....+1$$

$$=\frac{10\cdot 11}{2}=55$$

= – 55 + 385 + 55 = 385

8. Let f be a differentiable function satisfying $f(x)=\frac{2}{\sqrt{3}}\int_0^{\sqrt{3}} f\left(\frac{\lambda^2x}{3}\right)d\lambda$, x > 0 and f(1) = $\sqrt{3}$. If y=f(x) passes through the point (α, 6), then α is equal to ______.

Official Ans. by NTA (12)

Sol. Let, $\frac{\lambda^2x}{3}=t$

$$\Rightarrow \frac{2\lambda x}{3}d\lambda=dt$$

$$\Rightarrow d\lambda=\frac{3}{2}\cdot\frac{1\sqrt{x}}{x\cdot\sqrt{3}\sqrt{t}}dt$$

$$\Rightarrow d\lambda=\frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{x}}\cdot\frac{dt}{\sqrt{t}}$$

$$\text{So, } f(x)=\frac{1}{\sqrt{x}}\int_0^x\frac{f(t)}{\sqrt{t}}dt$$

$$\Rightarrow \sqrt{x}\cdot f'(x)+\frac{f(x)}{2\sqrt{x}}=\frac{f(x)}{\sqrt{x}}$$

$$\Rightarrow \sqrt{x}\cdot f'(x)=\frac{f(x)}{2\sqrt{x}}$$

$$\Rightarrow \frac{dy}{y}=\frac{dx}{2x}$$

$$\Rightarrow \ln y=\frac{1}{2}\ln x+c\Rightarrow f(x)=\sqrt{x}$$

$$\Rightarrow y=\sqrt{3x} \quad \{\text{as } f(1)=\sqrt{3}\}$$

So, $f(x)=\sqrt{3x}$

Now, f(α) = 6 ⇒ 36 = 3α

$\Rightarrow \alpha=12$

9. A common tangent T to the curves $C_1:\frac{x^2}{4}+\frac{y^2}{9}=1$ and $C_2:\frac{x^2}{42}-\frac{y^2}{143}=1$ does not pass through the fourth quadrant. If T touches C_1 at (x_1, y_1) and C_2 at (x_2, y_2), then $|2x_1 + x_2|$ is equal to _______.

Official Ans. by NTA (20)

Sol. Let common tangents are

$T_1 : y = mx \pm \sqrt{4m^2 + 9}$

& $T_2 : y = mx \pm \sqrt{42m^2 - 13}$

So, $4m^2 + 9 = 42m^2 - 143$

$\Rightarrow 38m^2 = 152$

$\Rightarrow m = \pm 2$

& $c = \pm 5$

For given tangent not pass through 4^{th} quadrant

$T : y = 2x + 5$

Now, comparing with $\frac{xx_1}{4} + \frac{yy_1}{9} = 1$

We get, $\frac{x_1}{8} = -\frac{1}{5} \Rightarrow x_1 = -\frac{8}{5}$

$\frac{xx_2}{42} - \frac{yy_2}{143} = 1$

$2x - y = -5$ we have

$x_2 = -\frac{84}{5}$

So, $|2x_1 + x_2| = \left|\frac{-100}{5}\right| = 20$

10. Let $\vec{a}, \vec{b}, \vec{c}$ be three non-coplanar vectors such that

$\vec{a} \times \vec{b} = 4\vec{c}$, $\vec{b} \times \vec{c} = 9\vec{a}$ and $\vec{c} \times \vec{a} = \alpha\vec{b}, \alpha > 0$.

If $|\vec{a}| + |\vec{b}| + |\vec{c}| = \frac{1}{36}$, then α is equal to ______.

Official Ans. by NTA (36)

Sol. $\vec{a} \times \vec{b} = 4\vec{c} \Rightarrow \vec{a} \cdot \vec{c} = 0 = \vec{b} \cdot \vec{c}$

$\vec{b} \times \vec{c} = 9\vec{a} \Rightarrow \vec{a} \cdot \vec{b} = 0 = \vec{a} \cdot \vec{c}$

$\therefore \vec{a}, \vec{b}, \vec{c}$ are mutually $\perp$ set of vectors.

$\Rightarrow |\vec{a}||\vec{b}| = 4|\vec{c}|$, $|\vec{b}||\vec{c}| = 9|\vec{a}|$ & $|\vec{c}||\vec{a}| = \alpha|\vec{b}|$

$\Rightarrow \frac{|\vec{a}|}{|\vec{c}|} = \frac{4}{9}\frac{|\vec{c}|}{|\vec{a}|}$

$\Rightarrow \frac{|\vec{c}|}{|\vec{a}|} = \frac{3}{2}$

$\therefore$ If $|a| = \lambda$, $|c| = \frac{3\lambda}{2}$ & $|b| = 6$

Now $|a| + |b| + |c| = \frac{1}{36}$

$\Rightarrow \frac{5}{2}\lambda + 6 = \frac{1}{36}$, $\lambda = \frac{-43}{18} = |a|$

which gives negative value of λ or $|a|$ which is NOT possible & hence data seems to be wrong.

But if $|\vec{a}| + |\vec{b}| + |\vec{c}| = 36$

$\frac{5}{2}\lambda + 6 = 36$

$\lambda = 12$

$\alpha = \frac{|\vec{c}||\vec{a}|}{|\vec{b}|} = \frac{3 \times 12}{2} \times \frac{12}{6}$

$\alpha = 36$