FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Velocity (v) and acceleration (a) in two systems of units 1 and 2 are related as $v_2 = \frac{n}{m^2}v_1$ and $a_2 = \frac{a_1}{mn}$ respectively. Here m and n are constants. The relations for distance and time in two systems respectively are:

(A) $\frac{n^3}{m^3}L_1 = L_2$ and $\frac{n^2}{m}T_1 = T_2$

(B) $L_1 = \frac{n^4}{m^2}L_2$ and $T_1 = \frac{n^2}{m}T_2$

(C) $L_1 = \frac{n^2}{m}L_2$ and $T_1 = \frac{n^4}{m^2}T_2$

(D) $\frac{n^2}{m}L_1 = L_2$ and $\frac{n^4}{m^2}T_1 = T_2$

Official Ans. by NTA (A)

Sol. $\frac{L_2}{T_2} = \frac{n}{m^2}\frac{L_1}{T_1}$

$$\frac{L_2}{T_2^2} = \frac{L_1}{T_1^2 \times mn}$$

$$\frac{n}{m^2} \times \frac{T_2}{T_1} = \frac{T_2^2}{T_1^2 \times mn}$$

$$\frac{n^2}{m} = \frac{T_2}{T_1}$$

$$\frac{L_2}{L_1} = \frac{n^4}{m^2} \times \frac{1}{mn}$$

$$\frac{L_2}{L_1} = \frac{n^3}{m^3}$$

2. A ball is spun with angular acceleration $\alpha = 6t^2 - 2t$ where t is in second and α is in rads^{-2}. At t = 0, the ball has angular velocity of 10 rads^{-1} and angular position of 4 rad. The most appropriate expression for the angular position of the ball is:

(A) $\frac{3}{2}t^4 - t^2 + 10t$

(B) $\frac{t^4}{2} - \frac{t^3}{3} + 10t + 4$

(C) $\frac{2t^4}{3} - \frac{t^3}{6} + 10t + 12$

(D) $2t^4 - \frac{t^3}{2} + 5t + 4$

Official Ans. by NTA (B)

Sol. $\frac{dw}{dt} = 6t^2 - 2t$

$$\int_{10}^{w} dw = 2t^3 - t^2$$

$$w = 10 + 2t^3 - t^2$$

$$\frac{d\theta}{dt} = 10 + 2t^3 - t^2$$

$$\int_4^{\theta} d\theta = 10 + 2t^3 - t^2$$

$$\int_4^{\theta} d\theta = 10t + \frac{t^4}{2} - \frac{t^3}{3}$$

$$\theta = 4 + 10t + \frac{t^4}{2} - \frac{t^3}{3}$$

3. A block of mass 2 kg moving on a horizontal surface with speed of 4 ms^{-1} enters a rough surface ranging from x = 0.5 m to x = 1.5 m. The retarding force in this range of rough surface is related to distance by $F = -kx$ where k = 12 Nm^{-1}. The speed of the block as it just crosses the rough surface will be:

(A) Zero (B) 1.5 ms^{-1}

(C) 2.0 ms^{-1} (D) 2.5 ms^{-1}

Official Ans. by NTA (C)

Sol. $a = \frac{-kx}{2} = \frac{-12x}{2} = -6x$

$$\frac{vdv}{dx} = -6x$$

$$\int_4^v vdv = -\int_{\frac{1}{2}}^{3/2} 6xdx$$

$$\frac{v^2 - 4^2}{2} = -\frac{6}{2}\left[\left(\frac{3}{2}\right)^2 - \left(\frac{1}{2}\right)^2\right]$$

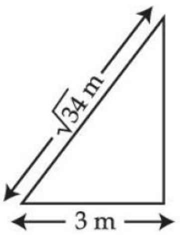
$$v^2 - 16 = -6\left(\frac{9}{4} - \frac{1}{4}\right)$$

$$v^2 = 16 - 6 \times 2 = 4$$

V = 2 m/s

4. A $\sqrt{34}$ m long ladder weighing 10 kg leans on a frictionless wall. Its feet rest on the floor 3 m away from the wall as shown in the figure. If F_f and F_w are the reaction forces of the floor and the wall, then ratio of F_w/F_f will be:

(Use g = 10 m/s^2)

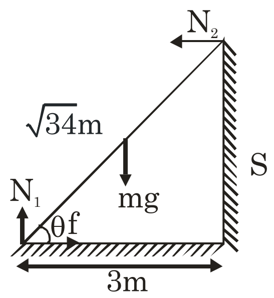


(A) $\frac{6}{\sqrt{110}}$ (B) $\frac{3}{\sqrt{113}}$

(C) $\frac{3}{\sqrt{109}}$ (D) $\frac{2}{\sqrt{109}}$

Official Ans. by NTA (C)

Sol.



$f = N_2$

$N_1 = mg$

$$N_2 \times \ell \sin\theta = mg\frac{\ell}{2}\cos\theta$$

$$N_2 = \frac{mg}{2}\cot\theta$$

$$\frac{F_w}{F_f} = \frac{\frac{mg}{2}\cot\theta}{\sqrt{(mg)^2 + \left(\frac{mg}{2}\cot\theta\right)^2}}$$

$$= \frac{1}{\sqrt{1 + \frac{4}{\cot^2\theta}}}$$

$$= \frac{3}{\sqrt{109}}$$

5. Water fall from a 40 m high dam at the rate of 9×10^4 kg per hour. Fifty percentage of gravitational potential energy can be converted into electrical energy. Using this hydroelectric energy number of 100W lamps, that can be lit, is:

(Take g = 10 ms^{-2})

(A) 25 (B) 50

(C) 100 (D) 18

Official Ans. by NTA (B)

Sol. $\frac{9 \times 10^4 \times g \times 40}{3600} \times 0.5 = n \times 100$

$$\frac{10^4 \times 0.5}{100} = n$$

100 × 0.5 = n

n = 50

6. Two objects of equal masses placed at certain distance from each other attracts each other with a force of F. If one-third mass of one object is transferred to the other object, then the new force will be :

(A) $\frac{2}{9}F$ (B) $\frac{16}{9}F$

(C) $\frac{8}{9}F$ (D) F

Official Ans. by NTA (C)

Sol. $F = \frac{Gm^2}{r^2}$

$$F' = \frac{G\left(\frac{4m}{3}\right)\times\left(\frac{2m}{3}\right)}{r^2}$$

$$F' = \frac{8}{9}F$$

7. A water drop of radius $1\mu m$ falls in a situation where the effect of buoyant force is negligible. Co-efficient of viscosity of air is $1.8\times10^{-5}\,Nsm^{-2}$ and its density is negligible as compared to that of water $10^6\,gm^{-3}$. Terminal velocity of the water drop is:

(Take acceleration due to gravity = 10 ms^{-2})

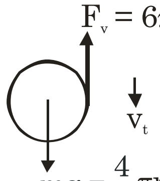
(A) $145.4\times10^{-6}\,ms^{-1}$ (B) $118.0\times10^{-6}\,ms^{-1}$

(C) $132.6\times10^{-6}\,ms^{-1}$ (D) $123.4\times10^{-6}\,ms^{-1}$

Official Ans. by NTA (D)

Sol.

$F_v = 6\pi\eta r v_t$



v_t

$mg = \frac{4}{3}\pi r^3\rho g$

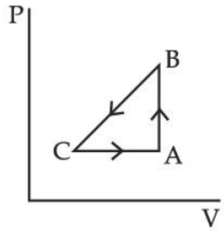
$$6\pi\eta r v_t = \frac{4}{3}\pi r^3 \rho g$$

$$v_t = \frac{4}{3}\times\frac{\pi r^3\rho g}{6\pi\eta r}$$

$$v_t = \frac{4}{3}\times\frac{\pi r^3\rho g}{6\pi\eta r} = \frac{2\times10^{-12}\times10^3\times10}{9\times1.8\times10^{-5}}$$

$= 123.4 \times 10^{-6}$ m/s

8. A sample of an ideal gas is taken through the cyclic process ABCA as shown in figure. It absorbs, 40 J of heat during the part AB, no heat during BC and rejects 60J of heat during CA. A work 50J is done on the gas during the part BC. The internal energy of the gas at A is 1560J. The work done by the gas during the part CA is:

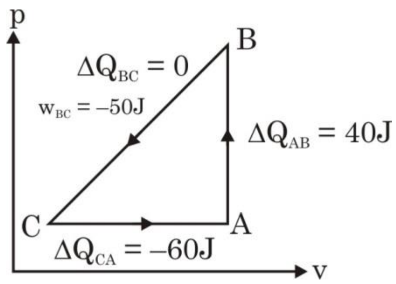


(A) 20 J (B) 30 J

(C) –30J (D) –60 J

Official Ans. by NTA (B)

Sol.



$\Delta Q_{cycle} = 40 - 60 = \Delta W$

$\Rightarrow \Delta W = -20J = W_{BC} + W_{CA}$

$\Rightarrow W_{CA} = -20J - W_{BC}$

$= -20 - (-50)$

$= 30$ J

9. What will be the effect on the root mean square velocity of oxygen molecules if the temperature is doubled and oxygen molecule dissociates into atomic oxygen?

(A) The velocity of atomic oxygen remains same

(B) The velocity of atomic oxygen doubles

(C) The velocity of atomic oxygen becomes half

(D) The velocity of atomic oxygen becomes four times

Official Ans. by NTA (B)

Sol. $V_{rms} = \sqrt{\dfrac{3RT}{M}}$

$T \to 2T$

$M \to \dfrac{M}{2}$

$V_{rms} \propto \sqrt{\dfrac{T}{M}}$

$\Rightarrow (V_{rms})_{atomic} = (V_{rms})_{molecular} \times \sqrt{\dfrac{2}{1/2}} = 2(V_{rms})_{molecular}$

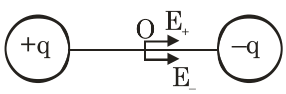
10. Two point charges A and B of magnitude $+8\times10^{-6}C$ and $-8\times10^{-6}C$ respectively are placed at a distance d apart. The electric field at the middle point O between the charges is $6.4\times10^{4}\,NC^{-1}$. The distance 'd' between the point charges A and B is:

(A) 2.0 m (B) 3.0 m

(C) 1.0 m (D) 4.0 m

Official Ans. by NTA (B)

Sol.



$E_0 = 2\times\dfrac{Kq}{(d/2)^2}$

$\Rightarrow E_0 = 8\dfrac{Kq}{d^2}$

$\Rightarrow d^2 = \dfrac{8\times9\times10^{9}\times8\times10^{-6}}{6.4\times10^{4}}$

d = 3 m

11. Resistance of the wire is measured as 2Ω and 3Ω at 10°C and 30°C respectively. Temperature co–coefficient of resistance of the material of the wire is :

(A) $0.033°C^{-1}$ (B) $-0.033°C^{-1}$

(C) $0.011°C^{-1}$ (D) $0.055°C^{-1}$

Official Ans. by NTA (A)

Sol. $R = R_0(1+\alpha\Delta T)$

$3 = R_0(1+\alpha(30-0))$

$2 = R_0(1+\alpha(10-0))$

$\dfrac{3}{2} = \dfrac{1+30\alpha}{1+10\alpha}$

$\alpha = \dfrac{1}{30} = 0.033$

12. The space inside a straight current carrying solenoid is filled with a magnetic material having magnetic susceptibility equal to 1.2×10^{-5}. What is fractional increase in the magnetic field inside solenoid with respect to air as medium inside the solenoid?

(A) 1.2×10^{-5} (B) 1.2×10^{-3}

(C) 1.8×10^{-3} (D) 2.4×10^{-5}

Official Ans. by NTA (A)

Sol. $\chi = 1.2\times10^{-5}$

$\mu_r = 1+\chi = 1+1.2\times10^{-5}$

Fractional Change

$= \dfrac{\Delta B}{B} = \dfrac{\mu_0\mu_r ni - \mu_0 ni}{\mu_0 ni} = (\mu_r - 1)$

$= 1.2 \times 10^{-5}$

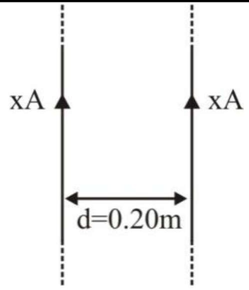
13. Two parallel, long wires are kept 0.20 m apart in vacuum, each carrying current of x A in the same direction. If the force of attraction per meter of each wire is $2\times10^{-6}N$, then the value of x is approximately:

(A) 1 (B) 2.4

(C) 1.4 (D) 2

Official Ans. by NTA (C)

Sol.



Force per unit length = $\frac{\mu_0 i_1 i_2}{2\pi d}$

$= \frac{\mu_0 \cdot x^2}{2\pi \times 0.2}$

$F = 2 \times 10^{-6} = \frac{4\pi \times 10^{-7} \times x^2}{2\pi \times 0.2}$

$\Rightarrow \; 10^{-6} = 10^{-7} \frac{x^2}{0.2}$

$\Rightarrow \; x^2 = 10 \times 0.2$

$= 2$

$\Rightarrow \; x = \sqrt{2} \approx 1.4$ Amp.

14. A coil is placed in a time varying magnetic field. If the number of turns in the coil were to be halved and the radius of wire doubled, the electrical power dissipated due to the current induced in the coil would be:

(Assume the coil to be short circuited.)

(A) Halved

(B) Quadrupled

(C) The same

(D) Doubled

Official Ans. by NTA (D)

Sol. $P = \frac{\varepsilon^2}{R} = \frac{\left(NA\frac{dB}{dt}\right)^2 \times A_C}{\rho\ell}$

$P' = \frac{\left(\frac{NA}{2}\frac{dB}{dt}\right)^2 \times 4A_C}{\rho\ell/2}$

$\Rightarrow P' = 2P$

15. An EM wave propagating in x-direction has a wavelength of 8 mm. The electric field vibrating y-direction has maximum magnitude of 60 Vm^{-1}. Choose the correct equations for electric and magnetic fields if the EM wave is propagating in vacuum :

(A) $E_y = 60\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 2\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{k}T$

(B) $E_y = 60\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 2\times10^{-7}\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{k}T$

(C) $E_y = 2\times10^{-7}\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times10^3\left(x-3\times10^8 t\right)\right]\hat{k}T$

(D) $E_y = 2\times10^{-7}\sin\left[\frac{\pi}{4}\times10^4\left(x-4\times10^8 t\right)\right]\hat{j}Vm^{-1}$

$B_z = 60\sin\left[\frac{\pi}{4}\times10^4\left(x-4\times10^8 t\right)\right]\hat{k}T$

Official Ans. by NTA (B)

Sol. $B_0 = \frac{E_0}{c} = \frac{60}{3\times10^8} = 2\times10^{-7}T$

$E \times B$ must be direction of propagation.

So, $B \rightarrow$ z-axis

$k = \frac{2\pi}{\lambda} = \frac{\pi}{4}\times10^3 m^{-1}$

$E_y = 60\sin\left[\frac{\pi}{4}\times10^3(x-3\times10^8 t)\right]\hat{j}\,Vm^{-1}$

$B_z = 2\times10^{-7}\sin\left[\frac{\pi}{4}\times10^3(x-3\times10^8 t)\right]k\,T$

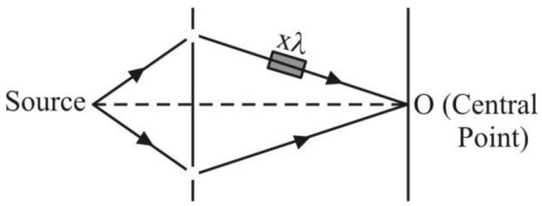
16. In young's double slit experiment performed using a monochromatic light of wavelength λ, when a glass plate $(\mu = 1.5)$ of thickness $x\lambda$ is introduced in the path of the one of the interfering beams, the intensity at the position where the central maximum occurred previously remains unchanged. The value of x will be:

(A) 3 (B) 2

(C) 1.5 (D) 0.5

Official Ans. by NTA (B)

Sol.



Path difference at O = $(\mu - 1)t$.

If the intensity at O remains (maximum) unchanged, path difference must be n λ.

$\Rightarrow (\mu - 1)t = n\lambda$

$(1.5 - 1)x\lambda = n\lambda$

$\Rightarrow \quad x = 2n$

For n = 1, x = 2

17. Let K_1 and K_2 be the maximum kinetic energies of photo–electrons emitted when two monochromatic beams of wavelength λ_1 and λ_2, respectively are incident on a metallic surface. If $\lambda_1 = 3\lambda_2$ then:

(A) $K_1 > \frac{K_2}{3}$ (B) $K_1 < \frac{K_2}{3}$

(C) $K_1 = \frac{K_2}{3}$ (D) $K_2 = \frac{K_1}{3}$

Official Ans. by NTA (B)

Sol. $\frac{hc}{\lambda_1} - \phi = K_1$

$\frac{hc}{\lambda_2} - \phi = K_2$

$\lambda_1 = 3\lambda_2$

$3K_1 = \frac{3hc}{\lambda_1} - 3\phi$

$3K_1 = \frac{hc}{\lambda_2} - 3\phi$

$3K_1 = K_2 - 2\phi$

$3K_1 < K_2$

$K_1 < \frac{K_2}{3}$

18. Following statements related to radioactivity are given below:

(A) Radioactivity is a random and spontaneous process and is dependent on physical and chemical conditions.

(B) The number of un–decayed nuclei in the radioactive sample decays exponentially with time.

(C) Slope of the graph of $\log_e$(no. of undecayed nuclei) Vs. time represents the reciprocal of mean life time (τ).

(D) Product of decay constant (λ) and half–life time $(T_{1/2})$ is not constant.

Choose the most appropriate answer from the options given below:

(A) (A) and (B) only

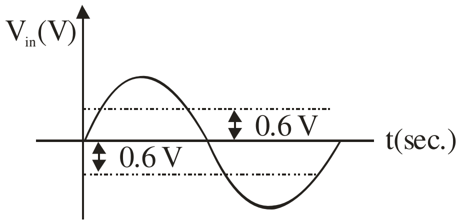
(B) (B) and (D) only

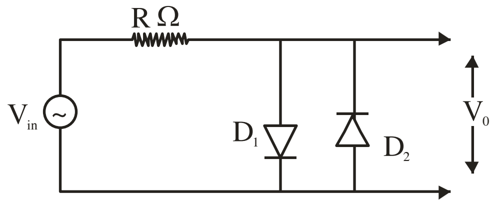
(C) (B) and (C) only

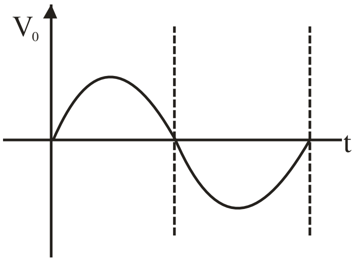
(D) (C) and (D) only

Official Ans. by NTA (C)

19. In the given circuit the input voltage V_{in} is shown in figure. The cut–in voltage of p–n junction diode (D_1 or D_2) is 0.6 V. Which of the following output voltage (V_0) waveform across the diode is correct?





(A)


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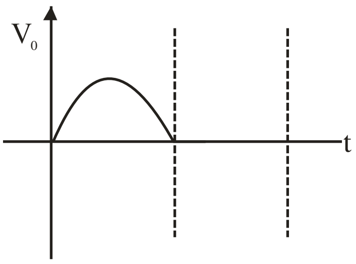

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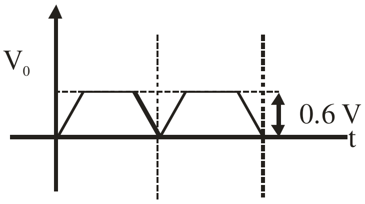

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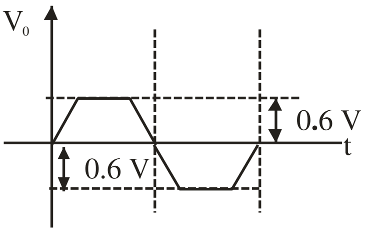

Official Ans. by NTA (D)

Sol. In +ve half cycle

$D_1 \rightarrow$ F.B.; $D_2 \rightarrow$ R.B.

0 – 0.6 V

V_{out} same as V_{in}

In –ve half cycle

$D_2 \rightarrow$ F.B.; $D_1 \rightarrow$ R.B.

20. Amplitude modulated wave is represented by

$$V_{AM} = 10\left[1+0.4\cos\left(2\pi\times10^4 t\right)\right]\cos\left(2\pi\times10^7 t\right).$$

The total bandwidth of the amplitude modulated wave is :

(A) 10 kHz (B) 20 MHz

(C) 20 kHz (D) 10 MHz

Official Ans. by NTA (C)

Sol. Bandwidth = 2 f_m

= 2 × 10^4 Hz = 20 × 10^3 Hz

= 20 kHz

SECTION-B

1. A student in the laboratory measures thickness of a wire using screw gauge. The readings are 1.22 mm, 1.23 mm, 1.19 mm and 1.20 mm. The percentage error is $\frac{x}{121}\%$. The value of x is ___

Official Ans. by NTA (150)

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Sol. $X = \frac{1.22mm + 1.23mm + 1.19mm + 1.20mm}{4}$

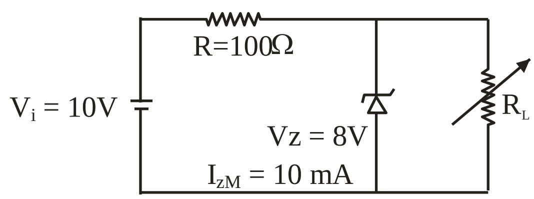
X = 1.21 mm

$$\Delta x = \frac{0.01+0.02+0.02+0.01}{4} = \frac{0.06}{4} = 0.015$$

Percentage error = $\frac{0.015}{1.21}\times100$

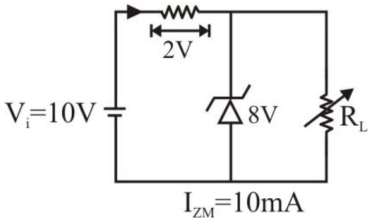
X = 150

2. A Zener of breakdown voltage V_Z = 8V and maximum zener current, I_{ZM} = 10 mA is subjected to an input voltage V_i = 10V with series resistance $R = 100\Omega$. In the given circuit R_L represents the variable load resistance. The ratio of maximum and minimum value of R_L is ______



Official Ans. by NTA (2)

Sol.



$I = \frac{2}{100} = 20\text{mA}$ $V_L = I_L R_L$ $8 = 10 \times 10^{-3} \times R_{L_{max}}$ $\frac{4}{5} \times 10^3 = R_{L_{max}}$ $\boxed{800 = R_{L_{max}}}$	$I = I_Z + I_L$ $I_L = 10$ mA If $\boxed{I_Z = 0}$ $I_{L_{max}} = 20\text{mA}$ $V_L = I_{L_{max}} \times R_{L_{min}}$ $\frac{8}{20} \times 10^3 = R_{L_{min}}$ $\boxed{400 = R_{L_{min}}}$

$$\frac{R_{L_{max}}}{R_{L_{min}}} = \frac{800}{400} = 2$$

3. In a Young's double slit experiment, an angular width of the fringe is 0.35° on a screen placed at 2 m away for particular wavelength of 450 nm. The angular width of the fringe, when whole system is immersed in a medium of refractive index 7/5, is $\frac{1}{\alpha}$. The value of α is ________

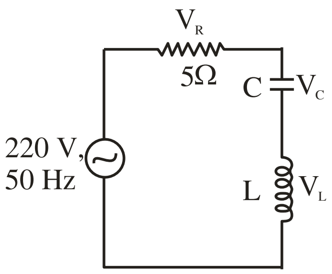
Official Ans. by NTA (4)

Sol. $\beta = \frac{0.35 \times 5}{7} = 0.25$

$\frac{1}{\alpha} = \frac{25}{100}$

$\boxed{\alpha = 4}$

4. In the given circuit, the magnitude of V_L and V_C are twice that of V_R. Given that f = 50 Hz, the inductance of the coil is $\frac{1}{K\pi}$mH. The value of K is _______



Official Ans. by NTA (0)

Sol. $V_L = V_C = 2V_R$

$X_L = X_C = 2R$

$X_L = 10\Omega$

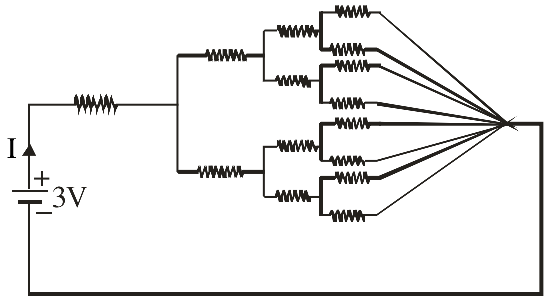
$\omega L = 10$

$2\pi f L = 10$

$L = \frac{10}{2\pi f} = \frac{1}{10\pi} H = \frac{1000}{10\pi} mH$

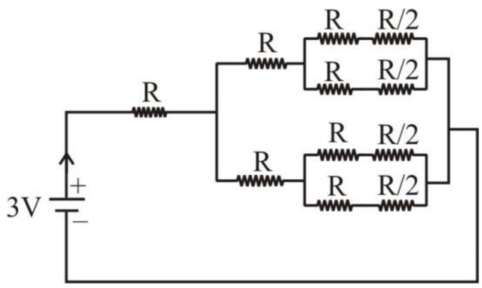
$L = \frac{1}{\frac{1}{100}\pi}$; $\quad K = \frac{1}{100} = 0.01 \approx 0$

5. All resistances in figure are 1Ω each. The value of current 'I' is $\frac{a}{5}$A. The value of a is _______



Official Ans. by NTA (8)

Sol.

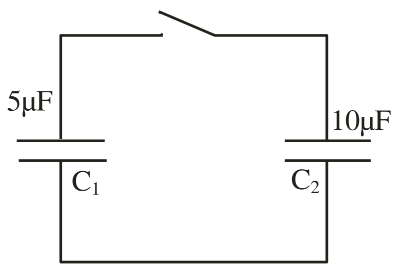


$R_{eq} = \frac{15R}{8} = \frac{15}{8}\Omega$

$I = \frac{3}{\frac{15}{8}} = \frac{8}{5}A$

$\therefore \ a = 8$

6. A capacitor C_1 of capacitance $5\mu F$ is charged to a potential of 30 V using a battery. The battery is then removed and the charged capacitor is connected to an uncharged capacitor C_2 of capacitance $10\mu F$ as shown in figure. When the switch is closed charge flows between the capacitors. At equilibrium, the charge on the capacitor C_2 is ________ μC.



Official Ans. by NTA (100)

Sol. Before closing the switch

$Q = C_1V_0 = 5 \times 30 = 150\mu C$

After closing the switch

$$V = \frac{Q}{C_1+C_2} = \frac{150}{10+5} = 10\text{ V}$$

$$Q_2 = C_2V = 10 \times 10 = 100\mu C$$

7. A tuning fork of frequency 340 Hz resonates in the fundamental mode with an air column of length 125 cm in a cylindrical tube closed at one end. When water is slowly poured in it, the minimum height of water required for observing resonance once again is ______cm.

(Velocity of sound in air is 340 ms^{-1})

Official Ans. by NTA (50)

Sol. Assumption : Ignore word "**fundamental mode**" in question.

$$\lambda = \frac{V}{f} = \frac{340}{340} = 1\text{m}$$

First resonating length = $\frac{\lambda}{4} = 25$ cm

Second resonating length = $\frac{3\lambda}{4} = 75$ cm

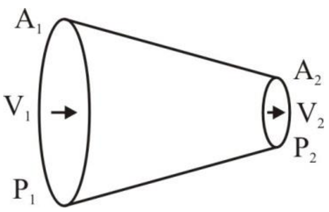
Third resonating length = $\frac{5\lambda}{4} = 125$ cm

Height of water required = 125 – 75 = 50 cm

8. A liquid of density 750 kgm^{-3} flows smoothly through a horizontal pipe that tapers in cross–sectional area from $A_1 = 1.2\times10^{-2}m^2$ to $A_2 = \frac{A_1}{2}$. The pressure difference between the wide and narrow sections of the pipe is 4500 Pa. The rate of flow of liquid is _______ $\times10^{-3}m^3s^{-1}$.

Official Ans. by NTA (24)

Sol.



$$A_2 = \frac{A_1}{2}$$

$$P_1 - P_2 = 4500\text{Pa}$$

$$P_1 + \frac{1}{2}\rho V_1^2 + \rho gh = P_2 + \frac{1}{2}\rho V_2^2 + \rho gh$$

$$P_1 - P_2 = \frac{1}{2}\rho\left(V_2^2 - V_1^2\right) \quad \ldots(1)$$

And $A_1V_1 = A_2V_2$

$$\Rightarrow \quad V_2 = 2V_1 \quad \ldots(2)$$

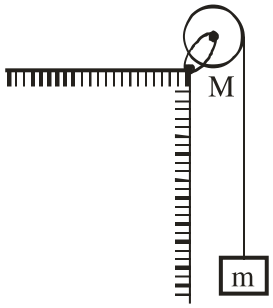
$$4500 = \frac{1}{2}\times750\times3V_1^2$$

$V_1 = 2$ m/s

Volume flow rate = $A_1V_1 = 24 \times 10^{-3}\ m^3s^{-1}$

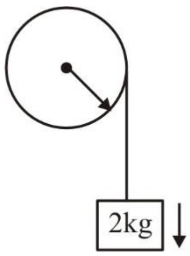
9. A uniform disc with mass $M = 4$ kg and radius $R = 10$ cm is mounted on a fixed horizontal axle as shown in figure. A block with mass $m = 2$ kg hangs from a massless cord that is wrapped around the rim of the disc. During the fall of the block, the cord does not slip and there is no friction at the axle. The tension in the cord is ________N.

(Take $g = 10\ ms^{-2}$)



Official Ans. by NTA (10)

Sol.



$2g - T = 2a$ ...(1)

$TR = \frac{MR^2}{2}\alpha$ ...(2)

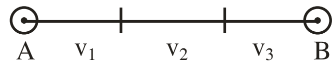
$\alpha = \frac{a}{R}$ ...(3)

$T = 2a$

$2g - T = 2a$

$T = g = 10N$

10. A car covers AB distance with first one–third at velocity $v_1\ ms^{-1}$, second one–third at $v_2\ ms^{-1}$ and last one–third at $v_3\ ms^{-1}$. If $v_3 = 3v_1$, $v_2 = 2v_1$ and $v_1 = 11\ ms^{-1}$ then the average velocity of the car is ________ ms^{-1}.



Official Ans. by NTA (18)

Sol. $<\vec{v}> = \frac{\text{Displacement}}{\text{time}}$

(Let displacement be l)

$$= \frac{\ell}{\left(\frac{\ell}{V_3} + \frac{\ell}{V_2} + \frac{\ell}{V_1}\right)\frac{1}{3}}$$

$$= \frac{3}{\frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3}} = \frac{3}{\frac{1}{11} + \frac{1}{22} + \frac{1}{33}}$$

$= 18$ m/s

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 6 : 00 PM**

CHEMISTRY **TEST PAPER WITH SOLUTION**

SECTION-A

1. Compound A contains 8.7% Hydrogen, 74% Carbon and 17.3% Nitrogen. The molecular formula of the compound is,

Given : Atomic masses of C, H and N are 12, 1 and 14 amu respectively.

The molar mass of the compound A is 162 g mol^{-1}.

(A) $C_4H_6N_2$ (B) C_2H_3N

(C) C_5H_7N (D) $C_{10}H_{14}N_2$

Official Ans. by NTA (D)

Sol.

C	74%	$\frac{74}{12}=6.16$	$\frac{6.16}{1.23}=5$
N	17.3%	$\frac{17.3}{14}=1.23$	$\frac{1.23}{1.23}=1$
H	8.7%	$\frac{8.7}{1}=8.7$	$\frac{8.7}{1.23}=7$

Emperical formula = C_5NH_7

Emperical weight = 81

Multiplying factor = $\frac{162}{81}=2$

Molecular formula = $C_{10}N_2H_{14}$

2. Consider the following statements :

(A) The principal quantum number 'n' is a positive integer with values of 'n' = 1, 2, 3, ….

(B) The azimuthal quantum number 'l' for a given 'n' (principal quantum number) can have values as 'l' = 0, 1, 2, …. n

(C) Magnetic orbital quantum number 'm_l' for a particular 'l' (azimuthal quantum number) has ($2l$ + 1) values.

(D) ±1/2 are the two possible orientations of electron spin.

(E) For l = 5, there will be a total of 9 orbital.

Which of the above statements are **correct**?

(A) (A), (B) and (C)

(B) (A), (C), (D) and (E)

(C) (A), (C) and (D)

(D) (A), (B), (C) and (D)

Official Ans. by NTA (C)

Sol. (A) Number of values of n = 1, 2, 3 … ∞

(B) Number of values of ℓ = 0 to (n – 1)

(C.) Number of values of $m = -\ell$ to $+\ell$

Total values = $2\ell + 1$

(D) Values of spin = $\pm\frac{1}{2}$

(E) For ℓ = 5 number of orbitals = $2\ell + 1 = 11$

3. In the structure of SF_4, the lone pair of electrons on S is in.

(A) equatorial position and there are two lone pair-bond pair repulsions at 90°

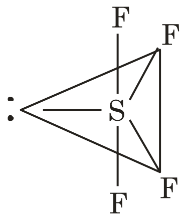
(B) equatorial position and there are three lone pair-bond pair repulsions at 90°

(C) axial position and there are three lone pair – bond pair repulsion at 90°.

(D) axial position and there are two lone pair – bond pair repulsion at 90°.

Official Ans. by NTA (A)

Sol.



sp^3d, See-Saw

4. A student needs to prepare a buffer solution of propanoic acid and its sodium salt with pH 4. The ratio of $\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$ required to make buffer is ……….

Given : $K_a(CH_3CH_2COOH) = 1.3 \times 10^{-5}$

(A) 0.03 (B) 0.13

(C) 0.23 (D) 0.33

Official Ans. by NTA (B)

Sol. $$pH = pK_a + \log\frac{[Salt]}{[Acid]}$$

$$4 = 5 - \log 1.3 + \log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]}$$

$$\log\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = \log 1.3 - 1 = \log\frac{1.3}{10}$$

$$\frac{[CH_3CH_2COO^-]}{[CH_3CH_2COOH]} = 0.13$$

5. Match List-I with List-II.

List-I		List-II	
(A)	Negatively charged sol	(I)	$Fe_2O_3 \cdot xH_2O$
(B)	Macromolecular colloid	(II)	CdS sol
(C)	Positively charged sol	(III)	Starch
(D)	Cheese	(IV)	a gel

Choose the correct answer from the options given below :

(A) (A) – (II), (B) – (III), (C) – (IV), (D) – (I)

(B) (A) – (II), (B) – (I), (C) – (III), (D) – (IV)

(C) (A) – (II), (B) – (III), (C) – (I), (D) – (IV)

(D) (A) – (I), (B) – (III), (C) – (II), (D) – (IV)

Official Ans. by NTA (C)

Sol. Negative charged sol = CdS (II)

Macromolecular colloid = starch (III)

Positively charged sol = $Fe_2O_3.xH_2O$ (I)

Cheese = gel (IV)

6. Match List-I with List-II.

List-I (Oxide)		List-II (Nature)	
(A)	Cl_2O_7	(I)	Amphoteric
(B)	Na_2O	(II)	Basic
(C)	Al_2O_3	(III)	Neutral
(D)	N_2O	(IV)	Acidic

Choose the **correct** answer from the options given below :

(A) (A) – (IV), (B) – (III), (C) – (I), (D) – (II)

(B) (A) – (IV), (B) – (II), (C) – (I), (D) – (III)

(C) (A) – (II), (B) – (IV), (C) – (III), (D) – (I)

(D) (A) – (I), (B) – (II), (C) – (IIII), (D) – (IV)

Official Ans. by NTA (B)

Sol. Cl_2O_7 Acidic

Na_2O Basic

Al_2O_3 Amphoteric

N_2O Neutral

7. In the metallurgical extraction of copper, following reaction is used :

$FeO + SiO_2 \rightarrow FeSiO_3$

FeO and $FeSiO_3$ respectively are.

(A) gangue and flux (B) flux and slag

(C) slag and flux (D) gangue and slag

Official Ans. by NTA (D)

Sol. FeO = Gangue

$FeSiO_3$ = Slag

8. Hydrogen has three isotopes : protium (^1H), deuterium (^2H or D) and tritium (^3H or T). They have nearly same chemical properties but different physical properties. They differ in

(A) number of protons

(B) atomic number

(C) electronic configuration

(D) atomic mass

Official Ans. by NTA (D)

Sol. They have different neutrons and mass number

9. Among the following basic oxide is :

(A) SO_3 (B) SiO_2

(C) CaO (D) Al_2O_3

Official Ans. by NTA (C)

Sol. SO_3, SiO_2 = Acidic

CaO = Basic

Al_2O_3 = Amphoteric

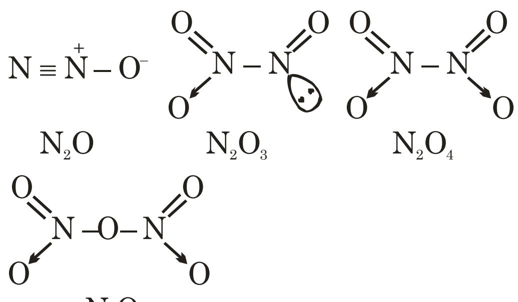
10. Among the given oxides of nitrogen; N_2O, N_2O_3, N_2O_4 and N_2O_5, the number of compound/(s) having N–N bond is :

(A) 1 (B) 2

(C) 3 (D) 4

Official Ans. by NTA (C)

Sol.



N_2O N_2O_3 N_2O_4 N_2O_5

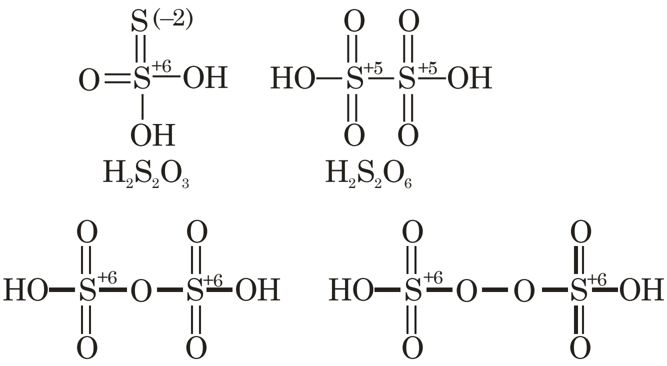
11. Which of the following oxoacids of sulphur contains "S" in two different oxidation states?

(A) $H_2S_2O_3$ (B) $H_2S_2O_6$

(C) $H_2S_2O_7$ (D) $H_2S_2O_8$

Official Ans. by NTA (A)

Sol.



$H_2S_2O_3$ $H_2S_2O_6$ $H_2S_2O_7$ $H_2S_2O_8$

12. Correct statement about photo-chemical smog is :

(A) It occurs in humid climate.

(B) It is a mixture of smoke, fog and SO_2

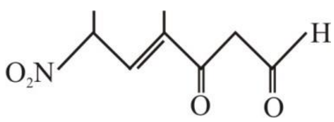
(C) It is reducing smog.

(D) It results from reaction of unsaturated hydrocarbons.

Official Ans. by NTA (D)

Sol. Photo chemical smog results from the action of sunlight on unsaturated hydro carbons and nitrogen oxide

13. The correct IUPAC name of the following compound is :



(A) 4-methyl-2-nitro-5-oxohept-3-enal

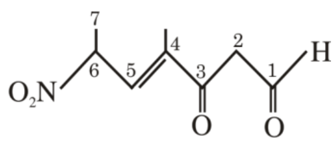
(B) 4-methyl-5-oxo-2-nitrohept-3-enal

(C) 4-methyl-6-nitro-3-oxohept-4-enal

(D) 6-formyl-4-methyl-2-nitrohex-3-enal

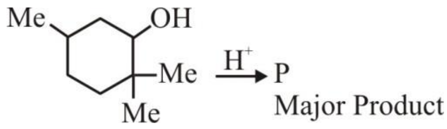
Official Ans. by NTA (C)

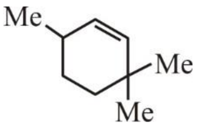
Sol.

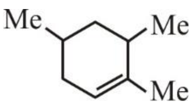


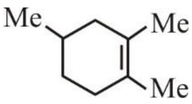
4-Methyl-6-nitro-3-oxohept-4-enal

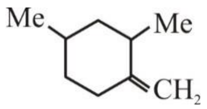
14. The major product (P) of the given reaction is (where, Me is $-CH_3$)



(A) 

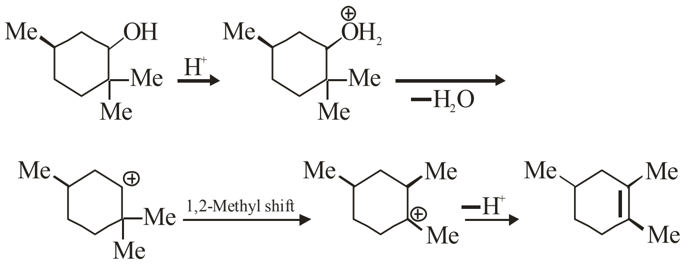
(B) 

(C) 

(D) 

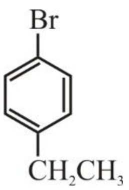
Official Ans. by NTA (C)

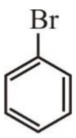
Sol.

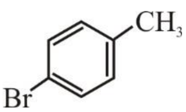


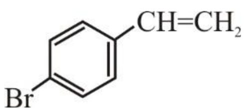
15. $A \xrightarrow[\text{(ii) } CN^- \\ \text{(iii) } H_2O/H^+]{\text{(i) } Cl_2,\Delta}$ 4-Bromophenyl acetic acid.

In the above reaction 'A' is

(A) 

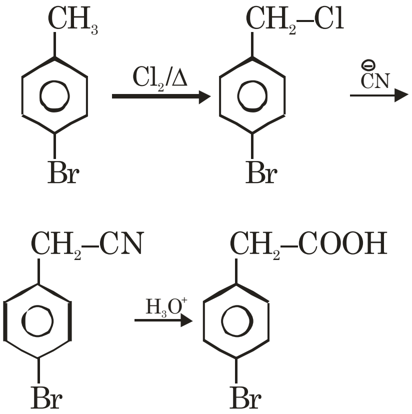
(B) 

(C) 

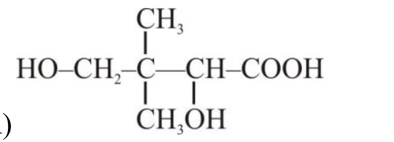
(D) 

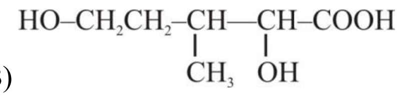
Official Ans. by NTA (C)

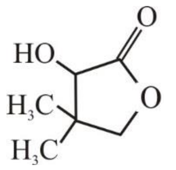
Sol.

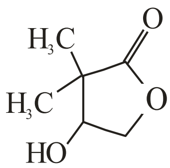


16. Isobutyraldehyde on reaction with formaldehyde and K_2CO_3 gives compound 'A'. Compound 'A' reacts with KCN and yields compound 'B', which on hydrolysis gives a stable compound 'C'. The compound 'C' is :

(A) 

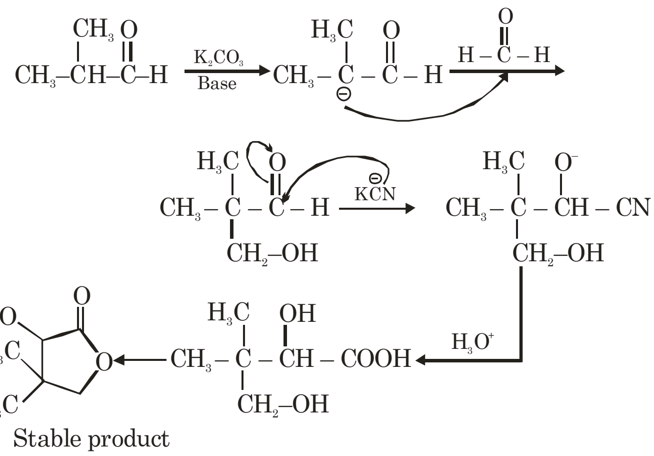
(B) 

(C) 

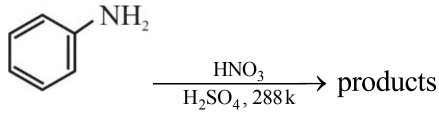
(D) 

Official Ans. by NTA (C)

Sol.



17. With respect to the following reaction, consider the given statements :



(A) o-Nitroaniline and p-nitroaniline are the predominant products

(B) p-Nitroaniline and m-nitroaniline are the predominant products

(C) HNO_3 acts as an acid

(D) H_2SO_4 acts as an acid

(A) (A) and (C) are correct statements.

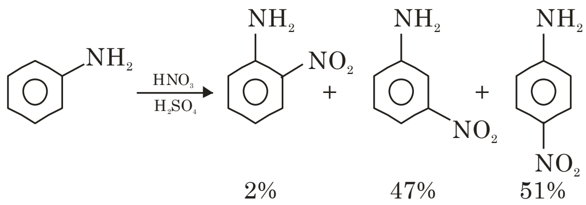
(B) (A) and (D) are correct statements.

(C) (B) and (D) are correct statements.

(D) (B) and (C) are correct statements.

Official Ans. by NTA (C)

Sol.



$$\underset{\text{Base}}{HNO_3} + \underset{\text{Acid}}{H_2SO_4} \rightarrow NO_2^+$$

18. Given below are two statements, one is Assertion (A) and other is Reason (R).

Assertion (A) : Natural rubber is a linear polymer of isoprene called cis-polyisoprene with elastic properties.

Reason (R) : The cis-polyisoprene molecules consist of various chains held together by strong polar interactions with coiled structure.

In the light of the above statements, choose the **correct** one from the options given below :

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A).

(C) (A) is true but (R) is false.

(D) (A) is false but (R) is true.

Official Ans. by NTA (C)

Sol. Natural rubber is linear polymer of isoprene (2-methyl-1,3-butadiene) and is also called cis-1,4-polyisoprene. The cis-polyisoprene molecules consists of various chains held together by weak Vander Waal's interactions and has a coiled structure

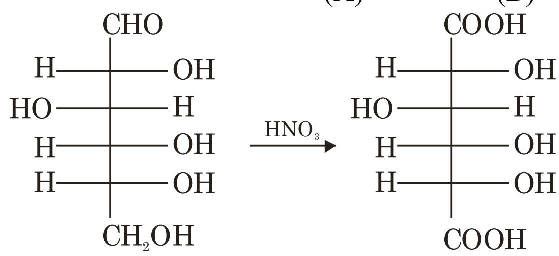
19. When sugar 'X' is boiled with dilute H_2SO_4 in alcoholic solution, two isomers 'A' and 'B' are formed. 'A' on oxidation with HNO_3 yields saccharic acid where as 'B' is laevorotatory. The compound 'X' is :

(A) Maltose (B) Sucrose

(C) Lactose (D) Strach

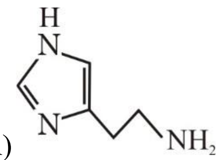
Official Ans. by NTA (B)

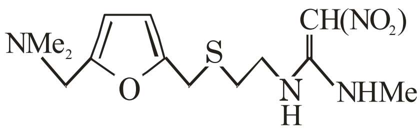
Sol. $$\underset{[\alpha]=66.6^\circ}{C_{12}H_{22}O_{11}} + H_2O \xrightarrow{H^+} \underset{\substack{\text{D-Glucose}\\ [\alpha]=+52.7^\circ \\ (A)}}{C_6H_{12}O_6} + \underset{\substack{\text{D-Fructose}\\ [\alpha]=-92.2^\circ \\ (B)}}{C_6H_{12}O_6}$$

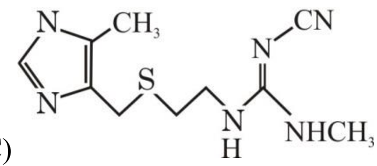


Sachharic acid

20. The drug tegamet is :

(A)


(B)


(C)


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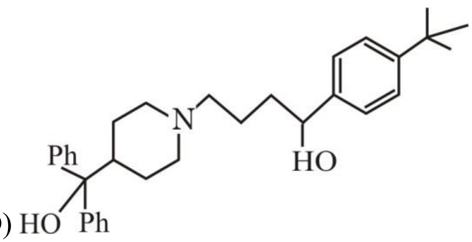
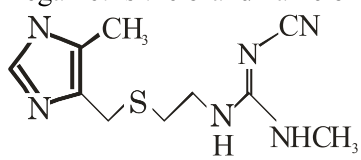


Official Ans. by NTA (C)

Sol. Tegamet is the brand name of Cimetidine



SECTION-B

1. 100 g of an ideal gas is kept in a cylinder of 416 L volume at 27°C under 1.5 bar pressure. The molar mass of the gas is ______ g mol^{-1}. (Nearest integer) (Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (4)

Sol. $1.5\times416=\frac{100}{M}\times0.083\times300$

M = 3.99

Ans. 4

2. For combustion of one mole of magnesium in an open container at 300 K and 1 bar pressure, $\Delta_C H^{\ominus}$ = –601.70 kJ mol^{-1}, the magnitude of change in internal energy for the reaction is _____ kJ. (Nearest integer)

(Given : R = 8.3 J K^{-1} mol^{-1})

Official Ans. by NTA (600)

Sol. $Mg(s)+\frac{1}{2}O_2(g)\rightarrow MgO(s)$

$\Delta H=\Delta U+\Delta n_g RT$

$-601.70\times10^3=\Delta U-\frac{1}{2}\times8.3\times300$

$-601.70\,kJ=\Delta U-1.245\,kJ$

$\Delta U=-600.455\,kJ$

Ans. 600

3. 2.5 g of protein containing only glycine ($C_2H_5NO_2$) is dissolved in water to make 500 mL of solution. The osmotic pressure of this solution at 300 K is found to be 5.03 × 10^{-3} bar. The total number of glycine units present in the protein is ____

(Given : R = 0.083 L bar K^{-1} mol^{-1})

Official Ans. by NTA (330)

Sol. $\pi=CRT$

$5.03\times10^{-3}=C\times0.083\times300$

$C=0.202\times10^{-3}M$

Moles of protein = 0.202 × 10^{-3} × 0.5

= 10^{-4} × 1.01

$1.01\times10^{-4}=\frac{2.5}{M}$

M(molar mass of protein) = 24752

$\therefore$ No. of glycine units = $\frac{24752}{75}=330.03$

4. For the given reactions

$Sn^{2+}+2e^-\rightarrow Sn$

$Sn^{4+}+4e^-\rightarrow Sn$

The electrode potentials are; $E^o_{Sn^{2+}/Sn}$ = –0.140 V and $E^o_{Sn^{4+}/Sn}$ = 0.010 V. The magnitude of standard electrode potential for Sn^{4+}/Sn^{2+} i.e. $E^o_{Sn^{4+}/Sn^{2+}}$ is ______ × 10^{-2} V. (Nearest integer)

Official Ans. by NTA (16)

Sol. $Sn^{2+}+2e^-\rightarrow Sn \qquad \Delta G_1^0=+2\times0.140\times F$

$Sn^{+4}+4e^-\rightarrow Sn \qquad \Delta G_2^0=-4\times0.01\times F$

$Sn^{+4}+2e^-\rightarrow Sn^{+2} \qquad \Delta G_3^0=-2\times E^0_{Sn^{+4}/Sn^{+2}}\times F$

$\Delta G_3^0=\Delta G_2^0-\Delta G_1^0$

$-2\times E^0\times F=-(0.04+0.28)\times F$

$E^0=0.16\text{ volt}=16\times10^{-2}\,V$

Ans 16

5. A radioactive element has a half life of 200 days. The percentage of original activity remaining after 83 days is ______. (Nearest integer)

(Given : antilog 0.125 = 1.333, antilog 0.693 = 4.93)

Official Ans. by NTA (75)

Sol. $t=\frac{t_{1/2}}{0.3}\log\frac{[A]_0}{[A]_t}$

$83=\frac{200}{0.3}\log\frac{[A]_0}{[A]_t}$

$0.125=\log\frac{[A]_0}{[A]_t}$

$\frac{[A]_0}{[A]_t}=1.333\simeq\frac{4}{3}$

$\therefore\frac{[A]_t}{[A]_0}\times100=\frac{3}{4}\times100=75\%$

Ans. 75

6. $[Fe(CN)_6]^{4-}$

$[Fe(CN)_6]^{3-}$

$[Ti(CN)_6]^{3-}$

$[Ni(CN)_4]^{2-}$

$[Co(CN)_6]^{3-}$

Among the given complexes, number of paramagnetic complexes is _____.

Official Ans. by NTA (2)

Sol. $[Fe(CN)_6]^{4-}$ Diamagnetic

$[Fe(CN)_6]^{3-}$ Paramagnetic (1 unpaired electron)

$[Ti(CN)_6]^{3-}$ Paramagnetic (1 unpaired electron)

$[Ni(CN)_4]^{2-}$ Diamagnetic

$[Co(CN)_6]^{3-}$ Diamagnetic

Ans. 2

7. (a) $CoCl_3{\cdot}4\ NH_3$

(b) $CoCl_3{\cdot}5NH_3$

(c) $CoCl_3{\cdot}.6NH_3$ and

(d) $CoCl(NO_3)_2{\cdot}5NH_3$

Number of complex(es) which will exist in cis-trans is/are

Official Ans. by NTA (1)

Sol. (a) $CoCl_3{\cdot}4\ NH_3 = [Co(NH_3)_4Cl_2]Cl$

Can exhibit G.I.

(b) $CoCl_3{\cdot}5NH_3 = [Co(NH_3)_5Cl]Cl_2$

Can't exhibit G.I.

(c) $CoCl_3{\cdot}.6NH_3 = [Co(NH_3)_6]Cl_3$

Can't exhibit G.I.

(d) $CoCl(NO_3)_2{\cdot}5NH_3 = [Co(NH_3)_5Cl](NO_3)_2$

OR

$= [Co(NH_3)_5(NO_3)]Cl(NO_3)$

Both can't exhibit G.I.

8. The complete combustion of 0.492 g of an organic compound containing 'C', 'H' and 'O' gives 0.793g of CO_2 and 0.442 g of H_2O. The percentage of oxygen composition in the organic compound is ___________. (nearest integer)

Official Ans. by NTA (46)

Sol. Mole of CO_2 = Moles of C = $\frac{0.793}{44}$

Weight of 'C' = $\frac{0.793}{44}\times 12 = 0.216$ gm

Moles of 'H' = $\frac{0.442}{18}\times 2$

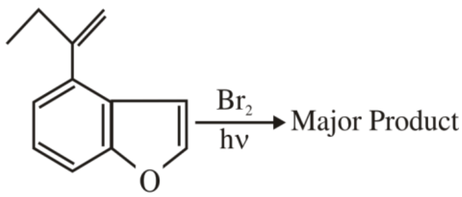
Weight of 'H' = $\frac{0.442}{18}\times 2\times 1 = 0.049$ gm

$\therefore$ Weight of 'O'=0.492–0.216–0.049= 0.227 gm

$\%$ of 'O' $= \frac{0.227}{0.492}\times 100 = 46.13\%$

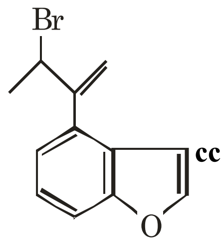
Ans. 46

9. The major product of the following reaction contains _______ bromine atom(s).



Official Ans. by NTA (1)

Sol.



No. of Br atoms = 1

10. 0.01 M $KMnO_4$ solution was added to 20.0 mL of 0.05 M Mohr's salt solution through a burette. The initial reading of 50 mL burette is zero. The volume of $KMnO_4$ solution left in the burette after the end point is _______ mL. (nearest integer)

Official Ans. by NTA (30)

Sol. $N_1V_1 = N_2V_2$

$0.01 \times 5 \times V_1 = 0.05 \times 1 \times 20$

$V_1 = 20$ ml used

$\therefore$ Volume left = 50 – 20 = 30 ml

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Tuesday 28th June, 2022) **TIME : 3 : 00 PM to 06 : 00 PM**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let $R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$ and $R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$. Then on N:

(A) Both R_1 and R_2 are equivalence relations

(B) Neither R_1 nor R_2 is an equivalence relation

(C) R_1 is an equivalence relation but R_2 is not

(D) R_2 is an equivalence relation but R_1 is not

Official Ans. by NTA (B)

Sol. $R_1 = \{(a,b) \in N \times N : |a-b| \le 13\}$

$R_2 = \{(a,b) \in N \times N : |a-b| \ne 13\}$.

For R_1 :

i) Reflexive relation

$(a,a) \in N \times N : |a-a| \le 13$

ii) Symmetric relation

$(a,b) \in R_1, (b,a) \in R_1 : |b-a| \le 13$

iii) Transitive relation

$(a,b) \in R_1, (b,c) \in R_1, (a,c) \in R_1 :$

$(1, 3) \in R_1$, $(3, 16) \in R_1$, but $(1, 16) \notin R_1$

For R_2 :

i) Reflexive relation

$(a,a) \in N \times N : |a-a| \ne 13$

ii) Symmetric relation

$(b,a) \in N \times N : |b-a| \ne 13$

iii) Transitive relation

$(a,b) \in R_2, (b,c) \in R_2, (a,c) \in R_2$

$(1, 3) \in R_2$, $(3, 14) \in R_2$, but $(1, 14) \notin R_2$

2. Let f(x) be a quadratic polynomial such that $f(-2) + f(3) = 0$. If one of the roots of $f(x) = 0$ is -1, then the sum of the roots of $f(x) = 0$ is equal to :

(A) $\frac{11}{3}$ (B) $\frac{7}{3}$

(C) $\frac{13}{3}$ (D) $\frac{14}{3}$

Official Ans. by NTA (A)

Sol. $f(-2) + f(3) = 0$

$f(x) = (x + 1)(ax + b)$

$f(-2) + f(3) = -1(-2a + b) + 4(3a + b) = 0$

$2a - b + 12a + 4b = 0$

$14a + 3b = 0$

$\frac{-b}{a} = \frac{14}{3}$

Sum of roots $= \left(-1 + \frac{-b}{a}\right) = -1 + \frac{14}{3} = \frac{11}{3}$

3. The number of ways to distribute 30 identical candies among four children C_1, C_2, C_3 and C_4 so that C_2 receives atleast 4 and atmost 7 candies, C_3 receives atleast 2 and atmost 6 candies, is equal to

(A) 205 (B) 615

(C) 510 (D) 430

Official Ans. by NTA (D)

Sol. $t_1 + t_2 + t_3 + t_4 = 30$

Coefficient of x^{30} in $(1 + x + x^2 + \ldots + x^{30})^2$

$(x^4 + x^5 + x^6 + x^7)(x^2 + x^3 + x^4 + x^5 + x^6)$

$x^6\left(\frac{1-x^{31}}{1-x}\right)^2 (1+x+x^2+x^3)(1+x+x^2+x^3+x^4)$

$x^6(1-x^{31})^2(1-x^4)(1-x^5)(1-x)^{-4}$

$x^6(1-x^4-x^5+x^9)(1+x^{62}-2x^{31}(1-x)^{-4})$

$x^6(1-x^4-x^5+x^9)(1-x)^{-4}$

Coefficient of x^n in $(1-x)^{-r}$ is ${}^{n+r-1}C_{r-1}$

$\Rightarrow {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3$

$2925 - 1771 - 1540 + 816$

$= 430$

OR

$x_2 \in [4,7], x_3 \in [2,6]$

$\Rightarrow t_1 + t_2 + t_3 + t_4 = 24$

total ways $=$

${}^{24+4-1}C_{4-1} - {}^{20+4-1}C_{4-1} - {}^{19+4-1}C_{4-1} + {}^{15+4-1}C_{4-1}$

$= {}^{27}C_3 - {}^{23}C_3 - {}^{22}C_3 + {}^{18}C_3 = 430$

4. The term independent of x in the expression of $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}, x \neq 0$ is

(A) $\frac{7}{40}$ (B) $\frac{33}{200}$

(C) $\frac{39}{200}$ (D) $\frac{11}{50}$

Official Ans. by NTA (B)

Sol. $\left(1-x^2+3x^3\right)\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

General term of $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ is

$${}^{11}C_r\left(\frac{5}{2}x^3\right)^{11-r}\left(-\frac{1}{5x^2}\right)^r$$

General term is ${}^{11}C_r\left(\frac{5}{2}\right)^{11-r}\left(-\frac{1}{5}\right)^r x^{33-5r}$

Now, term independent of x

$1 \times$ coefficient of x^0 in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

$-\ 1 \times$ coefficient of x^{-2} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$ +

$3 \times$ coefficient of x^{-3} in $\left(\frac{5}{2}x^3-\frac{1}{5x^2}\right)^{11}$

for coefficient of x^0 $\quad 33-5r=0$ not possible

for coefficient of x^{-2} $\quad 33-5r=-2$

$35=5r \Rightarrow r=7$

for coefficient of x^{-3} $\quad 33-5r=-3$

$36=5r$ not possible

So term independent of x is

$$(-1)\ {}^{11}C_7\left(\frac{5}{2}\right)^4\left(-\frac{1}{5}\right)^7=\frac{33}{200}$$

5. If n arithmetic means are inserted between a and 100 such that the ratio of the first mean to the last mean is 1 : 7 and a + n = 33, then the value of n is

(A) 21 (B) 22

(C) 23 (D) 24

Official Ans. by NTA (C)

Sol. $d=\frac{100-a}{n+1}$

$A_1=a+d$

$A_n=100-d$

$\Rightarrow \frac{A_1}{A_n}=\frac{1}{7} \Rightarrow \frac{a+d}{100-d}=\frac{1}{7}$

$\Rightarrow 7a+8d=100$

$\Rightarrow 7a+8\left(\frac{100-a}{n+1}\right)=100 \qquad \ldots.(1)$

$\because a+n=33 \qquad \ldots(2)$

Now, by Eq. (1) and (2)

$7n^2-132n-667=0$

$\boxed{n=23}$ and $n=\frac{-29}{7}$ reject.

6. Let $f, g: \mathbf{R} \to \mathbf{R}$ be functions defined by

$$f(x)=\begin{cases}[x] & , \ x<0 \\ |1-x| & , \ x \geq 0\end{cases} \text{ and}$$

$$g(x)=\begin{cases}e^x-x & , \ x<0 \\ (x-1)^2-1 & , \ x \geq 0\end{cases}$$

where [x] denote the greatest integer less than or equal to x. Then, the function fog is discontinuous at exactly :

(A) one point (B) two points

(C) three points (D) four points

Official Ans. by NTA (B)

Sol. Check continuity at x = 0 and also check continuity at those x where g(x) = 0

g(x) = 0 at x = 0, 2

$\text{fog}\left(0^+\right)=-1$

$\text{fog}(0)=0$

Hence, discontinuous at x = 0

$\text{fog}\left(2^+\right)=1$

$\text{fog}\left(2^-\right)=-1$

Hence, discontinuous at x = 2

7. Let $f : \mathbf{R} \to \mathbf{R}$ be a differentiable function such that $f\left(\frac{\pi}{4}\right) = \sqrt{2}, f\left(\frac{\pi}{2}\right) = 0$ and $f'\left(\frac{\pi}{2}\right) = 1$ and let $g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t\, f(t))dt$ for $x \in \left[\frac{\pi}{4}, \frac{\pi}{2}\right)$. Then $\lim_{x \to \left(\frac{\pi}{2}\right)^-} g(x)$ is equal to

(A) 2 (B) 3

(C) 4 (D) –3

Official Ans. by NTA (B)

Sol. $g(x) = \int_x^{\pi/4} (f'(t)\sec t + \tan t \sec t f(t))\, dt$

$g(x) = \int_x^{\pi/4} d(f(t) \cdot \sec t) = f(t)\sec t \Big|_x^{\pi/4}$

$g(x) = f\left(\frac{\pi}{4}\right)\sec\frac{\pi}{4} - f(x) \cdot \sec x$

$g(x) = 2 - f(x)\sec x = 2 - \left(\frac{f(x)}{\cos x}\right)$

$\lim_{x \to \left(\frac{\pi}{2}\right)^-} g(x) = 2 - \lim_{x \to \left(\frac{\pi}{2}\right)^-} \left(\frac{f(x)}{\cos x}\right)$

Using L'Hopital Rule

$= 2 - \lim_{x \to \left(\frac{\pi}{2}\right)^-} \frac{f'(x)}{(-\sin x)}$

$= 2 + \frac{f'\left(\frac{\pi}{2}\right)}{\sin\frac{\pi}{2}} = 2 + \frac{1}{1} = 3$

8. Let $f : \mathbf{R} \to \mathbf{R}$ be continuous function satisfying $f(x) + f(x + k) = n$, for all $x \in \mathbf{R}$ where $k > 0$ and n is a positive integer. If $I_1 = \int_0^{4nk} f(x)dx$ and $I_2 = \int_{-k}^{3k} f(x)dx$, then

(A) $I_1 + 2I_2 = 4nk$ (B) $I_1 + 2I_2 = 2nk$

(C) $I_1 + nI_2 = 4n^2k$ (D) $I_1 + nI_2 = 6n^2k$

Official Ans. by NTA (C)

Sol. $f(x) + f(x+k) = n$

$\Rightarrow f(x) = f(x + 2k)$

f(x) is periodic with period 2k

$I_1 = \int_0^{4nk} f(x)dx = 2n\int_0^{2k} f(x)dx$

$I_2 = \int_{-k}^{3k} f(x)dx = 2\int_0^{2k} f(x)dx$

Now,

$f(x) + f(x+k) = n$

$\Rightarrow \int_0^k f(x)dx + \int_0^k f(x+k)dx = nk$

$\Rightarrow \int_0^k f(x)dx + \int_k^{2k} f(x)dx = nk$

$\Rightarrow \int_0^{2k} f(x)dx = nk$

$\Rightarrow I_1 = 2n^2k,\ I_2 = 2nk$

$\Rightarrow I_1 + nI_2 = 4n^2k$

9. The area of the bounded region enclosed by the curve $y = 3 - \left|x - \frac{1}{2}\right| - |x+1|$ and the x-axis is

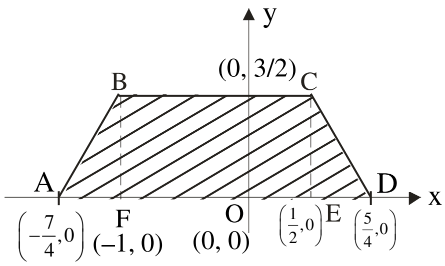
(A) $\frac{9}{4}$ (B) $\frac{45}{16}$

(C) $\frac{27}{8}$ (D) $\frac{63}{16}$

Official Ans. by NTA (C)

Sol. $y=\begin{cases}3+(x+1)+\left(x-\frac{1}{2}\right), & x<-1\\ 3-(x+1)+\left(x-\frac{1}{2}\right), & -1\le x<\frac{1}{2}\\ 3-(x+1)-\left(x-\frac{1}{2}\right), & \frac{1}{2}\le x\end{cases}$

$y=\begin{cases}\frac{7}{2}+2x, & x<-1\\ \frac{3}{2}, & -1\le x<\frac{1}{2}\\ \frac{5}{2}-2x, & \frac{1}{2}\le x\end{cases}$



Area bounded = ar ABF + ar BCEF + ar CDE

$=\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)+\left(\frac{3}{2}\right)\left(\frac{3}{2}\right)+\frac{1}{2}\left(\frac{3}{4}\right)\left(\frac{3}{2}\right)$

$=\frac{27}{8}$ sq. units.

10. Let x = x(y) be the solution of the differential equation $2ye^{x/y^2}dx+\left(y^2-4xe^{x/y^2}\right)dy=0$ such that x(1) = 0. Then, x(e) is equal to

(A) $e\log_e(2)$ (B) $-e\log_e(2)$

(C) $e^2\log_e(2)$ (D) $-e^2\log_e(2)$

Official Ans. by NTA (D)

Sol. $2ye^{x/y^2}dx+\left(y^2-4xe^{x/y^2}\right)dy=0$

$2e^{x/y^2}\left[ydx-2xdy\right]+y^2dy=0$

$2e^{x/y^2}\left[\frac{y^2dx-x\cdot(2y)dy}{y}\right]+y^2dy=0$

Divide by y^3

$2e^{x/y^2}\left[\frac{y^2dx-x\cdot(2y)dy}{y^4}\right]+\frac{1}{y}dy=0$

$2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\frac{1}{y}dy=0$

Integrating

$\int 2e^{x/y^2}d\left(\frac{x}{y^2}\right)+\int\frac{1}{y}dy=0$

$2e^{x/y^2}+\ell ny+c=0$

(0, 1) lies on it.

$2e^0+\ell n1+c=0\Rightarrow c=-2$

Required curve : $\boxed{2e^{x/y^2}+\ell ny-2=0}$

For x (e)

$2e^{x/e^2}+\ell ne-2=0\Rightarrow x=-e^2\log_e 2$

11. Let the slope of the tangent to a curve y = f(x) at (x, y) be given by 2 tanx (cosx – y). if the curve passes through the point $\left(\pi/4,0\right)$, then the value of $\int_0^{\pi/2} ydx$ is equal to

(A) $\left(2-\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (B) $2-\frac{\pi}{\sqrt{2}}$

(C) $\left(2+\sqrt{2}\right)+\frac{\pi}{\sqrt{2}}$ (D) $2+\frac{\pi}{\sqrt{2}}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}=2\tan x\cos x-2\tan x\cdot y$

$\frac{dy}{dx}+(2\tan x)y=2\sin x$

Integrating factor $=e^{\int 2\tan x dx}=\frac{1}{\cos^2 x}$

$y\left(\frac{1}{\cos^2 x}\right)=\int\frac{2\sin x}{\cos^2 x}dx$

$y\sec^2 x=\frac{2}{\cos x}+C$

$y = 2\cos x + C\cos^2 x$

Passes through $\left(\frac{\pi}{4}, 0\right)$

$0 = \sqrt{2} + \frac{C}{2} \Rightarrow C = -2\sqrt{2}$

$f(x) = 2\cos x - 2\sqrt{2}\cos^2 x$: Required curve

$\int_0^{\pi/2} y\,dx = 2\int_0^{\pi/2} \cos x\,dx - 2\sqrt{2}\int_0^{\pi/2} \cos^2 x\,dx$

$= [2\sin x]_0^{\pi/2} - 2\sqrt{2}\left[\frac{x}{2} + \frac{\sin 2x}{4}\right]_0^{\pi/2}$

$= 2 - \frac{\pi}{\sqrt{2}}$

12. Let a triangle be bounded by the lines $L_1 : 2x + 5y = 10$; $L_2 : -4x + 3y = 12$ and the line L_3, which passes through the point P(2, 3), intersect L_2 at A and L_1 at B. If the point P divides the line-segment AB, internally in the ratio 1 : 3, then the area of the triangle is equal to

(A) $\frac{110}{13}$ (B) $\frac{132}{13}$

(C) $\frac{142}{13}$ (D) $\frac{151}{13}$

Official Ans. by NTA (B)

Sol. Points A lies on L_2

$A\left(\alpha, 4 + \frac{4}{3}\alpha\right)$

Points B lies on L_1

$B\left(\beta, 2 - \frac{2}{5}\beta\right)$

Points P divides AB internally in the ratio 1 : 3

$\Rightarrow P(2,3) = P\left(\frac{3\alpha + \beta}{4}, \frac{3\left(4 + \frac{4}{3}\alpha\right) + 1\left(2 - \frac{2}{5}\beta\right)}{4}\right)$

$\Rightarrow \alpha = \frac{3}{13}, \beta = \frac{95}{13}$

Point $A\left(\frac{3}{13}, \frac{56}{13}\right), B\left(\frac{95}{13}, -\frac{12}{13}\right)$

Vertex C of triangle is the point of intersection of L_1 & L_2

$\Rightarrow C\left(-\frac{15}{13}, \frac{32}{13}\right)$

$$\text{area } \Delta ABC = \frac{1}{2}\left\|\begin{matrix} \frac{3}{13} & \frac{56}{13} & 1 \\ \frac{95}{13} & -\frac{12}{13} & 1 \\ -\frac{15}{13} & \frac{32}{13} & 1 \end{matrix}\right\|$$

$$= \frac{1}{2\times 13^3}\left\|\begin{matrix} 3 & 56 & 13 \\ 95 & -12 & 13 \\ -15 & 32 & 13 \end{matrix}\right\|$$

area $\Delta ABC = \frac{132}{13}$ sq. units.

13. Let $a > 0$, $b > 0$. Let e and ℓ respectively be the eccentricity and length of the latus rectum of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. Let e' and ℓ' respectively the eccentricity and length of the latus rectum of its conjugate hyperbola. If $e^2 = \frac{11}{14}\ell$ and $(e')^2 = \frac{11}{8}\ell'$, then the value of 77a+ 44b is equal to

(A) 100 (B) 110

(C) 120 (D) 130

Official Ans. by NTA (D)

Sol. $e = \sqrt{1 + \frac{b^2}{a^2}}$, $\ell = \frac{2b^2}{a}$

Given $e^2 = \frac{11}{14}\ell$

$1 + \frac{b^2}{a^2} = \frac{11}{14}\cdot\frac{2b^2}{a}$

$\frac{a^2 + b^2}{a^2} = \frac{11}{7}\cdot\frac{b^2}{a}$ ......(1)

Also $e' = \sqrt{1+\frac{a^2}{b^2}},\ \ell' = \frac{2a^2}{b}$

Given $(e')^2 = \frac{11}{8}\ell'$

$1+\frac{a^2}{b^2} = \frac{11}{8}.\frac{2a^2}{b}$

$\frac{a^2+b^2}{b^2} = \frac{11}{4}.\frac{a^2}{b}$ ....(2)

New (1) ÷ (2)

$\frac{b^2}{a^2} = \frac{4}{7}.\frac{b^3}{a^3}$

$\therefore 7a = 4b$ ...... (3)

From (2)

$$\frac{\frac{16b^2}{49}+b^2}{b^2} = \frac{11}{4}.\frac{16b^2}{49b}$$

$$\frac{65}{49} = \frac{11}{4}.\frac{16}{49}.b$$

$\therefore b = \frac{4\times 65}{11\times 16}$ ..... (4)

We have to find value of

77a + 44b

11(7a + 4b) = 11(4b + 4b) = 11×8b

$\therefore$ Value of 11×8b $= 11\times 8\times \frac{4\times 65}{16\times 11} = 130$

14. Let $\vec{a} = \alpha\hat{i}+2\hat{j}-\hat{k}$ and $\vec{b} = -2\hat{i}+\alpha\hat{j}+\hat{k}$, where $\alpha \in \mathbf{R}$. If the area of the parallelogram whose adjacent sides are represented by the vectors $\vec{a}$ and $\vec{b}$ is $\sqrt{15(\alpha^2+4)}$, then the value of $2|\vec{a}|^2+(\vec{a}\cdot\vec{b})|\vec{b}|^2$ is equal to

(A) 10 (B) 7

(C) 9 (D) 14

Official Ans. by NTA (D)

Sol. $\vec{a} = \alpha\hat{i}+2\hat{j}-\hat{k},\ \vec{b} = -2\hat{i}+\alpha\hat{j}+\hat{k},$

area of parallelogram $= |\hat{a}\times\hat{b}|$

$|\hat{a}\times\hat{b}| = \sqrt{(\alpha+2)^2+(\alpha-2)^2+(\alpha^2+4)^2}$

Given $|\hat{a}\times\hat{b}| = \sqrt{15(\alpha^2+4)}$

$2(\alpha^2+4)+(\alpha^2+4)^2 = 15(\alpha^2+4)$

$(\alpha^2+4)^2 = 13(\alpha^2+4)$

$\Rightarrow \alpha^2+4 = 13\ \therefore \alpha^2 = 9$

$2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$

$|\vec{a}|^2 = \alpha^2+4+1 = \alpha^2+5$

$|\vec{b}|^2 = 4+\alpha^2+1 = \alpha^2+5$

$\vec{a}.\vec{b} = -2\alpha+2\alpha-1 = -1$

$\therefore 2|\vec{a}|^2+(\vec{a}.\vec{b})|\vec{b}|^2$

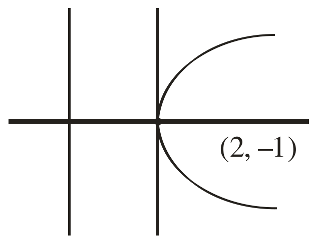
$2(\alpha^2+5)-1(\alpha^2+5) = \alpha^2+5 = 14$

15. If vertex of a parabola is (2, – 1) and the equation of its directrix is 4x – 3y = 21, then the length of its latus rectum is

(A) 2 (B) 8

(C) 12 (D) 16

Official Ans. by NTA (B)



Sol. 4x – 3y = 21

$a = \frac{|8+3-21|}{5} = \frac{10}{5} = 2$

$\therefore$ latus rectum = 4a = 8

16. Let the plane ax + by + cz = d pass through (2, 3, –5) and is perpendicular to the planes 2x + y – 5z = 10 and 3x + 5y – 7z = 12.

If a, b, c, d are integers d > 0 and gcd (|a|, |b|, |c|, d) = 1, then the value of a + 7b + c + 20d is equal to

(A) 18 (B) 20

(C) 24 (D) 22

Official Ans. by NTA (D)

Sol. DR'S normal of plane

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -5 \\ 3 & 5 & -7 \end{vmatrix} = 18\hat{i} - \hat{j} + 7\hat{k}$$

$\therefore$ eq^n of plane

$18x - y + 7z = d$

It passes through (2, 3, –5)

$36 - 3 - 35 = d \qquad \therefore d = -2$

$\therefore$ Eq^n of plane

$18x - y + 7z = -2$

$-18x + y - 7z = 2$

$\therefore$ a= –18, b= 1, c = –7, d= 2

$a + 7b + c + 20d = -18 + 7 - 7 + 40 = 22$

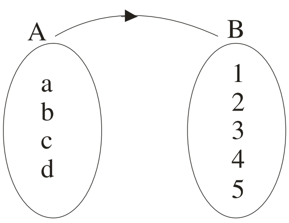
17. The probability that a randomly chosen one-one function from the set {a, b, c, d} to the set {1, 2, 3, 4, 5} satisfies f(a) + 2f(b) – f(c) = f(d) is :

(A) $\frac{1}{24}$ (B) $\frac{1}{40}$

(C) $\frac{1}{30}$ (D) $\frac{1}{20}$

Official Ans. by NTA (D)

Sol.



$n(s) = 5_{C_4} \times 4! = 120$

f(a) +	2f(b) =	f(c)	+ f(d)
5	2×1	3	4
4	2×2	3	5
1	2×3	2	5

$n(A) = 2! \times 3 = 6$

$$\therefore P(A) = \frac{n(A)}{n(s)} = \frac{6}{120} = \frac{1}{20}$$

18. The value of $\lim\limits_{n\to\infty} 6\tan\left\{\sum_{r=1}^{n} \tan^{-1}\left(\frac{1}{r^2+3r+3}\right)\right\}$ is equal to

(A) 1 (B) 2

(C) 3 (D) 6

Official Ans. by NTA (C)

Sol. $T_r = \tan^{-1}\left[\frac{(r+2)-(r+1)}{1+(r+2)(r+1)}\right]$

$= \tan^{-1}(r+2) - \tan^{-1}(r+1)$

$T_1 = \tan^{-1}3 - \tan^{-1}2$

$T_2 = \tan^{-1}4 - \tan^{-1}3$

$T_n = \tan^{-1}(n+2) - \tan^{-1}(n+1)$

$\overline{S_n = \tan^{-1}(n+2) - \tan^{-1}2} = \tan^{-1}\left(\frac{n+2-2}{1+2(n+2)}\right)$

$= \tan^{-1}\left(\frac{n}{2n+5}\right)$

$\lim\limits_{n\to\infty} 6\tan\left(\tan^{-1}\left(\frac{n}{2n+5}\right)\right)$

$= \lim\limits_{n\to\infty} \frac{6n}{2n+5} = \frac{6}{2} = 3$

19. Let $\vec{a}$ be a vector which is perpendicular to the vector $3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}$. If $\vec{a} \times (2\hat{i} + \hat{k}) = 2\hat{i} - 13\hat{j} - 4\hat{k}$, then the projection of the vector $\vec{a}$ on the vector $2\hat{i} + 2\hat{j} + \hat{k}$ is

(A) $\frac{1}{3}$ (B) 1

(C) $\frac{5}{3}$ (D) $\frac{7}{3}$

Official Ans. by NTA (C)

Sol. $\left(\vec{a} \times (2\hat{i} + \hat{k})\right) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$

$= \left(2\hat{i} - 13\hat{j} - 4\hat{k}\right) \times \left(3\hat{i} + \frac{1}{2}\hat{j} + 2\hat{k}\right)$

$$-(6+2)\vec{a}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 2 & -13 & -4\\ 3 & \frac{1}{2} & 2\end{vmatrix}$$

$$\vec{a}=3\hat{i}+2\hat{j}-5\hat{k}$$

Projection of $\vec{a}$ on vector $2\hat{i}+2\hat{j}+\hat{k}$ is

$$\vec{a}\cdot\frac{\left(2\hat{i}+2\hat{j}+\hat{k}\right)}{3}=\frac{5}{3}$$

20. If $\cot\alpha=1$ and $\sec\beta=-\frac{5}{3}$, where $\pi<\alpha<\frac{3\pi}{2}$ and $\frac{\pi}{2}<\beta<\pi$, then the value of $\tan(\alpha+\beta)$ and the quadrant in which $\alpha+\beta$ lies, respectively are

(A) $-\frac{1}{7}$ and IVth quadrant

(B) 7 and Ist quadrant

(C) – 7 and IVth quadrant

(D) $\frac{1}{7}$ and Ist quadrant

Official Ans. by NTA (A)

Sol. $\cot\alpha=1,\sec\beta=\frac{-5}{3},\cos\beta=\frac{-3}{5},\tan\beta=\frac{-4}{3}$

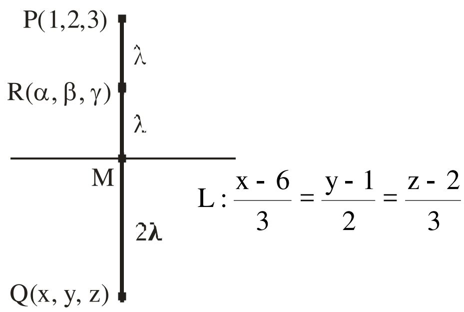
$$\tan(\alpha+\beta)=\frac{1-\frac{4}{3}}{1+\frac{4}{3}\times 1}=\frac{-1}{7}$$

SECTION-B

1. Let the image of the point P(1, 2, 3) in the line $L:\frac{x-6}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ be Q. let $R(\alpha,\beta,\gamma)$ be a point that divides internally the line segment PQ in the ratio 1 : 3. Then the value of $22(\alpha+\beta+\gamma)$ is equal to

Official Ans. by NTA (125)

Sol.



Let M be the mid-point of PQ

$\therefore M=(3\lambda+6,\ 2\lambda+1,\ 3\lambda+2)$

Now, $\overrightarrow{PM}=(3\lambda+5)\hat{i}+(2\lambda-1)\hat{j}+(3\lambda-1)\hat{k}$

$\because \overrightarrow{PM}\perp\left(3\hat{i}+2\hat{j}+3\hat{k}\right)$

$\therefore 3(3\lambda+5)+2(2\lambda-1)+3(3\lambda-1)=0$

$\lambda=\frac{-5}{11}$

$\therefore M\left(\frac{51}{11},\frac{1}{11},\frac{7}{11}\right)$

Since R is mid-point of PM

$22(\alpha+\beta+\gamma)=125$

2. Suppose a class has 7 students. The average marks of these students in the mathematics examination is 62, and their variance is 20. A student fails in the examination if he/she gets less than 50 marks, then in worst case, the number of students can fail is

Official Ans. by NTA (0)

)

Sol. $$20=\frac{\sum_{i=1}^{7}|x_i-62|^2}{7}$$

$\Rightarrow |x_1-62|^2+|x_2-62|^2+....+|x_7-62|^2=140$

If $x_1=49$

$|49-62|^2=169$

then,

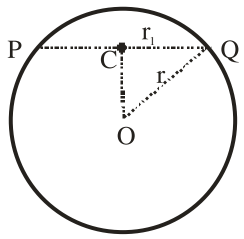
$|x_2-62|^2+....+|x_7-62|^2=$ Negative Number

which is not possible, therefore, no student can fail.

3. If one of the diameters of the circle $x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$ is a chord of the circle $\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$, then the value of r^2 is equal to

Official Ans. by NTA (10)

Sol.



PQ is diameter of circle

$$S: x^2+y^2-2\sqrt{2}x-6\sqrt{2}y+14=0$$

$$C\left(\sqrt{2},3\sqrt{2}\right), O\left(2\sqrt{2},2\sqrt{2}\right)$$

$$r_1=\sqrt{6}$$

$$S_1:\left(x-2\sqrt{2}\right)^2+\left(y-2\sqrt{2}\right)^2=r^2$$

Now in ΔOCQ

$$|OC|^2+|CQ|^2=|OQ|^2$$

$4+6=r^2$

$r^2=10$

4. If $\lim\limits_{x\to1}\dfrac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$, then the value of $(a-b)$ is equal to

Official Ans. by NTA (11)

Sol. $\lim\limits_{x\to1}\dfrac{\sin\left(3x^2-4x+1\right)-x^2+1}{2x^3-7x^2+ax+b}=-2$

For finite limit

$a+b-5=0 \qquad \ldots(1)$

Apply L'H rule

$$\lim_{x\to1}\frac{\cos(3x^2-4x+1)(6x-4)-2x}{(6x^2-14x+a)}=-2$$

For finite limit

$6-14+a=0$

$\boxed{a=8}$

From (1) $\boxed{b=-3}$

Now $(a-b)=11$

5. Let for $n = 1, 2, \ldots.., 50$, S_n be the sum of the infinite geometric progression whose first term is n^2 and whose common ratio is $\dfrac{1}{(n+1)^2}$. Then the value of $\dfrac{1}{26}+\sum\limits_{n=1}^{50}\left(S_n+\dfrac{2}{n+1}-n-1\right)$ is equal to

Official Ans. by NTA (41651)

$$S_n=\frac{n^2}{1-\frac{1}{(n+1)^2}}=\frac{n(n+1)^2}{(n+2)}$$

$$S_n=\frac{n(n^2+2n+1)}{(n+2)}$$

$$S_n=\frac{n[n(n+2)+1]}{(n+2)}$$

$$S_n=n\left[n+\frac{1}{n+2}\right]$$

$$S_n=n^2+\frac{n+2-2}{(n+2)}$$

$$S_n=n^2+1-\frac{2}{(n+2)}$$

Now $\dfrac{1}{26}+\sum\limits_{n=1}^{50}\left[(n^2-n)-2\left(\dfrac{1}{n+2}-\dfrac{1}{n+1}\right)\right]$

$$=\frac{1}{26}+\left[\frac{50\times51\times101}{6}-\frac{50\times51}{2}-2\left(\frac{1}{52}-\frac{1}{2}\right)\right]$$

$=41651$

6. If the system of linear equations

$2x-3y=\gamma+5,$

$\alpha x+5y=\beta+1,$ where $\alpha,\beta,\gamma\in\mathbf{R}$ has infinitely many solutions, then the value of $|9\alpha+3\beta+5\gamma|$ is equal to

Official Ans. by NTA (58)

Sol. $2x-3y=\gamma+5$

$\alpha x+5y=\beta+1$

Infinite many solution

$$\frac{\alpha}{2}=\frac{5}{-3}=\frac{\beta+1}{\gamma+5}$$

$$\alpha=\frac{-10}{3},\quad 5\gamma+25=-3\beta-3$$

$$9\alpha=-30,\quad 3\beta+5\gamma=-28$$

Now, $9\alpha+3\beta+5\gamma=-58$

$$|9\alpha+3\beta+5\gamma|=58$$

7. Let $A=\begin{pmatrix}1+i & 1\\ -i & 0\end{pmatrix}$ where $i=\sqrt{-1}$.

Then, the number of elements in the set

$\{n\in\{1,2,....,100\}: A^n=A\}$ is

Official Ans. by NTA (25)

Sol. $A=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}$

$$A^2=\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}\begin{bmatrix}1+i & 1\\ -i & 0\end{bmatrix}$$

$$A^2=\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}$$

$$A^4=\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}\begin{bmatrix}i & 1+i\\ -i+1 & -i\end{bmatrix}$$

$$A^4=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}=I$$

$$A^{4n+1}=A$$

$$n=1,5,9,......,97$$

$\Rightarrow$ total elements in the set is 25.

8. Sum of squares of modulus of all the complex numbers z satisfying $\overline{z}=iz^2+z^2-z$ is equal to

Official Ans. by NTA (2)

Sol. $z+\overline{z}=iz^2+z^2$

Consider $z = x + iy$

$2x = (i + 1)(x^2 - y^2 + 2xyi)$

$\Rightarrow 2x = x^2 - y^2 - 2xy$ and $x^2 - y^2 + 2xy = 0$

$\Rightarrow 2x = -4xy$

$\Rightarrow x = 0$ or $y=\frac{-1}{2}$

Case 1 : $x = 0 \Rightarrow y = 0$ here $z = 0$

Case 2 : $y=\frac{-1}{2}$

$$\Rightarrow 4x^2-4x-1=0$$

$$(2x-1)^2=2$$

$$2x-1=\pm\sqrt{2}$$

$$x=\frac{1\pm\sqrt{2}}{2}$$

Here $z=\frac{1+\sqrt{2}}{2}-\frac{i}{2}$ or $z=\frac{1-\sqrt{2}}{2}-\frac{i}{2}$

Sum of squares of modulus of z

$$=0+\frac{(1+\sqrt{2})^2+1}{4}+\frac{(1-\sqrt{2})^2+1}{4}=\frac{8}{4}=2$$

9. Let $S = \{1, 2, 3, 4\}$. Then the number of elements in the set $\{f : S \times S \to S : f$ is onto and $f(a, b) = f(b, a) \ge a\,\forall(a,b)\in S\times S\}$ is

Official Ans. by NTA (37)

Sol. (1, 1), (1, 4), (4, 1), (2, 4), (4, 2), (3, 4), (4, 3), (4, 4) – all have one choice for image.

(2, 1), (1, 2), (2, 2) – all have three choices for image

(3, 2), (2, 3), (3, 1), (1, 3), (3, 3) – all have two choices for image.

So the total functions = $3 \times 3 \times 2 \times 2 \times 2 = 72$

Case 1 : None of the pre-images have 3 as image

Total functions = $2 \times 2 \times 1 \times 1 \times 1 = 4$

Case 2 : None of the pre-images have 2 as image

Total functions = $2 \times 2 \times 2 \times 2 \times 2 = 32$

Case 3 : None of the pre-images have either 3 or 2 as image

Total functions = $1 \times 1 \times 1 \times 1 \times 1 = 1$

$\therefore$ Total onto functions = $72 - 4 - 32 + 1 = 37$

10. The maximum number of compound propositions, out of
$p \vee r \vee s,\ p \vee r \vee \sim s,\ p \vee \sim q \vee s,$
$\sim p \vee \sim r \vee s,\ \sim p \vee \sim r \vee \sim s,\ \sim p \vee q \vee \sim s,$
$q \vee r \vee \sim s,\ q \vee \sim r \vee \sim s,\ \sim p \vee \sim q \vee \sim s$
that can be made simultaneously true by an assignment of the truth values to p, q, r and s, is equal to

Official Ans. by NTA (9)

Sol. If we take

p	q	r	s
F	F	T	F

The truth value of all the propositions will be true.