FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS | **TEST PAPER WITH SOLUTON**

SECTION-A

1. The dimensions of $\left(\frac{B^2}{\mu_0}\right)$ will be :

(if μ_0 : permeability of free space and

B : magnetic field)

(A) $[M L^2 T^{-2}]$ (B) $[M L T^{-2}]$

(C) $[M L^{-1} T^{-2}]$ (D) $[M L^2 T^{-2} A^{-1}]$

Official Ans. by NTA (C)

Sol. $u = \frac{B^2}{2\mu_0}$

$u \rightarrow$ Energy per unit volume

$$\left[\frac{B^2}{\mu_0}\right] = [u] = \frac{[ML^2T^{-2}]}{[L^3]} = [ML^{-1}T^{-2}]$$

2. A NCC parade is going at a uniform speed of 9 km/h under a mango tree on which a monkey is sitting at a height of 19.6 m. At any particular instant, the monkey drops a mango. A cadet will receive the mango whose distance from the tree at time of drop is :

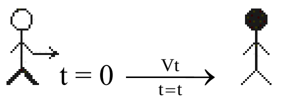
(Given g = 9.8 m/s^2)

(A) 5 m (B) 10 m

(C) 19.8 m (D) 24.5 m

Official Ans. by NTA (A)

Sol. Monkey



Time taken by mango $= \sqrt{\frac{2n}{g}}$

$= \sqrt{\frac{2\times 19.6}{9.8}} = 2$ second

Distance = vt

$= 9 \times \frac{5}{18} \times 2 = 5\text{m}$

3. In two different experiments, an object of mass 5 kg moving with a speed of 25 ms^{-1} hits two different walls and comes to rest within
(i) 3 second, (ii) 5 seconds, respectively.
Choose the correct option out of the following :

(A) Impulse and average force acting on the object will be same for both the cases.

(B) Impulse will be same for both the cases but the average force will be different.

(C) Average force will be same for both the cases but the impulse will be different.

(D) Average force and impulse will be different for both the cases.

Official Ans. by NTA (B)

Sol. Impulse = change in momentum

$I = \Delta P$

$F_{aug} = \frac{\Delta P}{\Delta t}$

$\Delta t_1 = 3 \qquad \Delta t_2 = 5$

$\Delta P_1 = \Delta P_2$

$I_1 = I_2$

F_{avg} in case (i) is more than (ii)

4. A balloon has mass of 10 g in air. The air escapes from the balloon at a uniform rate with velocity 4.5 cm/s. If the balloon shrinks in 5 s completely. Then, the average force acting on that balloon will be (in dyne).

(A) 3 (B) 9 (C) 12 (D) 18

Official Ans. by NTA (B)

Sol. $F = \frac{dm}{dt} v$

$= \frac{10g}{5s}\left(4.5\frac{cm}{s}\right) = 9\frac{gcm}{s^2} = 9$ dyne

5. If the radius of earth shrinks by 2% while its mass remains same. The acceleration due to gravity on the earth's surface will approximately :

(A) decrease by 2% (B) decrease by 4%

(C) increase by 2% (D) increase by 4%

Official Ans. by NTA (D)

Sol. $g = \frac{GM}{R^2}$

M = constant $g < \frac{1}{R^2}$

$100\frac{\Delta g}{g} = -2\frac{\Delta R}{R}100$

% change = – 2 (–2)

% change in g = 4%

increase by 4%

6. The force required to stretch a wire of cross-section 1 cm^2 to double its length will be :

(Given Yong's modulus of the wire = 2×10^{11} N/m^2)

(A) 1×10^7 N (B) 1.5×10^7 N

(C) 2×10^7 N (D) 2.5×10^7 N

Official Ans. by NTA (C)

Sol. $F = \gamma A \frac{\Delta \ell}{\ell}$

$= 2 \times 10^{11} \times 10^{-4}\left(\frac{2\ell - \ell}{\ell}\right)$

$= 2 \times 10^7$ N

7. A Carnot engine has efficiency of 50%. If the temperature of sink is reduced by 40°C, its efficiency increases by 30%. The temperature of the source will be :

(A) 166.7 K (B) 255.1 K

(C) 266.7 K (D) 367.7 K

Official Ans. by NTA (C)

Sol. $\eta = 1 - \frac{T_L}{T_H}$

$\frac{1}{2} = 1 - \frac{T_L}{T_H}$

$\frac{1}{2}(1 \cdot 3) = 1 - \left(\frac{T_L - 40}{T_H}\right)$

$\frac{1}{2}(1 \cdot 3) = \frac{1}{2} + \frac{40}{T_H}$ $T_H = 266.7\,K$

8. Given below are two statements :

Statement I : The average momentum of a molecule in a sample of an ideal gas depends on temperature.

Statement II : The rms speed of oxygen molecules in a gas is v. If the temperature is doubled and the oxygen molecules dissociate into oxygen atoms, the rms speed will become 2v.

In the light of the above statements, choose the correct answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

Official Ans. by NTA (D)

Sol. $[P_{avg} = 0]$ (due to random motion)

$v_{rms} = \sqrt{\frac{3RT}{M}}$

$T_{new} = 2T$

$M_{new} = \frac{M}{2}$

$\frac{v_{new}}{v} = \frac{\sqrt{\frac{2T}{M/2}}}{\sqrt{\frac{T}{M}}}$

$v_{new} = 2v$

9. In the wave equation

$$y = 0.5\sin\frac{2\pi}{\lambda}(400t - x)m$$

the velocity of the wave will be :

(A) 200 m/s (B) $200\sqrt{2}$ m/s

(C) 400 m/s (D) $400\sqrt{2}$ m/s

Official Ans. by NTA (C)

Sol. $y = 0.5\sin\left(\frac{2\pi}{\lambda}400t - \frac{2\pi}{\lambda}x\right)$

$$\omega = \frac{2\pi}{\lambda}400$$

$$K = \frac{2\pi}{\lambda}$$

$$v = \frac{\omega}{k} \qquad [v = 400 \text{ m/s}]$$

10. Two capacitors, each having capacitance $40\ \mu F$ are connected in series. The space between one of the capacitors is filled with dielectric material of dielectric constant K such that the equivalence capacitance of the system became $24\ \mu F$. The value of K will be :

(A) 1.5 (B) 2.5

(C) 1.2 (D) 3

Official Ans. by NTA (A)

Sol.

$$C_{eq} = \frac{C(KC)}{C + KC} = \frac{KC}{K+1}$$

$$24 = \frac{K40}{K+1}$$

$$[K = 1.5]$$

11. A wire of resistance R_1 is drawn out so that its length is increased by twice of its original length. The ratio of new resistance to original resistance is:

(A) 9 : 1 (B) 1 : 9

(C) 4 : 1 (D) 3 : 1

Official Ans. by NTA (A)

Sol. $R_1 = \rho\frac{L_1}{A_1}$

$$R_2 = \rho\left(\frac{3L_1}{A_1/3}\right) = 9\rho\frac{L_1}{A_1}$$

$$\therefore \frac{R_2}{R_1} = 9$$

12. The current sensitivity of a galvanometer can be increased by :

(A) decreasing the number of turns

(B) increasing the magnetic field

(C) decreasing the area of the coil

(D) decreasing the torsional constant of the spring

Choose the most appropriate answer from the options given below :

(A) (B) and (C) only (B) (C) and (D) only

(C) (A) and (C) only (D) (B) and (D) only

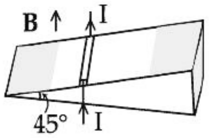
Official Ans. by NTA (D)

Sol. $i = \left(\frac{K}{NAB}\right)\theta$

$$\therefore \frac{d\theta}{di} = \frac{NAB}{K}$$

13. As shown in the figure, a metallic rod of linear density 0.45 kg m^{-1} is lying horizontally on a smooth incline plane which makes an angle of 45° with the horizontal. The minimum current flowing in the rod required to keep it stationary, when 0.15 T magnetic field is acting on it in the vertical upward direction, will be :

{Use $g = 10\ m/s^2$}

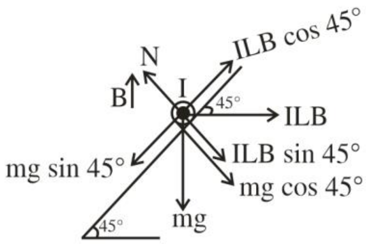


(A) 30 A (B) 15 A

(C) 10 A (D) 3 A

Official Ans. by NTA (A)

Sol.



$$mg\sin 45° = ILB\cos 45°$$

$$\therefore I = \left(\frac{m}{L}\right)\frac{g}{B}$$

$$= \frac{(0.45)(10)}{0.15} = 30\ A$$

14. The equation of current in a purely inductive circuit is $5\sin(49\pi t - 30°)$. If the inductance is 30 mH then the equation for the voltage across the inductor, will be :

$\left\{\text{Let } \pi = \frac{22}{7}\right\}$

(A) $1.47\sin(49\pi t - 30°)$ (B) $1.47\sin(49\pi t + 60°)$

(C) $23.1\sin(49\pi t - 30°)$ (D) $23.1\sin(49\pi t + 60°)$

Official Ans. by NTA (D)

Sol. $V_0 = i_0 X_L$

$= i_0(wL)$

$= (5)(49\pi)(30\times10^{-3})$

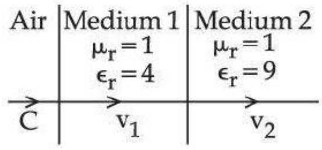
$= 23.1$

Voltage will lead current by 90°.

$\therefore V = 23.1\sin(49\pi t + 60°)$

15. As shown in the figure, after passing through the medium 1. The speed of light v_2 in medium 2 will be :

(Given $c = 3 \times 10^8\ ms^{-1}$)



(A) $1.0 \times 10^8\ ms^{-1}$ (B) $0.5 \times 10^8\ ms^{-1}$

(C) $1.5 \times 10^8\ ms^{-1}$ (D) $3.0 \times 10^8\ ms^{-1}$

Official Ans. by NTA (A)

Sol. $\frac{\mu_2}{\mu_{air}} = \frac{C}{v_2}$

$\therefore \frac{\sqrt{\mu_{r_2}\varepsilon_{r_2}}}{(1)} = \frac{C}{v_2}$

$\therefore \sqrt{(1)(9)} = \frac{C}{v_2}$

$\therefore v_2 = \frac{C}{3}$

16. In normal adjustment, for a refracting telescope, the distance between objective and eye piece is 30 cm. The focal length of the objective, when the angular magnification of the telescope is 2, will be:

(A) 20 cm (B) 30 cm

(C) 10 cm (D) 15 cm

Official Ans. by NTA (A)

Sol. $f_0 + f_e = 30$

$m = \frac{f_0}{f_e}$

$2 = \frac{f_0}{f_e} \Rightarrow f_0 = 2f_e$

So $f_0 + \frac{f_0}{2} = 30$

$f_0 = 20\,\text{cm}$

17. The equation $\lambda = \frac{1.227}{x}\text{nm}$ can be used to find the de-Brogli wavelength of an electron. In this equation x stands for :

Where,

m = mass of electron

P = momentum of electron

K = Kinetic energy of electron

V = Accelerating potential in volts for electron

(A) $\sqrt{mK}$ (B) $\sqrt{P}$

(C) $\sqrt{K}$ (D) $\sqrt{V}$

Official Ans. by NTA (D)

Sol. $\lambda = \frac{h}{m\nu}$ (de-Broglie's wavelength)

$\lambda \frac{h}{\sqrt{2m(K \cdot E)}}$

$h = \frac{h}{\sqrt{2mqV}}$

Putting the values of m ; q

We get $\lambda = \frac{1 \cdot 22}{\sqrt{V}}\text{nm}$

18. The half life period of a radioactive substance is 60 days. The time taken for $\frac{7}{8}$th of its original mass to disintegrate will be :

(A) 120 days (B) 130 days

(C) 180 days (D) 20 days

Official Ans. by NTA (C)

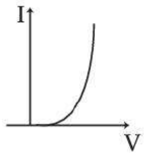
Sol. 7/8 disintegrates means 1/8 remains

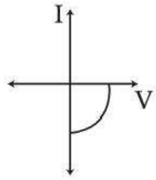
Or $\left(\frac{1}{2}\right)^3$

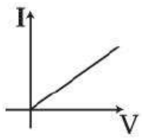
$\therefore$ 3 half lives

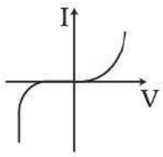
= 180 days

19. Identify the solar cell characteristics from the following options :

(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. Conceptual / theory

20. In the case of amplitude modulation to avoid distortion the modulation index (μ) should be :

(A) $\mu \le 1$ (B) $\mu \ge 1$

(C) $\mu = 2$ (D) $\mu = 0$

Official Ans. by NTA (A)

Sol. $\mu = \frac{A_m}{A_c}$

$\mu \le 1$ to avoid distortion

because $\mu > 1$ will result in interference between career frequency & message frequency.

SECTION-B

1. If the projection of $2\hat{i}+4\hat{j}-2\hat{k}$ on $\hat{i}+2\hat{j}+\alpha\hat{k}$ is zero. Then, the value of α will be

Official Ans. by NTA (5)

Sol. $\vec{a}\cdot\vec{b}=0$

$\therefore\ \vec{a}\cdot\vec{b}=0$

$\therefore\ 2\times1+4\times2-2\times\alpha=0$

$\therefore\ \alpha=\boxed{5}$

2. A freshly prepared radioactive source of half life 2 hours 30 minutes emits radiation which is 64 times the permissible safe level. The minimum time, after which it would be possible to work safely with source, will be ________ hours.

Official Ans. by NTA (15)

Sol. $A=A_0\times2^{-t/T}$

$\frac{A_0}{64}=A_0\times2^{-t/T}$

$\therefore\ t=6T=6\times2\cdot5=\boxed{15}\text{ hours}$

3. In a Young's double slit experiment, a laser light of 560 nm produces an interference pattern with consecutive bright fringes' separation of 7.2 mm. Now another light is used to produce an interference pattern with consecutive bright fringes' separation of 8.1 mm. The wavelength of second light is _________ nm.

Official Ans. by NTA (630)

Sol. $\beta\propto\lambda$

$\lambda_2=\frac{9}{8}\lambda_1$

$\therefore\ \beta_2=\frac{9}{8}\beta_1=\frac{9}{8}\times560=\boxed{630}\text{ nm}.$

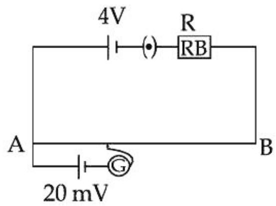
4. The frequencies at which the current amplitude in an LCR series circuit becomes $\frac{1}{\sqrt{2}}$ times its maximum value, are 212 rad s^{-1} and 232 rad s^{-1}. The value of resistance in the circuit is $R=5\Omega$. The self inductance in the circuit is ________ mH.

Official Ans. by NTA (250)

Sol. Band width $=232-212=\frac{R}{L}$

$\therefore\ L=\frac{5}{20}=\boxed{250}\text{ mH}$

5. As shown in the figure, a potentiometer wire of resistance 20Ω and length 300 cm is connected with resistance box (R.B.) and a standard cell of emf 4 V. For a resistance 'R' of resistance box introduced into the circuit, the null point for a cell of 20 mV is found to be 60 cm. The value of 'R' is __________ Ω.



Official Ans. by NTA (780)

Sol. $E=\frac{AC}{AB}\left(V_A-V_B\right)$

$\therefore\ 20\times10^{-3}=\frac{60}{300}\times\frac{4\times20}{R+20}$

$\therefore\ R=\boxed{780}\ \Lambda$

6. Two electric dipoles of dipole moments 1.2×10^{-30} cm and 2.4×10^{-30} cm are placed in two difference uniform electric fields of strengths 5×10^4 NC^{-1} and 15×10^4 NC^{-1} respectively. The ratio of maximum torque experienced by the electric dipoles will be $\frac{1}{x}$. The value of x is ______.

Official Ans. by NTA (6)

Sol. $|\tau|_{max} = PE$

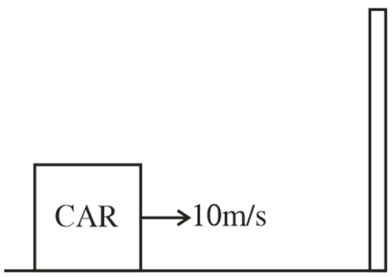
$$\frac{\tau_1}{\tau_2} = \frac{P_1E_1}{P_2E_2} = \frac{1\cdot2\times10^{-30}\times5\times10^4}{2\cdot4\times10^{-30}\times15\times10^4} = \frac{1}{6}$$

Hence x = 6

7. The frequency of echo will be ________ Hz if the train blowing a whistle of frequency 320 Hz is moving with a velocity of 36 km/h towards a hill from which an echo is heard by the train driver. Velocity of sound in air is 330 m/s.

Official Ans. by NTA (340)

Sol. The hill will be a secondary source.



f_1 = frequency of the car w.r.t. the hill

$$f_1 = \left(\frac{v}{v - v_s}\right) f = \left(\frac{330}{320}\right) \times 320 = 330\,Hz$$

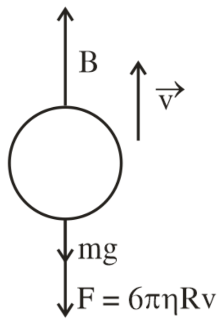
f_2 = Frequency of the sound reflected by hill w.r.t. the car (echo)

$$f_2 = \left(\frac{v + v_0}{v}\right) f_1 = \frac{(330+10)}{330} \times 330 = 340\,Hz$$

8. The diameter of an air bubble which was initially 2 mm, rises steadily through a solution of density 1750 kg m^{-3} at the rate of 0.35 cms^{-1}. The coefficient of viscosity of the solution is _______ poise (in nearest integer). (the density of air is negligible).

Official Ans. by NTA (11)

Sol. As the bubble is rising steadily the net force acting on it will be zero



(Because of density of air the value of mg can be neglected)

So $B = F \Rightarrow \frac{4\pi}{3} R^3 \rho g = 6\pi\eta R v$

Putting $R = 1mm = 10^{-3} m$

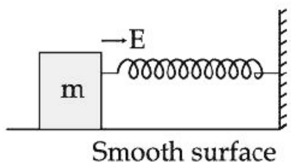
$\rho = 1\cdot75 \times 10^3 kg/m^3$

$g = 10\,m/s^2$

$v = 0.35 \times 10^{-2}\,m/s$

$$\eta = \frac{10}{9} \simeq 1\cdot11 \text{ SI unit} = 11 \text{poise}(CGS)$$

9. A block of mass 'm' (as shown in figure) moving with kinetic energy E compresses a spring through a distance 25 cm when, its speed is halved. The value of spring constant of used spring will be nE Nm^{-1} for n = __________.



Official Ans. by NTA (24)

Sol. Using work – energy theorem

$$W_{net} = (K_f - K_i)$$

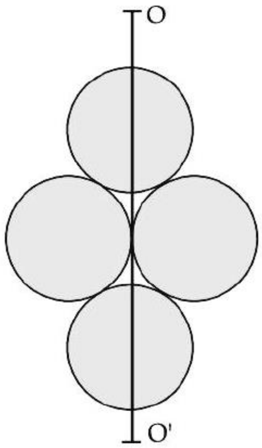
$$\Rightarrow -\frac{1}{2}Kx^2 = \frac{1}{2}m\left(\frac{v}{2}\right)^2 - \frac{1}{2}mv^2 = \frac{E}{4} - E$$

$$\Rightarrow \frac{1}{2}Kx^2 = \frac{3E}{4} \Rightarrow K = \frac{3E}{2x^2}$$

$$\Rightarrow K = \frac{3E}{2\times\left(\frac{1}{4}\right)^2} = 24E$$

n = 24

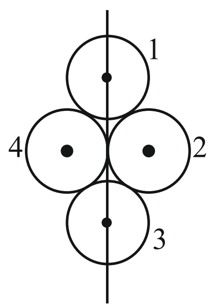
10. Four identical discs each of mass 'M' and diameter 'a' are arranged in a small plane as shown in figure. If the moment of inertia of the system about OO' is $\frac{x}{4}Ma^2$. Then, the value of x will be _____.



Official Ans. by NTA (3)

Sol. $I_1 = I_3 = \frac{MR^2}{4}$

$$I_2 = \frac{MR^2}{4} + MR^2 = \frac{5}{4}MR^2 = I_4$$



So $I = I_1 + I_2 + I_3 + I_4$

$$= \frac{MR^2}{2} + \frac{5}{2}MR^2$$

$= 3MR^2$, Putting $R = \frac{a}{2}$

$$I = \frac{3Ma^2}{4}, \text{So } x = 3$$

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY — TEST PAPER WITH SOLUTION

SECTION-A

1. Identify the incorrect statement from the following.

(A) A circular path around the nucleus in which an electron moves is proposed as Bohr's orbit.

(B) An orbital is the one electron wave function (Ψ) in an atom.

(C) The existence of Bohr's orbits is supported by hydrogen spectrum.

(D) Atomic orbital is characterised by the quantum numbers n and l only

Official Ans. by NTA (D)

Sol. Atomic orbital is characterised by n, l, m.

2. Which of the following relation is not correct ?

(A) $\Delta H = \Delta U - P\Delta V$ (B) $\Delta U = q + W$

(C) $\Delta S_{sys} + \Delta S_{surr} \geq 0$ (D) $\Delta G = \Delta H - T\Delta S$

Official Ans. by NTA (A)

Sol. If $U + Pv$ (By definition)

$\Delta 14 = \Delta U + \Delta(\Pr)$ at constant pressure

$\Delta H = \Delta U + P\Delta V$

3. Match List-I with List-II.

	List-I		List-II
(A)	$Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$	(I)	Primary battery
(B)	$Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$	(II)	Discharging of secondary battery
(C)	$2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$	(III)	Fuel cell
(D)	$2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$	(IV)	Charging of secondary battery

Choose the correct answer from the options given below :

(A) (A) – (I), (B) – (II), (C) – (III), (D) – (IV)

(B) (A) – (IV), (B) – (I), (C) – (II), (D) – (III)

(C) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

(D) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

Official Ans. by NTA (C)

Sol. (a) $Cd(s) + 2Ni(OH)_3(s) \rightarrow CdO(s) + 2Ni(OH)_2(s) + H_2O(l)$

Discharge of secondary Battery

(b) $Zn(Hg) + HgO(s) \rightarrow ZnO(s) + Hg(l)$

(Primary Battery Mercury cell**)**

(c) $2PbSO_4(s) + 2H_2O(l) \rightarrow Pb(s) + PbO_2(s) + 2H_2SO_4(aq)$

Charging of secondary Battery

(d) $2H_2(g) + O_2(g) \rightarrow 2H_2O(l)$ – Fuel cell

4. Match List-I with List-II.

	List-I Reaction		List-II Catalyst
(A)	$4NH_3(g) + 5O_2(g) \rightarrow 4NO(g) + 6H_2O(g)$	(I)	NO(g)
(B)	$N_2(g) + 3H_2(g) \rightarrow 2NH_3(g)$	(II)	$H_2SO_4(l)$
(C)	$C_{12}H_{22}O_{11}(aq) + H_2O(l) \rightarrow C_6H_{12}O_6$ (Glucose) + $C_6H_{12}O_6$ (Fructose)	(III)	Pt(s)
(D)	$2SO_2(g) + O_2(g) \rightarrow 2SO_3(g)$	(IV)	Fe(s)

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(B) (A) – (III), (B) – (II), (C) – (I), (D) – (IV)

(C) (A) – (III), (B) – (IV), (C) – (II), (D) – (I)

(D) (A) – (III), (B) – (II), (C) – (IV), (D) – (I)

Official Ans. by NTA (C)

Sol.

(a) $4NH_3(g) + 5O_2(g) \xrightarrow{Pt(s)} 4NO(g) + 6H_2O(g)$

Ostwald process 500 K

(b) $N_2 + 3H_2 \xrightarrow{Fe(s)} 2NH_3(g)$

Haber's process

(c) $C_{12}H_{22}O_{11}(aq.) + H_2O(\ell) \xrightarrow{H^+} \underset{(glucose)}{C_6H_{12}O_6} + \underset{(fructose)}{C_6H_{12}O_6}$

Inversion of sugar cane

(d) $2SO_2(g) + O_2(g) \xrightarrow{NO(g)} 2SO_3(g)$

5. In which of the following pairs, electron gain enthalpies of constituent elements are nearly the same or identical ?

(A) Rb and Cs (B) Na and K

(C) Ar and Kr (D) I and At

Choose the correct answer from the options given below :

(A) (A) and (B) only

(B) (B) and (C) only

(C) (A) and (C) only

(D) (C) and (D) only

Official Ans. by NTA (C)

Sol. Rb & Cs have nearly same electron gain enthalpy electron gain enthalpy = – 46 kj/ml

Ar & Kr have same ΔH_{eq}. Value is + 96 kj/ml

6. Which of the reaction is suitable for concentrating ore by leaching process ?

(A) $2Cu_2S + 3O_2 \rightarrow 2Cu_2O + 2SO_2$

(B) $Fe_3O_4 + CO \rightarrow 3FeO + CO_2$

(C) $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na[Al(OH)_4]$

(D) $Al_2O_3 + 6Mg \rightarrow 6MgO + 4Al$

Official Ans. by NTA (C)

Sol. $Al_2O_3 + 2NaOH + 3H_2O \rightarrow 2Na,\left[Al(OH)_4\right]$

Leaching.

7. The metal salts formed during softening of hardwater using Clark's method are :

(A) $Ca(OH)_2$ and $Mg(OH)_2$

(B) $CaCO_3$ and $Mg(OH)_2$

(C) $Ca(OH)_2$ and $MgCO_3$

(D) $CaCO_3$ and $MgCO_3$

Official Ans. by NTA (B)

Sol. Clark's Method Reaction

$$Ca(HCO_3)_2 + Ca(OH)_2 \rightarrow 2CaCO_3 + 2H_2O$$

$$Mg(HCO_3)_2 + 2Ca(OH)_2 \rightarrow 2CaCO_3 + Mg(OH)_2 + 2H_2O$$

8. Which of the following statement is incorrect ?

(A) Low solubility of LiF in water is due to its small hydration enthalpy.

(B) KO_2 is paramagnetic.

(C) Solution of sodium in liquid ammonia is conducting in nature.

(D) Sodium metal has higher density than potassium metal

Official Ans. by NTA (A)

Sol. Low solubility of LiF in water is due to high lattice enthalpy

9. Match List-I with List-II, match the gas evolved during each reaction.

	List-I		List-II
(A)	$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta}$	(I)	H_2
(B)	$KMnO_4 + HCl \rightarrow$	(II)	N_2
(C)	$Al + NaOH + H_2O \rightarrow$	(III)	O_2
(D)	$NaNO_3 \xrightarrow{\Delta}$	(IV)	Cl_2

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(B) (A) – (III), (B) – (I), (C) – (IV), (D) – (II)

(C) (A) – (II), (B) – (IV), (C) – (I), (D) – (III)

(D) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

Official Ans. by NTA (C)

Sol. $$(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + 4H_2O$$

$$KMnO_4 + HCl \rightarrow MnCl_2 + KCl + Cl_2 + H_2O$$

$$Al + NaOH + H_2O \rightarrow H_2 + Na\left[Al(OH)_4\right]$$

$$NaNO_3 \longrightarrow NaNO_2 + O_2$$

10. Which of the following has least tendency to liberate H_2 from mineral acids ?

(A) Cu (B) Mn

(C) Ni (D) Zn

Official Ans. by NTA (A)

Sol. Copper is least electropositive among the given metals and it lies below H in reactivity series

11. Given below are two statements :

Statement I : In polluted water values of both dissolved oxygen and BOD are very low.

Statement II : Eutrophication results in decrease in the amount of dissolved oxygen.

In the light of the above statements, choose the most appropriate answer from the options given below :

(A) Both Statement I and Statement II are true

(B) Both Statement I and Statement II are false

(C) Statement I is true but Statement II is false

(D) Statement I is false but Statement II is true

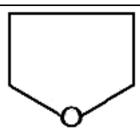
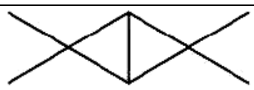
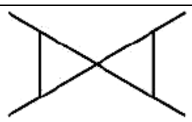
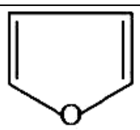
Official Ans. by NTA (D)

Sol. Since eutrophication is result of excessive growth of weed in water bodies, which consume dissolved oxygen of water bodies.

$\therefore$ Eutrophication decreases amount of dissolved oxygen in water bodies.

Polluted water has low value of dissolved oxygen, but high valueof BOD (Biological oxygen demand), since chemical and organic matter requires dissolved oxygen to get decompose.

12. Match List-I with List-II.

	List-I		List-II
(A)		(I)	Spiro compound
(B)		(II)	Aromatic compound
(C)		(III)	Non-planar Heterocyclic compound
(D)		(IV)	Bicyclo compound

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (I), (C) – (IV), (D) – (III)

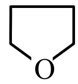
(B) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(C) (A) – (III), (B) – (IV), (C) – (I), (D) – (II)

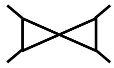
(D) (A) – (IV), (B) – (III), (C) – (II), (D) – (I)

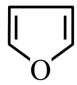
Official Ans. by NTA (C)

Sol.

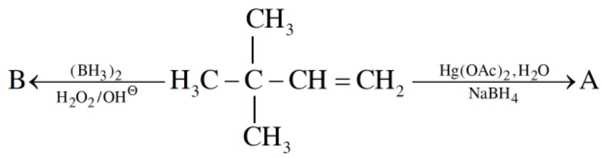

: Non-planar heterocyclic Compound


: Bicyclo Compound


: Spiro Compound


: Aromatic Compound

13. Choose the correct option for the following reactions.



(A) 'A' and 'B' are both Markovnikov addition products.

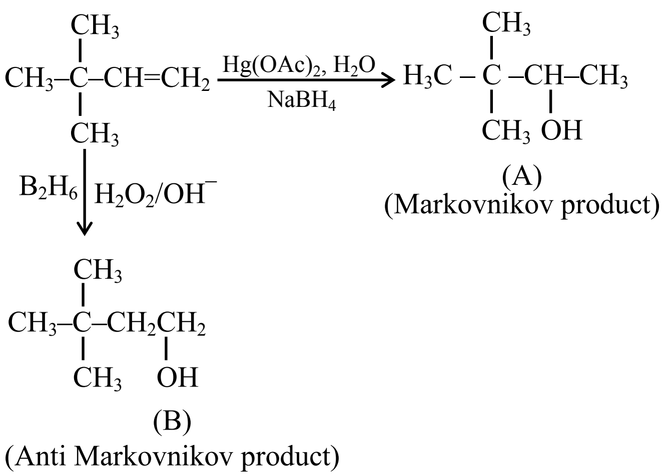
(B) 'A' is Markovnikov product and 'B' is anti-Markovnikov product.

(C) 'A' and 'B' are both anti-Markovnikov products.

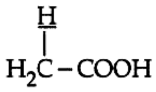
(D) 'B' is Markovnikov and 'A' is anti-Markovnikov product.

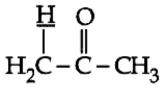
Official Ans. by NTA (B)

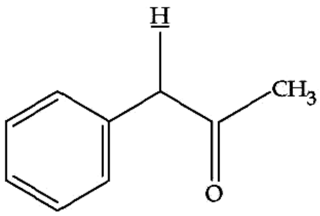
Sol.

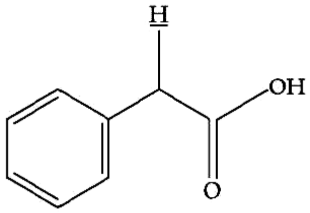


14. Among the following marked proton of which compound shows lowest pK_a value ?

(A) 

(B) 

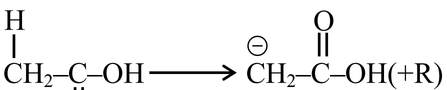
(C) 

(D) 

Official Ans. by NTA (C)

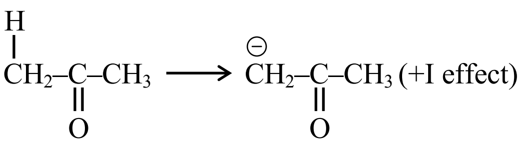
Sol.

(A)

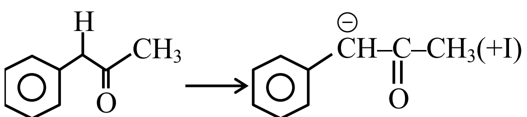


Less stable
(Cross resonance)

(B)



(C)

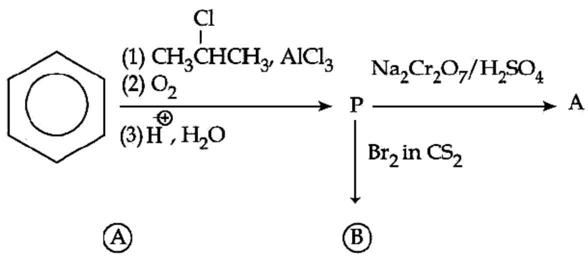


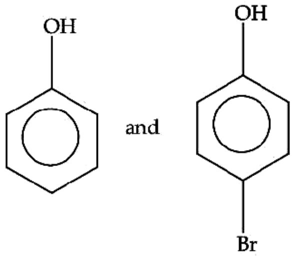
(most acidic)

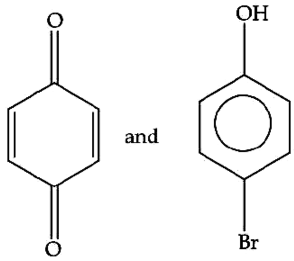
Resonance through benzene ring (most stable)

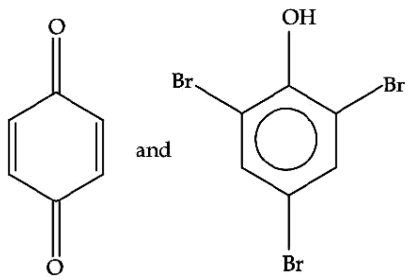
So it has least pK_a value.

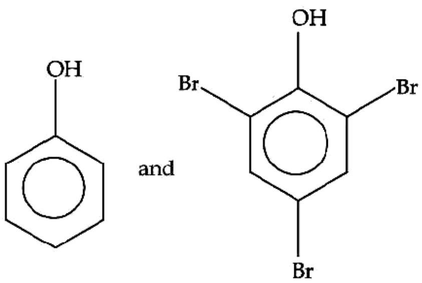
15. Identify the major product A and B for the below given reaction sequence.



(A) 

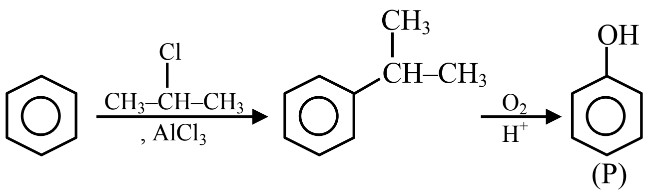
(B) 

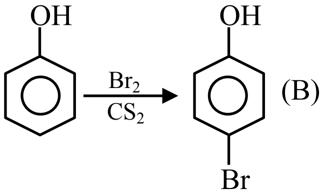
(C) 

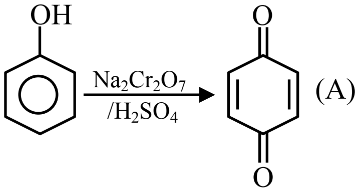
(D) 

Official Ans. by NTA (B)

Sol.







16. Identify the correct statement for the below given transformation.

$$CH_3-CH_2-CH_2-\underset{\oplus N(CH_3)_3}{CH}-CH_3 \xrightarrow[C_2H_5OH]{C_2H_5ONa} \underset{(Major)}{A} + \underset{(Minor)}{B}$$

(A) A - $CH_3CH_2CH = CH–CH_3$,
B – $CH_3CH_2CH_2CH = CH_2$,
Saytzeff products

(B) A - $CH_3CH_2CH = CH–CH_3$,
B – $CH_3CH_2CH_2CH = CH_2$,
Hafmann products

(C) A - $CH_3CH_2CH_2CH = CH_2$,
B – $CH_3CH_2CH = CHCH_3$,
Hofmann products

(D) A - $CH_3CH_2CH_2CH = CH_2$,
B – $CH_3CH_2CH = CHCH_3$,
Saytzeff products

Official Ans. by NTA (C)

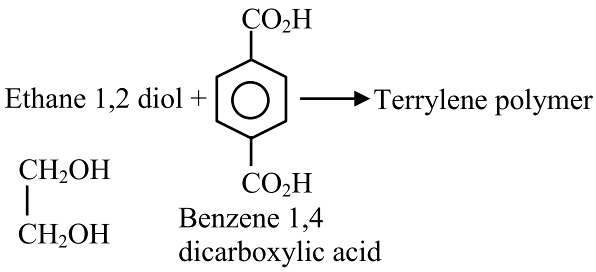
Sol.

$$CH_3CH_2CH_2\underset{\overset{+}{N}Me_3}{CH}–CH_3 \xrightarrow[EtOH]{EtO^-} \underset{(major)}{CH_3CH_2CH_2CH=CH_2} \ (A) + \underset{(minor)}{CH_3CH_2CH=CH–CH_3}$$

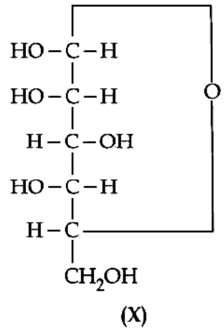
17. Terylene polymer is obtained by condensation of :
(A) Ethane-1, 2-diol and Benzene-1, 3 dicarboxylic acid
(B) Propane-1, 2-diol and Benzene-1, 4 dicarboxylic acid
(C) Ethane-1, 2-diol and Benzene-1, 4 dicarboxylic acid
(D) Ethane-1, 2-diol and Benzene-1, 2 dicarboxylic acid

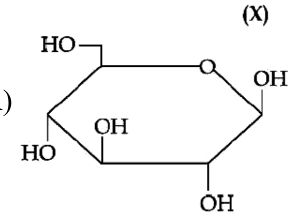
Official Ans. by NTA (C)

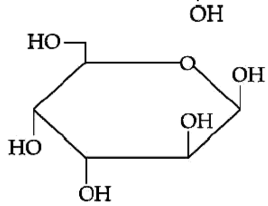
Sol.

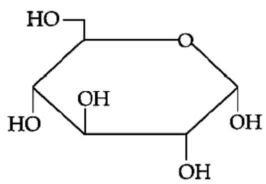


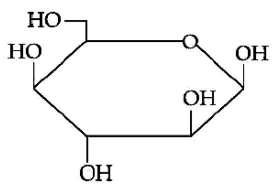
18. For the below given cyclic hemiacetal (X), the correct pyranose structure is :



(A) 

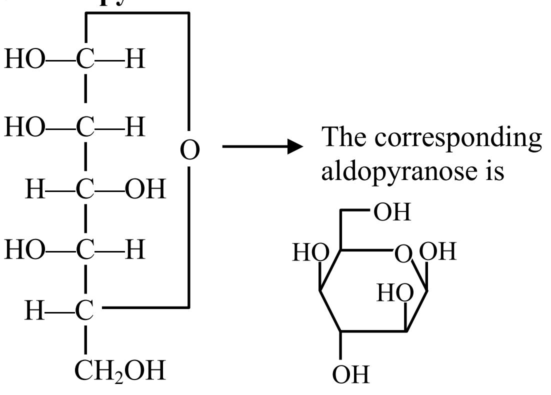
(B) 

(C) 

(D) 

Official Ans. by NTA (D)

Sol. Correct pyranose structure is



X(Hemiacetal)

19. Statements about Enzyme Inhibitor Drugs are given below :

(A) There are Competitive and Non-competitive inhibitor drugs.

(B) These can bind at the active sites and allosteric sites.

(C) Competitive Drugs are allosteric site blocking drugs.

(D) Non-competitive Drugs are active site blocking drugs.

Choose the correct answer from the options given below :

(A) (A), (D) only (B) (A), (C) only

(C) (A), (B) only (D) (A), (B), (C) only

Official Ans. by NTA (C)

Sol. Enzyme inhibitors can be competitive inhibitors (inhibit the attachment of substrate on active site of enzyme) and non-competitive inhibitor (changes the active site of enzyme after binding at allosteric site.)

20. For kinetic study of the reaction of iodide ion with H_2O_2 at room temperature :

(A) Always use freshly prepared starch solution.

(B) Always keep the concentration of sodium thiosulphate solution less than that of KI solution.

(C) Record the time immediately after the appearance of blue colour.

(D) Record the time immediately before the appearance of blue colour.

(E) Always keep the concentration of sodium thiosulphate solution more than that of KI solution.

Choose the correct answer from the options given below :

(A) (A), (B), (C) only

(B) (A), (D), (E) only

(C) (D), (E) only

(D) (A), (B), (E) only

Official Ans. by NTA (A)

Sol. The is recorded immediately after the blue colour appears.

$Na_2S_2O_3$ is kept in limited amount.

SECTION-B

1. In the given reaction,

$$X + Y + 3Z \rightleftarrows XYZ_3$$

if one mole of each of X and Y with 0.05 mol of Z gives compound XYZ_3. (Given : Atomic masses of X, Y and Z are 10, 20 and 30 amu, respectively). The yield of XYZ_3 is __________ g.

(Nearest integer)

Official Ans. by NTA (2)

Sol. $\underset{1mol}{X} + \underset{1mol}{Y} + \underset{0.05mol}{3Z} \rightleftharpoons \mathbf{XYZ_3}$

Z is L.R.

$$\frac{0.05}{3} = 1 \text{ mole of } XYZ_3$$

$$\text{Mass of } XYZ_3 = \frac{0.05}{3} \times (10 + 20 + 30 \times 3)$$

$$= 2g$$

2. An element M crystallises in a body centred cubic unit cell with a cell edge of 300 pm. The density of the element is 6.0 g cm^{-3}. The number of atoms present in 180 g of the element is ______ $\times 10^{23}$.

(Nearest integer)

Official Ans. by NTA (22)

Sol. M is body certred cubic , $\therefore Z = 2$

Let mass of 1 atom of M is A

Edge length = 300 pm

Density = $6 g/cm^3$

$$\therefore 6 g/cm^3 = \frac{Z \times A}{(300 \times 10^{-10})^3} = \frac{2 \times A}{27 \times 10^{-24}}$$

$$A = 81 \times 10^{-24} g$$

$\therefore$ Atomic mass = 48.6g

$$\therefore \text{Mole in } 180g = \frac{180}{48.6} = 3.7 \text{ moles}$$

Atoms of M = $3.7 \times 6 \times 10^{23}$

$= 22.22 \times 10^{23}$ atoms

3. The number of paramagnetic species among the following is ___________.

B_2, Li_2, C_2, C_2^-, O_2^{2-}, O_2^+ and He_2^+

Official Ans. by NTA (4)

Sol. Paramagnetic $B_2, C_2^-, O_2^+, He_2^+$

4. 150 g of acetic acid was contaminated with 10.2 g ascorbic acid ($C_6H_8O_6$) to lower down its freezing point by $(x \times 10^{-1})$°C. The value of x is ________. (Nearest integer) [Given $K_f = 3.9$ K kg mol^{-1}; Molar mass of ascorbic acid = 176 g mol^{-1}]

Official Ans. by NTA (15)

Sol. 150g CH_3COOH

10.2g ascorbic acid $\Rightarrow$ 0.058 moles

$$\Delta T_f = \left(x \times 10^{-1}\right)°C$$

$$\Delta T_f = k_f \cdot \text{molality}$$

$$= 3.9 \times \frac{0.058}{150} \times 1000$$

$$= 1.5°C$$

$$= 15 \times 10^{-1}°C$$

5. K_a for butyric acid (C_3H_7COOH) is 2×10^{-5}. The pH of 0.2 M solution of butyric acid is ___ $\times 10^{-1}$. (Nearest integer) [Given log 2 = 0.30]

Official Ans. by NTA (27)

Sol. K_a of Butyric acid $\Rightarrow 2 \times 10^{-5}$ PKa = 4.7

pH of 0.2 M solution

$$pH = \frac{1}{2}pK_a - \frac{1}{2}\log C$$

$$= \frac{1}{2}(4 \cdot 7)\frac{1}{2}\log(0.2)$$

= 2.35 + 0.35 = 2.7

pH = 27×10^{-1}

6. For the given first order reaction

$A \rightarrow B$

the half life of the reaction is 0.3010 min. The ratio of the initial concentration of reactant to the concentration of reactant at time 2.0 min will be equal to ___________. (Nearest integer)

Official Ans. by NTA (100)

Sol. $A \rightarrow B$ $\quad t_{1/2} = 0.3010$ min

A_0/A_t at time 2 min = ?

$$K = \frac{2.303}{t}\log\left[\frac{A_0}{A_t}\right]$$

$$\Rightarrow \frac{0 \cdot 693}{t_{\frac{1}{2}}} = \frac{2 \cdot 303}{2}\log\left(\frac{A_0}{A_t}\right)$$

Or $\frac{2.303 \times 0.3010}{0.3010} = \frac{2.303}{2}\log\frac{A_0}{A_t}$

$$\log\frac{A_0}{A_t} = 2$$

$$\therefore \frac{A_0}{A_t} = 10^2 = 100$$

7. The number of interhalogens from the following having square pyramidal structure is :

ClF_3, IF_7, BrF_5, BrF_3, I_2Cl_6, IF_5, ClF, ClF_5

Official Ans. by NTA (3)

Sol. Square pyramidal structures are

BrF_5, IF_5 and ClF_5.

8. The disproportionation of MnO_4^{2-} in acidic medium resulted in the formation of two manganese compounds A and B. If the oxidation state of Mn in B is smaller than that of A, then the spin-only magnetic moment (μ) value of B in BM is ___________. (Nearest integer)

Official Ans. by NTA (4)

Sol. $MnO_4^{2-} \xrightarrow{H^+} MnO_4^- + MnO_2$

No. of unpaired $\bar{e} = 3$

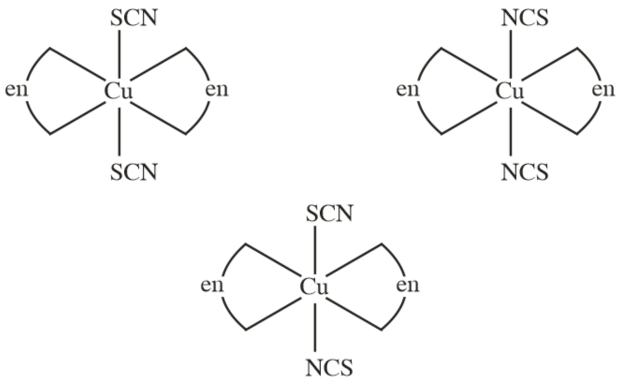
$\therefore \ \mu = \sqrt{15} = 3.877$

Nearest Integer = 4

9. Total number of relatively more stable isomer(s) possible for octahedral complex $[Cu(en)_2(SCN)_2]$ will be ___________.

Official Ans. by NTA (3)

Sol. $[Cu(en)_2(SCN)_2]$



10. On complete combustion of 0.492 g of an organic compound containing C, H and O, 0.7938 g of CO_2 and 0.4428 g of H_2O was produced. The % composition of oxygen in the compound is _____.

Official Ans. by NTA (46)

Sol. 0.492g of $C_xH_yO_z$

Gives 0.7938 g CO_2 = 0.018 moles

0.4428g H_2O = 0.0246 moles

So moles of C = 0.018 $\Rightarrow$ 0.216 g

Moles of H = 0.049 $\Rightarrow$ 0.049g

$\therefore$ wt. of Oxygen = 0.492 – 0.216 – 0.049

= 0.227g

% of Oxygen = $\frac{0.227}{0.492} \times 100$ 46 (approx.)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Thursday 28th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Let the solution curve of the differential equation $xdy = \left(\sqrt{x^2+y^2}+y\right)dx,\ x>0,$ intersect the line $x = 1$ at $y = 0$ and the line $x = 2$ at $y = \alpha$. Then the value of α is :

(A) $\frac{1}{2}$ (B) $\frac{3}{2}$ (C) $-\frac{3}{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (B)

Sol. $xdy = \left(\sqrt{x^2+y^2}+y\right)dx$

$xdy - ydx = \sqrt{x^2+y^2}\,dx$

$\frac{xdy-ydx}{x^2} = \sqrt{1+\frac{y^2}{x^2}}\cdot\frac{dx}{x}$

$\frac{d(y/x)}{\sqrt{1+\left(\frac{y}{x}\right)^2}} = \frac{dx}{x}$

$\ln\left(\frac{y}{x}+\sqrt{\left(\frac{y}{x}\right)^2+1}\right) = \ln x + R$

$\frac{y+\sqrt{y^2+x^2}}{x} = cx$

$y+\sqrt{y^2+x^2} = cx^2$

$x=1, y=0 \Rightarrow 0+1 = C \Rightarrow C=1$

Curve is $y+\sqrt{x^2+y^2} = x^2$

$x=2, y=\alpha$

$2+\sqrt{4+\alpha^2} = 4$

$4+\alpha^2 = 16+\alpha^2 = 8\alpha$

$\alpha = \frac{3}{2}$

2. Considering only the principal values of the inverse trigonometric functions, the domain of the function $f(x) = \cos^{-1}\left(\frac{x^2-4x+2}{x^2+3}\right)$ is :

(A) $\left(-\infty, \frac{1}{4}\right]$ (B) $\left[-\frac{1}{4}, \infty\right)$

(C) $\left(-\frac{1}{3}, \infty\right)$ (D) $\left(-\infty, \frac{1}{3}\right]$

Official Ans. by NTA (B)

Sol.

$\left|\frac{x^2+4x+2}{x^2+3}\right| \le 1$

$\Leftrightarrow \left(x^2-4x+2\right)^2 \le \left(x^2+3\right)^2$

$\Leftrightarrow \left(x^2-4x+2\right)^2 - \left(x^2+3\right)^2 \le 0$

$\Leftrightarrow \left(2x^2-4x+5\right)\left(-4x-1\right) \le 0$

$\Leftrightarrow -4x-1 \le 0 \rightarrow x \ge -\frac{1}{4}$

3. Let the vectors $\vec{a} = (1+t)\hat{i}+(1-t)\hat{j}+\hat{k}$, $\vec{b} = (1-t)\hat{i}+(1+t)\hat{j}+2\hat{k}$ and $\vec{c} = t\hat{i}-t\hat{j}+\hat{k}$, $t\in R$ be such that for $\alpha, \beta, \gamma \in R$, $\alpha\vec{a}+\beta\vec{b}+\gamma\vec{c} = \vec{0}$ $\Rightarrow \alpha=\beta=\gamma=0$. Then, the set of all values of t is :

(A) a non-empty finite set

(B) equal to N

(C) equal to $R - \{0\}$

(D) equal to R

Official Ans. by NTA (C)

Sol. By its given condition

: $\vec{a},\vec{b},\vec{c}$ are linearly independent vectors

$\Rightarrow \left[\bar{a}\,\bar{b}\,\bar{c}\right] \neq 0$ ..(i)

Now, $\left[\bar{a}\ \bar{b}\ \bar{c}\right]$

$$= \begin{vmatrix} 1+t & 1-t & 1 \\ 1-t & 1+t & 2 \\ t & -t & 1 \end{vmatrix}$$

$C_2 \to C_1 + C_2$

$$\begin{vmatrix} 1+t & 2 & 1 \\ 1-t & 2 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$$= 2\begin{vmatrix} 1+t & 1 & 1 \\ 1-t & 1 & 2 \\ t & 0 & 1 \end{vmatrix}$$

$= 2\left[(1+t)-(1-t)+t\right]$

$= 2[3t] = 6t$

$\left[\bar{a}\ \bar{b}\ \bar{c}\right] \neq 0 \Rightarrow t \neq 0$

4. Considering the principal values of the inverse trigonometric functions, the sum of all the solutions of the equation $\cos^{-1}(x) - 2\sin^{-1}(x) = \cos^{-1}(2x)$ is equal to :

(A) 0 (B) 1 (C) $\frac{1}{2}$ (D) $-\frac{1}{2}$

Official Ans. by NTA (A)

Sol. $\cos^{-1}x = 2\sin^{-1}x = \cos^{-1}2x$

$\cos^{-1}x - 2\left(\frac{\pi}{2} - \cos^{-1}x\right) = \cos^{-1}2x$

$\cos^{-1}x - \pi + 2\cos^{-1}x = \cos^{-1}2x$

$3\cos^{2}x = \pi + \cos^{-1}2x$ **...(1)**

$\cos\left(3\cos^{-1}x\right) = \cos\left(\pi + \cos^{-1}2x\right)$

$4x^3 - 3x = -2x$

$4x^3 = x \Rightarrow x = 0, \pm\frac{1}{2}$

All satisfy the original equation

$\text{sum} = -\frac{1}{2} \text{ to } + \frac{1}{2} = 0$

5. Let the operations $*, \odot \in \{\wedge, \vee\}$. If $(p * q) \odot (p \odot \sim q)$ is a tautology, then the ordered pair $(*, \odot)$ is :

(A) $(\vee, \wedge)$ (B) $(\vee, \vee)$ (C) $(\wedge, \wedge)$ (D) $(\wedge, \vee)$

Official Ans. by NTA (B)

Sol. Well check each option

For A $\pi = \text{v of } 0 = \Lambda$

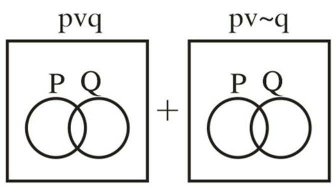
$(pvq) \wedge (pv \sim q)$

$\equiv pv(q \wedge \sim q)$

$\equiv pv(c) \equiv p$

For B : * = v, O = v

$(pvq) \vee (pv \sim q) \equiv t$ using Venn Diagrams



6. Let a vector $\vec{a}$ has a magnitude 9. Let a vector $\vec{b}$ be such that for every $(x, y) \in R \times R - \{(0,0)\}$, the vector $(x\vec{a} + y\vec{b})$ is perpendicular to the vector $(6y\,\vec{a} - 18x\,\vec{b})$. Then the value of $\left|\vec{a} \times \vec{b}\right|$ is equal to:

(A) $9\sqrt{3}$ (B) $27\sqrt{3}$ (C) 9 (D) 81

Official Ans. by NTA (B)

Sol. $\left|\vec{a}\right| = 9$ & $\left(x\vec{a} + y\vec{b}\right)\cdot\left(6y\,\vec{a} - 18x\,\vec{b}\right) = 0$

$\Rightarrow 6xy\left|\bar{a}\right|^2 - 18x^2\left(\bar{a}\cdot\bar{b}\right) + 6y^2\left(\bar{a}\cdot\bar{b}\right) - 18xy\left|\bar{b}\right|^2 = 0$

$\Rightarrow 6xy\left(\left|\bar{a}\right|^2 - 3\left|\bar{b}\right|^2\right) + \left(\bar{a}\cdot\bar{b}\right)\left(y^2 - 3x^2\right) = 0$

This should hold $\forall\, x, y \in R \times R$

$\therefore \left|\bar{a}\right|^2 = 3\left|\bar{b}\right|^2$ & $\left(\bar{a}\cdot\bar{b}\right) = 0$

Now $\left|\bar{a}\times\bar{b}\right|^2 = \left|\bar{a}\right|^2\left|\bar{b}\right|^2 - \left(\bar{a}\cdot\bar{b}\right)^2$

$= \left|\bar{a}\right|^2 \cdot \frac{\left|\bar{a}\right|^2}{3}$

$\therefore \left|\bar{a}\times\bar{b}\right| = \frac{\left|\bar{a}\right|^2}{\sqrt{3}} = \frac{81}{\sqrt{3}} = 27\sqrt{3}$

7. For $t \in (0, 2\pi)$, if ABC is an equilateral triangle with vertices A(sint, –cost), B(cost, sint) and C(a, b) such that its orthocentre lies on a circle with centre $\left(1, \frac{1}{3}\right)$, then $(a^2 - b^2)$ is equal to :

(A) $\frac{8}{3}$ (B) 8

(C) $\frac{77}{9}$ (D) $\frac{80}{9}$

Official Ans. by NTA (B)

Sol. $s \equiv \sin t, c \equiv \cos t$

Let orthocentre be (h,k)

Since it if an equilateral triangle hence orthocentre coincides with centroid.

$\therefore\ a + s + c = 3h, b + s - c = 3k$

$\therefore\ (3h - a)^2 + (3k - b)^2 = (s + c)^2 + (s - c)^2 = 2(s^2 + c^2) = 2$

$\therefore\ \left(h - \frac{a}{3}\right)^2 + \left(K - \frac{b}{3}\right)^2 = \frac{2}{9},$

circle centre at $\left(\frac{a}{3}, \frac{b}{3}\right)$

Gives, $\frac{a}{3} = 1, \frac{b}{3} = \frac{1}{3} \Rightarrow a = 3, b = 1$

$\therefore\ a^2 - b^2 = 8$

8. For $\alpha \in N$, consider a relation R on N given by $R = \{(x, y) : 3x + \alpha y \text{ is a multiple of } 7\}$. The relation R is an equivalence relation if and only if :

(A) $\alpha = 14$

(B) α is a multiple of 4

(C) 4 is the remainder when α is divided by 10

(D) 4 is the remainder when α is divided by 7

Official Ans. by NTA (D)

Sol. For R to be reflexive $\Rightarrow$ x R x

$\Rightarrow\ 3x + \alpha x = 7x \Rightarrow (3 + \alpha)x = 7K$

$\Rightarrow\ 3 + \alpha = 7\lambda \Rightarrow \alpha = 7\lambda - 3 = 7N + 4, K, \lambda, N \in I$

$\therefore$ when α divided by 7, remainder is 4.

R to be symmetric $xRy \Rightarrow yRx$

$3x + \alpha y = 7N_1, 3y + \alpha x = 7N_2$

$\Rightarrow (3 + \alpha)(x + y) = 7(N_1 + N_2) = 7N_3$

Which holds when $3 + \alpha$ is multiple of 7

$\therefore\ \alpha = 7N + 4$ (as did earlier)

R to be transitive

$xRy \ \&\ yRz \Rightarrow xRz$.

$3x + \alpha y = 7N_1 \ \&\ 3y + \alpha z = 7N_2$ and $3x + \alpha z = 7N_3$

$\therefore\ 3x + 7N_2 - 3y = 7N_3$

$\therefore\ 7N_1 - \alpha y + 7N_2 - 3y = 7N_3$

$\therefore\ 7(N_1 + N_2) - (3 + \alpha)y = 7N_3$

$\therefore\ (3 + \alpha)y = 7N$

Which is true again when $3 + \alpha$ divisible by 7, i.e. when α divided by 7, remainder is 4.

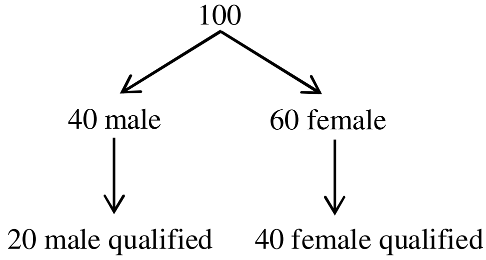
9. Out of 60% female and 40% male candidates appearing in an exam, 60% candidates qualify it. The number of females qualifying the exam is twice the number of males qualifying it. A candidate is randomly chosen from the qualified candidates. The probability, that the chosen candidate is a female, is :

(A) $\frac{3}{4}$ (B) $\frac{11}{16}$

(C) $\frac{23}{32}$ (D) $\frac{13}{16}$

Official Ans. by NTA (A)

Sol.



Probability that chosen candidate is female = $\frac{40}{60} = \frac{2}{3}$

10. If $y = y(x),\ x \in \left(0, \frac{\pi}{2}\right)$ be the solution curve of the differential equation

$$\left(\sin^2 2x\right)\frac{dy}{dx} + \left(8\sin^2 2x + 2\sin 4x\right)y =$$

$$2e^{-4x}\left(2\sin 2x + \cos 2x\right), \quad \text{with} \quad y\left(\frac{\pi}{4}\right) = e^{-\pi},$$

then $y\left(\frac{\pi}{6}\right)$ is equal to :

(A) $\frac{2}{\sqrt{3}}e^{-2\pi/3}$ (B) $\frac{2}{\sqrt{3}}e^{2\pi/3}$

(C) $\frac{1}{\sqrt{3}}e^{-2\pi/3}$ (D) $\frac{1}{\sqrt{3}}e^{2\pi/3}$

Official Ans. by NTA (A)

Sol. Given differential equation can be re-written as

$$\frac{dy}{dx} + \left(8 + 4\cot 2x\right)y = \frac{2e^{-4x}}{\sin^2 2x}\left(2\sin x + \cos 2x\right)$$

which is a linear diff. equation.

$$\text{I.f.} = e^{\int(8+4\cot 2x)dx} = e^{8x+2Cu(\sin 2x)}$$

$$= e^{8x}\cdot\sin^2 2x$$

$\therefore$ solution is

$$y\left(e^{8x}\cdot\sin^2 2x\right) = \int 2e^{4x}\left(2\sin 2x + \cos 2x\right)dx + C$$

$$= e^{4x}\cdot\sin 2x + C$$

Given $y\left(\frac{\pi}{4}\right) = e^{-\pi} \Rightarrow C = 0$

$$\therefore\ y = \frac{e^{-4x}}{\sin 2x}$$

$$\therefore\ y\left(\frac{\pi}{6}\right) = \frac{e^{-4\cdot\frac{\pi}{6}}}{\sin\left(2\cdot\frac{\pi}{6}\right)} = \frac{2}{\sqrt{3}}e^{-\frac{2\pi}{3}}$$

11. If the tangents drawn at the points P and Q on the parabola $y^2 = 2x - 3$ intersect at the point R(0, 1), then the orthocentre of the triangle PQR is :

(A) (0, 1) (B) (2, –1)

(C) (6, 3) (D) (2, 1)

Official Ans. by NTA (B)

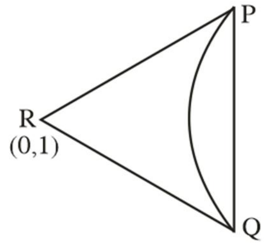
Sol. $y^2 = 2x - 3$ **...(i)**

Equation of chord of contact

PQ : r = 0

$yx1 = (x + 0) - 3$

$y = x - 3$ ...(2)



from (1) and (2)

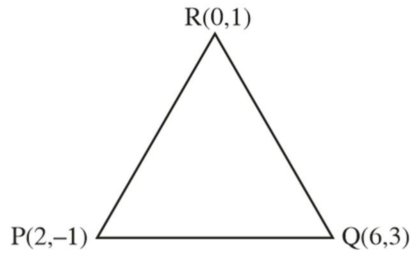
$$(x\cdot 3)^2 = 2x - 3$$

$$x^2 - 8x + 12 = 0$$

$$(x-2)(x-6) = 0$$

$$x = 2 \text{ or } 6$$

$$y = -1 \text{ or } 3$$



$$MPQ = \frac{1}{4} = 1$$

$$MQR = \frac{2}{6} = \frac{1}{3}$$

$$MPR = \frac{2}{6} = \frac{1}{3}$$

$$MPR = \frac{2}{-2} = -1$$

$$MPQ \times MPR = - \Rightarrow PQ \perp PR$$

Orthocentre = P (2,–1)

12. Let C be the centre of the circle $x^2 + y^2 - x + 2y = \frac{11}{4}$ and P be a point on the circle. A line passes through the point C, makes an angle of $\frac{\pi}{4}$ with the line CP and intersects the circle at the points Q and R. Then the area of the triangle PQR (in unit^2) is :

(A) 2 (B) $2\sqrt{2}$

(C) $8\sin\left(\frac{\pi}{8}\right)$ (D) $8\cos\left(\frac{\pi}{8}\right)$

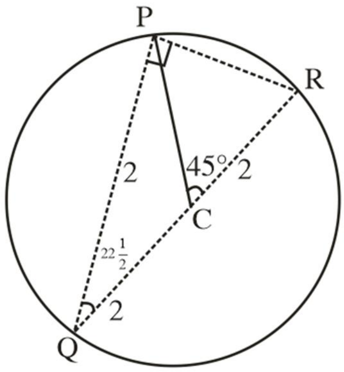
Official Ans. by NTA (B)

Sol. $x^2 + y^2 - x + 2y = \frac{11}{4}$

$$\left(x - \frac{1}{2}\right)^2 + (y+1)^2 = (2)^2$$

Or Δ PQR

$$PR = QK\sin 2 \geq \frac{1}{3}$$



$$= 4\cdot 6\sin\frac{\pi}{8}$$

$$PQ = QR\cos 22\frac{1}{2}$$

$$= 4\cos\frac{\pi}{8}$$

As $\Delta PQR = \frac{1}{2} PR \times PQ$

$$= \frac{1}{2}\left(4^2\sin\frac{\pi}{6}\right)\left(4\cos\frac{\pi}{8}\right)$$

$$= 4\sin\frac{\pi}{4} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$$

13. The remainder when $7^{2022} + 3^{2022}$ is divided by 5 is:

(A) 0 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (C)

Sol. $7^{2022} + 3^{2022}$

$$= (49)^{1011} + (9)^{1011}$$

$$= (50-1)^{1011} + (10-1)^{1011}$$

$$= 5\lambda - 1 + 5K - 1$$

$$= 5m - 2$$

Remainder $= 5 - 2 = 3$

14. Let the matrix $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ and the matrix $B_0 = A^{49} + 2A^{98}$. If $B_n = Adj(B_{n-1})$ for all $n \geq 1$, then $\det(B_4)$ is equal to :

(A) 3^{28} (B) 3^{30} (C) 3^{32} (D) 3^{36}

Official Ans. by NTA (C)

Sol. $A^2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix}$

$$= \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$$

$a \leftrightarrow R_2$

$$-\begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

$R_2 \leftrightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

$B_0 = A^{49} + 2A^{98}$

$= A + 2I$

$B_n = Adj(B_n - 1)$

$$B_4 = \text{Adj}(\text{Adj}(\text{Adj}(\text{Adj}B_0))$$

$$= |B_0|^{(n-1)^4}$$

$$= |B_0|^{16}$$

$$B_0 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{vmatrix} 2 & 1 & 0 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix}$$

$$= 2(4-0) - 1(0-1)$$

$$= 9$$

$$B_4 (9)^{16} = (3)^{32}$$

15. Let $S_1 = \left\{z_1 \in C : |z_1 - 3| = \frac{1}{2}\right\}$ and $S_2 = \{z_2 \in C : |z_2 - |z_2 + 1|| = |z_2 + |z_2 - 1||\}$. Then, for $z_1 \in S_1$ and $z_2 \in S_2$, the least value of $|z_2 - z_1|$ is :

(A) 0 (B) $\frac{1}{2}$ (C) $\frac{3}{2}$ (D) $\frac{5}{2}$

Official Ans. by NTA (C)

Sol. $|z_2 + |z_2 - 1||^2 = |z_2 - |z_2 + 1||^2$

$\Rightarrow |z_2 + |z_2 - 1||(\bar{z}_2 + |z_2 - 1|) = (z_2 - |z_2 + 1|)(\bar{z}_2 - (z_2 + 1))$

$\Rightarrow z_2|\bar{z}_2 + 12_2 - 1| - (\bar{z}_2 - |z_2 + 1|) + \bar{z}_2(|z_2 - 1| + |z_2 + 1|)$

$= |z_2 + 1|^2 = |z_2 - 1|^2$

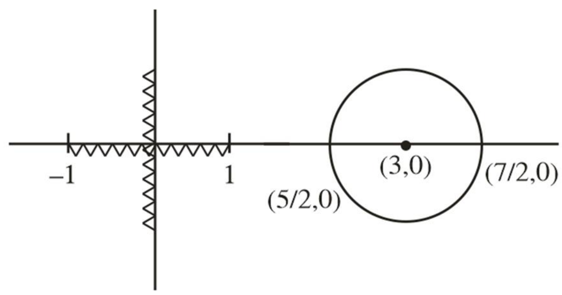
$\Rightarrow [z_2 + \bar{z}_2)(|z_2 - 1|) + (z_2 + 1|) = 2(z_2 + \bar{z}_2)$

$\Rightarrow (z_2 + \bar{z}_2)(|z_2 - 1| + |z_2 + 1| - 2) = 0$

$\therefore z_2 + \bar{z}_2 = 0$ or $|z_2 - 1| + |z_2 + 1| - 2 = 0$

$\therefore z_2$ lie on imaginary axis. Or on real axis with in [–1,1]

Also $|z_1 - 3| = \frac{1}{2}$ lie on circle having centre 3 and radius $\frac{1}{2}$.



Clearly $|z_1 - z_2|\text{min} = \frac{5}{2} - 1 = \frac{3}{2}$

16. The foot of the perpendicular from a point on the circle $x^2 + y^2 = 1$, $z = 0$ to the plane $2x + 3y + z = 6$ lies on which one of the following curves ?

(A) $(6x + 5y - 12)^2 + 4(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$

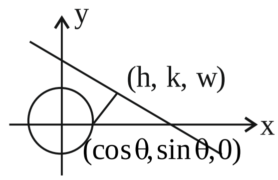
(B) $(5x + 6y - 12)^2 + 4(3x + 5y - 9)^2 = 1$, $z = 6 - 2x - 3y$

(C) $(6x + 5y - 14)^2 + 9(3x + 5y - 7)^2 = 1$, $z = 6 - 2x - 3y$

(D) $(5x + 6y - 14)^2 + 9(3x + 7y - 8)^2 = 1$, $z = 6 - 2x - 3y$

Official Ans. by NTA (B)

Sol.



$$\frac{h - \cos\theta}{2} = \frac{k - \sin\theta}{3} = \frac{w - 0}{1}$$

$$= \frac{-1(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$h = \cos\frac{-2(2\cos\theta + 3\sin\theta - 6)}{14}$$

$$= \frac{10\cos\theta - 6\sin\theta + 12}{14}$$

$$k = \sin\theta - \frac{3}{14}(2\cos\theta + 3\sin\theta - 6)$$

$$k = \frac{5\sin\theta - 6\cos\theta + 18}{14}$$

Elementary $\sin\theta$ and $\cos\theta$

$$(5h + 6k - 12)^2 + 4(3h + 5k - 9)^2 = 1$$

17. If the minimum value of $f(x)=\frac{5x^2}{2}+\frac{\alpha}{x^5}, x>0,$ is 14, then the value of α is equal to :

(A) 32 (B) 64

(C) 128 (D) 256

Official Ans. by NTA (C)

Sol. $\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{x^2}{2}+\frac{\alpha}{2x^5}+\frac{\alpha}{2x^5}$

$\ge 7\left(\frac{\alpha^2}{2^7}\right)^{\frac{1}{7}}$

$\frac{7\cdot(\alpha)^{2/7}}{2}=14$

$(\alpha^2)^{1/7}=2^2$

$\alpha=\left(2^2\right)^{7/2}=2^7$

$\alpha=128$

18. Let α, β and γ be three positive real numbers. Let $f(x)=\alpha x^5+\beta x^3+\gamma x, x\in R$ and $g:R\to R$ be such that $g(f(x)) = x$ for all $x\in R$. If $a_1, a_2, a_3,..., a_n$ be in arithmetic progression with mean zero, then the value of $f\left(g\left(\frac{1}{n}\sum_{i=1}^{n} f(a_i)\right)\right)$ is equal to :

(A) 0 (B) 3

(C) 9 (D) 27

Official Ans. by NTA (A)

Sol. Consider a case when $\alpha=\beta=0$ then

$f(x) = yx$

$g(x)=\frac{x}{y}$

$\frac{1}{n}\sum_{i=1}^{n} f(a_i)\Rightarrow\frac{y}{n}(a_1+a_2+.....+a_n)$

$=0$

$\Rightarrow f(g(0))\Rightarrow f(0)$

$\Rightarrow 0$

19. Consider the sequence $a_1, a_2, a_3, \ldots\ldots$ such that $a_1 = 1, a_2 = 2$ and $a_{n+2}=\frac{2}{a_{n+1}}+a_n$ for $n = 1, 2, 3, \ldots$.

If $\left(\frac{a_1+\frac{1}{a_2}}{a_3}\right)\cdot\left(\frac{a_2+\frac{1}{a_3}}{a_4}\right)\cdot\left(\frac{a_3+\frac{1}{a_4}}{a_5}\right).....\left(\frac{a_{30}+\frac{1}{a_{31}}}{a_{32}}\right)=2^{\alpha}\left({}^{61}C_{31}\right),$

then α is equal to :

(A) –30 (B) –31

(C) –60 (D) –61

Official Ans. by NTA (C)

Sol. $a_{n+2}\,a_{n+1}-a_{n+1}.a_n=2$

Series will satisfy

$a_1a_2, a_2a_3, a_3a_4, a_4a_5,$
1.2 2.2 2.3 2.4

$\frac{a_n+\frac{1}{a_{n+1}}}{a_{n+2}}=\frac{a_{n+2}-\frac{1}{a_{n+1}}}{a_{n+2}}$

$=1-\frac{1}{a_{n+1}a_{n+2}}$

$=1-\frac{1}{2(r+1)}$

$=\frac{2r+1}{2(r+1)}$

Now proof is given by

$=\prod_{r=1}^{30}\frac{(2r+1)}{2(r+1)}$

$=\frac{(1\cdot3\cdot5\cdot........\cdot61)}{2^{30}\cdot(2\cdot3\cdot.......\cdot31)}$

$\Rightarrow\frac{(1\cdot3\cdot5\cdot........\cdot61)}{\lfloor 31\cdot2^{30}}\times\frac{2^{30}\times\lfloor 30}{2^{30}\times\lfloor 30}$

$=\frac{\lfloor 61}{2^{60}\lfloor 31\cdot\lfloor 30}$

$\alpha=-60$

20. The minimum value of the twice differentiable function $f(x)=\int_0^x e^{x-t}f'(t)dt-(x^2-x+1)e^x,\ x\in R,$ is :

(A) $-\frac{2}{\sqrt{e}}$ (B) $-2\sqrt{e}$

(C) $-\sqrt{e}$ (D) $\frac{2}{\sqrt{e}}$

Official Ans. by NTA (A)

Sol. $f(x)=e^x\cdot\int_0^x\frac{f'(t)}{e^t}dt$

$f'(x)=e^x\cdot\int_0^x\frac{f'(t)}{e^t}dt+e^x\cdot\frac{f'(x)}{e^x}$

$-\left[(2x-1)\cdot e^x+(x^2-x+1)\cdot e^x\right]$

$\int_0^x\frac{f'(t)}{e^t}dt=x^2+x$

$\frac{f'(x)}{e^x}=2x+1$

$f'(x)=(2x+1)\cdot e^x$

$f'(x)=0\Rightarrow x=-\frac{1}{2}$

$f(x)=(2x+1)\cdot e^x-2e^x+C$

$f(0)=-1$

$-1=1-2+C$

$C=0$

$f(x)=e^x(2x-1)$

$f\left(-\frac{1}{2}\right)=\frac{-2}{\sqrt{e}}$

SECTION-B

1. Let S be the set of all passwords which are six to eight characters long, where each character is either an alphabet from {A, B, C, D, E} or a number from {1, 2, 3, 4, 5} with the repetition of characters allowed. If the number of passwords in S whose at least one character is a number from {1, 2, 3, 4, 5} is $\alpha\times5^6$, then α is equal to _____.

Official Ans. by NTA (7073)

Sol. Required no. = Total – no character from {1, 2, 3, 4, 5}

$=(10^6-5^6)+(10^7-5^7)+(10^8-5^8)$

$=10^6(1+10+100)-5^6(1+5+25)$

$=10^6\times111-5^6\times31$

$=2^6\times5^6\times111-5^6\times31$

$=5^6(2^6\times111-31)$

$=5^6\times\underbrace{7073}_{\alpha}$

$\therefore\alpha=7073$

2. Let P(–2, –1, 1) and $Q\left(\frac{56}{17},\frac{43}{17},\frac{111}{17}\right)$ be the vertices of the rhombus PRQS. If the direction ratios of the diagonal RS are $\alpha,-1,\beta,$ where both α and β are integers of minimum absolute values, then $\alpha^2+\beta^2$ is equal to __________.

Official Ans. by NTA (450)

Sol. $RS\equiv(\alpha,-1,\beta)$

DR of PQ $\equiv\left(\frac{56}{17}+2,\frac{43}{17}+1,\frac{111}{17}-1\right)$

$\equiv\left(\frac{90}{17},\frac{60}{17},\frac{94}{17}\right)$

$\frac{90}{17}\alpha+\frac{60}{17}(-1)+\frac{94}{17}\beta=0$

$90\alpha+94\beta=60$

$\beta=\frac{60-90\alpha}{94}$

$\beta=\frac{30(2-3\alpha)}{94}$

$\beta=-30\frac{(3\alpha-2)}{94}$

$\beta=\frac{-15}{47}(3\alpha-2)$

$\Rightarrow\frac{\beta}{-15}=\frac{3\alpha-2}{47}$

$\Rightarrow\beta=-15,\ \alpha=-15$

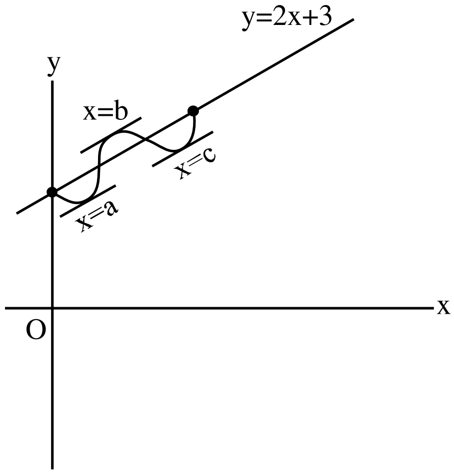
$\alpha^2+\beta^2=225+225$

$=450$

3. Let $f:[0,1]\to R$ be a twice differentiable function in (0, 1) such that f(0) = 3 and f(1) = 5. If the line y = 2x + 3 intersects the graph of f at only two distinct points in (0, 1), then the least number of points $x\in(0,1)$, at which $f''(x)=0$, is ______.

Official Ans. by NTA (2)

Sol.



$f'(a)=f'(b)=f'(c)=2$

$\Rightarrow f''(x)$ is zero

for atleast $x_1\in(a,b)$ & $x_2\in(b,c)$

4. If $\int_0^{\sqrt{3}}\frac{15x^3}{\sqrt{1+x^2+\sqrt{(1+x^2)^3}}}dx=\alpha\sqrt{2}+\beta\sqrt{3}$, where α,β are integers, then $\alpha+\beta$ is equal to

Official Ans. by NTA (10)

Sol. Put $1+x^2=t^2$

$2x\,dx=2t\,dt$

$X\,dx=t\,dt$

$\therefore\int_1^2\frac{15(t^2-1)t\,dt}{\sqrt{t^2+t^3}}$

$15\int_1^2\frac{t(t^2-1)}{t\sqrt{1+t}}dt$

Put $1+t=u^2$

$dt=2u\,du$

$15\int_{\sqrt{2}}^{\sqrt{3}}\frac{(u^2-1)^2-1}{u}\times 2u\,du$

$30\int_{\sqrt{2}}^{\sqrt{3}}(u^4-2u^2)du$

$30\left(\frac{u^5}{5}-\frac{2u^3}{3}\right)_{\sqrt{2}}^{\sqrt{3}}$

$30\left[\frac{1}{5}\left(\sqrt{3}^5-\sqrt{2}^5\right)-\frac{2}{3}\left(\sqrt{3}^3-\sqrt{2}^3\right)\right]$

$30\left[\frac{1}{5}\left(9\sqrt{3}-4\sqrt{2}\right)-\frac{2}{3}\left(3\sqrt{3}-2\sqrt{2}\right)\right]$

$30\left[-\frac{1}{5}\times\sqrt{3}+\frac{8}{15}\sqrt{2}\right]$

$-6\sqrt{3}+16\sqrt{2}=\alpha\sqrt{2}+\beta\sqrt{3}$

$\alpha=16,\beta=-6$

$\therefore\alpha+\beta=10$

5. Let $A=\begin{bmatrix}1&-1\\2&\alpha\end{bmatrix}$ and $B=\begin{bmatrix}\beta&1\\1&0\end{bmatrix}$, $\alpha,\beta\in R$. Let α_1 be the value of α which satisfies $(A+B)^2=A^2+\begin{bmatrix}2&2\\2&2\end{bmatrix}$ and α_2 be the value of α which satisfies $(A+B)^2=B^2$. Then $|\alpha_1-\alpha_2|$ is equal to ______.

Official Ans. by NTA (2)

Sol. $A+B=\begin{bmatrix}\beta+1&0\\3&\alpha\end{bmatrix}$

$(A+B)^2=\begin{bmatrix}\beta+1&0\\3&\alpha\end{bmatrix}\begin{bmatrix}\beta+1&0\\3&\alpha\end{bmatrix}$

$=\begin{bmatrix}(\beta+1)^2&0\\3(\beta+1)+3\alpha&\alpha^2\end{bmatrix}$

$A^2=\begin{bmatrix}1&-1\\2&\alpha\end{bmatrix}\begin{bmatrix}1&-1\\2&\alpha\end{bmatrix}$

$=\begin{bmatrix}-1&-1-\alpha\\2+2\alpha&\alpha^2-2\end{bmatrix}$

$\therefore\begin{bmatrix}1&-\alpha+1\\2\alpha+4&\alpha^2\end{bmatrix}=\begin{bmatrix}(\beta+1)^2&0\\3(\alpha+\beta+1)&\alpha^2\end{bmatrix}$

$\boxed{\alpha=1}=\alpha_1$

$B^2=\begin{bmatrix}\beta&1\\1&0\end{bmatrix}\begin{bmatrix}\beta&1\\1&0\end{bmatrix}$

$=\begin{bmatrix}\beta^2+1&\beta\\\beta&1\end{bmatrix}=\begin{bmatrix}(\beta+1)^2&0\\3(\beta+1)+3\alpha&\alpha^2\end{bmatrix}$

$\therefore\beta=0,\alpha=-1=\alpha_2$

$|\alpha_1-\alpha_2|=|1-(-1)|=2$

6. For $p, q \in R$, consider the real valued function $f(x) = (x - p)^2 - q, x \in R$ and $q > 0$. Let a_1, a_2, a_3 and a_4 be in an arithmetic progression with mean p and positive common difference. If $|f(a_i)| = 500$ for all i = 1, 2, 3, 4, then the absolute difference between the roots of f(x) = 0 is

Official Ans. by NTA (50)

Sol. $f(x) = 0 \Rightarrow (x - p)^2 - q = 0.$

Roots are $p + \sqrt{q}$, $p - \sqrt{q}$ absolute difference between roots $2\sqrt{q}$.

Now, $|f(a_i)| = 500$

Let a_1, a_2, a_3, a_4 are a_1 a + d, a + 2d, a + 3d

$|f(a_4)| = 500$

$|(a_1 - p)^2 - q| = 500$

$\Rightarrow (a_1 - p)^2 - q = 500$

$$\Rightarrow \frac{9}{4}d^2 - q = 500 \qquad \text{\_\_\_\_\_\_\_\_(1)}$$

and $|f(a_1)|^2 = |f(a_2)|^2$

$((a_1 - p)^2 - q)^2 = ((a_2 - p)^2 - q)^2$

$\Rightarrow ((a_1 - p)^2 - (a_2 - p)^2)((a_1 - p)^2 - q + (a_2 - p)^2 - q) = 0$

$$\Rightarrow \frac{9}{4}d^2 - q + \frac{d^2}{4} - q = 0$$

$$2q = \frac{10d^2}{4} \Rightarrow q = \frac{5d^2}{4}$$

$$\Rightarrow d^2 = \frac{4q}{5}$$

From equation (1) $\frac{9}{4}.\frac{4.q}{5} - q = 500$

$$\frac{4q}{5} = 500$$

$$\frac{4q}{5} = 500$$

and $2\sqrt{q} = 2 \times \frac{50}{2} = 50$

7. For the hyperbola H : $x^2 - y^2 = 1$ and the ellipse $E : \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > b > 0,$ let the

(1) eccentricity of E be reciprocal of the eccentricity of H, and

(2) the line $y = \sqrt{\frac{5}{2}}x + K$ be a common tangent of E and H.

Then $4(a^2 + b^2)$ is equal to ________.

Official Ans. by NTA (3)

Sol. $e_E = \sqrt{1 - \frac{b^2}{a^2}}$, $e_H = \sqrt{2}$

If $\Rightarrow e_E = \frac{1}{e_H}$

$$\Rightarrow \frac{a^2 - b^2}{a^2} = \frac{1}{2}$$

$2a^{2-2b}2 = a^2$

$a^2 = 2b^2$

and $y = \sqrt{\frac{5}{2}}x + k$ is tangent to ellipse then

$$K^2 = a^2 \times \frac{5}{2} + b^2 = \frac{3}{2}$$

$$6b^2 = \frac{3}{2} \Rightarrow b^2 = \frac{1}{4} \text{ and } a^2 = \frac{1}{2}$$

$\therefore 4.(a^2 + b^2) = 3$

8. Let $x_1, x_2, x_3, \ldots, x_{20}$ be in geometric progression with $x_1 = 3$ and the common ration $\frac{1}{2}$. A new data is constructed replacing each x_i by $(x_i - i)^2$. If $\overline{x}$ is the mean of new data, then the greatest integer less than or equal to $\overline{x}$ is _________.

Official Ans. by NTA (142)

Sol. $\sum x_0^1 = \frac{3\left(1-\left(\frac{1}{2}\right)\right)^{20}}{1\frac{-1}{2}} = 6\left(1-\frac{1}{2^{20}}\right)$

$= \sum_{i=1}^{20} (x_{i-i})^2$

$= \sum_{i=1}^{20} (x_i)^2 + (i)^2 - 2x_i i$

Now $= \sum_{i=1}^{20} (x_i)^2 = \frac{9\left(1-\left(\frac{1}{4}\right)\right)^{20}}{1-\frac{1}{4}} = 12\left(1-\frac{1}{2^{40}}\right)$

$\sum_{i=1}^{20} i^2 = \frac{1}{6} \times 20 \times 21 \times 41 = 2870$

$\sum_{i=1}^{20} x_i\ i = s = 3 + 2.3\frac{1}{2} + 3.3\frac{1}{2^2} + 4.3\frac{1}{2^3} + ...AGP$

$= 6\left(2-\frac{22}{2^{20}}\right)$

$\bar{x} = \frac{12-\frac{12}{2^{40}} + 2870 - 12\left(2-\frac{22}{2^{20}}\right)}{20}$

$\bar{x} = \frac{2858}{20} + \left(\frac{-12}{2^{40}} + \frac{22}{2^{20}}\right) \times \frac{1}{20}$

$[\bar{x}] = 142$

9. $\lim_{x\to 0}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{\frac{100}{x}}$

is equal to ____________.

Official Ans. by NTA (1)

Sol.

$\lim_{x\to 10}\left(\frac{(x+2\cos x)^3 + 2(x+2\cos x)^2 + 3\sin(x+2\cos x)}{(x+2)^3 + 2(x+2)^2 + 3\sin(x+2)}\right)^{x}$

Form 1^∞

$= e^{\lim_{x\to 0}\left[\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)-1\right]\times\frac{100}{x}}$

$= e^{\lim_{x\to 0}\left[\frac{100}{x}\left(\frac{(x+2\cos x)^3+2(x+2\cos x)^2+3\sin(x+2\cos x)-\left((x+2)^3+2(x+2)^2+3\sin(x+2)\right)}{(x+2)^3+2(x+2)^2+3\sin(x+2)}\right)\right]}$

$= e^{\lim_{x\to 0}\frac{100}{x}\left[\left(\frac{(x+2\cos x)^3+(x+2)^3+2(x+2\cos x)^2-2(x+2)^2+3\sin(x+2\cos x)-3\sin(x+2)}{8+8+3\sin^2}\right)\right]}$

$= e^{\frac{100}{16+3\sin^2}\lim_{x\to 0}\frac{3(x+2\cos x)^2\times(1+2\sin x)-3(x+2)^2-4(x+2\cos x)}{x(1-2\sin x)-4(x+2)+3\cos(x+2\cos x)\times(1-2\sin x)-3\cos(x+2)}}$

$= e^{\frac{100}{16+3\sin 2}}\left(\frac{12-3(4)+8\times 1-8+3\cos 2-3\cos 2}{1}\right)$

Using L'H rule.

$= e^o = 1$

10. The sum of all real values of x for which $\frac{3x^2-9x+17}{x^2+3x+10} = \frac{5x^2-7x+19}{3x^2+5x+12}$ is equal to ____.

Official Ans. by NTA (6)

Sol. $\frac{3x^2-9x+17}{x^2+3x+10} = \frac{5x^2-7x+19}{3x^2+5x+12}$

$\frac{x^2+3x+10+2x^2-12x+7}{x^2+3x+10} = \frac{3x^2+5x+12+2x^2-12x+7}{3x^2+5x+12}$

$1+\frac{2x^2-12x+7}{x^2+3x+10} = 1+\frac{2x^2-12x+7}{3x^2+5x+12}$

$(2x^2-12x+7)\left(\frac{1}{x^2+3x+10} - \frac{1}{3x^2+5x+12}\right) = 0$

$2x^2-12x+7=0$ OR $3x^2+5x+12 = x^2+3x+10$

$x = \frac{12\pm\sqrt{D}}{4}$ $\qquad 2x^2+2x+2=0$

$x^2+x+1=0$

Sum of Roots = 6 $\qquad$ No solution.