FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS **TEST PAPER WITH SOLUTION**

SECTION-A

1. The SI unit of a physical quantity is pascal-second. The dimensional formula of this quantity will be

(A) $[ML^{-1}T^{-1}]$ (B) $[ML^{-1}T^{-2}]$

(C) $[ML^{2}T^{-1}]$ (D) $[M^{-1}L^{3}T^{0}]$

Official Ans. by NTA (A)

Sol. Pascal second

$$\frac{F}{A}t = \frac{MLT^{-2}}{L^2}T = ML^{-1}T^{-1}$$

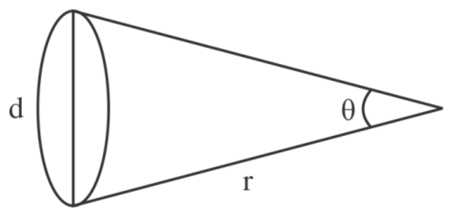
2. The distance of the Sun from earth is 1.5×10^{11} m and its angular diameter is (2000) s when observed from the earth. The diameter of the Sun will be :

(A) 2.45×10^{10} m (B) 1.45×10^{10} m

(C) 1.45×10^{9} m (D) 0.14×10^{9} m

Official Ans. by NTA (C)

Sol.



$$\theta = \frac{d}{r}$$

$$\frac{2000}{60\times60}\times\frac{\pi}{180} = \frac{d}{1.5\times10''}$$

$$\Rightarrow d = \frac{2000}{60\times60}\times\frac{\pi}{180}\times1.5\times10''$$

$$= \frac{\pi\times1.5}{3\times6\times18}\times10'' = 1.45\times10^9$$

3. When a ball is dropped into a lake from a height 4.9 m above the water level, it hits the water with a velocity v and then sinks to the bottom with the constant velocity v. It reaches the bottom of the lake 4.0 s after it is dropped. The approximate depth of the lake is :

(A) 19.6 m (B) 29.4 m

(C) 39.2 m (D) 73.5 m

Official Ans. by NTA (B)

Sol. $V^2 = 2 \times 9.8 \times 4.9$

$V = 9.8$ m/s

Depth = distance travelled in 3 seconds

$= 9.8 \times 3 = 29.4$ m

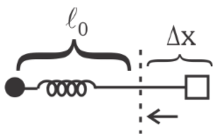
4. One end of a massless spring of spring constant k and natural length l_0 is fixed while the other end is connected to a small object of mass m lying on a frictionless table. The spring remains horizontal on the table. If the object is made to rotate at an angular velocity ω about an axis passing through fixed end, then the elongation of the spring will be:

(A) $\frac{k - m\omega^2 l_0}{m\omega^2}$ (B) $\frac{m\omega\ l_0}{k + m\omega^2}$

(C) $\frac{m\omega^2 l_0}{k - m\omega^2}$ (D) $\frac{k + m\omega^2 l_0}{m\omega^2}$

Official Ans. by NTA (C)

Sol.



$$K\,\Delta x = m(\ell_0 + \Delta x)w^2$$

$$K\,\Delta x = m\,\ell_0\,w^2 + mw^2\,\Delta x$$

$$\Delta x = \frac{m\ell_0 w^2}{k - mw^2}$$

5. A stone tide to a string of length L is whirled in a vertical circle with the other end of the string at the centre. At a certain instant of time, the stone is at its lowest position and has a speed u. The magnitude of change in its velocity, as it reaches a position where the string is horizontal, is $\sqrt{x(u^2 - gL)}$. The value of x is

(A) 3 (B) 2

(C) 1 (D) 5

Official Ans. by NTA (B)

Sol. $v = \sqrt{u^2 - 2gL}$

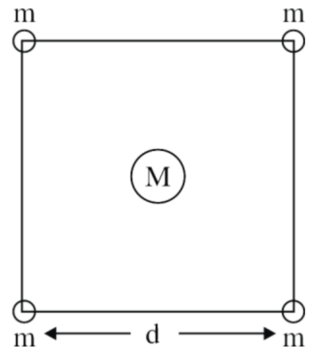
$$\Delta v = \sqrt{u^2 + v^2}$$

$$\Delta v = \sqrt{u^2 + v^2 - 2gL}$$

$$\Delta v = \sqrt{2u^2 - 2gL}$$

$$\Delta v = \sqrt{2\left(u^2 - gL\right)} \quad x = 2$$

6. Four spheres each of mass m form a square of side d (as shown in figure). A fifth sphere of mass M is situated at the centre of square. The total gravitational potential energy of the system is :



(A) $-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$

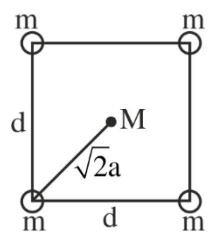
(B) $-\frac{Gm}{d}\left[(4+\sqrt{2})M + 4\sqrt{2}m\right]$

(C) $-\frac{Gm}{d}\left[3m^2 + 4\sqrt{2}M\right]$

(D) $-\frac{Gm}{d}\left[6m^2 + 4\sqrt{2}M\right]$

Official Ans. by NTA (A)

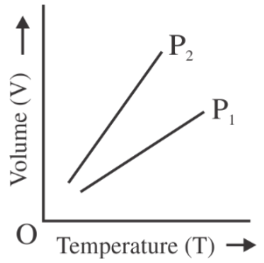
Sol.



$$-\frac{Gm^2}{d} \times 4 - \frac{Gm^2}{\sqrt{2}d} \times 2 - \frac{GMm}{d} \times 4\sqrt{2}$$

$$-\frac{Gm}{d}\left[(4+\sqrt{2})m + 4\sqrt{2}M\right]$$

7. For a perfect gas, two pressures P_1 and P_2 are shown in figure. The graph shows:



(A) $P_1 > P_2$

(B) $P_1 < P_2$

(C) $P_1 = P_2$

(D) Insufficient data to draw any conclusion

Official Ans. by NTA (A)

Sol. $PV = nRT$

$$\frac{V}{T} = \frac{nR}{P}$$

$$\frac{nR}{P_1} < \frac{nR}{P_2}$$

$$P_2 < P_1$$

8. According to kinetic theory of gases,

A. The motion of the gas molecules freezes at 0ºC

B. The mean free path of gas molecules decreases if the density of molecules is increased.

C. The mean free path of gas molecules increases if temperature is increased keeping pressure constant.

D. Average kinetic energy per molecule per degree of freedom is $\frac{3}{2}k_BT$ (for monoatomic gases)

Choose the most appropriate answer from the options given below:

(A) A and C only (B) B and C only

(C) A and B only (D) C and D only

Official Ans. by NTA (B)

Sol. $\lambda = \dfrac{kT}{\sqrt{2}\pi d^2 P}$

9. A lead bullet penetrates into a solid object and melts. Assuming that 40% of its kinetic energy is used to heat it, the initial speed of bullet is:

(Given, initial temperature of the bullet = 127ºC,

Melting point of the bullet = 327ºC,

Latent heat of fusion of lead = 2.5×10^4 J Kg^{-1},

Specific heat capacity of lead = 125J/kg K)

(A) 125 ms^{-1} (B) 500 ms^{-1}

(C) 250 ms^{-1} (D) 600 ms^{-1}

Official Ans. by NTA (B)

Sol. $m \times 125 \times 200 + m \times 2.5 \times 10^4 = \frac{1}{2}mv^2 \times \frac{40}{100}$

V = 500 m/s

10. The equation of a particle executing simple harmonic motion is given by $x = \sin\pi\left(t + \frac{1}{3}\right)$ m.

At t = 1s, the speed of particle will be

(Given : $\pi = 3.14$)

(A) 0 cm s^{-1} (B) 157 cm s^{-1}

(C) 272 cm s^{-1} (D) 314 cm s^{-1}

Official Ans. by NTA (B)

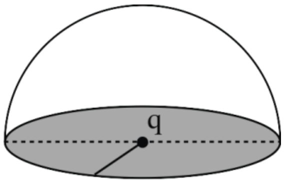
Sol. $x = \sin\pi\left(t + \frac{1}{3}\right)$

$$x = \sin\left(\pi t + \frac{\pi}{3}\right)$$

$$V = \frac{dx}{dt} = \cos\left(\pi t + \frac{\pi}{3}\right)\pi$$

$$= -\pi \times \frac{1}{2} = 157 \text{ cm/s}$$

11. If a charge q is placed at the centre of a closed hemispherical non-conducting surface, the total flux passing through the flat surface would be :

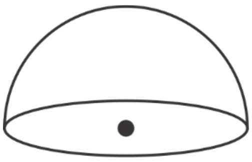


(A) $\frac{q}{\varepsilon_0}$ (B) $\frac{q}{2\varepsilon_0}$

(C) $\frac{q}{4\varepsilon_0}$ (D) $\frac{q}{2\pi\varepsilon_0}$

Official Ans. by NTA (B)

Sol.



Total flux through complete spherical surface is $\frac{q}{\varepsilon_0}$.

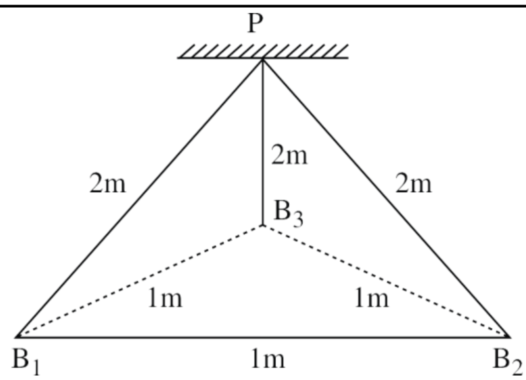
So the flux through curved surface will be $\frac{q}{2\varepsilon_0}$.

The flux through flat surface will be zero.

Remark : Electric flux through flat surface is zero but no option is given, option is available for electric flux passing through curved surface.

12. Three identical charged balls each of charge 2C are suspended from a common point P by silk threads of 2m each (as shown in figure). They form an equilateral triangle of side 1m.

The ratio of net force on a charged ball to the force between any two charged balls will be :

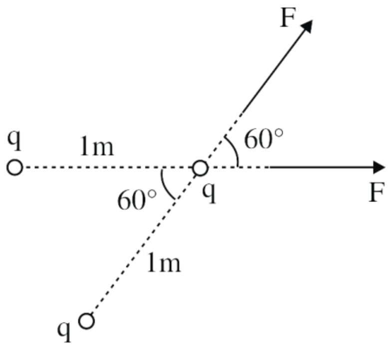


(A) 1 : 1 (B) 1 : 4

(C) $\sqrt{3}:2$ (D) $\sqrt{3}:1$

Official Ans. by NTA (D)

Sol.



$$F = \frac{k(2)(2)}{(1)^2}$$

(F = Force between two charges).

$F = 4k$

$$F_{net} = 2F\cos 30° = 2\cdot F\cdot\frac{\sqrt{3}}{2} = F\sqrt{3}$$

(F_{net} = Net electrostatic force on one charged ball)

$$\frac{F_{net}}{F} = \frac{\sqrt{3}F}{F} = \left(\sqrt{3}\right)$$

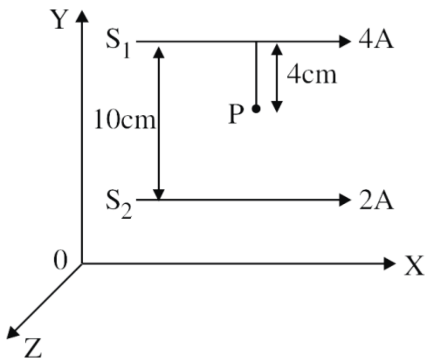
Remark: Net force on any one of the ball is zero. But no option given in options.

13. Two long parallel conductors S_1 and S_2 are separated by a distance 10 cm and carrying currents of 4A and 2A respectively. The conductors are placed along x-axis in X-Y plane. There is a point P located between the conductors (as shown in figure).

A charge particle of 3π coulomb is passing through the point P with velocity

$\vec{v} = (2\hat{i} + 3\hat{j})m/s$; where $\hat{i}$ & $\hat{j}$ represents unit vector along x & y axis respectively.

The force acting on the charge particle is $4\pi \times 10^{-5}(-x\hat{i} + 2\hat{j})N$. The value of x is :

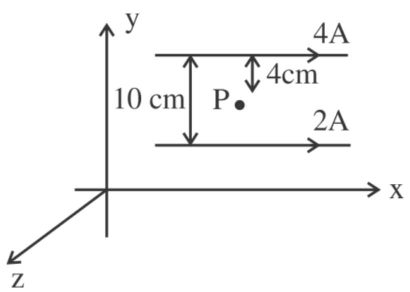


(A) 2 (B) 1

(C) 3 (D) –3

Official Ans. by NTA (C)

Sol.



$$B_{net} = B_1 - B_2 = \frac{\mu_0 \times 4}{2\pi[.04]} - \frac{\mu_0 \times 2}{2\pi[.06]}$$

$$\vec{B}_{net} = \frac{\mu_0}{2\pi}\left[\frac{200}{3}\right](-\hat{k})$$

$$\vec{F} = q\left[\vec{v} \times \vec{B}\right]$$

$$= [3\pi]\left[(2\hat{i} + 3\hat{j}) \times \left(\frac{\mu_0}{2\pi}\right)\left(\frac{200}{3}\right) - \hat{k}\right]$$

$$= 3\pi \times \frac{\mu_0}{2\pi}\left(\frac{200}{3}\right)\left[2 \times \hat{j} - 3\left(\hat{i}\right)\right]$$

$$= \left(4\pi \times 10^{-7}\right)(100)\left(-3\hat{i} + 2\hat{j}\right)$$

$$= 4\pi \times 10^{-5} \times \left[-3\hat{i} + 2\hat{j}\right]$$

14. If L, C and R are the self inductance, capacitance and resistance respectively, which of the following does not have the dimension of time ?

(A) RC (B) $\frac{L}{R}$

(C) $\sqrt{LC}$ (D) $\frac{L}{C}$

Official Ans. by NTA (D)

Sol. $\left(\frac{L}{C}\right)$ does not have dimension of time.

RC, $\frac{L}{R}$ are time constant while $\sqrt{LC}$ is reciprocal of angular frequency or having dimension of time.

15. Given below are two statements:

Statement I : A time varying electric field is a source of changing magnetic field and vice-versa. Thus a disturbance in electric or magnetic field creates EM waves.

Statement II : In a material medium. The EM wave travels with speed $v = \frac{1}{\sqrt{\mu_0 \varepsilon_0}}$.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both statement I and statement II are true.

(B) Both statement I and statement II are false.

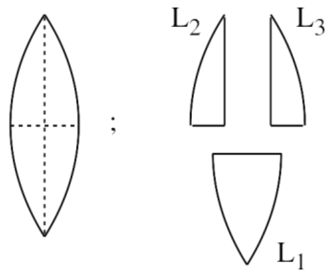
(C) Statement I is correct but statement II is false.

(D) Statement I is incorrect but statement II is true.

Official Ans. by NTA (C)

Sol. The statement II is wrong as the velocity of em wave in a medium is $\frac{1}{\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}}$.

16. A convex lens has power P. It is cut into two halves along its principal axis. Further one piece (out of the two halves) is cut into two halves perpendicular to the principal axis (as shown in figure). Choose the incorrect option for the reported pieces.



(A) Power of $L_1 = \frac{P}{2}$

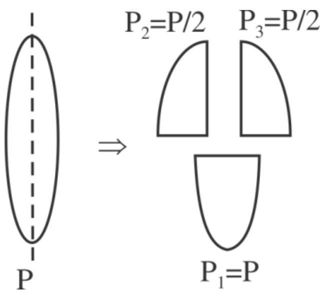
(B) Power of $L_2 = \frac{P}{2}$

(C) Power of $L_3 = \frac{P}{2}$

(D) Power of $L_1 = P$

Official Ans. by NTA (A)

Sol.



17. If a wave gets refracted into a denser medium, then which of the following is true?

(A) wavelength speed and frequency decreases.

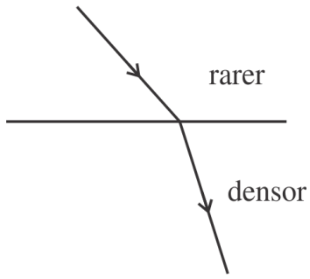
(B) wavelength increases, speed decreases and frequency remains constant.

(C) wavelength and speed decreases but frequency remains constant.

(D) wavelength, speed and frequency increases.

Official Ans. by NTA (C)

Sol.



No change in frequency but speed and wave-length decreases.

18. Given below are two statements:

Statement I : In hydrogen atom, the frequency of radiation emitted when an electron jumps from lower energy orbit (E_1) to higher energy orbit (E_2), is given as $hf = E_1 - E_2$.

Statement-II : The jumping of electron from higher energy orbit (E_2) to lower energy orbit (E_1) is associated with frequency of radiation given as $f = (E_2 - E_1)/h$

This condition is Bohr's frequency condition.

In the light of the above statements, choose the correct answer from the options given below:

(A) Both statement I and statement II are true.

(B) Both statement I and statement II are false

(C) Statement I is correct but statement II is false

(D) Statement I is incorrect but statement II is true.

Official Ans. by NTA (D)

Sol. When electron jump from lower to higher energy level, energy absorbed so statement-I incorrect.

When electron jump from higher to lower energy level, energy of emitted photon

$E = E_2 - E_1$

$hf = E_2 - E_1 \Rightarrow f = \frac{E_2 - E_1}{h}$

so statement-II is correct.

19. For a transistor to act as a switch, it must be operated in

(A) Active region

(B) Saturation state only

(C) Cut-off state only

(D) Saturation and cut-off state

Official Ans. by NTA (D)

Sol. Transistor act as a switch in saturation and cut of region.

20. We do not transmit low frequency signal to long distances because

(a) The size of the antenna should be comparable to signal wavelength which is unreal solution for a signal of longer wavelength.

(b) Effective power radiated by a long wavelength baseband signal would be high.

(c) We want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signal can be sent to long distances by superimposing with a high frequency wave as well.

Therefore, the most suitable options will be :

(A) All statements are true

(B) (a), (b) and (c) are true only

(C) (a), (c) and (d) are true only

(D) (b), (c) and (d) are true only

Official Ans. by NTA (C)

Sol. (a) For low frequency or high wavelength size of antenna required is high.

(b) E P R is low for longer wavelength.

(c) yes we want to avoid mixing up signals transmitted by different transmitter simultaneously.

(d) Low frequency signals sent to long distance by superimposing with high frequency.

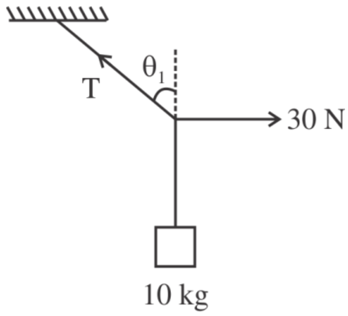
SECTION-B

1. A mass of 10 kg is suspended vertically by a rope of length 5m from the roof. A force of 30 N is applied at the middle point of rope in horizontal direction. The angle made by upper half of the rope with vertical is $\theta = \tan^{-1}(x \times 10^{-1})$. The value of x is ______ .

(Given $g = 10\ m/s^2$)

Official Ans. by NTA (3)

Sol.



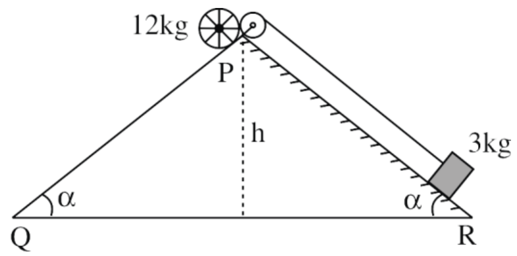
$T \sin\theta = 30$

$T \cos\theta = 100$

$\Rightarrow$ $\tan\theta = 0.3$

2. A rolling wheel of 12 kg is on an inclined plane at position P and connected to a mass of 3 kg through a string of fixed length and pulley as shown in figure. Consider PR as friction free surface.

The velocity of centre of mass of the wheel when it reaches at the bottom Q of the inclined plane PQ will be $\frac{1}{2}\sqrt{xgh}$ m/s. The value of x is ______ .



Official Ans. by NTA (3)

Sol. Net loss in PE = Gain in KE

$$12\,gh - 3gh = \frac{1}{2}3v^2 + \frac{1}{2}12v^2 + \frac{1}{2}\left[12r^2\right]\left(\frac{v}{r}\right)^2$$

$$9gh = \frac{1}{2}\left[3+12+12\right]v^2$$

$$v^2 = \frac{2gh}{3} \Rightarrow v = \frac{1}{2}\sqrt{\frac{8}{3}gh}$$

$$x = \frac{8}{3} \simeq 3$$

3. A diatomic gas ($\gamma = 1.4$) does 400 J of work when it is expanded isobarically. The heat given to the gas in the process is _______ J.

Official Ans. by NTA (1400)

Sol. $Q = nC_p\Delta T = \frac{n\nu}{\nu-1}R\Delta T$

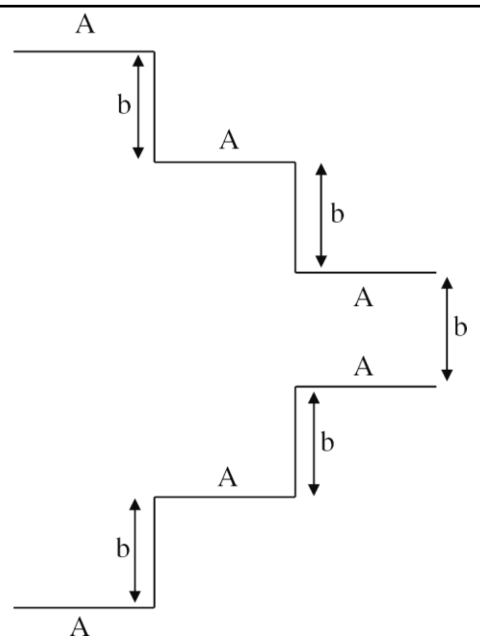
$$Q = \frac{\nu}{\nu-1}\omega = \frac{1.4}{0.4}\times 400 = 1400\text{ J}$$

4. A particle executes simple harmonic motion. Its amplitude is 8 cm and time period is 6s. The time it will take to travel from its position of maximum displacement to the point corresponding to half of its amplitude, is ________ s.

Official Ans. by NTA (1)

Sol. $t = \frac{\Delta\phi}{\omega} = \frac{\pi/2 - \pi/6}{2\pi/6} = \frac{\pi/3}{\pi/3} = 1\text{ sec}$

5. A paralle plate capacitor is made up of stair like structure with a palte area A of each stair and that is connected with a wire of length b, as shown in the figure. The capacitance of the arrangement is $\frac{x}{15}\frac{\varepsilon_0 A}{b}$. The value of x is _______ .



Official Ans. by NTA (23)

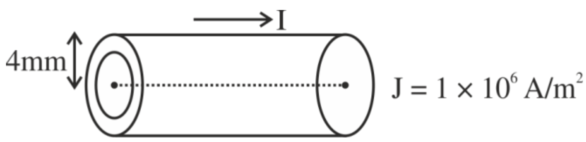
Sol. Parallel combination

$$c_{eq} = \varepsilon_0 A\left[\frac{1}{5b}+\frac{1}{3b}+\frac{1}{b}\right] = \frac{23}{15}\frac{\varepsilon_0 A}{b}$$

6. The current density in a cylindrical wire of radius r = 4.0 mm is 1.0×10^6 A/m^2. The current through the outer portion of the wire between radial distances r/2 and r is $x\pi$ A; where x is _______ .

Official Ans. by NTA (12)

Sol.



$$I = \int JdA$$

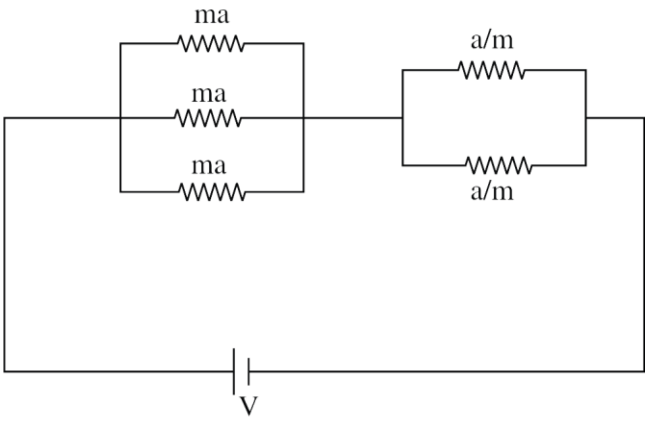
$$= \int 10^6 \times 2\pi x dx$$

$$= 10^6 \times 2\pi . \left.\frac{x^2}{2}\right]_{\frac{r}{2}}^{r}$$

$$= \pi \times 10^6\left[r^2 - \frac{r^2}{4}\right] = 12\pi$$

$x = 12$

7. In the given circuit 'a' is an arbitrary constant. The value of m for which the equivalent circuit resistance is minimum, will be $\sqrt{\frac{x}{2}}$. The value of x is _____ .



Official Ans. by NTA (3)

Sol. $R = \left(\frac{ma}{3}\right) + \left(\frac{a}{2m}\right)$

$$\frac{dR}{dm} = \frac{a}{3} - \frac{a}{2m^2} = 0$$

$$\frac{a}{3} = \frac{a}{2m^2}$$

$$m^2 = \frac{3}{2}$$

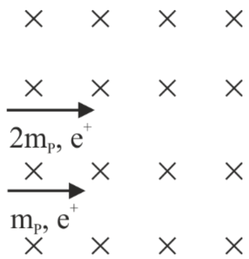
$$m = \sqrt{\frac{3}{2}}$$

$x = 3$

8. A deuteron and a proton moving with equal kinetic energy enter into to a uniform magnetic field at right angle to the field. If r_d and r_p are the radii of their circular paths respectively, then the ratio $\frac{r_d}{r_p}$ will be $\sqrt{x} : 1$ where x is ______ .

Official Ans. by NTA (2)

Sol.



$$R = \frac{mv}{q_B}$$

$$R_D = \frac{(2m_P)\,v_D}{e\,B}$$

$$R_P = \frac{(m_P)\,v_P}{e\,B}$$

$$\frac{R_D}{R_P} = \frac{2v_D}{v_P} = \frac{2v_D}{\sqrt{2}v_D} = \frac{\sqrt{2}}{1}$$

$$\frac{1}{2}(2mp)v_D^2 = \frac{1}{2}m_P.v_P^2$$

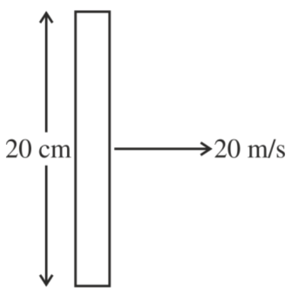
$$\sqrt{2}v_D = v_P$$

$x = 2$

9. A metallic rod of length 20 cm is palced in North-South direction and is moved at a constant speed of 20 m/s towards East. The horizontal component of the Earth's magnetic field at that place is 4×10^{-3} T and the angle of dip is 45°. The emf induced in the rod is _______ mV.

Official Ans. by NTA (16)

Sol.



$B_H = 4 \times 10^{-3}\,T$

$\theta \rightarrow 45°$

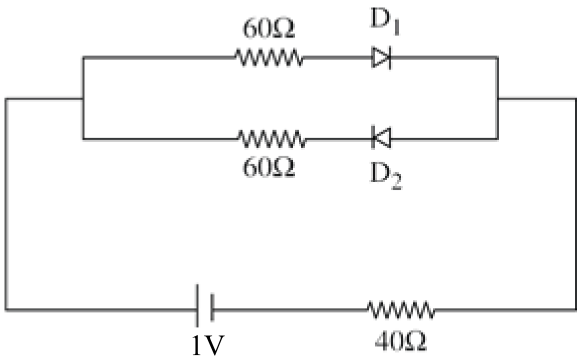
$B_V = B_H$

$\in = (\vec{V} \times \vec{B}) \cdot \vec{\ell}$

$= ((4 \times 10^{-3})(20))\dfrac{20}{100}$

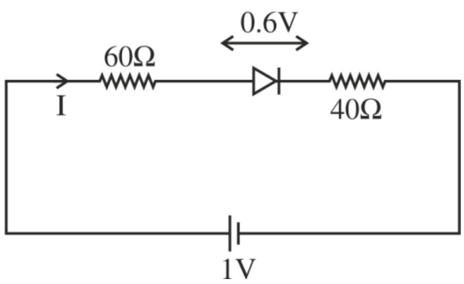
$= 16 \times 10^{-3}$ V $= 16$ mV

10. The cut-off voltage of the diodes (shown in figure) in forward bias is 0.6 V. The current through the resister of 40 Ω is _______ mA.



Official Ans. by NTA (4)

Sol.



$1 - I(60) - 0.6 - I(40) = 0$

$\dfrac{0.4}{100} = I$

$I = 4$ mA

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

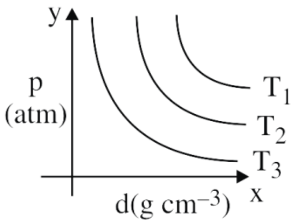
(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY | TEST PAPER WITH SOLUTIONS

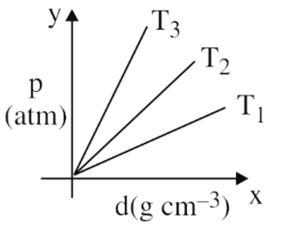
SECTION-A

1. Which amongst the given plots is the correct plot for pressure (p) vs density (d) for an ideal gas ?

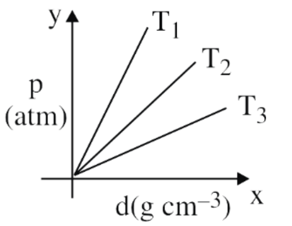
(A) $T_3 > T_2 > T_1$



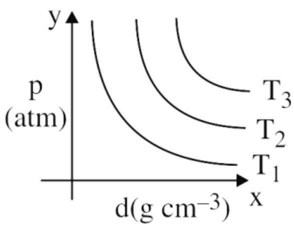
(B) $T_3 > T_2 > T_1$



(C) $T_3 > T_2 > T_1$



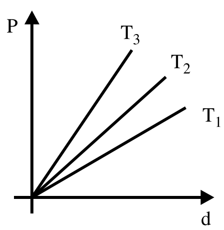
(D) $T_3 > T_2 > T_1$



Official Ans. by NTA (B)

Sol. P vs d :

$$P = \left(\frac{RT}{M}\right) d$$



$T_3 > T_2 > T_1$

2. Identify the **incorrect** statement for PCl_5 from the following.

(A) In this molecule, orbitals of phosphorous are assumed to undergo sp^3d hybridization.

(B) The geometry of PCl_5 is trigonal bipyramidal.

(C) PCl_5 has two axial bonds stronger than three equatorial bonds.

(D) The three equatorial bonds of PCl_5 lie in a plane.

Official Ans. by NTA (C)

Sol. In PCl_5, axial bonds are weaker than equatorial.

3. **Statement I : Leaching of gold** with cyanide ion in absence of air / O_2 leads to cyano complex of Au(III).

Statement II : Zinc is oxidized during the displacement reaction carried out for gold extraction.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are correct

(B) Both Statement I and Statement II are incorrect

(C) Statement I is correct but Statement II is incorrect

(D) Statement I is incorrect but Statement II is correct

Official Ans. by NTA (D)

Sol. Statement-1 : wrong, Au^+ is correct, not Au^{+3}

Statement-2 : correct

4. The correct order of increasing intermolecular hydrogen bond strength is

(A) $HCN < H_2O < NH_3$

(B) $HCN < CH_4 < NH_3$

(C) $CH_4 < HCN < NH_3$

(D) $CH_4 < NH_3 < HCN$

Official Ans. by NTA (C)

Sol. Order of H-Bonding

$CH_4 < HCN < NH_3$

NCH . . . NCH

H_2NH . . . NH_3

5. The correct order of increasing ionic radii is

(A) $Mg^{2+} < Na^{+} < F^{-} < O^{2-} < N^{3-}$

(B) $N^{3-} < O^{2-} < F^{-} < Na^{+} < Mg^{2+}$

(C) $F^{-} < Na^{+} < O^{2-} < Mg^{2+} < N^{3-}$

(D) $Na^{+} < F^{-} < Mg^{2+} < O^{2-} < N^{3-}$

Official Ans. by NTA (A)

Sol. $N^{-3} > O^{-2} > F^{-} > Na^{+} > Mg^{+2}$ (Radii)

(Isoelectronic species)

6. The gas produced by treating an aqueous solution of ammonium chloride with sodium nitrite is

(A) NH_3 (B) N_2

(C) N_2O (D) Cl_2

Official Ans. by NTA (B)

Sol. $NH_4Cl + NaNO_2 \rightarrow NH_4NO_2 + NaCl$

$$NH_4NO_2 \downarrow N_2 + 2H_2O$$

7. Given below are two statements: one is labelled as Assertion A and the other is labelled as Reason R.

Assertion A : Flourine forms one oxoacid.

Reason R : Flourine has smallest size amongst all halogens and is highly electronegative

In the light of the above statements, choose the most appropriate answer from the options given below.

(A) Both A and R are correct and R is the correct explanation of A.

(B) Both A and R are correct but R is NOT the correct explanation of A.

(C) A is correct but R is not correct.

(D) A is not correct but R is correct

Official Ans. by NTA (A)

Sol. Both A and R are correct and R is the correct explanation of A.

8. In 3d series, the metal having the highest M^{2+}/M standard electrode potential is

(A) Cr (B) Fe

(C) Cu (D) Zn

Official Ans. by NTA (C)

Sol. $Cr^{+2}/Cr \rightarrow -0.90$ V

$Fe^{+2}/Fe \rightarrow -0.44$ V

$Cu^{+2}/Cu \rightarrow +0.34$ V

$Zn^{+2}/Zn \rightarrow -0.76$ V

So Ans. Cu^{+2}/Cu

9. The 'f' orbitals are half and completely filled, respectively in lanthanide ions

(Given: Atomic no. Eu, 63; Sm, 62; Tm, 69; Tb, 65; Yb, 70; Dy, 66]

(A) Eu^{2+} and Tm^{2+} (B) Sm^{2+} and Tm^{3+}

(C) Tb^{4+} and Yb^{2+} (D) Dy^{3+} and Yb^{3+}

Official Ans. by NTA (C)

Sol. $Tb \rightarrow 4f^9 6s^2$

$Tb^{+4} \rightarrow 4f^7$

$Yb \rightarrow 4f^{14} 6s^2$

$Yb^{+2} \rightarrow 4f^{14}$

10. Arrange the following coordination compounds in the increasing order of magnetic moments. (Atomic numbers: Mn = 25; Fe = 26)

(A) $[FeF_6]^{3-}$

(B) $[Fe(CN)_6]^{3-}$

(C) $[MnCl_6]^{3-}$ (high spin)

(D) $[Mn(CN)_6]^{3-}$

(A) A < B < D < C (B) B < D < C < A

(C) A < C < D < B (D) B < D < A < C

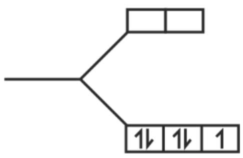
Official Ans. by NTA (B)

Sol. (A) $[FeF_6]^{3-}$

$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 5

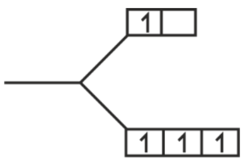
(B) $[Fe(CN)_6]^{3-}$



$Fe^{+3} \rightarrow 3d^5\ 4s^0$

n = 1

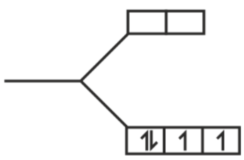
(C) $[MnCl_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 4

(D) $[Mn(CN)_6]^{3-}$



$Mn^{+3} \rightarrow 3d^4\ 4s^0$

n = 2

$\mu \Rightarrow A > C > D > B$

11. On the surface of polar stratospheric clouds, hydrolysis of chlorine nitrate gives A and B while its reaction with HCl produces B and C. A, B and C are, respectively

(A) HOCl, HNO_3, Cl_2

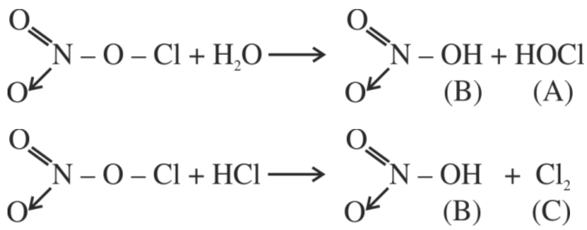
(B) Cl_2, HNO_3, HOCl

(C) $HClO_2$, HNO_2, HOCl

(D) HOCl, HNO_2, Cl_2O

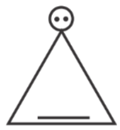
Official Ans. by NTA (A)

Sol.

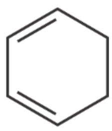


12. Which of the following is most stable?

(A) 

(B) 

(C) 

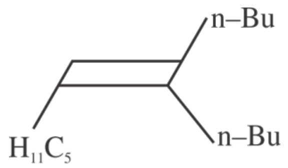
(D) 

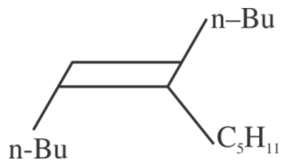
Official Ans. by NTA (A)

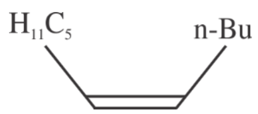
Sol.  is most stable as it is aromatic.

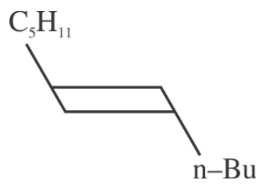
13. What will be the major product of following sequence of reactions?

$$\text{n}-\text{Bu}-\equiv \xrightarrow[\text{(ii) Lindlar cat, } H_2]{\text{(i) n}-\text{BuLi, n}-C_5H_{11}Cl}$$

(A) 

(B) 

(C) 

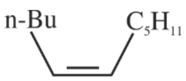
(D) 

Official Ans. by NTA (C)

Sol. $\text{n}-\text{Bu}-C\equiv CH \xrightarrow[\text{(acid base reaction)}]{\text{nBuLi}}$

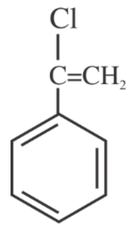
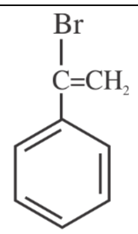
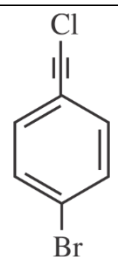
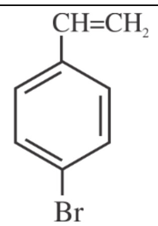
$\text{n}-\text{Bu}-C\equiv C^{-}Li^{+}$

$\text{n}-C_5H_{11}Cl \downarrow$ (SN reaction)

 $\xleftarrow[\text{Lindlar's Catalyst}]{H_2}$ $\text{n}-\text{Bu}-C\equiv C-C_5H_{11}$

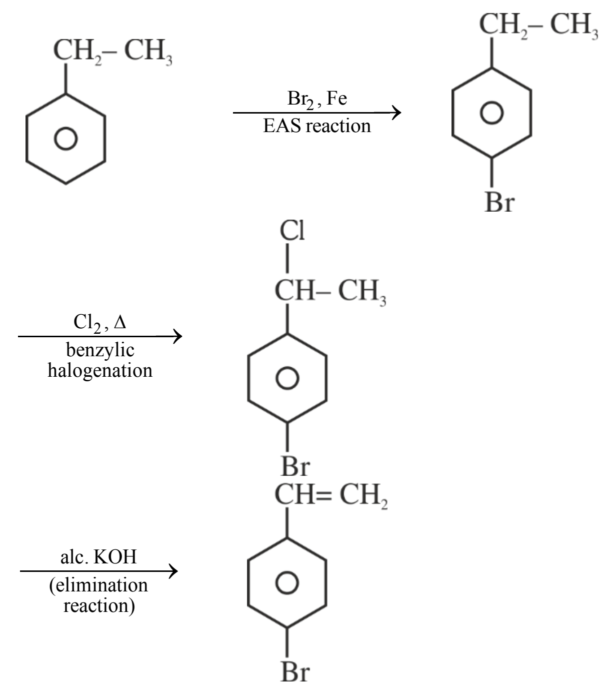
14. Product 'A' of following sequence of reactions is

$$\text{Ethylbenzene} \xrightarrow[\text{(b) } Cl_2,\Delta \text{ (c) alc. KOH}]{\text{(a) } Br_2.Fe} \text{'A'(Major product)}$$

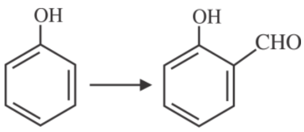
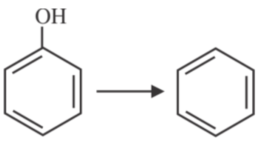
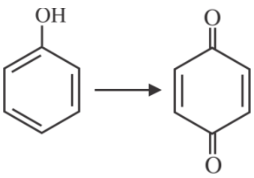
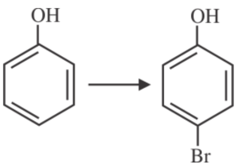
(A)	
(B)	
(C)	
(D)	

Official Ans. by NTA (D)

Sol.



15. Match List I with List II

List I	List II
A. 	I. Br_2 in CS_2
B. 	II. $Na_2Cr_2O_7/H_2SO_4$
C. 	III. Zn
D. 	IV. $CHCl_3/NaOH$

Choose the correct answer from the options given below:

(A) A-IV, B-III, C-II, D-I

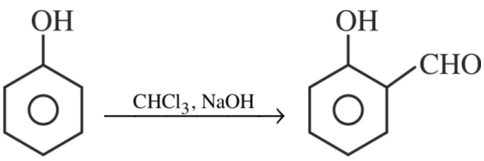
(B) A-IV, B-III, C-I, D-II

(C) A-II, B-III, C-I, D-IV

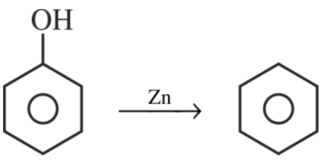
(D) A-IV, B-II, C-III, D-I

Official Ans. by NTA (A)

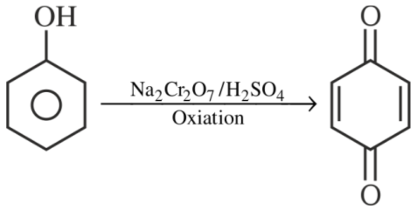
Sol. (A)



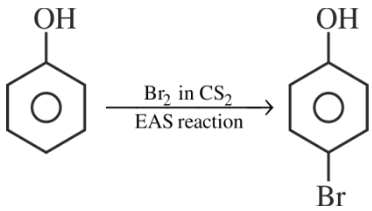
(B)



(C)



(D)



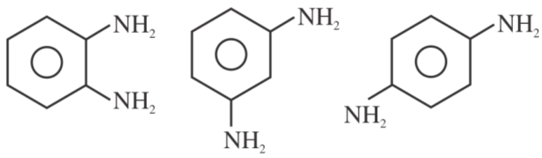
16. Decarboxylation of all six possible forms of diaminobenzoic acids $C_6H_3(NH_2)_2COOH$ yields three products A, B and C. Three acids give a product 'A', two acids gives a product 'B' and one acid give a product 'C'. The melting point of product 'C' is

(A) 63ºC (B) 90ºC

(C) 104ºC (D) 142ºC

Official Ans. by NTA (D)

Sol.



B A C

M.P. 142ºC

17. Which is true about Buna-N?

(A) It is a linear polymer of 1, 3-butadiene.

(B) It is obtained by copolymerization of 1, 3-butadiene and styrene.

(C) It is obtained by copolymerization of 1, 3-butadiene and acrylonitrile.

(D) The suffix N in Buna-N stands for its natural occurrence

Official Ans. by NTA (C)

Sol. It is copolymerization of 1, 3-butadiene and acrylonitrile.

18. Given below are two statements.

Statments I: Maltose has two α-D-glucose units linked at C_1 and C_4 and is a reducing sugar.

Statement II: Maltose has two monosaccharides: α-D-glucose and β-D-glucose linked at C_1 and C_6 and it is a non-reducing sugar.

In the light of the above statements, choose the correct answer from the options given below.

(A) Both Statement I and Statement II are true

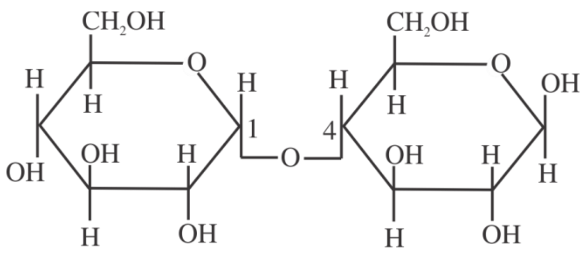
(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (C)

Sol.



Maltose

19. Match List I with List Ii

List I	List II
A. Antipyretic	I. Reduces pain
B. Analgesic	II. Reduces stress
C. Tranquilizer	III. Reduces fever
D. Antacid	IV. Reduces acidity (Stomach)

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-III, B-I, C-IV, D-II

(C) A-I, B-IV, C-II, D-III

(D) A-I, B-III, C-II, D-IV

Official Ans. by NTA (A)

Sol.

A. Antipyretic	Reduces fever
B. Analgesic	Reduces pain
C. Tranquilizer	Reduces stress
D. Antacid	Reduces acidity (Stomach)

20. Match List I with List II

List I (Anion)	List II (Gas evolved on reaction with dil. H_2SO_4)
A. CO_3^{2-}	I. Colourless gas which turns lead acetate paper black
B. S^{2-}	II. Colourless gas which turns acidified potassium dichromate solution green.
C. SO_3^{2-}	III. Brown fumes which turns acidified KI solution containing starch blue.
D. NO_2^-	IV. Colourless gas evolved with brisk effervescence, which turns lime water milky.

Choose the correct answer from the options given below:

(A) A-III, B-I, C-II, D-IV

(B) A-II, B-I, C-IV, D-III

(C) A-IV, B-I, C-III, D-II

(D) A-IV, B-I, C-II, D-III

Official Ans. by NTA (D)

Sol. CO_3^{2-} will give CO_2(g) which will turns lime water milky.

S^{2-} will give H_2S (g), will turns lead acetate paper black

SO_3^{2-} will give SO_2 (g), which will turns acidified potassium dichromate solution green.

NO_2^- will give brown NO_2(g) will turn KI solution blue.

SECTION-B

1. 116 g of a substance upon dissociation reaction, yields 7.5 g of hydrogen, 60g of oxygen and 48.5 g of carbon. Given that the atomic masses of H, O and C are 1, 16 and 12 respectively. The data agrees with how many formulae of the following?

(A) CH_3COOH (B) HCHO

(C) CH_3OOCH_3 (D) CH_3CHO

Official Ans. by NTA (2)

Sol. $\%H = \frac{7.5}{116} \times 100 = 6.5$

$\%O = \frac{60}{116} \times 100 = 51.7$

$\%C = \frac{48.5}{116} \times 100 = 41.8$

Relative atomicities = $H \Rightarrow 6.5$

$O \Rightarrow \frac{51.7}{16} = 3.25$

$C \Rightarrow \frac{41.8}{12} = 3.5$

Emperically formula is approx.. CH_2O

(A) $C_2H_4O_2$ (B) CH_2O relate to this formula.

2. Consider the following set of quantum numbers

	n	l	m_l
A.	3	3	-3
B.	3	2	-2
C.	2	1	+1
D.	2	2	+2

The number of correct sets of quantum numbers is _______

Official Ans. by NTA (2)

Sol. Quantum no. of set (B) and (C) can be correct.

(A) and (D) are wrong as $n = \ell$ is not possible.

3. BeO reacts with HF in presence of ammonia to give [A] which on thermal decomposition produces [B] and ammonium fluoride. Oxidation state of Be in [A] is_______

Official Ans. by NTA (2)

Sol.

$$BeO + HF + NH_3 \rightarrow \underset{[A]}{(NH_4)_2[BeF_4]}$$

$$\underset{[A]}{(NH_4)_2[BeF_4]} \xrightarrow{\Delta} NH_4F + \underset{[B]}{BeF_2}$$

Oxidation state of Be in A is (+2)

4. When 5 moles of He gas expand isothermally and reversibly at 300 K from 10 litre to 20 litre, the magnitude of the maximum work obtained is _____ J. [nearest integer] (Given: R = 8.3 J $K^{-1}mol^{-1}$ and log 2 = 0.3010)

Official Ans. by NTA (8630)

Sol. n = 5 mol

T = 300 K

$V_1 = 10$ L

$V_2 = 20$ L

$$w = -nRT\,\ell n\frac{V_2}{V_1}$$

$$= -5 \times 8.3 \times 300 \times \ell n\frac{20}{10}$$

$= -8630.38$ J

5. A solution containing 2.5×10^{-3} kg of a solute dissolved in 75×10^{-3} kg of water boils at 373.535 K. The molar mass of the solute is ______ g mol^{-1}. [nearest integer] (Given: K_b (H_2O) = 0.52 K Kg mol^{-1}, boiling point of water = 373.15K)

Official Ans. by NTA (45)

Sol. $w = 2.5$ g $\quad K_b = 0.52$

$w_{solvent} = 75$ g $\quad$ M = Mol. Wt. of solute

$T'_B = 373.535\,K$

$T_B^o = 373.15\,K$

$\Delta T_B = 0.385 = K_b$ molality

$$0.385 = 0.52 \times \left(\frac{2.5}{M} \times \frac{1000}{75}\right)$$

$M = 45$ g mol^{-1}

6. pH value of 0.001 M NaOH solution is______.

Official Ans. by NTA (11)

Sol. 0.001 M NaOH

$[OH^-] = 10^{-3}$

pOH = 3

pH = 11

7. For the reaction taking place in the cell:

$Pt(s) \mid H_2(g) \mid H^+(aq) \parallel Ag^+(aq) \mid Ag(s)$

$E^o_{Cell} = +0.5332$ V.

The value of $\Delta_f G^0$ is ________ kJ mol^{-1}. (in nearest integer)

Official Ans. by NTA (51)

Sol. $\frac{1}{2}H_2 + Ag^+ \rightarrow H^+ + Ag$

$\Delta G^\circ = -nE^\circ F$

$= -1 \times 0.5332 \times 96500$ J

$= -51.35$ kJ

$(n = 2 \text{ for } H_2 + 2Ag^+ \rightarrow 2H^+ + 2Ag)$

8. It has been found that for a chemical reaction with rise in temperature by 9K the rate constant gets doubled. Assuming a reaction to be occurring at 300 K, the value of activation energy is found to be ______ kJ mol^{-1}. [nearest integer]

(Given ln 10 = 2.3, R = 8.3 $JK^{-1}mol^{-1}$, log2 = 0.30)

Official Ans. by NTA (59)

Sol. $$\log_{10}\frac{K_2}{K_1} = \frac{E_a}{2.303R}\left(\frac{1}{300} - \frac{1}{309}\right)$$

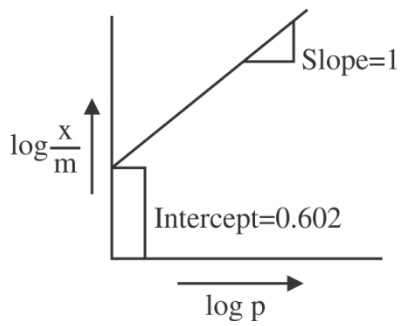
$$0.3 = \frac{E_a}{2.303 \times 8.3}\left(\frac{9}{300 \times 309}\right)$$

$$E_a = \frac{0.3 \times 2.303 \times 8.3 \times 300 \times 309}{9}$$

$= 59065.04$ J

$E_a = 59.06$ kJ

9.



If the initial pressure of a gas is 0.03 atm, the mass of the gas adsorbed per gram of the adsorbent is ______ $\times 10^{-2}$g.

Official Ans. by NTA (12)

Sol. $$\frac{x}{m} = kP^{\frac{1}{n}}$$

$$\log\frac{x}{m} = \log k + \frac{1}{n}\log P$$

From graph

$$\text{Slope} = \frac{1}{n} = 1 \Rightarrow n = 1$$

Intercept = log k = 0.602

k = 4

$$\frac{x}{m} = 4 \times (0.03)^{\frac{1}{1}}$$

$$\frac{x}{m} = 12 \times 10^{-2}$$

10. 0.25 g of an organic compound containing chlorine gave 0.40 g of silver chloride in Carius estimation. The percentage of chlorine present in the compound is ______. [in nearest integer]

(Given: Molar mass of Ag is 108 g mol^{-1} and that of Cl is 35.5 g mol^{-1})

Official Ans. by NTA (40)

Sol. wt. of organic compound = 0.25 g

$$\text{mass of Cl} = \frac{35.5}{143.5} \times 0.4\text{g}$$

mass % of Cl in the organic compound

$$= \frac{35.5 \times 0.4}{143.5 \times 0.25} \times 100$$

= 39.58%

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Monday 27th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. The number of points of intersection of $|z-(4+3i)|=2$ and $|z|+|z-4|=6$, $z \in C$ is :

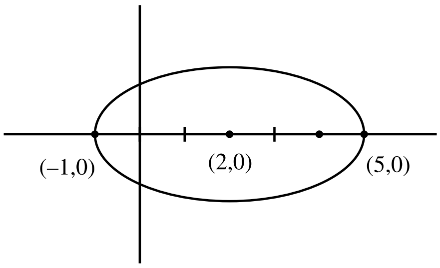
(A) 0 (B) 1

(C) 2 (D) 3

Official Ans. by NTA (C)

Sol. $C : (x-4)^2 + (y-3)^2 = 4$

$E : \frac{(x-2)^2}{9} + \frac{y^2}{5} = 1$



Lower Extremity of vertical diameter of circle $\rightarrow (4,1)$

Put in ellipse $\Rightarrow \frac{(4-2)^2}{9} + \frac{1}{5} - 1$

$= \frac{4}{9} + \frac{1}{5} - 1$

$= \frac{29}{45} - 1 < 0$

Two Solutions

Answer (C)

2. Let $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$, $a \in R$. Then the sum of which the squares of all the values of a for $2f'(10) - f'(5) + 100 = 0$ is :

(A) 117 (B) 106

(C) 125 (D) 136

Official Ans. by NTA (C)

Sol. $f(x) = \begin{vmatrix} a & -1 & 0 \\ ax & a & -1 \\ ax^2 & ax & a \end{vmatrix}$

$f(x) = a \begin{vmatrix} 1 & -1 & 0 \\ x & a & -1 \\ x^2 & ax & a \end{vmatrix}$

$= a\,[1\,(a^2 + ax) + 1\,(ax + x^2)]$

$\Rightarrow f(x) = a\,(x + a)^2$

so, $f'(x) = 2a\,(x + a)$

as, $2f'(10) - f'(5) + 100 = 0$

$\Rightarrow 2 \times 2a\,(10 + a) - 2a\,(5 + a) + 100 = 0$

$\Rightarrow 40a + 4a^2 - 10a - 2a^2 + 100 = 0$

$2a^2 + 30a + 100 = 0$

$\Rightarrow a^2 + 15a + 50 = 0$

$(a + 10)\,(a + 5) = 0$

$a = -10$ or $a = -5$

Required $= (-10)^2 + (-5)^2 = 125$

3. Let for some real numbers α and β, $a = \alpha - i\beta$. If the system of equations $4ix + (1+i)y = 0$ and $8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$ has more than one solution then $\frac{\alpha}{\beta}$ is equal to :

(A) $-2+\sqrt{3}$ (B) $2-\sqrt{3}$

(C) $2+\sqrt{3}$ (D) $-2-\sqrt{3}$

Official Ans. by NTA (B)

Sol. $a = \alpha - i\beta$; $\alpha \in R$; $\beta \in R$

$4ix + (1 + i)\, y = 0$ and

$$8\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)x + \bar{a}y = 0$$

$$\begin{vmatrix} 4i & 1+i \\ 8e^{i2\pi/3} & \bar{a} \end{vmatrix} = 0$$

$\Rightarrow 4i\bar{a} - (1 + i)\, 8e^{i2\pi/3} = 0$

$\Rightarrow 4i\,(\alpha + i\beta) - 8\,(1 + i)\left(\dfrac{-1+i\sqrt{3}}{2}\right) = 0$

$\Rightarrow i\alpha - \beta + 1 + \sqrt{3} + i\,(1 - \sqrt{3}) = 0$

$\Rightarrow \beta = \sqrt{3} + 1$

$\alpha = \sqrt{3} - 1$

So, $\dfrac{\alpha}{\beta} = \dfrac{\sqrt{3}-1}{\sqrt{3}+1} = 2 - \sqrt{3}$

4. Let A and B be two 3 × 3 matrices such that $AB = I$ and $|A| = \frac{1}{8}$ then $|\text{adj}(B\,\text{adj}(2A))|$ is equal to

(A) 16 (B) 32

(C) 64 (D) 128

Official Ans. by NTA (C)

Sol. $AB = i$

$|\text{adj}\,(B\ \text{adj}\,(2A)| = |B\ \text{adj}\,(2A)|^2$

$= |B|^2\ |\text{adj}\,(2A)|^2$

$= |B|^2\ (|2A|^2)^2 = |B|^2\ (2^6\ |A|^2)^2$

$|A| = \frac{1}{8}$ and $|AB| = 1 \Rightarrow |A|\,|B| = 1$

$\Rightarrow \frac{1}{8}\,|B| = 1$

$\Rightarrow |B| = 8$

required value = 64

5. Let $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots$ then 4S is equal to

(A) $\left(\frac{7}{3}\right)^2$ (B) $\frac{7^3}{3^2}$

(C) $\left(\frac{7}{3}\right)^3$ (D) $\frac{7^2}{3^3}$

Official Ans. by NTA (C)

Sol. $S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots$

Considering infinite sequence,

$$S = 2 + \frac{6}{7} + \frac{12}{7^2} + \frac{20}{7^3} + \frac{30}{7^4} + \ldots\ldots\ldots$$

$$\frac{S}{7} = \frac{2}{7} + \frac{6}{7^2} + \frac{12}{7^3} + \frac{20}{7^4} + \ldots\ldots\ldots$$

$$\Rightarrow \frac{6S}{7} = 2 + \frac{4}{7} + \frac{6}{7^2} + \frac{8}{7^3} + \frac{10}{7^4} + \ldots\ldots$$

$$\Rightarrow \frac{6S}{7^2} = \frac{2}{7} + \frac{4}{7^2} + \frac{6}{7^3} + \frac{8}{7^4} + \ldots\ldots\ldots$$

$$\frac{6S}{7}\left(1 - \frac{1}{7}\right) = 2 + \frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3} + \ldots\ldots\ldots$$

$$\Rightarrow \frac{6^2S}{7^2} = \frac{2}{1-\frac{1}{7}} = \frac{2}{6} \times 7$$

$$\Rightarrow S = \frac{2 \times 7^3}{6^3} \Rightarrow 4S = \frac{7^3}{3^3} = \left(\frac{7}{3}\right)^3$$

6. If $a_1, a_2, a_3 \ldots$ and $b_1, b_2, b_3 \ldots$ are A.P. and $a_1 = 2$, $a_{10} = 3$, $a_1b_1 = 1 = a_{10}b_{10}$ then a_4b_4 is equal to

(A) $\frac{35}{27}$ (B) 1

(C) $\frac{27}{28}$ (D) $\frac{28}{27}$

Official Ans. by NTA (D)

Sol. $a_1, a_2, a_3 \ldots$ A.P. ; $a_1 = 2$; $a_{10} = 3$; $d_1 = \frac{1}{9}$

$b_1, b_2, b_3, \ldots$ A.P. ; $b_1 = \frac{1}{2}$; $b_{10} = \frac{1}{3}$; $d_2 = \frac{-1}{54}$

[Using $a_1b_1 = 1 = a_{10}b_{10}$; d_1 & d_2 are common differences respectively]

$$a_4.b_4 = (2 + 3d_1)\left(\frac{1}{2} + 3d_2\right)$$

$$= \left(2 + \frac{1}{3}\right)\left(\frac{1}{2} - \frac{1}{18}\right)$$

$$= \left(\frac{7}{3}\right)\left(\frac{8}{18}\right) = \frac{28}{27}$$

7. If m and n respectively are the number of local maximum and local minimum points of the function $f(x) = \int_0^{x^2} \frac{t^2 - 5t + 4}{2 + e^t}\,dt$, then the ordered pair (m, n) is equal to

(A) (3, 2) (B) (2, 3)

(C) (2, 2) (D) (3, 4)

Official Ans. by NTA (B)

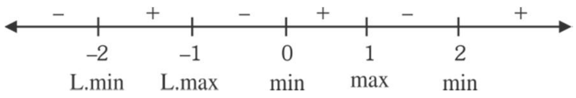
Sol. m = L · max

N = L · min

$$f(x)=\int_0^{x^2}\frac{t^2-5t+4}{2+e^t}dt$$

$$f'(x)=\frac{(x^4-5x^2+4)2x}{2+e^{x^2}}=\frac{2x(x^2-1)(x^2-4)}{2+e^{x^2}}$$

$$=\frac{2x(x-1)(x+1)(x-2)(x+2)}{2+e^{x^2}}$$



so, m = 2 and n = 3

8. Let f be a differentiable function in $\left(0,\frac{\pi}{2}\right)$.

If $\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x$ then $\frac{1}{\sqrt{3}}f'\left(\frac{1}{\sqrt{3}}\right)$ is equal to :

(A) $6-9\sqrt{2}$ (B) $6-\frac{9}{\sqrt{2}}$

(C) $\frac{9}{2}-6\sqrt{2}$ (D) $\frac{9}{\sqrt{2}}-6$

Official Ans. by NTA (B)

Sol. At right hand vicinity of x = 0 given equation does not satisfy

$\because$ LHS $=\int_{1^-}^{1} t^2 f(t)dt=0$, RHS $=\lim_{x\to 0^+}(\sin^3 x+\cos x)=1$

LHS ≠ RHS hence data given in question is wrong hence BONUS

Correct data should have been

$$\int_{\cos x}^{1} t^2 f(t)dt=\sin^3 x+\cos x-1$$

Calculation for option

differentiating both sides

$-\cos^2 x\, f(\cos x).(-\sin x) = 3\sin^2 x.\cos x-\sin x$

$\Rightarrow f(\cos x) = 3\tan x - \sec^2 x$

$\Rightarrow f'(\cos x)(-\sin x) = 3\sec^2 x - 2\sec^2 x\tan x$

$\Rightarrow f'(\cos x)\cos x = \frac{2}{\cos^2 x}-\frac{3}{\sin x.\cos x}$

When $\cos x = \frac{1}{\sqrt{3}}$; $\sin x = \frac{\sqrt{2}}{\sqrt{3}}$

$$\therefore f'\left(\frac{1}{\sqrt{3}}\right)\frac{1}{\sqrt{3}}=6-\frac{9}{\sqrt{2}}.$$

9. The integral $\int_0^1\frac{1}{7^{\left[\frac{1}{x}\right]}}dx$, where [.] denotes the greatest integer function is equal to

(A) $1+6\log_e\left(\frac{6}{7}\right)$ (B) $1-6\log_e\left(\frac{6}{7}\right)$

(C) $\log_e\left(\frac{7}{6}\right)$ (D) $1-7\log_e\left(\frac{6}{7}\right)$

Official Ans. by NTA (A)

Sol. $$\int_0^1\frac{1}{7^{\left[\frac{1}{x}\right]}}dx=-\int_1^0\frac{1}{7^{\left[\frac{1}{x}\right]}}dx$$

$$=(-1)\left[\int_1^{1/2}\frac{1}{7}dx+\int_{1/2}^{1/3}\frac{1}{7^2}dx+\int_{1/3}^{1/4}\frac{1}{7^3}dx+......\infty\right]$$

$$=\left(\frac{1}{7}+\frac{1}{2.7^2}+\frac{1}{3.7^3}+...\infty\right)-\left(\frac{1}{7.2}+\frac{1}{7^2.3}+\frac{1}{7^2.4}...\infty\right)$$

$$=-\ln\left(1-\frac{1}{7}\right)-7\left(\frac{1}{7^2.2}+\frac{1}{7^3.3}+\frac{1}{7^4.4}+.....\infty\right)$$

$$\left[\text{as } \ln(1+x)=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}....\infty\right]$$

$$\left[\text{as } \ln(1-x)=-\left(x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}....\infty\right)\right]$$

$$=-\ln\frac{6}{7}-7\left(-\ln\left(1-\frac{1}{7}\right)-\frac{1}{7}\right)$$

$$=6\ln\frac{6}{7}+1$$

10. If the solution curve of the differential equation $((\tan^{-1}y)-x)dy=(1+y^2)dx$ passes through the point (1, 0) then the abscissa of the point on the curve whose ordinate is tan (1) is :

(A) 2e (B) $\frac{2}{e}$

(C) 2 (D) $\frac{1}{e}$

Official Ans. by NTA (B)

Sol. $$\frac{dx}{dy}+\frac{x}{1+y^2}=\frac{\tan^{-1}y}{1+y^2}$$

$$\text{I.f}=e^{\int\frac{1}{1+y^2}dy}=e^{\tan^{-1}y}$$

$$xe^{\tan^{-1}y}=\int\frac{\tan^{-1}y}{1+y^2}e^{\tan^{-1}y}dy$$

$x \cdot e^{\tan^{-1} y} = \left(\tan^{-1} y - 1\right)e^{\tan^{-1} y} + c$

$\because$ (1, 0) lies exit c = 2.

For y = tan1 $\Rightarrow x = \frac{2}{e}$

11. If the equation of the parabola, whose vertex is at (5, 4) and the directrix is $3x+y-29=0$, is $x^2+ay^2+bxy+cx+dy+k=0$ then a + b + c +d + k is equal to

(A) 575 (B) –575

(C) 576 (D) –576

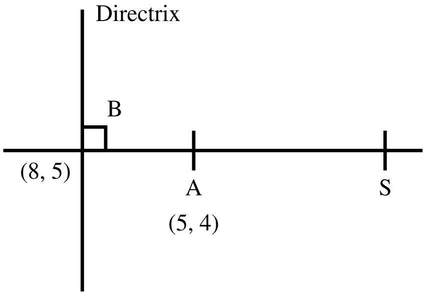
Official Ans. by NTA (D)

Sol. Vertex (5,4)

Directrix : 3x + y – 29 = 0

Co-ordinates of B (foot of directrix)

$\frac{x-5}{3} = \frac{y-4}{1} = -\left(\frac{15+4-29}{10}\right) = 1$



x = 8, y = 5

S = (2, 3) (focus)

Equation of parabola

PS = PM

so equation is

$x^2 + 9y^2 - 6xy + 134x - 2y - 711 = 0$

a + b + c + d + k = 9 – 6 + 134 – 2 – 711 = – 576

12. The set of values of k for which the circle C : $4x^2+4y^2-12x+8y+k=0$ lies inside the fourth quadrant and the point $\left(1, -\frac{1}{3}\right)$ lies on or inside the circle C is :

(A) An empty set (B) $\left(6, \frac{95}{9}\right]$

(C) $\left[\frac{80}{9}, 10\right)$ (D) $\left(9, \frac{92}{9}\right]$

Official Ans. by NTA (D)

Sol. C : $4x^2 + 4y^2 - 12x + 8y + k = 0$

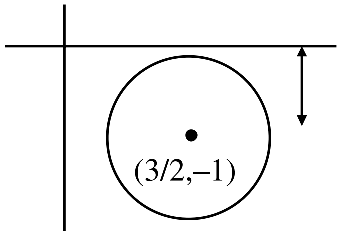
$\Rightarrow x^2 + y^2 - 3x + 2y + \left(\frac{k}{4}\right) = 0$

Centre $\left(\frac{3}{2}, -1\right)$; $r = \sqrt{\frac{13-k}{2}}$ $\Rightarrow k \le 13$ ... (1)

(i) Point $\left(1, \frac{-1}{3}\right)$ lies on or inside circle C

$\Rightarrow S_1 \le 0 \Rightarrow k \le \frac{92}{9}$ ... (2)

(ii) C lies in 4th quadrant



r < 1

$\Rightarrow \frac{\sqrt{13-k}}{2} < 1$

$\Rightarrow k < 9$ .... (3)

Hence (1) ∩ (2) ∩ (3) $\Rightarrow k \in \left(9, \frac{92}{9}\right]$

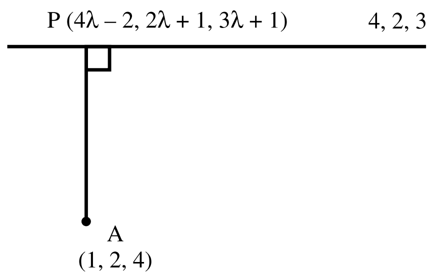
13. Let the foot of the perpendicular from the point (1, 2, 4) on the line $\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3}$ be P. Then the distance of P from the plane $3x+4y+12z+23=0$

(A) 5 (B) $\frac{50}{13}$

(C) 4 (D) $\frac{63}{13}$

Official Ans. by NTA (A)

Sol.



$$\frac{x+2}{4} = \frac{y-1}{2} = \frac{z+1}{3} = \lambda$$

$(x, y, z) = (4\lambda - 2, 2\lambda + 1, 3\lambda - 1)$

$\overrightarrow{AP} = (4\lambda - 3)\hat{i} + (2\lambda - 1)\hat{j} + (3\lambda - 5)\hat{k}$

$\vec{b} = 4\hat{i} + 2\hat{j} + 3\hat{k}$

$\overrightarrow{AP} \cdot \vec{b} = 0$

$4(4\lambda - 3) + 2(2\lambda - 1) + 3(3\lambda - 5) = 0$

$29\lambda = 12 + 2 + 15 = 29$

$\lambda = 1$

$P = (2, 3, 2)$

$3x + 4y + 12z + 23 = 0$

$$d = \left|\frac{6+12+24+23}{\sqrt{9+16+144}}\right|$$

$$d = \left|\frac{65}{13}\right| = 5$$

14. The shortest distance between the lines $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$ and $\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$ is :

(A) $\frac{18}{\sqrt{5}}$ (B) $\frac{22}{3\sqrt{5}}$

(C) $\frac{46}{3\sqrt{5}}$ (D) $6\sqrt{3}$

Official Ans. by NTA (A)

Sol. $\frac{x-3}{2} = \frac{y-2}{3} = \frac{z-1}{-1}$

$\frac{x+3}{2} = \frac{y-6}{1} = \frac{z-5}{3}$

$A = (3, 2, 1)$ $B = (-3, 6, 5)$

$\vec{n_1} = 2\hat{i} + 3\hat{j} - \hat{k}$

$\vec{n_2} = 2\hat{i} + \hat{j} - 3\hat{k}$

$\overrightarrow{BA} = 6\hat{i} - 4\hat{j} - 4\hat{k}$

$$\text{SHORTEST DISTANCE} = \frac{\left[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}\right]}{\left|\vec{n_1} \times \vec{n_2}\right|}$$

$$\vec{n_1} \times \vec{n_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & -1 \\ 2 & 1 & 3 \end{vmatrix}$$

$= 10\hat{i} - 8\hat{j} - 4\hat{k}$

$[\overrightarrow{BA}\ \vec{n_1}\ \vec{n_2}] = 60 + 32 + 16 = 108$

$|\vec{n_1} \times \vec{n_2}| = \sqrt{100+64+16} = \sqrt{180}$

$$\text{S.D} = \frac{108}{\sqrt{180}} = \frac{108}{6\sqrt{5}} = \frac{18}{\sqrt{5}}$$

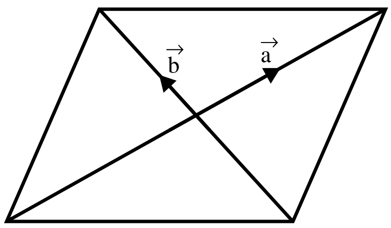
15. Let $\vec{a}$ and $\vec{b}$ be the vectors along the diagonal of a parallelogram having area $2\sqrt{2}$. Let the angle between $\vec{a}$ and $\vec{b}$ be acute. $|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$. If $\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$, then an angle between $\vec{b}$ and $\vec{c}$ is :

(A) $\frac{\pi}{4}$ (B) $-\frac{\pi}{4}$

(C) $\frac{5\pi}{6}$ (D) $\frac{3\pi}{4}$

Official Ans. by NTA (D)

Sol.



$$\text{Area} = \frac{1}{2}\left|\vec{a} \times \vec{b}\right| = 2\sqrt{2} \Rightarrow \left|\vec{a} \times \vec{b}\right| = 4\sqrt{2}$$

$|\vec{a}| = 1$ and $|\vec{a}.\vec{b}| = |\vec{a} \times \vec{b}|$

$\Rightarrow \cos\theta = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{4}$

$$\therefore \left|\vec{a} \times \vec{b}\right| = 4\sqrt{2} \Rightarrow |\vec{a}||\vec{b}|\sin\frac{\pi}{4} = 4\sqrt{2}$$

$\Rightarrow |\vec{b}| = 8$

Now, $\vec{c} = 2\sqrt{2}\left(\vec{a} \times \vec{b}\right) - 2\vec{b}$

$$|\vec{c}| = \sqrt{(2\sqrt{2})^2\left|\vec{a} \times \vec{b}\right|^2 + \left(2|\vec{b}|\right)^2} = 16\sqrt{2}$$

Now, $\vec{b}.\vec{c} = -2|\vec{b}|^2$

$\Rightarrow 8 \times 16\sqrt{2} \times \cos\alpha = -2.64$

$$\Rightarrow \cos\alpha = -\frac{1}{\sqrt{2}} \Rightarrow \alpha = \frac{3\pi}{4}$$

16. The mean and variance of the data4, 5, 6, 6, 7, 8, x, y where x < y are 6, and $\frac{9}{4}$ respectively. Then $x^4 + y^2$ is equal to

(A) 162 (B) 320
(C) 674 (D) 420

Official Ans. by NTA (B)

Sol. mean $\bar{x} = \frac{4+5+6+6+7+8+x+y}{8} = 6$

$\Rightarrow x + y = 48 - 36 = 12$

Variance

$= \frac{1}{8}(16+25+36+36+49+64+x^2+y^2) - 36 = \frac{9}{4}$

$\Rightarrow x^2 + y^2 = 80$

$\therefore x = 4; y = 8$

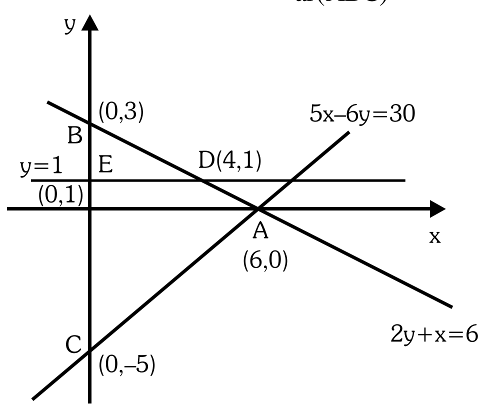
$x^4 + y^2 = 256 + 64 = 320$

17. If a point A(x, y) lies in the region bounded by the y-axis, straight lines 2y + x = 6 and 5x – 6y = 30, then the probability that y < 1 is :

(A) $\frac{1}{6}$ (B) $\frac{5}{6}$
(C) $\frac{2}{3}$ (D) $\frac{6}{7}$

Official Ans. by NTA (B)

Sol. Required probability $= \frac{ar(ADEC)}{ar(ABC)}$



$= 1 - \frac{ar(BDE)}{ar(ABC)}$

$= 1 - \frac{\frac{1}{2}\times 2\times 4}{\frac{1}{2}\times 8\times 6} = 1 - \frac{1}{6} = \frac{5}{6}$

18. The value of $\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right)$ is

(A) $\frac{26}{25}$ (B) $\frac{25}{26}$
(C) $\frac{50}{51}$ (D) $\frac{52}{51}$

Official Ans. by NTA (A)

Sol. $\tan^{-1}\frac{1}{1+n+n^2} = \tan^{-1}\left(\frac{(n+1)-n}{1+n(n+1)}\right)$

$= \tan^{-1}(n+1) - \tan^{-1} n$

so, $\sum_{n=1}^{50}(\tan^{-1}(n+1) - \tan^{-1} n)$

$= \tan^{-1} 51 - \tan^{-1} 1$

$\cot\left(\sum_{n=1}^{50}\tan^{-1}\left(\frac{1}{1+n+n^2}\right)\right) = \cot(\tan^{-1} 51 + \tan^{-1} 1)$

$= \frac{1}{\tan(\tan^{-1}51 - \tan^{-1}1)} = \frac{1+51\times 1}{51-1} = \frac{52}{50} = \frac{26}{25}$

19. $\alpha = \sin 36°$ is a root of which of the following equation

(A) $10x^4 - 10x^2 - 5 = 0$ (B) $16x^4 + 20x^2 - 5 = 0$

(C) $16x^4 - 20x^2 + 5 = 0$ (D) $16x^4 - 10x^2 + 5 = 0$

Official Ans. by NTA (C)

Sol. $\cos 72° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 1 - 2\sin^2 36° = \frac{\sqrt{5}-1}{4}$

$\Rightarrow 4 - 8\alpha^2 = \sqrt{5} - 1$

$\Rightarrow 5 - 8\alpha^2 = \sqrt{5}$

$\Rightarrow (5 - 8\alpha^2)^2 = 5$

$\Rightarrow 25 + 64\alpha^4 - 80\alpha^2 = 5$

$\Rightarrow 64\alpha^4 - 80\alpha^2 + 20 = 0$

$\Rightarrow 16\alpha^4 - 20\alpha^2 + 5 = 0$

20. Which of the following statement is a tautology?

(A) $((\sim q)\wedge p)\wedge q$

(B) $((\sim q)\wedge p)\wedge(p\wedge(\sim p))$

(C) $((\sim q)\wedge p)\vee(p\vee(\sim p))$

(D) $(p\wedge q)\wedge(\sim(p\wedge q))$

Official Ans. by NTA (C)

Sol. (A) $(\sim q \wedge p)\wedge q = (\sim q\wedge q)\wedge p = f$

(B) $(\sim q\wedge p)\wedge(p\wedge \sim p) = \sim q\wedge(p\wedge\sim p) = f$

(C) $(\sim q\wedge p)\vee(p\vee\sim p) = (\sim q\wedge p)\vee(t) = t$

(D) $(p\wedge q)\wedge(\sim(p\wedge q)) = f$

SECTION-B

1. Let S = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}. Define

$f: S \to S$ as $f(n)=\begin{cases} 2n, & \text{if } n=1,2,3,4,5 \\ 2n-11 & \text{if } n=6,7,8,9,10 \end{cases}$.

Let g : S → S be a function such that $fog(n)=\begin{cases} n+1 & ,\text{if } n \text{ is odd} \\ n-1 & ,\text{if } n \text{ is even} \end{cases}$, then

g(10) ((g(1) + g(2) + g(3) + g(4) + g(5)) is equal to:

Official Ans. by NTA (190)

Sol. $f^{-1}(n)=\begin{cases} \dfrac{n}{2} & ; \quad n=2,4,6,8,10 \\ \dfrac{n+11}{2} & ; \quad n=1,3,5,7,9 \end{cases}$

$f(g(n))=\begin{cases} n+1 & ; \quad n \in \text{odd} \\ n-1 & ; \quad n \in \text{even} \end{cases}$

$\Rightarrow \quad g(n)=\begin{cases} f^{-1}(n+1) & ; \quad n \in \text{odd} \\ f^{-1}(n-1) & ; \quad n \in \text{even} \end{cases}$

$\therefore \quad g(n)=\begin{cases} \dfrac{n+1}{2} & ; \quad n \in \text{odd} \\ \dfrac{n+10}{2} & ; \quad n \in \text{even} \end{cases}$

$g(10) \cdot [g(1)+g(2)+g(3)+g(4)+g(5)]$

$= 10 \cdot [1+6+2+7+3] = 190$

2. Let α, β be the roots of the equation $x^2-4\lambda x+5=0$ and α, γ be the roots of the equation $x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+7+3\lambda\sqrt{3}=0$.

If $\beta+\gamma=3\sqrt{2}$, then $(\alpha+2\beta+\gamma)^2$ is equal to :

Official Ans. by NTA (98)

Sol. $x^2-4\lambda x+5=0 \langle^{\alpha}_{\beta}$

$x^2-\left(3\sqrt{2}+2\sqrt{3}\right)x+\left(7+3\lambda\sqrt{3}\right)=0 \langle^{\alpha}_{\gamma}$

$\alpha+\beta=4\lambda$

$\alpha+\gamma=3\sqrt{2}+2\sqrt{3}$

$\beta+\lambda=3\sqrt{2} \qquad \alpha\gamma=7+3\lambda\sqrt{3}$

$\therefore \quad \alpha=2\lambda+\sqrt{3} \qquad \alpha\beta=5$

$\beta=2\lambda-\sqrt{3} \qquad 4\lambda^2=8 \Rightarrow \lambda=\sqrt{2}$

$\therefore \quad (\alpha+2\beta+\lambda)^2=\left(4\alpha+3\sqrt{2}\right)^2=\left(7\sqrt{2}\right)^2=98$

3. Let A be a matrix of order 2 × 2, whose entries are from the set {0, 1, 2, 3, 4, 5}. If the sum of all the entries of A is a prime number p, 2 < p < 8, then the number of such matrices A is :

Official Ans. by NTA (180)

Sol. Let $A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

$a+b+c+d=p$, $p \in \{3, 5, 7\}$

Case-(i)

$a+b+c+d=3$; a, b, c, d ∈ {0, 1, 2, 3}

No. of ways $= {}^{3+4-1}C_{4-1}={}^{6}C_3=56$ .... (1)

Case-(ii)

$a+b+c+d=5$; a, b, c, d ∈ {0, 1, 2, 3, 4, 5}

No. of ways $= {}^{5+4-1}C_{4-1}={}^{8}C_3=56$ .... (2)

Case-(iii)

$a+b+c+d=7$

No. of ways = total ways when a, b, c, d ∈ {0, 1, 2, 3, 4, 5, 6, 7}– total ways when a, b, c, d ∉ {6, 7}

No of ways $={}^{7+4-1}C_{4-1}=\left(\dfrac{\lfloor 4}{\lfloor 3}+\dfrac{\lfloor 4}{\lfloor 2}\right)$

$={}^{10}C_3-16=104$ .... (3)

Hence total no. of ways = 180

4. If the sum of the coefficients of all the positive powers of x, in the binomial expansion of $\left(x^n+\dfrac{2}{x^5}\right)^7$ is 939, then the sum of all the possible integral values of n is :

Official Ans. by NTA (57)

Sol. coefficients and there cumulative sum are :

Coefficient	Commulative sum
$x^{7n} \to {}^7C_0$	1
$x^{6n-5} \to 2\cdot{}^7C_1$	$1+14$
$x^{5n-10} \to 2^2\cdot{}^7C_2$	$1+14+84$
$x^{4n-15} \to 2^3\cdot{}^7C_3$	$1+14+84+280$
$x^{3n-20} \to 2^4\cdot{}^7C_4$	$1+4+84+280+560=939$
$x^{2n-25} \to 2^5\cdot{}^7C_5$	

$3n-20 \ge 0 \cap 2n-25<0 \cap n \in I$

$\therefore \quad 7 \le n \le 12$

Sum = 7 + 8 + 9 + 10 + 11 + 12 = 57

5. Let [t] denote the greatest integer ≤ t and {t} denote the fractional part of t. Then integral value of α for which the left hand limit of the function $f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$ at x = 0 is equal to $\alpha-\dfrac{4}{3}$ is ____

Official Ans. by NTA (3)

Sol. $f(x)=[1+x]+\dfrac{\alpha^{2[x]+\{x\}}+[x]-1}{2[x]+\{x\}}$

$\lim\limits_{x\to0^-} f(x)=\alpha-\dfrac{4}{3} \Rightarrow 0+\dfrac{\alpha^{-1}-2}{-1}=\alpha-\dfrac{4}{3}$

$\Rightarrow 2-\dfrac{1}{\alpha}=\alpha-\dfrac{4}{3}$

$\Rightarrow \alpha+\dfrac{1}{\alpha}=\dfrac{10}{3}$

$\Rightarrow \alpha = 3;\ \alpha \in I$

6. If $y(x)=(x^{x^x}),\ x>0$ then $\dfrac{d^2x}{dy^2}+20$ at x = 1 is equal to:

Official Ans. by NTA (16)

Sol. $y = (x) = (x^x)^x$

$ln\ y(x) = x^2 \cdot ln\ x$

$\dfrac{1}{y(x)} \cdot y'(x) = \dfrac{x^2}{x} + 2x \cdot ln\ x$

$y'(x) = y(x)\,[x + 2x\ ln\ x]$

$y(1) = 1;\ y'(1) = 1$

$y''(x) = y'(x)\,[x + 2x \cdot ln\,(x)]$

$+ y\,(x)\,[1 + 2(1 + ln\ x)]$

$y''(1) = 1\,[1 + 0] + 1\,(1 + 2) = 4$

$\dfrac{d^2y}{dx^2} = -\left(\dfrac{dy}{dx}\right)^3 \cdot \dfrac{d^2x}{dy^2}$

$\Rightarrow 4 = -\dfrac{d^2x}{dy^2}$

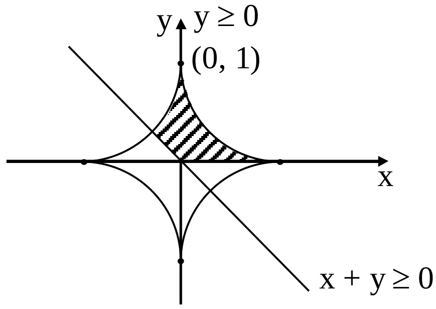
$\dfrac{d^2x}{dy^2} = -4$

Ans. –4 + 20 = 16

7. If the area of the region $\left\{(x,y): x^{\frac{2}{3}}+y^{\frac{2}{3}} \le 1\ x+y \ge 0,\ y \ge 0\right\}$ is A, then $\dfrac{256A}{\pi}$ is

Official Ans. by NTA (36)

Sol.



$A=\dfrac{3}{2}\int_0^1 (1-x^{2/3})^{3/2}dx$

Let $x = \sin^3\theta$

$A=\dfrac{3}{2}\int_0^{\pi/2}\left(1-\sin^2\theta\right)^{3/2}.3\sin^2\theta\cos\theta\, d\theta$

$=\dfrac{3}{2}\int_0^{\pi/2} 3\sin^2\theta\cos^4\theta\, d\theta$

$=\dfrac{9}{2}\int_0^{\pi/2} \sin^2\theta\cos^4\theta\, d\theta$

$A=\dfrac{9}{2}\times\dfrac{1.3.1}{(2+4)(4)(2)}.\dfrac{\pi}{2}$

$\Rightarrow A=\dfrac{9\pi}{64} \Rightarrow \dfrac{64A}{\pi}=9$

$\Rightarrow \dfrac{256A}{\pi}=36$ **Ans.**

8. Let v be the solution of the differential equation $(1-x^2)dy=\left(xy+(x^3+2)\sqrt{1-x^2}\right)dx,\ -1<x<1$ and y (0) = 0 if $\int_{-\frac{1}{2}}^{\frac{1}{2}}\sqrt{1-x^2}\,y(x)dx=k$ then k^{-1} is equal to :

Official Ans. by NTA (320)

Sol. $(1-x^2)\dfrac{dy}{dx} = xy + (x^3+2)\sqrt{1-x^2}$

$\Rightarrow \dfrac{dy}{dx} + \left(\dfrac{-x}{1-x^2}\right)y = \dfrac{x^3+2}{\sqrt{1-x^2}}$

$\text{IF} = e^{\int \frac{-x}{1-x^2}dx} = \sqrt{1-x^2}$

$y(x)\cdot\sqrt{1-x^2} = \dfrac{x^4}{4} + 2x + c$

$y(0) = 0 \Rightarrow c = 0$

$\sqrt{1-x^2}\,y(x) = \dfrac{x^4}{4} + 2x$

required value $= \displaystyle\int_{-1/2}^{1/2}\left(\frac{x^4}{4}+2x\right)dx - \frac{1}{4}\cdot 2\int_0^{1/2} x^4 dx$

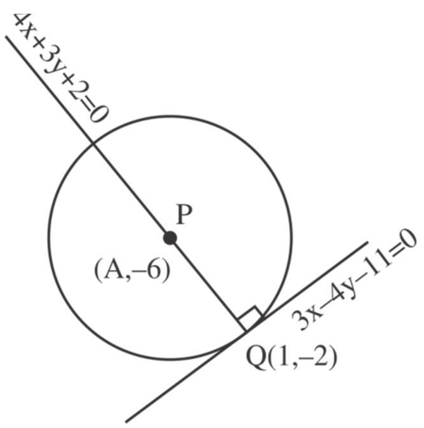
$= \dfrac{1}{10}\left(x^5\right)_0^{1/2} = \dfrac{1}{320}$

$k^{-1} = 320$

9. Let a circle C of radius 5 lie below the x-axis. The line $L_1 = 4x+3y-2$ passes through the centre P of the circle C and intersects the line $L_2: 3x-4y-11=0$ at Q. The line L_2 touches C at the point Q. Then the distance of P from the line $5x-12y+51=0$ is

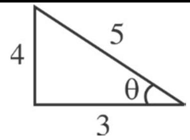
Official Ans. by NTA (11)

Sol.



$4x + 3y + 2 = 0$

$3x - 4y - 11 = 0$



$\dfrac{x}{-25} = \dfrac{y}{50} = \dfrac{1}{-25}$

$\dfrac{x-1}{\cos\theta} = \dfrac{y+2}{\sin\theta} = \pm 5$

$y = -2 + 5\left(-\dfrac{4}{5}\right) = -6$

$x = 1 + 5\left(\dfrac{3}{5}\right) = 4$

Req. distance

$\left|\dfrac{5(4)-12(-6)+51}{13}\right|$

$= \left|\dfrac{20+72+51}{13}\right|$

$= \dfrac{143}{13} = 11$

10. Let $S = \{E, E_2 .... E_8\}$ be a sample space of random experiment such that $P(E_n) = \dfrac{n}{36}$ for every n = 1, 2….8. Then the number of elements in the set $\left\{A \subset S : P(A) \geq \dfrac{4}{5}\right\}$ is ______

Official Ans. by NTA (19)

Sol. $P(A') < \dfrac{1}{5} = \dfrac{36}{180}$

5 times the sum of missing number should be less than 36.

If 1 digit is missing = 7

If 2 digit is missing = 9

If 3 digit is missing = 2

If 0 digit is missing = 1

Alternate

A is subset of S hence

A can have elements:

type 1 : { }

type 2: $\{E_1\}, \{E_2\}, \ldots\ldots \{E_8\}$

type 3: $\{E_1, E_2\}, \{E_1, E_3\}\ldots\ldots \{E_1, E_8\}$

$\vdots$

$\vdots$

type 6: $\{E_1, E_2, \ldots\ldots. E_5\}, \ldots\ldots.\{E_4, E_5, E_6, E_7, E_8\}$

type 7: $\{E_1, E_2, \ldots\ldots. E_6\}, \ldots\ldots.\{E_3, E_4, \ldots\ldots\ldots.. E_8\}$

type 8: $\{E_1, E_2, \ldots\ldots. E_7\}\{E_2, E_3, \ldots\ldots\ldots E_8\}$

type 9: $\{E_1, E_2, \ldots\ldots. E_8\}$

As $P(A) \geq \frac{4}{5}$;

Note : Type 1 to Type 4 elements can not be in set A as maximum probability of type 4 elements.

$\{E_5, E_6, E_7, E_8\}$ is $\frac{5}{36}+\frac{6}{36}+\frac{7}{36}+\frac{8}{36}=\frac{13}{18}<\frac{4}{5}$

Now for Type 5 acceptable elements let's call probability as P_5

$$P_5 = \frac{n_1+n_2+n_3+n_4+n_5}{36} \leq \frac{4}{5}$$

$\Rightarrow n_1 + n_2 + n_3 + n_4 + n_5 \geq 28.8$

Hence, 2 possible ways $\{E_5, E_6, E_7, E_8, \mathbf{E_3}$ **or** $\mathbf{E_4}\}$

$P_6 = n_1 + n_2 + n_3 + n_4 + n_5 + n_6 \geq 28.8$

$\Rightarrow$ 9 possible ways

$P_7 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots. + n_7 \geq 288$

$\Rightarrow$ 7 possible ways

$P_8 \Rightarrow n_1 + n_2 + \ldots\ldots\ldots. + n_8 \geq 28.8$

$\Rightarrow$ 1 possible way

Total = 19