JEE–MAIN EXAMINATION – JANUARY, 2023	
(Held On Thursday 29th January, 2023)	TIME : 3 : 00 PM to 6 : 00 PM

Physics

SECTION - A

1. Substance A has atomic mass number 16 and half-life of 1 day. Another substance *B* has atomic mass number 32 and half life of $\frac{1}{2}$ day. If both *A* and *B* simultaneously start undergo radio activity at the same time with initial mass 320 g each, how many total atoms of A and B combined would be left after 2 days.

(1) 3.38×10^{24} (2) 1.69×10^{24} (3) 6.76×10^{24} (4) 6.76×10^{23}

Sol. **(1)**

$$(N_0)_A = \frac{320}{16} = 20 \text{ moles}$$

$$(N_0)_B = \frac{320}{32} = 10 \text{ moles}$$

$$N_A = \frac{(N_0)_A}{2^{n_1}} = \frac{20}{4} = 5$$

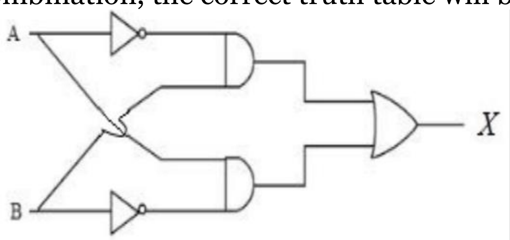
$$N_B = \frac{(N_0)_B}{2^{n_2}} = \frac{10}{(2)^{\frac{2}{0.5}}} = \frac{10}{2^4} = 0.625$$

Total N = 5.625 moles

No. of atoms $= (N)(N_A)$

$= 5.625 \times 6.023 \times 10^{23} = (3.38 \times 10^{24})$

2. For the given logic gates combination, the correct truth table will be



(1)

A	B	X
0	0	1
0	1	0
1	0	0
1	1	0

(2)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	1

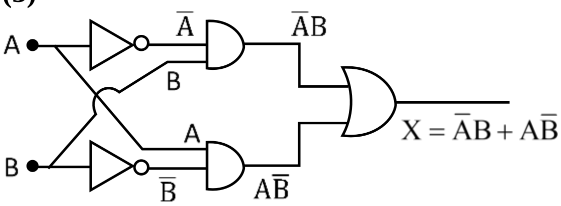
(3)

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

(4)

A	B	X
0	0	1
0	1	0
1	0	1
1	1	0

Sol. **(3)**



From Bodean Algebra :

$X = \bar{A}B + A\bar{B}$

The correct truth table will be

A	B	X
0	0	0
0	1	1
1	0	1
1	1	0

3. The time taken by an object to slide down 45° rough inclined plane is n times as it takes to slide down a perfectly smooth 45° incline plane. The coefficient of kinetic friction between the object and the incline plane is:

(1) $\sqrt{1-\frac{1}{n^2}}$ (2) $1+\frac{1}{n^2}$ (3) $1-\frac{1}{n^2}$ (4) $\sqrt{\frac{1}{1-n^2}}$

Sol. **(3)**

Acceleration on the smooth inclined plane

$$a_1 = g\sin\theta = \frac{g}{\sqrt{2}}$$

Acceleration on the rough inclined plane

$$a_2 = g\sin\theta - \mu g\cos\theta = \frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}(K=\mu)$$

Given that:

$$t_2 = nt_1 \quad \text{and} \quad \frac{1}{2}a_1t_1^2 = \frac{1}{2}a_2t_2^2$$

$$a_1t_1^2 = a_2t_2^2$$

$$\frac{g}{\sqrt{2}}t_1^2 = \left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)(n^2t_1^2)$$

$$\frac{g}{\sqrt{2}} = n^2\left(\frac{g}{\sqrt{2}} - \frac{Kg}{\sqrt{2}}\right)$$

$$K = 1 - \frac{1}{n^2}$$

4. Heat energy of 184 kJ is given to ice of mass 600 g at −12°C. Specific heat of ice is 2222.3 J $kg^{-1}C^{-1}$ and latent heat of ice in 336 $kJkg^{-1}$

A. Final temperature of system will be 0°C.
B. Final temperature of the system will be greater than 0°C.
C. The final system will have a mixture of ice and water in the ratio of 5: 1.
D. The final system will have a mixture of ice and water in the ratio of 1:5.
E. The final system will have water only.

Choose the correct answer from the options given below:

(1) A and D Only (2) A and E Only (3) A and C Only (4) B and D Only

Sol. **(1)**

Heat energy given $= 184KJ = 184\times10^3 J$

Amount of heat required to raise the temperature

$$\theta_1 = ms_{ice}\Delta T = 0.6\times2222.3\times12$$
$$=16000.56 \text{ J}$$

Remaining heat $\theta_2 = 184000 - 16000.56 = 167999.44$ J

For melting at 0°C heat required = mL_f

= 0.6 × 336000

= (201600) J needed

∴ 100% ice is not melted

Amount of ice melted

167999.44 = m × 336000

m = mass of water = 0.4999 Kg

Mass of ice = 0.1001

$$\text{Ratio} = \frac{0.1001}{0.4999} \approx 1:5$$

5. Identify the correct statements from the following:

A. Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is negative.

B. Work done by gravitational force in lifting a bucket out of a well by a rope tied to the bucket is negative.

C. Work done by friction on a body sliding down an inclined plane is positive.

D. Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity in zero.

E. Work done by the air resistance on an oscillating pendulum in negative.

Choose the correct answer from the options given below:

(1) B, D and E only (2) A and C Only (3) B and D only (4) B and E only

Sol. **(4)**

→ Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket is positive

→ Work done by friction on a body sliding down an inclined plane is negative

→ Work done by a applied force on a body moving on a rough horizontal plane with uniform velocity is positive

6. A scientist is observing a bacteria through a compound microscope. For better analysis and to improve its resolving power he should. (Select the best option)

(1) Increase the refractive index of the medium between the object and objective lens

(2) Decrease the diameter of the objective lens

(3) Increase the wave length of the light

(4) Decrease the focal length of the eye piece.

Sol. **(1)**

$$\text{R.P} = \frac{2\mu \sin\theta}{1.22\lambda}$$

$\mu \uparrow, \text{R.P} \uparrow$

$D \downarrow, \theta \downarrow, \text{R.P} \downarrow$

$\lambda \uparrow, \text{R.P} \downarrow$

R.P is independent of focal length of eye piece

7. With the help of potentiometer, we can determine the value of emf of a given cell. The sensitivity of the potentiometer is

(A) directly proportional to the length of the potentiometer wire

(B) directly proportional to the potential gradient of the wire

(C) inversely proportional to the potential gradient of the wire

(D) inversely proportional to the length of the potentiometer wire

Choose the correct option for the above statements:

(1) A only (2) C only (3) A and C only (4) B and D only

Sol. **(3)**

If on displacing the jockey slightly from the null point position, the galvanometer shows a large deflection, than the potentiometer is said to be sensitive. The sensitivity of the potentiometer depends upon the potential gradient along the wire. The smaller potential gradient greater will be sensitivity.

Sensitivity ↑, potential gradient ↓, length ↑

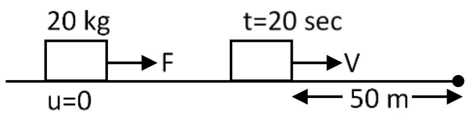
Sensitivity $\propto$ length

$$\text{Sensitivity} \propto \frac{1}{\text{Potential gradient}}$$

8. A force acts for 20 s on a body of mass 20 kg, starting from rest, after which the force ceases and then body describes 50 m in the next 10 s. The value of force will be:

(1) 40 N (2) 5 N (3) 20 N (4) 10 N

Sol. **(2)**



$50 = V \times 10$

$V = 5\ ms^{-1}$

$V = 0 + a \times 20$

$5 = a \times 20$

$a = \frac{1}{4} ms^{-2}$

$F = ma = 20 \times \frac{1}{4} = 5N$

9. The modulation index for an A.M. wave having maximum and minimum peak-to-peak voltages of 14 mV and 6 mV respectively is:

(1) 0.4 (2) 0.6 (3) 0.2 (4) 1.4

Sol. **(1)**

$\mu =$ Modulating index $= \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$

$= \frac{14 - 6}{14 + 6}$

$= 0.4$

10. Given below are two statements:

Statement I: Electromagnetic waves are not deflected by electric and magnetic field.

Statement II: The amplitude of electric field and the magnetic field in electromagnetic waves are related to each other as $E_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} B_0$.

In the light of the above statements, choose the correct answer from the options given below :

(1) Statement I is true but statement II is false

(2) Both Statement I and Statement II are false

(3) Statement I is false but statement II is true

(4) Both Statement I and Statement II are true

Sol. **(1)**

Statement -I is correct as
EMW are neutral

Statement – II is wrong

$E_0 = \sqrt{\frac{1}{\mu_0 \in_0}} B_0$

11. A square loop of area 25 cm^2 has a resistance of 10Ω. The loop is placed in uniform magnetic field of magnitude 40.0 T. The plane of loop is perpendicular to the magnetic field. The work done in pulling the loop out of the magnetic field slowly and uniformly in 1.0sec, will be

(1) 1.0×10^{-3} J (2) 2.5×10^{-3} J (3) 5×10^{-3} J (4) 1.0×10^{-4} J

Sol. **(1)**

l = 5 cm

t = 1 sec

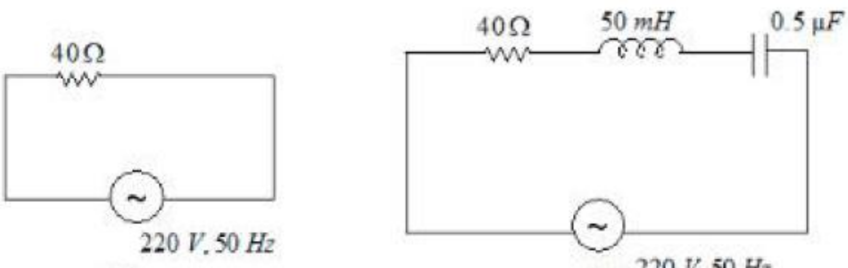
$$V = \frac{0.05}{1} = 0.05 ms^{-1}$$

$$I = \frac{40 \times 0.05 \times 0.05}{10} = \frac{BLV}{R} = 0.01A$$

$$F = BIL = 40 \times 0.010.05 = 0.02N$$

$$W = F\,\ell = 0.02 \times 0.05 = 1 \times 10^{-3} J$$

12. For the given figures, choose the correct options:



(a) (b)

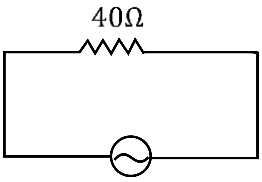
(1) At resonance, current in (b) is less than that in (a)

(2) The rms current in circuit (b) can never be larger than that in (a)

(3) The rms current in figure(a) is always equal to that in figure (b)

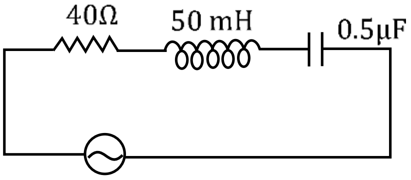
(4) The rms current in circuit (b) can be larger than that in (a)

Sol. **(2)**



220V, 50 Hz

fig (a)



220V, 50 Hz

fig (a)

$$I_{rms} = \frac{220}{40} = 5.5A$$

X_L is not equal to X_C, so rms current

In (b) can never be large than (a)

13. A fully loaded boeing aircraft has a mass of 5.4×10^5 kg. Its total wing area is $500\ \text{m}^2$. It is in level flight with a speed of 1080 km/h. If the density of air ρ is $1.2\ \text{kg m}^{-3}$, the fractional increase in the speed of the air on the upper surface of the wing relative to the lower surface in percentage will be. $(g = 10\ \text{m/s}^2)$

(1) 16 (2) 10 (3) 8 (4) 6

Sol. **(2)**

$$P_2A - P_1A = 5.4\times10^5\times g$$

$$P_2 - P_1 = \frac{5.4\times10^6}{500} = 10.8\times10^3$$

$$P_2 + 0 + \frac{1}{2}\rho v_2^2 = P_1 + 0 + \frac{1}{2}\rho v_1^2$$

$$P_2 - P_1 = \frac{1}{2}\rho\left(v_1^2 - v_2^2\right) = \frac{1}{2}\rho\left(v_1 + v_2\right)\left(v_1 - v_2\right)$$

$$10.8\times10^3 = \frac{1}{2}\times1.2\times\left(v_1 - v_2\right)\times2\times3\times10^2$$

$$v_1 - v_2 = 30$$

$$\frac{v_1 - v_2}{v}\times100 = \frac{30}{300}\times100 = 10\%$$

14. The ratio of de-Broglie wavelength of an α particle and a proton accelerated from rest by the same potential is $\frac{1}{\sqrt{m}}$, the value of m is-

(1) 16 (2) 4 (3) 2 (4) 8

Sol. **(4)**

$$\frac{\lambda_\alpha}{\lambda_p} = \frac{\dfrac{h}{\sqrt{2m_\alpha q_\alpha v}}}{\dfrac{h}{\sqrt{2m_p q_p v}}}$$

$$\frac{\lambda_\alpha}{\lambda_p} = \sqrt{\frac{1}{8}}$$

M = 8

15. The time period of a satellite of earth is 24 hours. If the separation between the earth and the satellite is decreased to one fourth of the previous value, then its new time period will become.

(1) 4 hours (2) 6 hours (3) 3 hours (4) 12 hours

Sol. **(3)**

$$T^2 \propto R^3$$

$$\frac{T_1^2}{T_2^2} = \frac{R_1^3}{R_2^3} \Rightarrow \left(\frac{T_1}{T_2}\right)^2 = \left(\frac{R}{\frac{R}{4}}\right)^3$$

$$\frac{T_1^2}{T_2^2} = 64$$

$$T_2^2 = \frac{T_1^2}{64}$$

$$T_2 = \frac{T_1}{8} = \frac{24}{8} = 3$$

16. The electric current in a circular coil of four turns produces a magnetic induction 32 T at its centre. The coil is unwound and is rewound into a circular coil of single turn, the magnetic induction at the centre of the coil by the same current will be :

(1) 16 T (2) 2 T (3) 8 T (4) 4 T

Sol. **(2)**

$$B=\frac{\mu_o i}{2R}\times 4$$

$$B'=\frac{\mu_o i}{2R'}$$

$$R'=4R$$

$$B'=\frac{\mu_0 i}{8R}$$

$$\frac{B'}{B}=\frac{1}{16}$$

$$B'=2T$$

17. A point charge 2×10^{-2}C is moved from P to S in a uniform electric field of $30NC^{-1}$ directed along positive x-axis. If coordinates of P and S are (1,2,0)m and (0,0,0)m respectively, the work done by electric field will be

(1) 1200 mJ (2) −1200 mJ (3) −600 mJ (4) 600 mJ

Sol. (3)

$$W_E=q\vec{E}.\vec{S} \;=2\times10^{-2}\times(-30)$$

$$=-0.6J\;=-600mJ$$

18. An object moves at a constant speed along a circular path in a horizontal plane with center at the origin. When the object is at = +2 m, its velocity is $-4\hat{j}$m/s.

The object's velocity (v) and acceleration (a) at $x=-2$ m will be

(1) $v=-4\hat{\imath}\frac{m}{s}, a=-8\hat{\jmath}\ m/s^2$ (2) $v=4\hat{\imath}\frac{m}{s}, a=8\hat{\jmath}\ m/s^2$

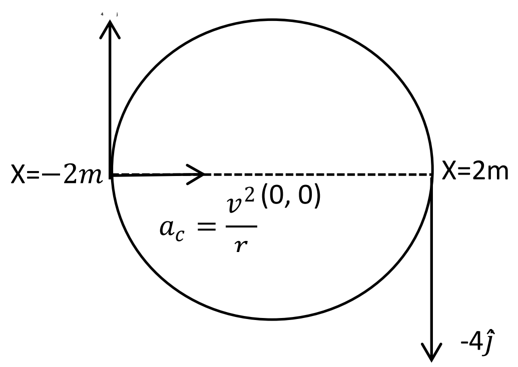
(3) $v=4\hat{\jmath}\frac{m}{s}, a=8\hat{\imath}\ m/s^2$ (4) $v=-4\hat{\jmath}\frac{m}{s}, a=8\hat{\imath}\ m/s^2$

Sol. (3)

$$a_c=\frac{v^2}{r}=\frac{4^2}{2}=8ms^{-2}$$

$$\vec{v}=4\hat{j}$$

$$\vec{a_c}=8i$$



19. At 300 K the rms speed of oxygen molecules is $\sqrt{\frac{\alpha+5}{\alpha}}$ times to that of its average speed in the gas. Then, the value of α will be

(used $= \frac{22}{7}$)

(1) 28 (2) 24 (3) 32 (4) 27

Sol. (1)

$$\sqrt{\frac{3RT}{M}} = \sqrt{\frac{\alpha+5}{\alpha}}\sqrt{\frac{8}{\pi}\frac{RT}{M}}$$

$$3 = \left(\frac{\alpha+5}{\alpha}\right)\left(\frac{8}{\pi}\right)$$

$$\alpha = 28$$

20. The equation of a circle is given by $x^2 + y^2 = a^2$, where a is the radius. If the equation is modified to change the origin other than (0,0), then find out the correct dimensions of A and B in a new equation : $(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$. The dimensions of t is given as $[T^{-1}]$.

(1) $A = [LT], B = [L^{-1}\ T^{-1}]$
(2) $A = [L^{-1}\ T^{-1}], B = [LT]$
(3) $A = [L^{-1}\ T], B = [LT^{-1}]$
(4) $A = [L^{-1}\ T^{-1}], B = [LT^{-1}]$

Sol. (1)

$$(x - At)^2 + \left(y - \frac{t}{B}\right)^2 = a^2$$

$$A = L^1T^1$$

$\frac{t}{B}$ is in meter

$$\frac{t}{B} = L$$

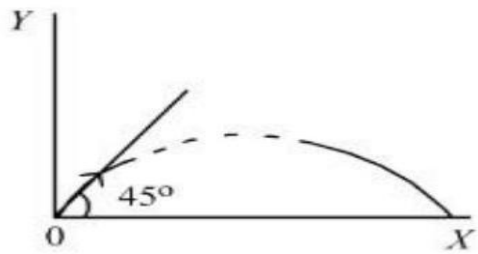
$$\frac{T^{-1}}{B} = L$$

$$B = T^{-1}L^{-1}$$

SECTION - B

21. A particle of mass 100 g is projected at time $t = 0$ with a speed $20\ ms^{-1}$ at an angle 45° to the horizontal as given in the figure. The magnitude of the angular momentum of the particle about the starting point at time $t = 2$ s is found to be $\sqrt{K}\ kgm^2/s$. The value of K is _________.

(Take $g = 10\ ms^{-2}$)



Sol. **800**

Use $\Delta L = \int_0^t \tau dt$

$L_0 = \int_0^2 (mg)(v_x t)dt$

$= (mgv_x)\frac{t^2}{2}$

$= (0.1)(10)(10)(\sqrt{2}) \times \frac{2^2}{2}$

$= 20\sqrt{2}$

$= \sqrt{800}$

22. Unpolarised light is incident on the boundary between two dielectric media, whose dielectric constants are 2.8 (medium -1) and 6.8 (medium -2), respectively. To satisfy the condition, so that the reflected and refracted rays are perpendicular to each other, the angle of incidence should be $\tan^{-1}\left(1+\frac{10}{\theta}\right)^{\frac{1}{2}}$ the value of θ is ________.
(Given for dielectric media, $\mu_r = 1$)

Sol. 7

$\mu_1 = \sqrt{2.8}$

$\mu_2 = \sqrt{6.8}$

$\mu \sin i = \mu_2 \cos i$

$\tan i = \frac{\mu_2}{\mu_1} = \sqrt{\frac{6.8}{2.8}}$

$\tan i = \left(\frac{2.8+4}{2.8}\right)^{\frac{1}{2}}$

$i = \tan^{-1}\left(1+\frac{10}{7}\right)^{\frac{1}{2}}$

$\theta = 7$

23. A particle of mass 250 g executes a simple harmonic motion under a periodic force $F = (-25x)$N. The particle attains a maximum speed of 4 m/s during its oscillation. The amplitude of the motion is ________ cm.

Sol. **(40)**

$F = ma$

$-25x = \frac{250}{100}a$

$a = -100x$

$\omega^2 = 100$

$\omega = 100$

$A\omega = 4$

$A = \frac{4}{10} = 0.4m$

$A = 40cm$

24. A car is moving on a circular path of radius 600 m such that the magnitudes of the tangential acceleration and centripetal acceleration are equal. The time taken by the car to complete first quarter of revolution, if it is moving with an initial speed of 54 km/hr is $t(1-e^{-\pi/2})s$. The value of t is ________.

Sol. **(40)**

$$\frac{dv}{dt}=\frac{v^2}{R}$$

$$\frac{vdv}{dx}=\frac{v^2}{R}$$

$$\frac{dv}{dx}=\frac{v}{R}$$

$$\int_{15}^{v}\frac{dv}{v}=\int_0^x\frac{dx}{R}$$

$$\frac{v}{15}=\frac{x}{R}$$

$$\frac{v}{15}=e^{\frac{x}{R}}$$

$$v=15e^{\frac{x}{R}}$$

$$\frac{dx}{dt}=15e^{\frac{x}{R}}$$

$$\int_0^{\frac{\pi R}{2}}e^{-\frac{x}{R}}dx=15\int_0^{to}dt$$

$$t_0=40\left(1-e^{-\frac{\pi}{2}}\right)s$$

$$\boxed{t=40}$$

25. When two resistances R_1 and R_2 connected in series and introduced into the left gap of a meter bridge and a resistance of 10Ω is introduced into the right gap, a null point is found at 60 cm from left side. When R_1 and R_2 are connected in parallel and introduced into the left gap, a resistance of 3Ω is introduced into the right-gap to get null point at 40 cm from left end. The product of R_1R_2 is ________ Ω^2

Sol. (30)

$$\frac{R_1+R_2}{10}=\frac{60}{40}$$

$$R_1+R_2=15 \quad \ldots\ldots\ldots\ldots (1)$$

$$\frac{R_1R_2}{(R_1+R_2)x3}=\frac{40}{60}$$

$$R_1R_2=30$$

26. In an experiment of measuring the refractive index of a glass slab using travelling microscope in physics lab, a student measures real thickness of the glass slab as 5.25 mm and apparent thickness of the glass slab as 5.00 mm. Travelling microscope has 20 divisions in one cm on main scale and 50 divisions on vernier scale is equal to 49 divisions on main scale. The estimated uncertainty in the measurement of refractive index of the slab is $\frac{x}{10} \times 10^{-3}$, where x is ________.

Sol. **(41)**

$$\mu = \frac{h}{h^1} = \frac{\text{Real depth}}{\text{Apparent depth}}$$

Least Count = M.S.D. – V.S.D

$$= \text{M.S.D.} - \frac{49}{50}\text{M.S.D}$$

$$= \left(\frac{50-49}{50}\right)\text{M.S.D}$$

$$= \frac{1}{50}\text{M.S.D}$$

$$= \frac{1}{50} \times \frac{1}{20}\text{cm}$$

$$= \frac{1}{1000}\text{cm}$$

$$= \frac{10}{1000}\text{mm} = 0.01\text{mm}$$

$$\ln\mu = \ln h - \ln h'$$

$$\frac{d\mu}{\mu} = \frac{dh}{h} + \frac{dh'}{h'}$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h'}\right]$$

$$d\mu = \mu\left[\frac{dh}{h} + \frac{dh'}{h}\right] = \frac{5.25}{5.00}\left[\frac{0.01}{5.25} + \frac{0.01}{5.00}\right]$$

$$= \frac{41}{10} \times 10^{-3}$$

27. An inductor of inductance 2μH is connected in series with a resistance, a variable capacitor and an AC source of frequency 7kHz. The value of capacitance for which maximum current is drawn into the circuit is $\frac{1}{x}$ F, where the value of x is _______. (Take $\pi = \frac{22}{7}$)

Sol. **(3872)**

For Maximum current is drawn

$$x_L = x_C$$

$$\omega L = \frac{1}{\omega C}$$

$$2\pi f L = \frac{1}{2\pi f c}$$

$$C=\frac{1}{4\pi^2f^2L}=\frac{1}{4\times\pi^2\times49\times10^6\times2\times10^{-6}}$$
$$C=\frac{1}{3872}F$$
$$X=3872$$

28. A null point is found at 200 cm in potentiometer when cell in secondary circuit is shunted by 5Ω. When a resistance of 15Ω is used for shunting, null point moves to 300 cm. The internal resistance of the cell is ________ Ω.

Sol. **(5)**

Potential Gradient $=\frac{\Delta V}{L}$

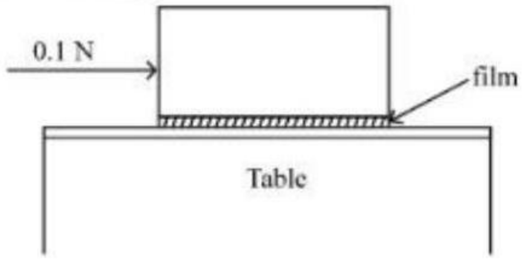
$$E-Ir=\left(\frac{\Delta v}{L}\right)x$$
$$\frac{ER}{R+r}=\left(\frac{\Delta V}{L}\right)x$$
$$\frac{E\times5}{5+r}=\frac{\Delta V}{L}\times200 \qquad \text{.........} \ (1)$$
$$\frac{E\times15}{15+r}=\frac{\Delta V}{L}\times300 \qquad \text{.........} \ (2)$$
$$=r=5\omega$$

29. For a charged spherical ball, electrostatic potential inside the ball varies with r as $V=2ar^2+b$. Here, a and b are constant and r is the distance from the center. The volume charge density inside the ball is $-\lambda a\varepsilon$. The value of λ is _________.
ε = permittivity of the medium

Sol. **(12)**

$$E=-\frac{dv}{dr}=-4ar$$
By the Gauss' theorem
$$\oint\vec{E}.\overrightarrow{dA}=\frac{q_{inside}}{\varepsilon}$$
$$E\times4\pi r^2=\frac{\rho\times\frac{4}{3}\pi r^3}{\varepsilon}$$
$$E=\frac{\rho r}{3\varepsilon}=-4ar$$
$$\rho=-12a\varepsilon$$

30. A metal block of base area $0.20\ m^2$ is placed on a table, as shown in figure. A liquid film of thickness 0.25 mm is inserted between the block and the table. The block is pushed by a horizontal force of 0.1 N and moves with a constant speed. If the viscosity of the liquid is 5.0×10^{-3}Pl, the speed of block is _________ $\times 10^{-3}$ m/s.



Sol. **(25)**

$$|F| = \eta A \frac{\Delta v}{\Delta h}$$

$$0.1 = 5 \times 10^{-3} \times 0.2 \times \frac{v}{0.25 \times 10^{-3}}$$

$$v = 0.025 ms^{-1}$$

$$v = 25 \times 10^{-3} ms^{-1}$$

Chemistry

SECTION - A

31. According to MO theory the bond orders for $O_2{}^{2-}$, CO and NO^+ respectively, are

(1) 1, 2 and 3 (2) 1,3 and 2 (3) 2,3 and 3 (4) 1, 3 and 3

Sol. **4**

Molecules	Total No. of e^-	Bond order
O_2^{-2}	18	1
CO	14	3
NO^+	14	3

32. A doctor prescribed the drug Equanil to a patient. The patient was likely to have symptoms of which disease?

(1) Hyperacidity (2) Anxiety and stress

(3) Depression and hypertension (4) Stomach ulcers

Sol. **3**

Equanil is a tranquiliger, used for treatment of depression and hypertension.

33. Reaction of propanamide with $Br_2/KOH(aq)$ produces :

(1) Propylamine (2) Ethylnitrile (3) Propanenitrile (4) Ethylamine

Sol. **4**

$$CH_3CH_2-\overset{\overset{\large O}{||}}{C}-NH_2 \xrightarrow{Br_2/KOH} CH_3-CH_2-NH_2+H_2O+KBr$$

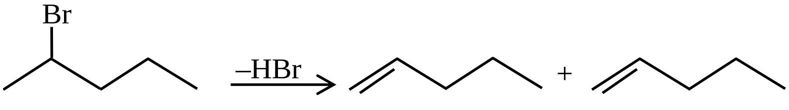
Hoffmann's bromamide reaction

34. The one giving maximum number of isomeric alkenes on dehydrohalogenation reaction is (excluding rearrangement)

(1) 2-Bromopropane (2) 2-Bromo-3,3-dimethylpentane

(3) 1-Bromo-2-methylbutane (4) 2-Bromopentane

Sol. **4**



2–bromo pentane (Cis+trons)

35. An indicator ' X ' is used for studying the effect of variation in concentration of iodide : on the rate of reaction of iodide ion with H_2O_2 at room temp. The indicator ' X ' forms blue colored complex with compound ' A ' present in the solution. The indicator ' X ' and compound 'A' respectively are

(1) Methyl orange and H_2O_2

(2) Starch and iodine

(3) Starch and H_2O_2

(4) Methyl orange and iodine

Sol. **2**

$$I^- + H_2O_2 \longrightarrow \underset{(A)}{I_2} + H_2O$$

$$I_2 + \underset{\text{(Indicator)}}{\text{Starch}} \longrightarrow \text{Blue}$$

36. The major component of which of the following ore is sulphide based mineral?

(1) Siderite (2) Sphalerite (3) Malachite (4) Calamine

Sol. **2**

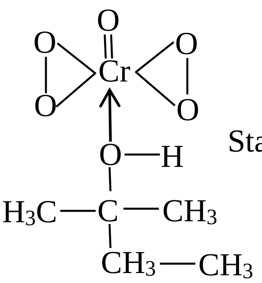
Zinc blade	Sphalerite	$\rightarrow$	Zns
	Siderite	$\rightarrow$	$feCO_3$
	Malachite	$\rightarrow$	$CuCO_3 \cdot CuCOHl_2$
	Calamine	$\rightarrow$	$ZnCO_3$

37. A solution of C_rO_5 in amyl alcohol has a _______ colour.

(1) Green (2) Orange-Red (3) Yellow (4) Blue

Sol. **4**

Blue



Stablize by L.P.

38. The set of correct statements is :

(i) Manganese exhibits +7 oxidation state in its oxide.

(ii) Ruthenium and Osmium exhibit +8 oxidation in their oxides.

(iii) Sc shows +4 oxidation state which is oxidizing in nature.

(iv) Cr shows oxidising nature in +6 oxidation state.

(1) (ii) and (iii) (2) (i), (ii) and (iv) (3) (ii), (iii) and (iv) (4) (i) and (iii)

Sol. **2**

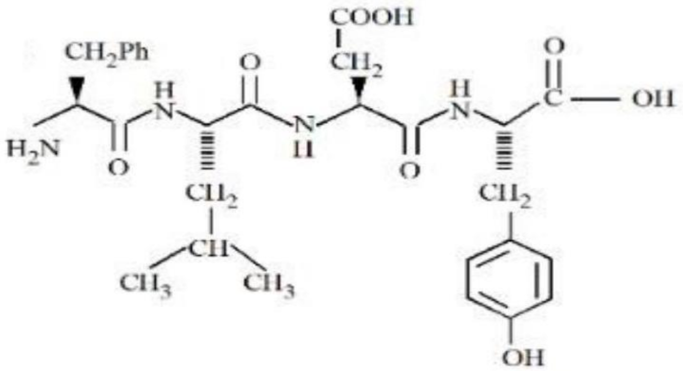
(i) Mn_2O_7

(ii) RuO_4 & OsO_4

(iii) Sc (+4) oxidation state not possible in oxidizing nature

(iv) Cr show oxidizing nature in +6 oxidation state

39. Following tetrapeptide can be represented as



(F, L, D, Y, I, Q, P are one letter codes for amino acids)

(1) PLDY (2) FIQY (3) YQLF (4) FLDY

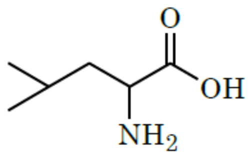
Sol. **4**

Hydrolysis of the given tetrapeptide will give the following:

$Ph–CH_2–CH–COOH$ (with NH_2 on CH)

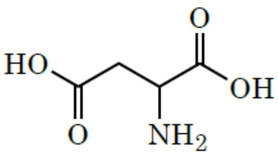
(F)

(Phenylalanine)



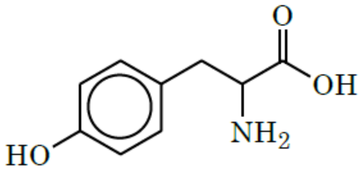
(L)

(Leucine)



(D)

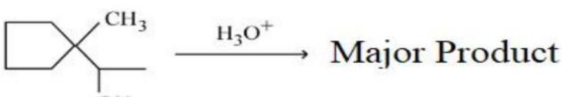
(Aspartic acid)



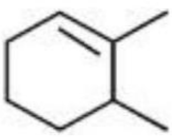
(Y)

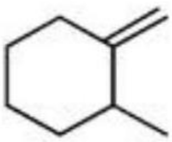
(Tyrosine)

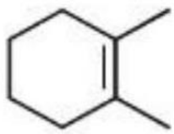
40. Find out the major product for the following reaction.



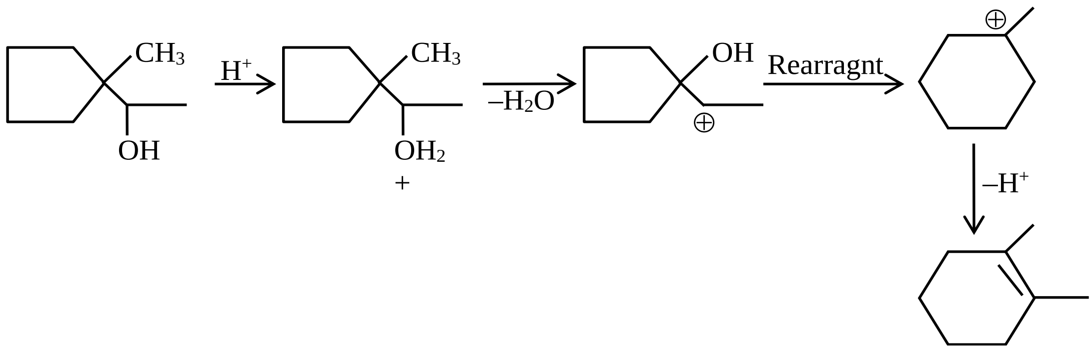
1. 

2. 

3. 

4. 

Sol. **4**



41.

List I	List II
A. van't Hoff factor, i	I. Cryoscopic constant
B. k_f	II. Isotonic solutions
C. Solution with same with same osmotic pressure	III. $\frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$
D. Azeotropes	IV. Solutions with same composition of vapour above it

Choose the correct answer from the options given below :

(1) A-I, B-III, C-II, D-IV (2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV (4) A-III, B-II, C-I, D-IV

Sol. **3**

(A) van’t Hoff factor, i

$$i = \frac{\text{Normal molar mass}}{\text{Abnormal molar mass}}$$

(B) k_f = Cryoscopic constant

(C) Solutions with same osmotic pressure are known as isotonic solutions.

(D) Solutions with same composition of vapour over them are called Azeotrope.‘

42. Correct order of spin only magnetic moment of the following complex ions is:

(Given At.no. Fe: 26, Co:27)

(1) $[FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-}$ (2) $[FeF_6]^{3-} > [CoF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

(3) $[Co(C_2O_4)_3]^{3-} > [CoF_6]^{3-} > [FeF_6]^{3-}$ (4) $[CoF_6]^{3-} > [FeF_6]^{3-} > [Co(C_2O_4)_3]^{3-}$

Sol. **2**

Complex	Central Metal E.C.	No. Of unpaired e^-	$\mu = \sqrt{n(n+2)}$ B.M.
(i) $[Fef_6]^{-3}$	$Fe^{+3} \rightarrow 3d^5 \rightarrow t_2g^{1,1,1}, eg^{1,1}$	5	$\sqrt{35}$ Br
(ii) $[Cof_6]^{-3}$	$CO^{+3} \rightarrow 3d^6 \rightarrow t_2g^{2,1,1}, eg^{1,1}$	4	$\sqrt{24}$ Br
(iii) $[Co(C_2O_4)_3]^{-3}$	$CO^{+3} \rightarrow 3d^6 \rightarrow t_2g^{2,2,2}, eg^{0,0}$	0	0 Br

43. Match List I with List II

List I	List II
A. Elastomeric polymer	I. Urea formaldehyde resin
B. Fibre Polymer	II. Polystyrene
C. Thermosetting Polymer	III. Polyester
D. Thermoplastic Polymer	IV. Neoprene

Choose the correct answer from the options given below :

(1) A-II, B-III, C-I, D-IV
(2) A-IV, B-III, C-I, D-II
(3) A-IV, B-I, C-III, D-II
(4) A-II, B-I, C-IV, D-III

Sol. **2**

Neoprene : Elastomer

Polyester Fibre

Polstyrene : THermolastic

Urea–Formaldhyde Resin: Thermosetting polymer

44. The concentration of dissolved Oxygen in water for growth of fish should be more than $\underline{X}$ ppm and Biochemical Oxygen Demand in clean water should be less than $\underline{Y}$ ppm. X and Y in ppm are, respectively.

1.
X	Y
4	8

2.
X	Y
6	5

3.
X	Y
4	15

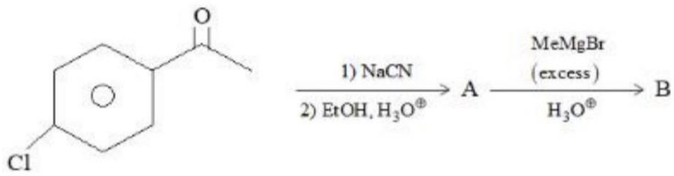
4.
X	Y
6	12

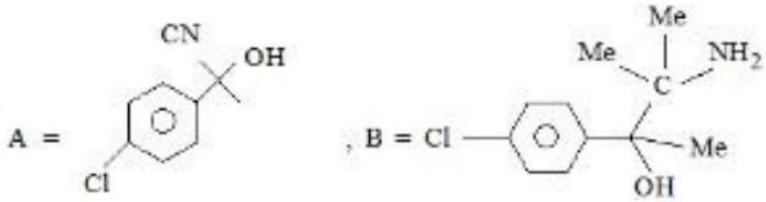
Sol. 2

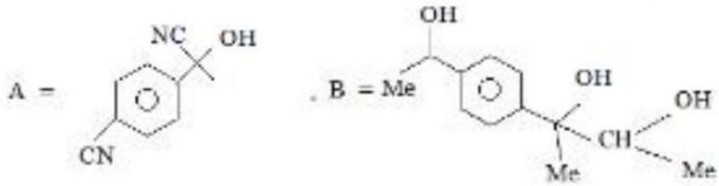
→ BOD value of water of water is in the range –3–5 (Less than 5)

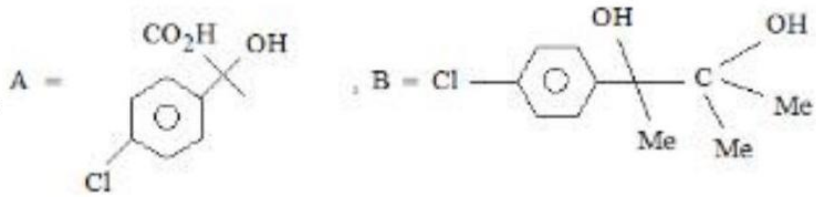
→ dissolve oxygen in water for growth of wish → Less than (6)

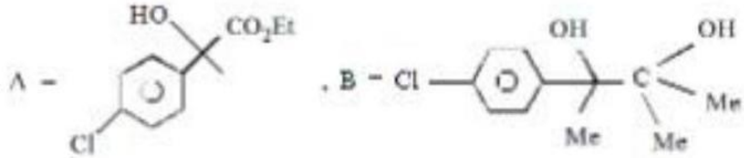
45. Find out the major products from the following reaction sequence.



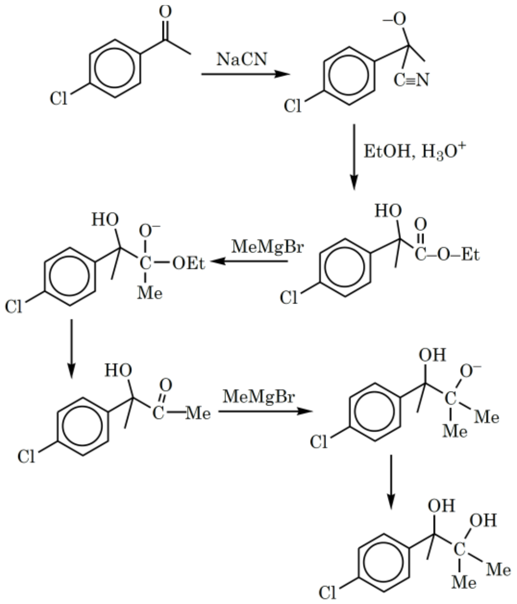
1. 

2. 

3. 

4. 

Sol. 4



46. When a hydrocarbon A undergoes combustion in the presence of air, it requirs 9.5 equivalents of oxygen and produces 3 equivalents of water. What is the molecular formula of A ?

(1) C_9H_9 (2) C_8H_6 (3) C_9H_6 (4) C_6H_6

Sol. **2**

$$C_xH_y + \left(x + \frac{y}{4}\right)O_2 \longrightarrow xCO_2 + \frac{y}{2}H_2O$$

Number of equivalents of O_2 = Number of equivalents of H_2O

Number of equivalents of H_2O = $\frac{y}{2} = 3$

y = 6

Number of equivalents of O_2 = x + $\frac{y}{4} = 9.5$

$x + \frac{6}{4} = 9.5$

x = 9.5 – 1.5 = 8

$C_xH_y = C_8H_6$

47. Given below are two statements:

Statement I : Nickel is being used as the catalyst for producing syn gas and edible fats.

Statement II : Silicon forms both electron rich and electron deficient hydrides.

In the light of the above statements, choose the most appropriate answer from the options given below:

(1) Statement I is correct but statement II is incorrect

(2) Both the statements I and II are incorrect

(3) Statement I is incorrect but statement II is correct

(4) Both the statements I and II are correct

Sol. **1**

(i) $\underset{(g)}{CH_4} + \underset{(g)}{H_2O} \xrightarrow[Ni]{1270K} CO + 3H_2(g)$

Ni used as a catalyst

(ii) Si neither formed e– deficient hydride nor electron rich species.

48. Which of the following relations are correct?

(A) $\Delta U = q + p\Delta V$ (B) $\Delta G = \Delta H - T\Delta S$ (C) $\Delta S = \frac{q_{rev}}{T}$ (D) $\Delta H = \Delta U - \Delta nRT$

Choose the most appropriate answer from the options given below:

(1) B and D Only (2) A and B Only

(3) B and C Only (4) C and D Only

Sol. **3**

Only (B) and (C) are correct.

(B) $G = H - TS$

At constant T

$\Delta G = \Delta H - T\Delta S$

(A) First law is given by

$\Delta U = Q + W$

If we apply constant P and reversible work.

$\Delta U = Q - P\Delta V$

(C) By definition of entropy change

$$dS = \frac{q_{rev}}{T}$$

At constant T

$$\Delta S = \frac{q_{rev}}{T}$$

(D) $H = U + PV$

For ideal gas

$H = U + nRT$

At constant T

$\Delta H = \Delta U + \Delta nRT$

49. Given below are two statements :

Statement I : The decrease in first ionization enthalpy from B to Al is much larger than that from Al to Ga.

Statement II : The d orbitals in Ga are completely filled.

In the light of the above statements, choose the most appropriate answer from the options given below

(1) Statement I is incorrect but statement II is correct

(2) Both the statements I and II are correct

(3) Both the statements I and II are incorrect

(4) Statement I is correct but statement II is incorrect

Sol. **1**

$$\begin{array}{lccccc} & B > & Tl > & Ga > & Al > & I \\ \text{Ionisation enthalpy} \rightarrow & \downarrow & \downarrow & \downarrow & \downarrow & \downarrow \\ & 801 & 589 & 579 & 577 & 558 \end{array}$$

$\rightarrow {}_{31}Ga \rightarrow [Ar]4s^2, 3d^{10}, 4p^1$

Ga $\rightarrow$ completely filled d orbital.

50. Match List I and List II

List I	List II
A. Osmosis	I. Solvent molecules pass through semi permeable membrane towards solvent side.
B. Reverse osmosis	II. Movement of charged colloidal particles under the influence of applied electric potential towards oppositely charged electrodes.
C. Electro osmosis	III. Solvent molecules pass through semi permeable membrane towards solution side.
D. Electrophoresis	IV. Dispersion medium moves in an electric field.

Choose the correct answer from the options given below :

(1) A-I, B-III, C-IV, D-II

(2) A-III, B-I, C-IV, D-II

(3) A-III, B-I, C-II, D-IV

(4) A-I, B-III, C-II, D-IV

Sol. **2**

(i) Electro osmosis: When movement of colloidal particles is prevented by some suitable means (porous diaphragm or semi permeable membranes), it is observed that the D.M. begins to move in an electric field. This phenomenon is termed electrosmosis.

(ii) Solvent molecules pass through semi-permeable membrane towards solvent side is termed as reverse osmosis.

(iii) When an electric potential is applied across two platinum electrodes dipping in a colloidal solution, the colloidal particles move towards move towards one or the other electrode. The movement of colloidal particles under an applied electric potential is called electrophoresis.

(iv) Solvent molecules pass through semipermeable membrane towards the solution side is termed as osmosis.

51. Assume that the radius of the first Bohr orbit of hydrogen atom is 0.6Å. The radius of the third Bohr orbit of $He^{+}$is ________ picometer. (Nearest Integer)

Sol. **(270)**

$$r \propto \frac{n^2}{Z}$$

$$r_{He^+} = r_H \times \frac{n^2}{Z}$$

$$r_{He^+} = 0.6 \times \frac{(3)^2}{2}$$

$$= 2.7 \text{ Å}$$

$$r_{He^+} = 270 \text{ pm}$$

52. Total number of acidic oxides among

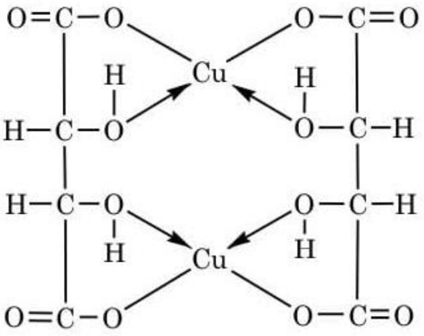
$N_2O_3, NO_2, N_2O, Cl_2O_7, SO_2, CO, CaO, Na_2O$ and NO is ________

Sol. **4**

Acidic oxide $\rightarrow$ $N_2O_3, NO_2, Cl_2O_7, SO_2$

53. The denticity of the ligand present in the Fehling's reagent is ________

Sol. **4**



Copper tartrate complex

Denticity = 2

54. The equilibrium constant for the reaction

$Zn(s) + Sn^{2+}(aq) \rightleftharpoons Zn^{2+}(aq) + Sn(s)$ is 1×10^{20} at 298 K. The magnitude of standard electrode potential of Sn/Sn^{2+} if $E^\circ_{Zn^{2+}/Zn} = -0.76V$ is ________ $\times 10^{-2}$ V (Nearest integer).

Given : $\frac{2.303RT}{F} = 0.059$ V

Sol. **17**

Given

$Zn(s) + Sn^{2+}(aq.) \rightleftharpoons Zn^{2+}(aq.) + Sn(s)$

$K_C = 1 \times 10^{20}$

$E^\circ_{Zn^{2+}/Zn} = -0.76V$

$E_{cell} = E^\circ_{cell} - \frac{0.059}{n}\log_{10}K_c$

$0 = E^\circ_{cell} - \frac{0.059}{2} \times 20$

$E^\circ_{cell} = 0.59$

$E^\circ_{cell} = \underset{(RP)}{E^\circ_{Cathode}} - \underset{(RP)}{E^\circ_{Anode}}$

$0.59 - E^\circ_{Sn^{2+}/Sn} - E^\circ_{Zn^{2+}/Zn}$

$0.59 = E^\circ_{Sn^{2+}/Sn} - (-0.76)$

$E^\circ_{Sn^{2+}/Sn} = 0.17$

$E^\circ_{Sn/Sn^{2+}} = 17 \times 10^{-2}$

55. The volume of HCl, containing $73\ \mathrm{g\ L^{-1}}$, required to completely neutralise NaOH obtained by reacting $0.69\ \mathrm{g}$ of metallic sodium with water, is _______ mL. (Nearest Integer)

(Given : molar Masses of Na, Cl, O, H, are 23,35.5,16 and 1 $\mathrm{g\ mol^{-1}}$ respectively)

Sol. **15**

Mole of Na = $\frac{0.69}{23} = 3 \times 10^{-2}$

$Na + H_2O \longrightarrow NaOH + \frac{1}{2}H_2$

By using POAC

Moles of NaOH = 3×10^{-2}

NaOH reacts with HCl

No. of equivalent of NaOH = No. of equivalent of HCl

$3 \times 10^{-2} \times 1 = \frac{73}{36.5} \times V(\text{in L}) \times 1$

$V = 1.5 \times 10^{-2}$ L

Volume of HCl = 15 ml.

56. For conversion of compound A → B, the rate constant of the reaction was found to be $4.6 \times 10^{-5}\ \mathrm{L\ mol^{-1}\ s^{-1}}$. The order of the reaction is _________.

Sol. **2**

As unit of rate constant is $(\text{conc.})^{1-n}$ time^{-1}

Put n = 2

then $\mathrm{L\ mol^{-1}\ s^{-1}}$

So order of the reaction is 2.

57. On heating, $LiNO_3$ gives how many compounds among the following? _______

$Li_2O, N_2, O_2, LiNO_2, NO_2$

Sol. **3**

$4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$

58. When 0.01 mol of an organic compound containing 60% carbon was burnt completely, 4.4 g of CO_2 was produced. The molar mass of compound is ________ $gmol^{-1}$ (Nearest integer).

Sol. **200**

Let M is the molar mass of the compound (g/mol)

mass of compound = 0.01 M gm

mass of carbon = 0.01 M × $\frac{60}{100}$

mass of carbon = $\frac{0.01M}{12} \times \frac{60}{100}$

moles of CO_2 from combustion = $\frac{4.4}{44}$ = moles of carbon

$$\frac{0.01M}{12} \times \frac{60}{100} = \frac{4.4}{44}$$

$$M = \frac{4.4}{44} \times \frac{100}{60} \times \frac{12}{0.01} = 200 \text{ gm/mol}$$

59. At 298 K

$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g), K_1 = 4 \times 10^5$

$N_2(g) + O_2(g) \rightleftharpoons 2NO(g), K_2 = 1.6 \times 10^{12}$

$H_2(g) + \frac{1}{2}O_2(g) \rightleftharpoons H_2O(g), K_3 = 1.0 \times 10^{-13}$

Based on above equilibria, the equilibrium constant of the reaction,

$2NH_3(g) + \frac{5}{2}O_2(g) \rightleftharpoons 2NO(g) + 3H_2O(g)$ is ________ $\times 10^{-33}$ (Nearest integer).

Sol. **4**

Reverse equation (1) So $K_1^1 = \frac{1}{K_1}$ … (a)

+

Add equation …. (2) $K_1^1 = K_2$ ….(b)

+

Multiply equation (3) by (3) $K_3^1 = k_3^3$ ….(c)

Add (a), (b) & (c)

$$K_c^1 = \frac{K_2 \times K_3^3}{K_1} = \frac{1.6 \times 10^{12} \times 1 \times 10^{-39}}{4 \times 10^5}$$

$\Rightarrow 4 \times 10^{-33}$

60. A metal M forms hexagonal close-packed structure. The total number of voids in 0.02 mol of it is ________ $\times 10^{21}$ (Nearest integer).(Given $\mathrm{N_A} = 6.02 \times 10^{23}$)

Sol. **(36)**

One unit cell of hcp contains = 18 voids

No. of voids in 0.02 mol of hcp

$$\frac{18}{6} \times 6.02 \times 10^{23} \times 0.02$$

$\approx 3.6 \times 10^{22}$

$\approx 36 \times 10^{21}$

Mathematics

SECTION - A

61. The statement $B \Rightarrow ((\sim A) \vee B)$ is equivalent to :

(1) $A \Rightarrow (A \Leftrightarrow B)$
(2) $A \Rightarrow ((\sim A) \Rightarrow B)$
(3) $B \Rightarrow (A \Rightarrow B)$
(4) $B \Rightarrow ((\sim A) \Rightarrow B)$

Sol. **1, 3 or 4**

$B \Rightarrow (\sim A) VB$

A	**B**	**~ A**	**~ AVB**	**$B \Rightarrow (\sim A)$ VB**
T	T	F	T	T
T	F	F	F	T
F	T	T	T	T
F	F	T	T	T

$A \Rightarrow B$	**$\sim A \Rightarrow B$**	**$B \Rightarrow A \Rightarrow B$**	**$A \Rightarrow ((\sim A) \Rightarrow B)$**	**$B \Rightarrow ((\sim A) \Rightarrow B)$**
T	T	T	T	T
F	T	T	T	T
T	T	T	T	T
T	F	T	T	T

62. The value of the integral $\int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$ is

(1) $\tan^{-1} 2 - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(2) $\tan^{-1} \frac{1}{2} + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$
(3) $\tan^{-1} \frac{1}{2} - \frac{1}{3}\tan^{-1} 8 + \frac{\pi}{3}$
(4) $\tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{3}$

Sol.

$$I = \int_1^2 \left(\frac{t^4+1}{t^6+1}\right) dt$$

$$\Rightarrow \int \frac{t^4+1-t^2+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int \frac{(t^4-t^2+1)+t^2}{(t^2+1)(t^4-t^2+1)} dt$$

$$\Rightarrow \int_1^2 \left[\frac{t^4-t^2+1}{(t^2+1)(t^4-t^2+1)} + \frac{t^2}{t^6+1}\right] dt$$

$$\Rightarrow \int_1^2 \frac{1}{t^2+1} dt + \frac{1}{3}\int_1^2 \frac{3t^2}{(t^3)^2+1} dt$$

$$\Rightarrow \left[\tan^{-1} t + \frac{1}{3}\tan^{-1}(t^3)\right]_1^2$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1}(8) - \tan^{-1}(1) - \frac{1}{3}\tan^{-1}(1)$$

$$\Rightarrow \tan^{-1} 2 + \frac{1}{3}\tan^{-1} 8 - \frac{\pi}{4} - \frac{\pi}{4} . \frac{1}{3}$$

$$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{3\pi+\pi}{12}$$

$$\Rightarrow \tan^{-1}2+\frac{1}{3}\tan^{-1}8-\frac{\pi}{3}$$

63. The set of all values of λ for which the equation $\cos^2 2x - 2\sin^4 x - 2\cos^2 x = \lambda$ has a real solution x, is

(1) $[-2,-1]$ (2) $\left[-1,-\frac{1}{2}\right]$ (3) $\left[-\frac{3}{2},-1\right]$ (4) $\left[-2,-\frac{3}{2}\right]$

Sol.

$$\cos^2 2x - 2\left(\frac{1-\cos 2x}{2}\right)^2-(1+\cos 2x)=\lambda$$

$$\Rightarrow \quad \cos^2 2x - 2\left(\frac{1-\cos^2 2x-2\cos 2x}{4}\right)-1-\cos 2x=\lambda$$

Let $\cos 2x = t$

$\Rightarrow \quad 2t^2 - 1 - t^2 + 2t - 2 - 2t = 2\lambda$

$\Rightarrow \quad t^2 - 3 = 2\lambda \qquad \because\ 0 \le t^2 \le 1$

$\Rightarrow \quad t^2 = 2\lambda + 3$

$0 \le 2\lambda + 3 \le 1$

$-3 \le 2\lambda \le -2$

$\frac{-3}{2} \le \lambda \le -1$

64. Let R be a relation defined on $\mathbb{N}$ as a R b if $2a + 3b$ is a multiple of $5, a, b \in \mathbb{N}$.
Then R is

(1) an equivalence relation (2)transitive but not symmetric

(3) not reflexive (4) symmetric but not transitive

Sol.

Reflexive

Let $a \in N$

a R a $\Rightarrow$ 2a + 3a is a multiple of 5

$\Rightarrow$ 5a which is a multiple of 5

$\Rightarrow$ R is reflexive

Symmetric

Let $a, b \in N$

a R b $\Rightarrow 2a + 3b = 5\lambda_1 \qquad \lambda_1 \in N$

b R a $\Rightarrow 2b + 3a = 5\lambda_2 \qquad \lambda_2 \in N$

On Adding

$(2a + 3b) + (2b + 3a) = 5(\lambda_1 + \lambda_2)$

$5a + 5b = 5(\lambda_1 + \lambda_2)$

$\Rightarrow$ Both sides are multiple of 5

$\Rightarrow$ R is symmetric

Transitive
Let a, b, c $\in$ N
a R b $\Rightarrow$ $2a + 3b = 5\lambda_1$ …(1)
b R c $\Rightarrow$ $2a + 3c = 5\lambda_2$ …(2)
$2a + 5b + 3c = 5(\lambda_1 + \lambda_2)$
$\Rightarrow (2a + 3c) = 5(\lambda_1 + \lambda_2 - b)$
2a + 3c is divisible by 5
$\Rightarrow$ a R c is true
$\Rightarrow$ R is transitive
R is Equivalence Relation

65. Consider a function $f: \mathbb{N} \to \mathbb{R}$, satisfying
$f(1) + 2f(2) + 3f(3) + \cdots + xf(x) = x(x+1)f(x); x \geq 2$ with $f(1) = 1$.
Then $\frac{1}{f(2022)} + \frac{1}{f(2028)}$ is equal to
(1) 8100 (2) 8400 (3) 8000 (4) 8200

Sol. **1**
$f(1) + 2f(2) + 3f(3) + \ldots\ldots + xf(x) = x^2f(x) + xf(x)$
$\Rightarrow f(1) + 2f(2) + 3f(3) + \ldots\ldots + (x-1)\, f(x-1) = x^2\, f(x)$
x = 2 $\quad f(1) = 2^2\, f(2) \Rightarrow f(2) = \dfrac{1}{4}$
x = 3 $\quad f(1) + 2f(2) = 3^2\, f(3)$
$$\Rightarrow f(3) = \frac{1}{9}\left(1+\frac{2}{4}\right) = \frac{1}{9}\times\frac{3}{2} = \frac{1}{6}$$
x = 4 $\quad f(1) + 2f(2) + 3f(3) = 4^2 f(4)$
$$\Rightarrow f(4) = \left(1+\frac{1}{2}+\frac{1}{2}\right).\frac{1}{16} \Rightarrow f(4) = \frac{1}{8}$$
x = 5 $\quad f(1) + 2f(2) + 3f(3) + 4f(4) = 5^2 f(5)$
$$f(5) = \left(1+\frac{1}{2}+\frac{1}{2}+\frac{1}{2}\right)\frac{1}{25} = \frac{5}{2}.\frac{1}{25} = \frac{1}{10}$$
In general $\quad f(x) = \dfrac{1}{2x}$
$$\therefore \quad \frac{1}{f(2022)} + \frac{1}{f(2028)}$$
$$\frac{1}{\frac{1}{2\times 2022}} + \frac{1}{\frac{1}{2\times 2028}}$$
$\Rightarrow$ 2[2022 + 2028]
$\Rightarrow$ 2×4050
$\Rightarrow$ 8100

66. If $\vec{a} = \hat{i} + 2\hat{k}, \vec{b} = \hat{i} + \hat{j} + \hat{k}, \vec{c} = 7\hat{i} - 3\hat{j} + 4\hat{k}, \vec{r} \times \vec{b} + \vec{b} \times \vec{c} = \vec{0}$ and $\vec{r} \cdot \vec{a} = 0$.
Then $\vec{r} \cdot \vec{c}$ is equal to
(1) 32 (2) 30 (3) 36 (4) 34

Sol. **4**

$\vec{r}\times\vec{b}+\vec{b}\times\vec{c}=0$

$\Rightarrow\ \vec{r}\times\vec{b}-\vec{c}\times\vec{b}=0$

$\Rightarrow\ \left(\vec{r}-\vec{c}\right)\times\vec{b}=0$

$\vec{r}-\vec{c}\ ||\ \vec{b}$

$\vec{r}-\vec{c}=\lambda\vec{b}$

$\vec{r}=\lambda\vec{b}+\vec{c}$

$=\ \lambda(i+j+k)+(7i-3j+4k)$

$= i(\lambda + 7) + j(\lambda - 3) + k(\lambda + 4)$

$\vec{r}.\vec{a}=0$

$\Rightarrow (7 + \lambda) + 2(\lambda + 4) = 0$

$\Rightarrow 3\lambda = -15 \quad \Rightarrow \lambda = -5$

$\therefore\ \vec{r}=2i-8j-k$

$\vec{r}.\vec{c}\ = (2i - 8j - k) . (7i - 3j + 4k)$

$= 14 + 24 - 4\ = 34$

67. The shortest distance between the lines $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}$ and $\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}$

(1) $5\sqrt{3}$ (2) $2\sqrt{3}$ (3) $3\sqrt{3}$ (4) $4\sqrt{3}$

Sol. **4**

$L_1=\frac{x-1}{2}=\frac{y+8}{-7}=\frac{z-4}{5}=\lambda$

$L_2\ \frac{x-1}{2}=\frac{y-2}{1}=\frac{z-6}{-3}=\mu$

$S.D.=\left|\frac{(\vec{b}-\vec{a}).(\vec{b}_1\times\vec{b}_2)}{\left|\vec{b}_1\times\vec{b}_2\right|}\right|$ $\qquad \vec{a}=i-8j+4k$

$\vec{b}=i+2j+6k$

$$\vec{b}_1\times\vec{b}_2=\begin{vmatrix} i & j & k \\ 2 & -7 & 5 \\ 2 & 1 & -3 \end{vmatrix}$$

$= i(21-5) - j(-6 -10) + k(2 + 14)$

$= 16i + 16j + 16 k$

$\left|\vec{b}_1\times\vec{b}_2\right|=|16(i+j+k)|$

$=\ 16\times\sqrt{3}$

$\vec{b}-\vec{a}=(10j+2k)$

$S.D. = \left|\frac{(10j+2k).16(i+j+k)}{16\sqrt{3}}\right|$

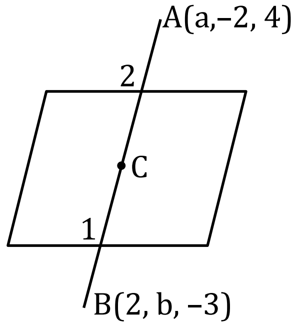
$=\left|\frac{16(10+2)}{16\sqrt{3}}\right| = \frac{12}{\sqrt{3}}\Rightarrow\frac{12}{\sqrt{3}}\times\frac{\sqrt{3}}{\sqrt{3}}$

$=\frac{12\sqrt{3}}{3}=4\sqrt{3}$

68. The plane $2x-y+z=4$ intersects the line segment joining the points $A(a,-2,4)$ and $B(2,b,-3)$ at the point C in the ratio $2:1$ and the distance of the point C from the origin is $\sqrt{5}$. If $ab<0$ and P is the point $(a-b,b,2b-a)$ then CP^2 is equal to

(1) $\frac{97}{3}$ (2) $\frac{17}{3}$ (3) $\frac{16}{3}$ (4) $\frac{73}{3}$

Sol. **2**



C divides AB in 2 : 1

$$C\left(\frac{4+a}{3},\frac{2b-2}{3},\frac{-6+4}{3}\right)$$

$$C\left(\frac{a+4}{3},\frac{2b-2}{3},\frac{-2}{3}\right)$$

C lies in the plane

$$\therefore\ 2\left(\frac{a+4}{3}\right)-\left(\frac{2b-2}{3}\right)+\left(\frac{-2}{3}\right)=4$$

$\Rightarrow 2a-2b=4$

$a-b=2$

$\because OC=\sqrt{5}$

$OC^2=5$

$$C\left(\frac{b+6}{3},\frac{2b-2}{3},\frac{-2}{3}\right)$$

$$\Rightarrow\left(\frac{b+6}{3}\right)^2+\left(\frac{2b-2}{3}\right)^2+\left(\frac{-2}{3}\right)^2=5$$

$\Rightarrow 5b^2+4b-1=0$

$\Rightarrow 5b^2+5b-b-1=0$

$\Rightarrow 5b(b+1)-1(b+1)=0$

$b=-1\ \&\ \frac{1}{5}$

$a=1$

$\because ab<0$

$\therefore\ a=1, b=-1$

Now,

$$C\left(\frac{5}{3},\frac{-4}{3},\frac{-2}{3}\right)$$

$P(a-b,\ b,\ 2b-a)$

$(2,-1,-3)$

$$CP^2=\left(\frac{5}{3}-2\right)^2+\left(\frac{-4}{3}+1\right)^2+\left(\frac{-2}{3}+3\right)^2=\frac{17}{3}$$

69. The value of the integral $\int_{1/2}^{2}\frac{\tan^{-1}x}{x}dx$ is equal to

(1) $\frac{\pi}{2}\log_e 2$ (2) $\pi\log_e 2$ (3) $\frac{1}{2}\log_e 2$ (4) $\frac{\pi}{4}\log_e 2$

Sol.

Let $x=\frac{1}{t}$

$dx=-\frac{1}{t^2}dt$

$$I=\int_{2}^{1/2}\frac{\tan^{-1}\left(\frac{1}{t}\right)}{\frac{1}{t}}\times-\frac{1}{t^2}dt$$

$$\Rightarrow \int_{1/2}^{2} \frac{\cot^{-1}(t)}{t} dt$$

$$2I = \int_{1/2}^{2} \frac{\tan^{-1} x + \cot^{-1} x}{x} dx$$

$$\Rightarrow \int_{1/2}^{2} \frac{\pi/2}{x} dx$$

$$\Rightarrow \frac{\pi}{2}[\ell n\, x]_{1/2}^{2}$$

$$\Rightarrow \frac{\pi}{2}\left(\ell n\, 2 - \ell n \frac{1}{2}\right)$$

$$\Rightarrow \frac{\pi}{2}(\ell n\, 2 + \ell n\, 2)$$

$$2I = \pi \ell n 2$$

$$\Rightarrow I = \frac{\pi}{2} \ell n\, 2$$

70. The letters of the word OUGHT are written in all possible ways and these words are arranged as in a dictionary, in a series. Then the serial number of the word TOUGH is

(1) 84 (2) 79 (3) 89 (4) 86

Sol. **3**

G H O T U

The words start from G $\lfloor\underline{4}$

The words start from H $\lfloor\underline{4}$ 24×3

The words start from O $\lfloor\underline{4}$

T

Start form TG $\lfloor\underline{3}$

TH $\lfloor\underline{3}$ 6×2

TO

TOG $\lfloor\underline{2}$ 2×2

TOH $\lfloor\underline{2}$

TOU

TOUG $\lfloor\underline{1}$ 1

89

71. The set of all values of $t \in \mathbb{R}$, for which the matrix

$$\begin{bmatrix} e^t & e^{-t}(\sin t - 2\cos t) & e^{-t}(-2\sin t - \cos t) \\ e^t & e^{-t}(2\sin t + \cos t) & e^{-t}(\sin t - 2\cos t) \\ e^t & e^{-t}\cos t & e^{-t}\sin t \end{bmatrix}$$ is invertible, is

(1) $\mathbb{R}$ (2) $\left\{k\pi + \frac{\pi}{4}, k \in \mathbb{Z}\right\}$ (3) $\{k\pi, k \in \mathbb{Z}\}$ (4) $\left\{(2k+1)\frac{\pi}{2}, k \in \mathbb{Z}\right\}$

Sol. **(1)**

$$|A| = \begin{vmatrix} e^t & e^{-t}(s-2c) & e^{-t}(-2s-c) \\ e^t & e^{-t}(2s+c) & e^{-t}(s-2c) \\ e^t & e^{-t}c & e^{-t}s \end{vmatrix}$$

$$\Rightarrow \ = e^t.e^{-t}.e^{-t}\begin{vmatrix} 1 & s-2c & -2s-c \\ 1 & 2s+c & s-2c \\ 1 & c & s \end{vmatrix}$$

$R_1 \rightarrow R_1 - R_2$ & $R_2 \rightarrow R_2 - R_3$

$$= e^t\begin{vmatrix} 0 & -s-3c & -3s-c \\ 0 & 2s & -2c \\ 1 & c & s \end{vmatrix}$$

$\Rightarrow e^{-t}[1(2sc + 6c^2 + 6s^2 + 2sc)]$

$\Rightarrow e^{-t}[4sc + 6(c^2 + s^2)] \ = e^{-t}(6 + 2\sin 2t)$

$\because 2\sin 2t \in [-2, 2]$

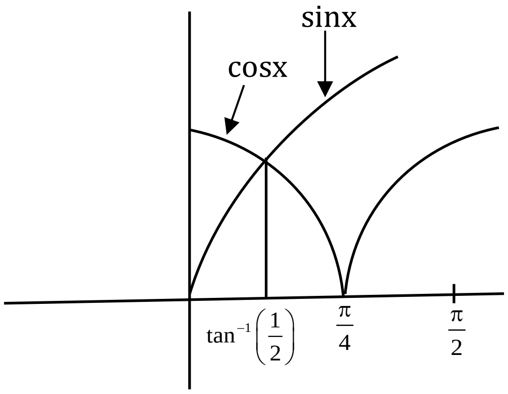
$\therefore e^{-t}(6 + 2\sin 2t) \neq 0 \quad \forall\, t \in R$

72. The area of the region $A = \left\{(x, y): |\cos x - \sin x| \leq y \leq \sin x, 0 \leq x \leq \frac{\pi}{2}\right\}$ is

(1) $\sqrt{5} + 2\sqrt{2} - 4.5$ (2) $1 - \frac{3}{\sqrt{2}} + \frac{4}{\sqrt{5}}$ (3) $\frac{3}{\sqrt{5}} - \frac{3}{\sqrt{2}} + 1$ (4) $\sqrt{5} - 2\sqrt{2} + 1$

Sol. **(4)**

$|\cos x - \sin x|$



A = area under the curve

y = sinx & above the curve |cosx – sinx|

$A = \int_0^{\pi/2} (\sin x - |\cos x - \sin x|)\, dx$

When 0 to $\frac{\pi}{4}$

$|\cos x - \sin x| = \cos x - \sin x$

$\sin x = \cos x - \sin x$

$2\sin x = \cos x$

$\tan x = \frac{1}{2}$

$x = \tan^{-1}\left(\frac{1}{2}\right)$

when $\frac{\pi}{4}$ to $\frac{\pi}{2}$

$|\cos x - \sin x| = \sin x - \cos x$

$\sin x = \sin x - \cos x$

$\cos x = 0$

$x = \frac{\pi}{2}$

$$A = \int_{\tan^{-1}(1/2)}^{\pi/4} \{\sin x - (\cos x - \sin x)\}\, dx \ + \int_{\pi/4}^{\pi/2} \{\sin x + (\cos x - \sin x)\}\, dx$$

$$\Rightarrow \int_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}(2\sin x-\cos x)dx + \int_{\pi/4}^{\pi/2}\cos x\, dx$$

$$\Rightarrow (-2\cos x-\sin x)_{\tan^{-1}\left(\frac{1}{2}\right)}^{\pi/4}+(\sin x)_{\pi/4}^{\pi/2}$$

$$\Rightarrow \left(-2\times\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right)-\left\{-2\cos\left(\tan^{-1}\frac{1}{2}\right)-\sin\left(\tan^{-1}\frac{1}{2}\right)\right\}+\left(1-\frac{1}{\sqrt{2}}\right)$$

$$\Rightarrow -\frac{3}{\sqrt{2}}+2\times\frac{2}{\sqrt{5}}+\frac{1}{\sqrt{5}}+1-\frac{1}{\sqrt{2}} = \frac{-4}{\sqrt{2}}+\frac{5}{\sqrt{5}}+1 = -2\sqrt{2}+\sqrt{5}+1$$

73. The number of 3 digit numbers, that are divisible by either 3 or 4 but not divisible by 48, is

(1) 507 (2) 432 (3) 472 (4) 400

Sol. (2)

Nos div. by 3

102, 105, 108 ……… 999

A.P. a = 102 d = 3 ℓ = 999

$$n=\frac{\ell-a}{d}+1=\frac{999-102}{3}+1 = 300$$

Numbers div. by 4

100, 104, 108 ……… 996

A.P. a = 100 d = 4 ℓ = 996

$$n=\frac{996-100}{4}+1=\frac{896}{4}+1 = 224 + 1 = 225$$

Numbers div. by 12

108, 120, ……… 996

A.P. a = 108 d = 12 ℓ = 996

$$n=\frac{996-108}{12}+1=\frac{888}{12}+1 = 74 + 1 = 75$$

Numbers div. by 48

144, 192, ……… 960

A.P. a = 144 d = 48 ℓ = 960

$$n=\frac{996-144}{12}+1=\frac{816}{48}+1 = 17 + 1 = 18$$

$\therefore$ No. Div. by 354 but not by 48

300 + 225 – 75 – 18

= 450 – 18 = 432

74. If the lines $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}$ and $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}$ intersect at the point P, then the distance of the point P from the plane $z=a$ is :

(1) 28 (2) 16 (3) 10 (4) 22

Sol. **1**

Let $\frac{x-1}{1}=\frac{y-2}{2}=\frac{z+3}{1}=\lambda$

P(λ + 1, 2λ + 2, λ – 3)

& $\frac{x-a}{2}=\frac{y+2}{3}=\frac{z-3}{1}=\mu$

P(2μ + a, 3μ – 2, μ + 3)

λ +1 = 2μ + a	2λ + 2 = 3μ – 2	λ – 3 = μ + 3
22 + 1 = 2 × 16 + a	2(μ + 6) + 2 = 3μ –2	λ = μ + 6
a = 23 – 32	2μ + 12 = 3μ – 4	
a = – 9	μ = 16	
	∴ λ = 22	

∴P(23, 46, 19) Plane is z = a

z = – 9

The distance of p from z = –9 is 19 – (–9) = 28

75. Let $y=y(x)$ be the solution of the differential equation $x\log_e x\frac{dy}{dx}+y=x^2\log_e x,(x>1)$. If $y(2)=2$, then $y(e)$ is equal to

(1) $\frac{1+e^2}{2}$ (2) $\frac{4+e^2}{4}$ (3) $\frac{2+e^2}{2}$ (3) $\frac{1+e^2}{4}$

Sol. **2**

D.E. $\frac{dy}{dx}+\frac{1}{x\log x}y=x$

Linear diff. eq^n $\frac{dy}{dx}+py=Q$

If = $e^{\int Pdx}$

$= e^{\int\frac{1}{x\log x}dx}$ logx = t

$\frac{1}{x}dx=dt$

$= e^{\int\frac{1}{t}dt}=e^{\ell nt}=t$

I.F. = ℓnx

Solution of DE.

y. If = $\int q(\text{If})\,dx+c$

y.ℓnx = $\int x.(\ell n\,x)\,dx+c$

$= \ell nx\,.\,\frac{x^2}{2}\ell nx-\frac{x^2}{4}+C$

At x = 2, y = 2

$2\ell n2=\frac{4}{2}\ell n2-\frac{4}{4}+C \Rightarrow C=1$

∴ $y\ell nx=\frac{x^2}{2}\ell n\,x-\frac{x^2}{4}+1$

At x = e

$$y(e)\,\ell ne = \frac{e^2}{2}\ell n\, e - \frac{e^2}{4} + 1$$

$$y(e) = \frac{e^2}{4} + 1$$

76. Let f and g be twice differentiable functions on $\mathbb{R}$ such that

$$f''(x) = g''(x) + 6x$$
$$f'(1) = 4g'(1) - 3 = 9$$
$$f(2) = 3g(2) = 12.$$

Then which of the following is NOT true?

(1) There exists $x_0 \in (1, 3/2)$ such that $f(x_0) = g(x_0)$

(2) $|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

(3) If $-1 < x < 2$, then $|f(x) - g(x)| < 8$

(4) $g(-2) - f(-2) = 20$

Sol. **3**

Let $F(x) = f(x) - g(x)$

Given $f'(x) = g''(x) + 6x$

$$f'(x) = g'(x) + \frac{6x^2}{2} + c_1$$

x = 1 $f'(1) = g'(1) + 3 \times (1)^2 + c_1$

$9 = 3 + 3 + c_1$

$c_1 = 3$

$\therefore$ $f'(x) = g'(x) + 3x^2 + 3$

$$f(x) = g(x) + \frac{3x^3}{3} + 3x + c_2$$

x = 2 $f(2) = g(2) + (2)^3 + 3(2) + c_2$

$12 = 4 + 8 + 6 + c_2$

$c_2 = -6$

$\therefore$ $f'(x) = g(x) + x^3 + 3x - 6$

$= x^3 + 3x - 6$

Option (1)

$x_0 \in \left(1, \frac{3}{2}\right)$ such that $f(x_0) = 9(x_0)$

$\because$ $F(1) = f(1) - g(1)$ $F\left(\frac{3}{2}\right) = f\left(\frac{3}{2}\right) - g\left(\frac{3}{2}\right)$

$= 1 + 3 - 6 = -2$ $= (2)^3 + 3(2) - 6$

$= 8 + 6 - 6 = 8$

$\because$ $F(1)\ F\left(\frac{3}{2}\right) < 0$

$\Rightarrow$ At least one root of F(x) = 0 lies in $\left(1, \frac{3}{2}\right)$

$\Rightarrow$ $f(x) - g(x) = 0$

$\Rightarrow$ $f(x) = g(x)$

Option (2)

$|f'(x) - g'(x)| < 6 \Rightarrow -1 < x < 1$

$F'(x) = x^3 + 3x - 6$

$F'(x) = 3x^2 + 3$

$f'(x) - g'(x) = 3x^2 + 3$

$|f'(x) - g'(x)| < 6$

$\Rightarrow \quad 3x^2 + 3 < 6$

$\Rightarrow \quad 3x^2 <x\ 3$

$x^2 < 1$

$\Rightarrow \quad x \in (-1, 1)$

Option (3)

If $-1 < x < 2$ then $|f(x) - g(x)| < 8$

$F(x) = x^3 + 3x - 6$

$F(-1) = -1 - 3 - 6 = -10$ But $|f'(x) - g'(x)| < 10$

$F(2) = (2)^3 + 3(2) - 6 = 8$

Option is not true

Option (4)

$g(-2) - f(-2) = 20$

$F(-2) = f(-2) - g(-2)$

$= (-2)^3 + 3(-2) - 6$

$-8 - 6 - 6 = -20$

$g(-2) - f(-2) = 20$

77. If the tangent at a point P on the parabola $y^2 = 3x$ is parallel to the line $x + 2y = 1$ and the tangents at the points Q and R on the ellipse $\frac{x^2}{4} + \frac{y^2}{1} = 1$ are perpendicular to the line $x - y = 2$, then the area of the triangle PQR is :

(1) $\frac{3}{2}\sqrt{5}$ (2) $3\sqrt{5}$ (3) $\frac{9}{\sqrt{5}}$ (4) $5\sqrt{3}$

Sol. **2**

$x + 2y = 1$

$m = -\frac{1}{2}$

$y^2 = 3x$

$T_p : y = -\frac{1}{2}x + \frac{\frac{3}{4}}{-\frac{1}{2}}$

$y = -\frac{x}{2} - \frac{3}{2}$

$2y + x + 3 = 0 \quad \ldots(1)$

$x - y = 2$

$m = 1$

slope of tangent at Q & R is -1

$E : \frac{x^2}{4} + \frac{y^2}{1} = 1$

$y = -x \pm \sqrt{(-1)^2 4 + 1}$

$y = -x \pm \sqrt{5}$

$x + y = \sqrt{5} \quad \ldots(2) \qquad x + y = -\sqrt{5} \quad \ldots(3)$

Point P :

T = O

$yy_1 = \frac{3}{2}(x + x_1)$

$3x - 2yy_1 + 3x_1 = 0$

Comparison with (1)

$\frac{3}{1} = \frac{-2y_1}{2} = \frac{3x_1}{3}$

$y_1 = -3$, $x_1 = 3$

Point Q :

$\frac{xx_2}{4} + \frac{yy_2}{1} = 1$

$xx_2 + 4yy_2 - 4 = 0$

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{-\sqrt{5}}$

$x_2 = \frac{4}{\sqrt{5}} - y_2 = \frac{1}{\sqrt{5}}$

Point R:

$\frac{x_2}{1} = \frac{4y_2}{1} = \frac{-4}{\sqrt{5}}$

$x_2 = \frac{-4}{\sqrt{5}}$, $y = \frac{1}{\sqrt{5}}$

Area of ΔPQR

$$= \frac{1}{2}\begin{vmatrix} 3 & -3 & 1 \\ \frac{4}{\sqrt{5}} & \frac{1}{\sqrt{5}} & 1 \\ -\frac{4}{\sqrt{5}} & -\frac{1}{\sqrt{5}} & 1 \end{vmatrix}$$

$$\Rightarrow \frac{1}{2}\left[3\left(\frac{1}{\sqrt{5}} + \frac{1}{\sqrt{5}}\right) + 3\left(\frac{4}{\sqrt{5}} + \frac{4}{\sqrt{5}}\right) + 1\left(-\frac{4}{5} + \frac{4}{5}\right)\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{6}{\sqrt{5}} + \frac{24}{\sqrt{5}}\right]$$

$$\Rightarrow \frac{1}{2} \times \frac{30}{\sqrt{5}} = \frac{5 \times 3}{\sqrt{5}} = 3\sqrt{5}$$

78. Let $\vec{a} = 4\hat{\imath} + 3\hat{\jmath}$ and $\vec{b} = 3\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$. If $\vec{c}$ is a vector such that $\vec{c} \cdot (\vec{a} \times \vec{b}) + 25 = 0, \vec{c} \cdot (\hat{\imath} + \hat{\jmath} + \hat{k}) = 4$, and projection of $\vec{c}$ on $\vec{a}$ is 1 , then the projection of $\vec{c}$ on $\vec{b}$ equals

(1) $\frac{1}{5}$ (2) $\frac{5}{\sqrt{2}}$ (3) $\frac{3}{\sqrt{2}}$ (4) $\frac{1}{\sqrt{2}}$

Sol. **(2)**

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$\vec{c}\ .\ \vec{a} \times \vec{b}$

$$\begin{vmatrix} c_1 & c_2 & c_3 \\ 4 & 3 & 0 \\ 3 & -4 & 5 \end{vmatrix} = -25$$

$\Rightarrow c_1(15 - 0) - c_2(20 - 0) + c_3(-16-9) = -25$

$\Rightarrow 15c_1 - 20c_2 - 25c_3 = -25$

$\Rightarrow 3c_1 - 4c_2 - 5c_3 = -5$ …(2)

$\vec{c} \cdot (i + j + k) = 4$

$c_1 + c_2 + c_3 = 4$ …(i)

Proj. of $\vec{c}$ on $\vec{a} = \dfrac{\vec{a}.\vec{c}}{|\vec{a}|} = 1$

$\Rightarrow \dfrac{(4\hat{i}+3\hat{j})(c_1\hat{i}+c_2\hat{j}+c_3\hat{k})}{\sqrt{16+9}} = 1$

$\Rightarrow 4c_1 + 3c_2 = 5$

$\Rightarrow 4c_1 = 5 - 3c_2$

$\Rightarrow c_1 = \dfrac{5-3c_2}{4}$ ...(3)

Eq^n. (1) & (3)

$\dfrac{5-3c_2}{4} + c_2 + c_3 = 4$

$5 - 3c_2 + 4c_2 + 4c_3 = 16$

$c_2 + 4c_3 = 11$ ...(4)

Eq^n. (2) & (3)

$3\left(\dfrac{5-3c_2}{4}\right) - 4c_2 - 5c_3 = -5$

$15 - 9c2 - 16c2 - 20c_3 = -20$

$-25c_2 - 20c_3 = -35$ ...(5)

Eq^n. (4) & (5)

$-25c_2 - 20c_3 = -35$

$-25(11 - 4c_3) - 20c_3 = -35$

$5(11 - 4c_3) + 4c_3 = 7$

$55 - 20c_3 + 4c_3 = 7$

$-16c_3 = -48$

$c_3 = 3$

$c_2 = 11 - 4c_3$

$c_2 = 11 - 4 \times 3$

$= 11 - 12$

$c_2 = -1$

$c_1 = \dfrac{5-3c_2}{4}$

$= \dfrac{5-3(-1)}{4}$

$c_1 = 2$

$\vec{c} = 2\hat{i} - \hat{j} + 3\hat{k}$

Projection of $\vec{c}$ on $\vec{b}$ = $\left|\dfrac{\vec{c}.\vec{b}}{|\vec{b}|}\right|$

$\Rightarrow \left|\dfrac{(2\hat{i}-\hat{j}+3\hat{k}).(3\hat{i}-4\hat{j}+5\hat{k})}{\sqrt{9+16+25}}\right|$

$\Rightarrow \left|\dfrac{6+4+15}{5\sqrt{2}}\right| = \dfrac{25}{5\sqrt{2}} = \dfrac{5}{\sqrt{2}}$

79. Let $S = \{w_1, w_2, \dots\dots\}$ be the sample space associated to a random experiment. Let $P(w_n) = \frac{P(w_{n-1})}{2}, n \geq 2$. Let $A = \{2k + 3l : \boldsymbol{k}, l \in \mathbb{N}\}$ and $B = \{w_n : n \in A\}$. Then $P(\ B)$ is equal to

(1) $\frac{3}{64}$ (2) $\frac{1}{16}$ (3) $\frac{1}{32}$ (4) $\frac{3}{32}$

Sol. **1**

A = {5, 7, 8, 9, 10, 11 ......}

$P(W_1) + P(W_2) + P(W_3) + \dots\dots P(W_n) = 1$

$P(W_1) + \dfrac{P(W_1)}{2} + \dfrac{P(W_2)}{2^2} + \dots\dots = 1$

$\Rightarrow P(W_1).\left(\dfrac{1}{1-1/2}\right) = 1$

$$\boxed{P(W_1)=\frac{1}{2}} \qquad P(W_n)=\frac{1}{2}\cdot\left(\frac{1}{2}\right)^{n-1}=\frac{1}{2^n}$$

$\because$ B = {W_n : $n \in A$}

= {W_5, W_7, W_8, …….}

P(B) = $P(W_5) + P(W_7) + P(W_8) + P(W_9) + P(W_{10}) + P(W_{11})$

$$=\frac{1}{2^5}+\frac{1}{2^7}+\frac{1}{2^8}+.....$$

$$=\frac{1}{32}+\frac{\frac{1}{2^7}}{1-\frac{1}{2}}$$

$$=\frac{1}{32}+\frac{1}{2^7}\times 2$$

$$=\frac{1}{32}+\frac{1}{64}=\frac{2+1}{64}=\frac{3}{64}$$

80. Let K be the sum of the coefficients of the odd powers of x in the expansion of $(1+x)^{99}$. Let a be the middle term in the expansion of $\left(2+\frac{1}{\sqrt{2}}\right)^{200}$. If $\frac{{}^{200}C_{99}\,K}{a}=\frac{2^l\,m}{n}$, where m and n are odd numbers, then the ordered pair (l, n) is equal to

(1) $(50,51)$ (2) $(50,101)$ (3) $(51,99)$ (4) $(51,101)$

Sol. **(2)**

$$K=\frac{(1+1)^{99}}{2}=2^{98}$$

$$a={}^{200}C_{100}2^{100}\times\frac{1}{\left(\sqrt{2}\right)^{100}}$$

$$a={}^{200}C_{100}2^{50}$$

$$\frac{{}^{200}C_{99}.K}{a}=\frac{{}^{200}C_{99}.2^{98}}{{}^{200}C_{100}.2^{50}}$$

$$\because \frac{{}^{200}C_{99}}{{}^{200}C_{100}}=\frac{\lfloor 200}{\lfloor 99 \lfloor 101}\times\frac{\lfloor 100 \lfloor 100}{\lfloor 200}$$

$$=\frac{100}{101}$$

$$\therefore \frac{{}^{200}C_{99}K}{a}=\frac{100}{101}\times 2^{48}$$

$$=\frac{25\times 2^{50}}{101}$$

$\ell = 50$, n = 101

$(\ell, n) = (50, 101)$

Section B

81. The total number of 4-digit numbers whose greatest common divisor with 54 is 2 , is

Sol. **3000**

$54 = 2 \times 3^3$

4 digit even numbers are $\begin{array}{cccc} 9 & 10 & 10 & 5 \\ \hline \multicolumn{1}{|c|}{} & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{} & \multicolumn{1}{c|}{} \\ \hline \end{array}$

4500

4 digit even numbers which are multiple of 3

Are the numbers which are multiple of 6

$= \dfrac{9000}{6} = 1500$

$\therefore$ The no. which has GCD with 54 us 2 is 4500 – 1500 = 3000

82. Let $a_1 = b_1 = 1$ and $a_n = a_{n-1} + (n-1), b_n = b_{n-1} + a_{n-1}, \forall n \geq 2$. If $S = \sum_{n=1}^{10} \frac{b_n}{2^n}$ and $T = \sum_{n=1}^{8} \frac{n}{2^{n-1}}$, then $2^7(2S - T)$ is equal to

Sol. **461**

$$T = \frac{1}{2^0} + \frac{2}{2^1} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots\ldots\ldots\ldots \frac{8}{2^7} \quad \ldots..(1)$$

$$S = \frac{b_1}{2} + \frac{b_2}{2^2} + \frac{b_3}{2^3} + \frac{b_4}{2^4} + \ldots\ldots\ldots\ldots \frac{b_{10}}{2^{10}}$$

$$\frac{S}{2} = \frac{b_1}{2^2} + \frac{b_2}{2^3} + \frac{b_3}{2^4} + \ldots\ldots\ldots\ldots \frac{b_9}{2^{10}} + \frac{b_{10}}{2^{11}} \text{ (Subtract)}$$

$$\frac{S}{2} = \frac{b_1}{2} + \left(\frac{b_2 - b_1}{2^2}\right) + \left(\frac{b_3 - b_2}{2^3}\right) + \left(\frac{b_4 - b_3}{2^4}\right) + \ldots\ldots + \left(\frac{b_{10} - b_9}{2^{10}}\right) - \frac{b_{10}}{2^{11}}$$

$$\frac{S}{2} = \frac{b_1}{2} + \frac{a_1}{2^2} + \frac{a_2}{2^3} + \frac{a_3}{2^4} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{11}}$$

$$\Rightarrow S = b_1 + \frac{a_1}{2} + \frac{a_2}{2^2} + \frac{a_3}{2^3} + \ldots\ldots\ldots\ldots\ldots\ldots\ldots \frac{a_9}{2^{10}} - \frac{b_{10}}{2^{10}}$$

$$S = \left(b_1 - \frac{b_{10}}{2^{10}}\right) + \left(\frac{a_1}{2} + \frac{a_2}{2^2} + \ldots\ldots\ldots \frac{a_9}{2^9}\right)$$

$$\Rightarrow \frac{S}{2} = \left(\frac{b_1}{2} - \frac{b_{10}}{2^{11}}\right) + \left(\frac{a_1}{2^2} + \frac{a_2}{2^3} + \ldots\ldots\ldots \frac{a_8}{2^9} + \frac{a_9}{2^{10}}\right) \text{ (Subtract)}$$

$$\frac{S}{2} = \frac{b_1}{2} - \frac{b_{10}}{2^{11}} + \left(\frac{a_1}{2} - \frac{a_9}{2^{10}}\right) + \left(\frac{1}{2^2} + \frac{2}{2^3} + \ldots\ldots\ldots\ldots\ldots \frac{8}{2^9}\right)$$

$$= \frac{b_1}{2} + \frac{a_1}{2} - \left(\frac{b_{10} + 2a_9}{2^{11}}\right) + \frac{T}{4} \qquad \text{from } \ldots\ldots\ldots(1)$$

Now $2s = 2(a_1 + b_1) - \left(\dfrac{b_{10} + 2a_9}{2^9}\right) + T$

$2S - T = 2(a_1 + b_1) - \left(\dfrac{b_{10} + 2a_9}{2^9}\right)$

$$\boxed{2^7(2S-T)=2^8(a_1+b_1)-\frac{b_{10}+2a_9}{4}} \qquad \ldots\ldots\ldots\ldots\ldots (2)$$

$\because a_n - a_{n-1} = n-1$

$a_2 - a_1 = 1$

$a_3 - a_2 = 2$

$a_4 - a_3 = 3$

……

……

$a_9 - a_8 = 8$

$a_9 - a_1 = 1 + 2 + 3 + \ldots\ldots\ldots\ldots +8$

$\boxed{a_9 = 36+1=37}$

and $b_n = b_{n-1} = a_{n-1}$

$b_{10} - b_1 = a_1 + a_2 + a_3 + \ldots\ldots + a_9$

$b_{10} - 1 = 1 + 2 + 4 + 7 + 11 + 16 + 22 + 29 + 37$

$b_{10} - 1 = 29$

$\boxed{b_{10} = 130}$

$$\boxed{\begin{aligned}&2^7(2S-T)=2^8\times(1+1)-\frac{130+2\times37}{4}\\&=2^9-\frac{102}{2}\\&=512-51=461\end{aligned}}$$

83. A triangle is formed by the tangents at the point $(2,2)$ on the curves $y^2 = 2x$ and $x^2 + y^2 = 4x$, and the line $x + y + 2 = 0$. If r is the radius of its circumcircle, then r^2 is equal to

Sol. **10**

Tangent at $y^2 = 2x$

T: $2y = 2\left(\frac{x+2}{2}\right)$

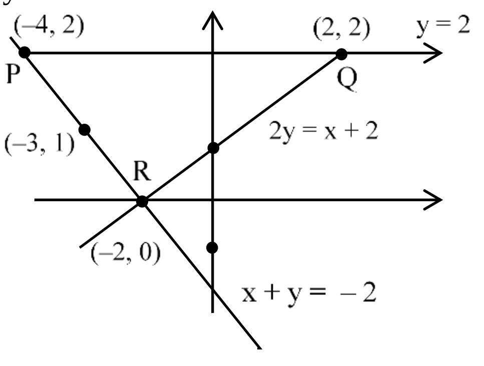
$2y = x + 2$

Tangent at $x^2 + y^2 = 4x$

$2x + 2y = \frac{4\times(x+2)}{2}$

$2x + 2y = 2x + 4$

$y = 2$



$M_{PR} = -1$
Slope of $\perp^{r\ell}$ Bisector = 1
$y - 1 = 1(x + 3)$
$y = x + 3 + 1$
$y = x + 4$
$\perp^{r\ell}$ Bisector of PQ
$x = -1$
$\therefore$ Centre is
$y = -1 + 4 = 3$
$(-1, 3)$
Radius: $r = \sqrt{(-1+4)^2 + (3-2)^2}$
$= \sqrt{9+1}$
$= \sqrt{10}$
$r^2 = 10$

84. Let $\alpha_1, \alpha_2, \ldots, \alpha_7$ be the roots of the equation $x^7 + 3x^5 - 13x^3 - 15x = 0$ and $|\alpha_1| \geq |\alpha_2| \geq \cdots \geq |\alpha_7|$. Then $\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$ is equal to

Sol. **3**

$\alpha_1, \alpha_2, \ldots \alpha_7$
$x^7 + 3x^5 - 13x^3 - 15x = 0$
$x(x^6 - 3x^4 - 13x^2 - 15) = 0$
$|\alpha_1| \geq |\alpha_2| \geq \ldots \geq |\alpha_7|$
$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_3\alpha_6 = ?$
$\boxed{\alpha_7 = 0}$
$\Rightarrow x(x^6 + 3x^4 - 13x^2 - 15) = 0$
$x = 0\ x^6 + 3x^4 - 13x^2 - 15 = 0$
$\Rightarrow t^3 + 3t^2 - 13t - 15 = 0$
$\Rightarrow (t - 3)(t^2 + 6t + 5) = 0$
$t = 3, t = -5, -1$
$x = 0,\ x = \pm\sqrt{3},\ x = \pm\sqrt{5}i,\ x = \pm i$
$\alpha_1 = \sqrt{5}i$
$\alpha_2 = -\sqrt{5}i$
$\alpha_3 = \sqrt{3}$
$\alpha_4 = -\sqrt{3}$
$\alpha_5 = i$
$\alpha_6 = -i$
$\alpha_7 = 0$
$\alpha_1\alpha_2 = 5,\ \alpha_3\alpha_4 = 3,\ \alpha_5\alpha_6 = 1$
$\alpha_1\alpha_2 - \alpha_3\alpha_4 + \alpha_5\alpha_6$
$\Rightarrow 5 - 3 + 1 = 3$

85. Let $X = \{11,12,13, \dots ,40,41\}$ and $Y = \{61,62,63, \dots ,90,91\}$ be the two sets of observations. If $\bar{x}$ and $\bar{y}$ are their respective means and σ^2 is the variance of all the observations in $X \cup Y$, then $|\bar{x} + \bar{y} - \sigma^2|$ is equal to

Sol. **603**

$$\bar{x} = \frac{11+12+........+41}{31} \qquad \bar{y} = \frac{61+62+63+.....+91}{31}$$

$$= \frac{\frac{31}{2}(11+41)}{31} = \frac{52}{2} = 26 \qquad = \frac{\frac{31}{2}(61+91)}{31} = \frac{152}{2} = 76$$

$$\sigma^2 = \frac{\sum x_i^2 + \sum y_i^2}{31+31} - \bar{x}^2$$

$$= \frac{\left(\sum_{n=1}^{41} n^2 - \sum_{n=1}^{10} n^2\right) + \left(\sum_{n=1}^{91} n^2 - \sum_{n=1}^{60} n^2\right)}{62} - \left(\frac{31\times 26 + 76\times 31}{62}\right)^2$$

$$\Rightarrow \frac{\frac{41\times 42\times 83}{6} - \frac{10\times 11\times 21}{6} + \frac{91\times 92\times 183}{6} - \frac{60\times 61\times 121}{6}}{62} - (51)^2$$

$$\Rightarrow \frac{7(41\times 83 - 55) + 61(91\times 46 - 1210)}{62}$$

$$\Rightarrow \frac{7(3403 - 55) + 61(4186 - 1210)}{62}$$

$$\Rightarrow \frac{7\times 3348 + 61\times 2976}{62} - 2601$$

$\Rightarrow 3306 - 2601 = 705 \Rightarrow \sigma^2 = 705$

$\therefore |\bar{x} + \bar{y} - \sigma^2| = |26 + 76 - 705| = 603$

86. If the equation of the normal to the curve $y = \frac{x-a}{(x+b)(x-2)}$ at the point $(1, -3)$ is $x - 4y = 13$, then the value of $a + b$ is equal to

Sol. **– 6**

(1, –3) is on the curve

$$\therefore -3 = \frac{1-a}{(1+b)(1-2)} \Rightarrow -3 = \frac{1-a}{(-1)(1+b)}$$

$\Rightarrow 3 + 3b = 1 - a \Rightarrow a + 3b = -2$

$a = -2 - 3b$

$\ell n y = \ell n (x - a) - \ell n(x + b) - \ell n(x - 2)$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x-a} - \frac{1}{x+b} - \frac{1}{x-2}$$

$$\left.\frac{dy}{dx}\right|_{(1,-3)} = -3\left(\frac{1}{1-a} - \frac{1}{1+b} - \frac{1}{1-2}\right) = -4$$

$$\Rightarrow \left(\frac{1}{1+2+3b} - \frac{1}{1+b} + 1\right) = \frac{4}{12} = \frac{1}{3}$$

$\Rightarrow \dfrac{1}{3(b+1)} - \dfrac{1}{b+1} = \dfrac{1}{3} - 1$

$\Rightarrow \dfrac{1-3}{3(b+1)} = -\dfrac{2}{3}$

$\Rightarrow$ b + 1 = 3

b = 2

a = – 2– 3b	a + b
a = –2 – 3 × 2	$\Rightarrow$ –8 + 2
a = – 2 – 6 = – 8	$\Rightarrow$ – 6

87. Let A be a symmetric matrix such that $|A| = 2$ and $\begin{bmatrix} 2 & 1 \\ 3 & \frac{3}{2} \end{bmatrix} A - \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$.

If the sum of the diagonal elements of A is s, then $\frac{\beta s}{\alpha^2}$ is equal to

Sol. **5**

A be a symmetric matrix such that |A| = 2 and $\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix} A = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$A = \begin{bmatrix} a & b \\ b & d \end{bmatrix}$ $\qquad |A| = ad - b^2 = 2$

$\begin{bmatrix} 2 & 1 \\ 3 & 3/2 \end{bmatrix}\begin{bmatrix} a & b \\ b & d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

$\begin{bmatrix} 2a+b & 2b+d \\ 3a+3/2\,b & 3b+3/2\,d \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ \alpha & \beta \end{bmatrix}$

2a + b = 1	2b + d = 2
b = 1 – 2a	d = 2 – 2b
	= 2 – 2(1 – 2a)
	= 2 – 2 +4a

$ad - b^2 = 2$

$a.4a - (1-2a)^2 = 2$

$\Rightarrow 4a^2 - 1 - 4a^2 + 4a = 2$

4a = 3

$a = \dfrac{3}{4}$

$b = 1 - 2 \times \dfrac{3}{4}$

$= \dfrac{-1}{2}$

$d = 4 \times \dfrac{3}{4} = 3$

Now $\alpha = 3a + \dfrac{3}{2}b$

$= \dfrac{9}{4} + \dfrac{3}{2} \cdot \left(\dfrac{-1}{2}\right)$

$= \dfrac{9-3}{4} = \dfrac{6}{4} = \dfrac{3}{2}$

$\beta = 3b + \dfrac{3}{2}d$

$3 \times \left(\dfrac{-1}{2}\right) + \dfrac{3}{2} \times 3$

$\dfrac{-3+9}{2} = 3$

$$A=\begin{bmatrix} 3/4 & -1/2 \\ -1/2 & 3 \end{bmatrix} \quad s=\frac{3}{4}+3=\frac{15}{4}$$

$$\frac{Bs}{\alpha^2}=\frac{3\times\frac{15}{4}}{\frac{9}{4}}=5$$

88. Let $\alpha = 8 - 14i, A = \left\{z \in \mathbb{C}: \frac{\alpha z - \bar{\alpha}\bar{z}}{z^2-(\bar{z})^2-112i} = 1\right\}$ and $B = \{z \in \mathbb{C}: |z + 3i| = 4\}$.
Then $\sum_{z\in A\cap B}(Re\,z - Im\,z)$ is equal to

Sol. **14**

$\alpha = 8 - 14i \quad A = \left\{z \in c: \dfrac{\alpha z - \overline{\alpha}\,\overline{z}}{z^2-(\overline{z})^2-112i}=1\right\}$

$z^2 - \overline{z}^2$

$\Rightarrow (z+\overline{z})(z-\overline{z}) \Rightarrow 2x\cdot 2yi = 4xyi$

$\alpha z = (8-14i)(x+iy)$

$= 8x - 8iy + 14xi + 14y$

$= (8x + 14y) + i(8y - 14x)$

$\overline{\alpha}\,\overline{z} = (8 + 14i)(x - iy)$

$= 8x - 8iy + 14ix + 14y$

$= (8x + 14y) + i(14x - 8y)$

$\dfrac{\alpha z - \overline{\alpha}\,\overline{z}}{z^2 - \overline{z}^2 - 112i} = 1$

$\Rightarrow \dfrac{(16y - 28x)i}{4xyi - 112i} = 1$

$\Rightarrow 4y - 7x = xy - 28$

$\Rightarrow 4y - 7x - xy + 28 = 0$

$\Rightarrow y(4 - x) - 7(x - 4) = 0$

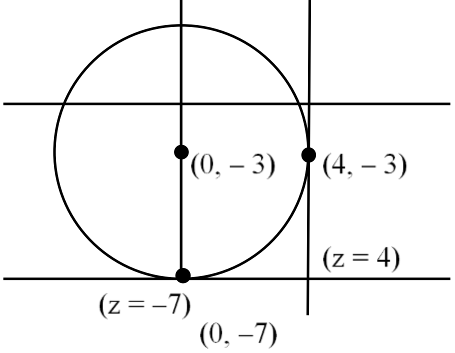
$(x - 4)(-y - 7) = 0$

$x = 4 \quad y = -7$

$z = 4$ or $z = -7i$

$B \Rightarrow |z + 3i| = 4$

$x^2 + (y - 3)^2 = 16$



$\sum Re(z) - Im(z)$

$= (0 + 4) - (-7 - 3)$
$= 4 + 10 = 14$

89. A circle with centre (2,3) and radius 4 intersects the line $x + y = 3$ at the points P and Q. If the tangents at P and Q intersect at the point $S(\alpha, \beta)$, then $4\alpha - 7\beta$ is equal to

Sol. **11**

$(x-2)^2 + (y-3)^2 = 16$
$x^2 + y^2 - 4x - 6y - 3 = 0$
let the chord of contact w.r. to the point $S(\alpha,\beta)$ is
$T = 0$
$\alpha x + \beta y - 2(x + \alpha) - 3(y + \beta) - 3 = 0$
$x(\alpha - 2) + y(\beta - 3) - 2\alpha - 3\beta - 3 = 0$
Comparison with $x + y = 3$

$$\frac{\alpha-2}{1} = \frac{\beta-3}{1} = \frac{-2\alpha-3\beta-3}{-3}$$

$\alpha - 2 = \beta - 3$ $\quad$ $-3\beta + 9 = -2\alpha - 3\beta - 3$
$\alpha - \beta = -1$ $\quad$ $2\alpha = -3-9$
$\beta = \alpha + 1$ $\quad$ $\alpha = \frac{-12}{2}$
$= -6 + 1 = -5$ $\quad$ $\alpha = -6$
$4\alpha - 7\beta$
$4(-6) - 7(-5)$
$\Rightarrow -24 + 35 = 11$

90. Let $\{a_k\}$ and $\{b_k\}, k \in \mathbb{N}$, be two G.P.s with common ratios r_1 and r_2 respectively such that $a_1 = b_1 = 4$ and $r_1 < r_2$. Let $c_k = a_k + b_k, k \in \mathbb{N}$. If $c_2 = 5$ and $c_3 = \frac{13}{4}$ then $\sum_{k=1}^{\infty} c_k - (12a_6 + 8b_4)$ is equal to

Sol. **9**

$a_1 = 4$ $\quad$ GP $4, 4r_1, 4r_1^2$ - -
$b_1 = 4$ $\quad$ GP $4, 4r_2, 4r_2^2$ - -
$C_2 = a_2 + b_2$ $\quad$ $C_3 = a_3 + b_3$
$5 = 4r_1 + 4r_2$ $\quad$ $\frac{13}{4} = 4r_1^2 + 4r_2^2$
$\frac{5}{4} = r_1 + r_2$ …(1) $\quad$ $r_1^2 + r_2^2 = \frac{13}{16}$ … (2)

$$\frac{25}{16} = r_1^2 + r_2^2 + 2r_1r_2$$

$$\frac{25}{16} = \frac{13}{16} + 2r_1r_2$$

$$\Rightarrow r_1r_2 = \frac{12}{16\times 2} = \frac{3}{8}$$

Now $r_1 + \frac{3}{8r_1} = \frac{5}{4}$

$8r_1^2 + 3 = 10\, r_1$

$\Rightarrow 8r_1^2 - 10r_1 + 3 = 0$

$r_1 = \frac{3}{4}$, $r_1 = \frac{1}{2}$

$r_2 = \frac{1}{2}$ $r_2 = \frac{3}{4}$

$\because r_1 < r_2$

$\therefore \begin{array}{l} r_1 = \frac{1}{2} \\ r_2 = \frac{3}{4} \end{array}$

Now $C_k = a_k + b_k$

$\sum_{k=1}^{\infty} C_k = \frac{4}{1-r_1} + \frac{4}{1-r_2}$

$= \frac{4}{1-\frac{1}{2}} + \frac{4}{1-\frac{3}{4}}$

= 8 + 16 = 24

$\sum_{k=1}^{\infty} C_k - (12a_6 + 8b_4) \Rightarrow 24 - \left\{12 \times 4\left(\frac{1}{2}\right)^5 + 8 \times 4\left(\frac{3}{4}\right)^3\right\}$

$= 24 - \left(12 \times \frac{1}{8} + 8 \times \frac{27}{16}\right)$

$= 24 - \left\{\frac{3}{2} + \frac{27}{2}\right\}$

= 24 – 15

= 9