FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

PHYSICS — TEST PAPER WITH SOLUTION

SECTION-A

1. Given below are two statements : One is labelled as Assertion (A) and other is labelled as Reason (R).

Assertion (A) : Time period of oscillation of a liquid drop depends on surface tension (S), if density of the liquid is p and radius of the drop is r, then T = $k\sqrt{pr^3/S^{3/2}}$ is dimensionally correct, where K is dimensionless.

Reason (R) : Using dimensional analysis we get R.H.S. having different dimension than that of time period.

In the light of above statements, choose the correct answer from the options given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is false but (R) is true

Official Ans. by NTA (D)

Sol. $T = k\sqrt{\frac{\rho r^3}{S^{3/2}}}$

Dimensions of RHS = $\frac{\left[M^{1/2}L^{-3/2}\right]\left[L^{3/2}\right]}{\left[MT^{-2}\right]^{3/4}} = M^{1/8}\,L^0\,T^{3/2}$

Dimensions of L.H.S $\neq$ Dimensions of R.H.S

$\therefore$ option (D)

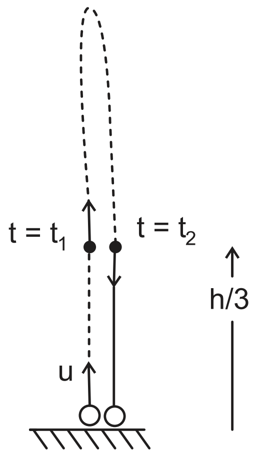
2. A ball is thrown up vertically with a certain velocity so that, it reaches a maximum height h. Find the ratio of the times in which it is at height $\frac{h}{3}$ while going up and coming down respectively.

(A) $\frac{\sqrt{2}-1}{\sqrt{2}+1}$ (B) $\frac{\sqrt{3}-\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

(C) $\frac{\sqrt{3}-1}{\sqrt{3}+1}$ (D) $\frac{1}{3}$

Official Ans. by NTA (B)

Sol.



Max. Height = h = $\frac{u^2}{2g}$

$\Rightarrow u = \sqrt{2gh}$

$S = ut + \frac{1}{2}at^2$

$\frac{h}{3} = \sqrt{2gh}\,t + \frac{1}{2}(-g)t^2$

$\frac{gt^2}{2} - \sqrt{2gh}\,t + \frac{h}{3} = 0$ (Roots are t_1 & t_2)

$$\frac{t_2}{t_1} = \frac{\sqrt{2gh}+\sqrt{2gh-4\times\frac{g}{2}\times\frac{h}{3}}}{\sqrt{2gh}-\sqrt{2gh-4\times\frac{g}{2}\times\frac{h}{3}}} = \frac{\sqrt{2gh}+\sqrt{\frac{4gh}{3}}}{\sqrt{2gh}-\sqrt{\frac{4gh}{3}}} = \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}-\sqrt{2}}$$

3. If $t=\sqrt{x}+4$, then $\left(\frac{dx}{dt}\right)_{t=4}$ is:

(A) 4 (B) Zero

(C) 8 (D) 16

Official Ans. by NTA (B)

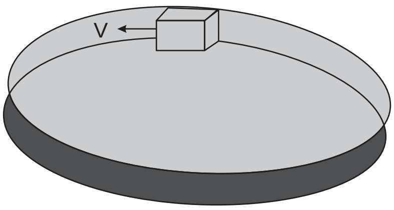
Sol. $t=\sqrt{x}+4$

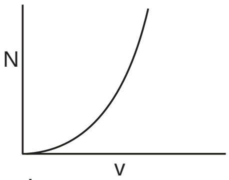
$\Rightarrow x=(t-4)^2=t^2-8t+16$

$\Rightarrow \frac{dx}{dt}=2t-8$

$\Rightarrow \left.\frac{dx}{dt}\right|_{t=4}=2\times4-8=0$

4. A smooth circular groove has a smooth vertical wall as shown in figure. A block of mass m moves against the wall with a speed v. Which of the following curve represents the correct relation between the normal reaction on the block by the wall (N) and speed of the block (v) ?



(A)


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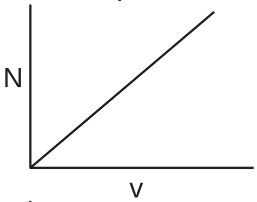
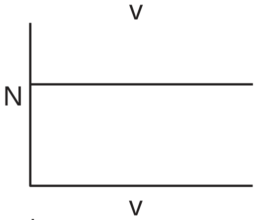
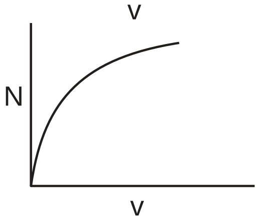


(C)


(D)


Official Ans. by NTA (A)

Sol. $N=\frac{mv^2}{r}$

Curve is parabola

$Y=kx^2$

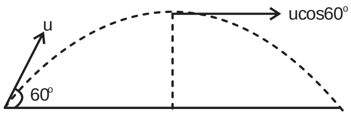
5. A ball is projected with kinetic energy E, at an angle of 60^0 to the horizontal. The kinetic energy of this ball at the highest point of its flight will become :

(A) Zero (B) $\frac{E}{2}$

(C) $\frac{E}{4}$ (D) E

Official Ans. by NTA (C)

Sol.



$E=\frac{1}{2}mu^2$

At Highest point, Velocity $V=u\cos60^o=\frac{u}{2}$

$\therefore$ K.E at topmost point $=\frac{1}{2}m\left(\frac{u}{2}\right)^2=\frac{E}{4}$

6. Two bodies of mass 1 kg and 3 kg have position vectors $\hat{i}+2\hat{j}+\hat{k}$ and $-3\hat{i}-2\hat{j}+\hat{k}$ respectively. The magnitude of position vector of centre of mass of this system will be similar to the magnitude of vector :

(A) $\hat{i}-2\hat{j}+\hat{k}$ (B) $-3\hat{i}-2\hat{j}+\hat{k}$

(C) $-2\hat{i}+2\hat{k}$ (D) $-2\hat{i}-\hat{j}+2\hat{k}$

Official Ans. by NTA (A)

Sol. $\overrightarrow{r_{com}}=\frac{m_1\vec{r_1}+m_2\vec{r_2}}{m_1+m_2}=\frac{1(\hat{i}+2\hat{j}+\hat{k})+3(-3\hat{i}-2\hat{j}+\hat{k})}{1+3}$

$=-2\hat{i}-\hat{j}+\hat{k}$

$|2\hat{i}-\hat{j}+\hat{k}|=\sqrt{(2)^2+(1)^2+(1)^2}=\sqrt{6}$

7. Given below are two statements : One is labelled as Assertion (A) and the other is labelled as Reason (R).

Assertion (A) : Clothes containing oil or grease stains cannot be cleaned by water wash.

Reason (R) : Because the angle of contact between the oil/ grease and water is obtuse. In the light of the above statements, choose the correct answer from the option given below.

(A) Both (A) and (R) are true and (R) is the correct explanation of (A)

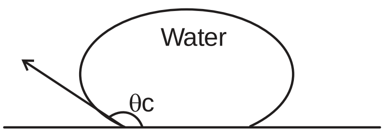
(B) Both (A) and (R) are true but (R) is not the correct explanation of (A)

(C) (A) is true but (R) is false

(D) (A) is true but (R) is true

Official Ans. by NTA (A)

Sol.



$\theta c > 90^{o}$

For water oil interface

8. If the length of a wire is made double and radius is halved of its respective values. Then, the Young's modules of the material of the wire will :

(A) Remains same

(B) Become 8 times its initial value

(C) Become $\frac{1^{th}}{4}$ of its initial value

(D) Become 4 times its initial value

Official Ans. by NTA (A)

Sol. Y depends on material of wire

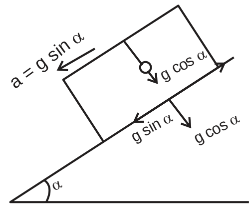
9. The time period of oscillation of a simple pendulum of length L suspended from the roof of a vehicle, which moves without friction down an inclined plane of inclination α, is given by :

(A) $2\pi\sqrt{L/(g\cos\alpha)}$ (B) $2\pi\sqrt{L/(g\sin\alpha)}$

(C) $2\pi\sqrt{L/g}$ (D) $2\pi\sqrt{L/(g\tan\alpha)}$

Official Ans. by NTA (A)

Sol. $g_{cff} = g\cos\alpha$

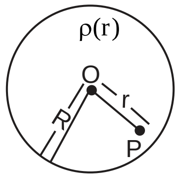


$$T = 2\pi\sqrt{\frac{L}{g\cos\alpha}}$$

10. A spherically symmetric charge distribution is considered with charge density varying as

$$\rho(r) = \begin{cases} \rho_0\left(\frac{3}{4} - \frac{r}{R}\right) & \text{for } r \le R \\ \text{Zero} & \text{for } r > R \end{cases}$$

Where, r(r < R) is the distance from the centre O (as shown in figure). The electric field at point P will be :

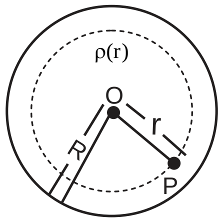


(A) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$ (B) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{3}{4} - \frac{r}{R}\right)$

(C) $\frac{\rho_0 r}{4\varepsilon_0}\left(1 - \frac{r}{R}\right)$ (D) $\frac{\rho_0 r}{5\varepsilon_0}\left(1 - \frac{r}{R}\right)$

Official Ans. by NTA (C)

Sol. By Gauss law



$$\oint \vec{E}.d\vec{s} = \frac{Q_{in}}{\varepsilon_o}$$

$$E.4\pi r^2 = \frac{\int_0^r \rho_o\left(\frac{3}{4} - \frac{r}{R}\right)4\pi r^2 dr}{\varepsilon_0}$$

$$E4\pi r^2 = \frac{\rho_o 4\pi}{\varepsilon_o}\left(\frac{3}{4}\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$$E r^2 = \frac{\rho_o r^3}{4\varepsilon_o}\left\{1 - \frac{r}{R}\right\}$$

$$E = \frac{\rho_o r}{4\varepsilon_o}\left\{1 - \frac{r}{R}\right\}$$

11. Given below are two statements.

Statement I : Electric potential is constant within and at the surface of each conductor.

Statement II : Electric field just outside a charged conductor is perpendicular to the surface of the conductor at every point.

In the light of the above statements, choose the most appropriate answer from the options give below.

(A) Both statement I and statement II are correct
(B) Both statement I and statement II are incorrect
(C) Statement I is correct but statement II is incorrect
(D) Statement I is incorrect but and statement II is correct

Official Ans. by NTA (A)

Sol. (Properties of conductor)

Statement – I, true as body of conductor acts as equipotential surface.

Statement – 2 True, as conductor is equipotential. Tangential component of electric field should be zero. Therefore electric field should be perpendicular to surface.

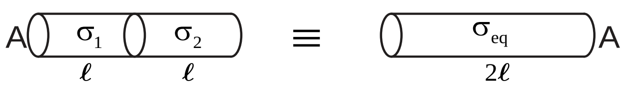
12. Two metallic wires of identical dimensions are connected is series. If σ_1 and σ_2 are the conductivities of the these wires respectively, the effective conductivity of the combination is :

(A) $\frac{\sigma_1 \sigma_2}{\sigma_1+\sigma_2}$ (B) $\frac{2\sigma_1 \sigma_2}{\sigma_1+\sigma_2}$

(C) $\frac{\sigma_1+\sigma_2}{2\sigma_1 \sigma_2}$ (D) $\frac{\sigma_1+\sigma_2}{\sigma_1 \sigma_2}$

Official Ans. by NTA (B)

Sol.



Let length of wire be 'ℓ'

Area of wire as 'A'

For equivalent wire length = 2ℓ & area will be A

Thermal resistance

$R_{eq} = R_1 + R_2$

$$\frac{2\ell}{\sigma_{eq} A} = \frac{\ell}{\sigma_1 A} + \frac{\ell}{\sigma_1 A}$$

$$\frac{2\ell}{\sigma_{eq}} = \frac{\ell}{\sigma_1} + \frac{\ell}{\sigma_2} \Rightarrow \sigma_{eq} = \frac{2\sigma_1\sigma_2}{\sigma_1+\sigma_2}$$

13. An alternating emf E = 440 sin $100\pi t$ is applied to a circuit containing an inductance of $\frac{\sqrt{2}}{\pi}$ H. If an a.c. ammeter is connected in the circuit, its reading will be :

(A) 4.4 A (B) 1.55 A
(C) 2.2 A (D) 3.11 A

Official Ans. by NTA (C)

Sol. $E = 440 \operatorname{Sin} 100\pi t$, $L = \frac{\sqrt{2}}{\pi} H$

$$X_L = \omega L = 100\pi \frac{\sqrt{2}}{\pi} = 100\sqrt{2}\ \Omega$$

$$\text{Peak current } I_0 = \frac{E_0}{X_L} = \frac{440}{100\sqrt{2}} = 2.2\sqrt{2}\ A$$

AC ammeter reads RMS value therefore reading will be I_{rms}

$$I_{rms} = \frac{I_0}{\sqrt{2}} = 2.2\ A$$

14. A coil of inductance 1 H and resistance 100 Ω is connected to a battery of 6 V. Determine approximately :

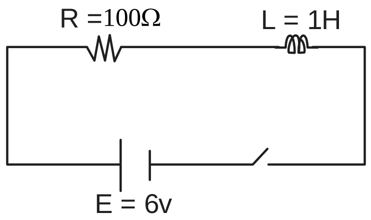
(a) The time elapsed before the current acquires half of its steady – state value

(b) The energy stored in the magnetic field associated with the coil at an instant 15 ms after the circuit is switched on. (Given In2 = 0.693, $e^{-3/2} = 0.25$)

(A) t = 10 ms; U = 2 mJ
(B) t = 10 ms; U = 1 mJ
(C) t = 7 ms; U = 1 mJ
(D) t = 7 ms; U = 2 mJ

Official Ans. by NTA (C)

Sol. Given circuit is R – L growth circuit



$$i = \frac{E}{R}(1-e^{-t/\tau})$$

$$i = \frac{E}{2R} = \frac{E}{R}(1-e^{-t/\tau})$$

Solving $t = \tau \ln 2$

$$t = \frac{1}{R}\ln 2 = \frac{1}{100}0.693 = 0.00693$$

$= 7$ ms

$$i(15ms) = \frac{E}{R}(1-e^{-\frac{15}{10}})$$

$$i = \frac{6}{100}(1-1/4) = \frac{3}{4}\times\frac{6}{100}$$

$$U = \frac{1}{2}LI^2,$$

by solving we get U = 1 mJ.

15. Match List – I with List – II

List – I	List - II
(a) UV rays	(i) Diagnostic tool in medicine
(b) X-rays	(ii) Water purification
(c) Microwave	(iii) Communication, Radar
(d) Infrared wave	(iv) Improving visibility in foggy days

Choose the correct answer from the options given below :

(A) (a)–(iii), (b)-(ii), (c)-(i), (d)-(iv)

(B) (a)–(ii), (b)-(i), (c)-(iii), (d)-(iv)

(C) (a)–(ii), (b)-(iv), (c)-(iii), (d)-(i)

(D) (a)–(iii), (b)-(i), (c)-(ii), (d)-(iv)

Official Ans. by NTA (B)

Sol. (a) uv rays – used for water purification

(b) x-rays used for diagnosing fracture

(c) Microwaves are used for mobile and radar communication

(d) Infrared waves show less scattering therefore used in foggy days

(a – ii), (b – i), (c – iii), (d – iv)

16. The kinetic energy of emitted electron is E when the light incident on the metal has wavelength λ. To double the kinetic energy, the incident light must have wavelength :

(A) $\frac{hc}{E\lambda - hc}$ (B) $\frac{hc\lambda}{E\lambda + hc}$

(C) $\frac{h\lambda}{E\lambda + hc}$ (D) $\frac{hc\lambda}{E\lambda - hc}$

Official Ans. by NTA (B)

Sol. $E = \frac{hc}{\lambda} - \phi$ – (i)

$2E = \frac{hc}{\lambda'} - \phi$ – (ii)

(ii) – (i)

$$E = hc\left(\frac{1}{\lambda'} - \frac{1}{\lambda}\right)$$

$$\Rightarrow \lambda' = \frac{hc\lambda}{E\lambda + hc}$$

17. Find the ratio of energies of photons produced due to transition of an election of hydrogen atom from its(i) second permitted energy level to the first level, and (ii) the highest permitted energy level to the first permitted level.

(A) 3 : 4 (B) 4 : 3

(C) 1 : 4 (D) 4 : 1

Official Ans. by NTA (A)

Sol. $E_n = \frac{-13.6}{n^2}$ev

$$\Rightarrow \frac{E_2 - E_1}{E_\infty - E_1} = \frac{13.6(1-\frac{1}{4})}{13.6} = \frac{3}{4}$$

18. Find the modulation index of an AM wave having 8 V variation where maximum amplitude of the AM wave is 9 V.

(A) 0.8 (B) 0.5

(C) 0.2 (D) 0.1

Official Ans. by NTA (A)

Sol. Modulation index: $m = \frac{A_m}{A_c}$

Given $2A_m = 8$

$A_m + A_c = 9 \Rightarrow A_c = 5$

$\therefore m = \frac{4}{5} = 0.8$

19. A travelling microscope has 20 divisions per cm on the main scale while its Vernier scale has total 50 divisions and 25 Vernier scale divisions are equal to 24 main scale divisions, what is the least count of the travelling microscope ?

(A) 0.001 cm (B) 0.002 mm

(C) 0.002 cm (D) 0.005 cm

Official Ans. by NTA (C)

Sol. 1 MSD = $\frac{1}{20}$ cm

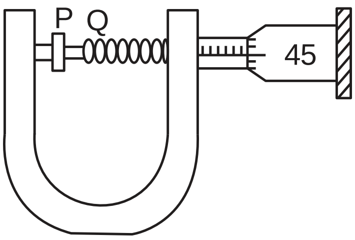
1 VSD = $\frac{24}{25}$ MSD $= \frac{24}{25} \times \frac{1}{20}$ cm

$\therefore \text{Least count} = \frac{1}{20}\left(1 - \frac{24}{25}\right)$ cm

$= \frac{1}{20} \times \frac{1}{25} = \frac{1}{500}$ cm

= 0.002 cm

20. In an experiment to find out the diameter of wire using screw gauge, the following observation were noted :



(a) Screw moves 0.5 mm on main scale in one complete rotation

(b) Total divisions on circular scale = 50

(c) Main scale reading is 2.5 mm

(d) 45^{th} division of circular scale is in the pitch line

(e) Instrument has 0.03 mm negative error

Then the diameter of wire is :

(A) 2.92 mm (B) 2.54 mm

(C) 2.98 mm (D) 3.45 mm

Official Ans. by NTA (C)

Sol. MSR = 2.5 mm

CSR $= 45 \times \frac{0.5}{50}$ mm

= 0.45 mm

Diameter reading = MSR + CSR – zero error

= 2.5 + 0.45 – (–0.03)

= 2.98 mm

SECTION-B

1. An object is projected in the air with initial velocity u at an angle θ. The projectile motion is such that the horizontal range R, is maximum. Another object is projected in the air with a horizontal range half of the range of first object. The initial velocity remains same in both the case. The value of the angle of projection, at which the second object is projected, will be _______degree.

Official Ans. by NTA (15)

Sol. $R_{max} = \frac{u^2 \sin 2(45^\circ)}{g} = \frac{u^2}{g}$

$\frac{R}{2} = \frac{u^2}{2g} = \frac{u^2 \sin 2\theta}{g}$

$\sin 2\theta = \frac{1}{2}$

$2\theta = 30^\circ, 150^\circ$

$\theta = 15^\circ, 75^\circ$

Ans. 15, 75

2. If the acceleration due to gravity experienced by a point mass at a height h above the surface of earth is same as that of the acceleration due to gravity at a depth αh (h << R_e) from the earth surface. The value of α will be _________.

(use R_e = 6400 km)

Official Ans. by NTA (2)

Sol. $g\left(1 - \frac{2h}{R}\right) = g\left(1 - \frac{d}{R}\right)$

$\frac{2h}{R} = \frac{d}{R}$

$\alpha h = d$

$\alpha = 2$

3. The pressure P_1 and density d_1 of diatomic gas $\left(\gamma = \frac{7}{5}\right)$ changes suddenly to $P_2(>P_1)$ and d_2 respectively during an adiabatic process. The temperature of the gas increases and becomes ________ times of its initial temperature.

(given $\frac{d_2}{d_1} = 32$)

Official Ans. by NTA (4)

Sol. $PV^{\gamma} = \text{const}$ $\qquad d = \frac{m}{v}$

$$p\left(\frac{m}{d}\right)^{\gamma} = \text{const}$$

$$\frac{p}{d^{\gamma}} = \text{const} \qquad \frac{d_2}{d_1} = 32$$

$$\frac{p_1}{p_2} = \left(\frac{d_1}{d_2}\right)^{\gamma} = \left(\frac{1}{32}\right)^{7/5} = \frac{1}{128}$$

$$\frac{T_1}{T_2} = \frac{P_1V_1}{P_2V_2} = \frac{1}{128}32 = \frac{1}{4}$$

4. One mole of a monoatomic gas is mixed with three moles of a diatomic gas. The molecular specific heat of mixture at constant volume is $\frac{\alpha^2}{4}R$ J/mol K; then the value of α will be _______. (Assume that the given diatomic gas has no vibrational mode.)

Official Ans. by NTA (3)

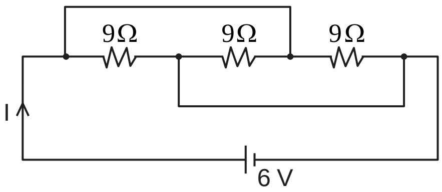
Sol. $C_V / \text{mix} = \frac{n_1Cv_1 + n_2Cv_2}{n_1 + n_2}$

$$= \frac{1.\frac{3R}{2} + 3.\frac{5R}{2}}{1+3}$$

$$= \frac{9R}{4} = \frac{\alpha^2}{4}R$$

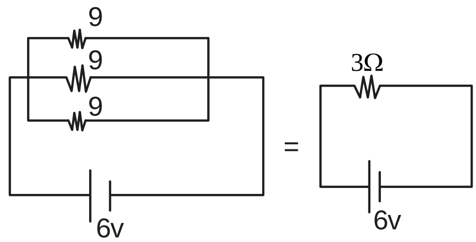
$\alpha = 3$

5. The current I flowing through the given circuit will be __________ A.



Official Ans. by NTA (2)

Sol. Equivalent circuit



$I = 6/3 = 2$ A

6. A closely wounded circular coil of radius 5 cm produces a magnetic field of 37.68×10^{-4} T at its center. The current through the coil is _________ A. [Given, number of turns in the coil is 100 and $\pi = 3.14$]

Official Ans. by NTA (3)

Sol. $B_{centre} = \frac{N\mu_0 I}{2R}$

$$37.68 \times 10^{-4} = \frac{100 \times 4\pi \times 10^{-7} \times I}{2 \times 5 \times 10^{-2}}$$

$I = 3A$

7. Two light beams of intensities 4I and 9I interfere on a screen. The phase difference between these beams on the screen at point A is zero and at point B is π. The difference of resultant intensities, at the point A and B, will be _________ I.

Official Ans. by NTA (24)

Sol. $I_{net} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

I_{max} for $\phi = 0$ & I_{min} for $\phi = \pi$

$$I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2 = \left(\sqrt{9I} + \sqrt{4I}\right)^2 = 25\,I$$

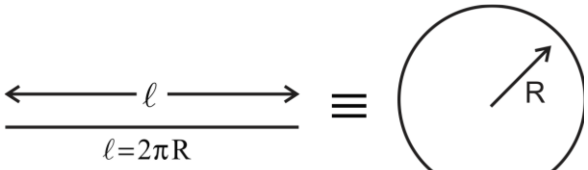
$$I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2 = \left(\sqrt{9I} - \sqrt{4I}\right)^2 = I$$

$$I_{max} - I_{min} = 25\,I - I = 24\,I$$

8. A wire of length 314 cm carrying current of 14 A is bent to form a circle. The magnetic moment of the coil is _______ A-m^2. [Given $\pi = 3.14$]

Official Ans. by NTA (11)

Sol.



$\frac{314}{100} = 2\pi R \qquad R = 0.5$ m

Magnetic Moment = IA

$= 14 \times \pi R^2$

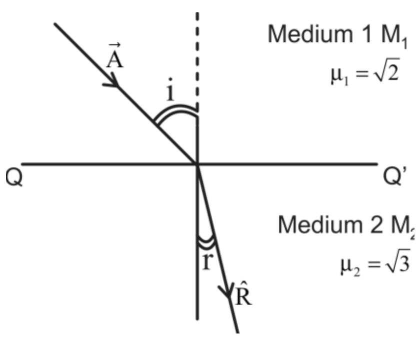
$= 14 \times (3.14) \times \frac{1}{4}$

$= 10.99 \approx 11.00$

9. The X-Y plane be taken as the boundary between two transparent media M_1 and M_2. M_1 in $Z \geq 0$ has a refractive index of $\sqrt{2}$ and M_2 with $Z < 0$ has a refractive index of $\sqrt{3}$. A ray of light travelling in M_1 along the direction given by the vector $\vec{A} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$, is incident on the plane of separation. The value of difference between the angle of incident in M_1 and the angle of refraction in M_2 will be ___________ degree.

Official Ans. by NTA (15)

Sol. $\vec{A} = 4\sqrt{3}\hat{i} - 3\sqrt{3}\hat{j} - 5\hat{k}$



$\mu_1 \sin i = \mu_2 \sin r$

As incident vector A makes i angle with normal z-axis & refracted vector R makes r angle with normal z – axis with help of direction cosine

$$i = \cos^{-1}\left(\frac{A_Z}{A}\right) = \cos^{-1}\left(\frac{5}{\sqrt{(4\sqrt{3})^2 + (3\sqrt{3})^2 + 5^2}}\right)$$

$$= \cos^{-1}\left(\frac{5}{10}\right) \Rightarrow i = 60^\circ$$

$$\sqrt{2}\sin 60 = \sqrt{3} \times \sin r$$

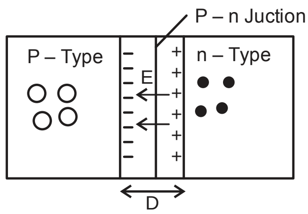
$$r = 45^\circ$$

Difference between $i \,\&\, r = 60 - 45 = 15$

10. If the potential barrier across a p-n junction is 0.6 V. Then the electric field intensity, in the depletion region having the width of 6×10^{-6}m, will be _________ $\times 10^5$ N/C.

Official Ans. by NTA (1)

Sol.



$$E = \frac{V}{d} = \frac{\text{Potential barrier Across Junction}}{\text{width of Depletion layer}}$$

$$= \frac{0.6V}{6\times10^{-6}\,m} = 1\times10^5\,V/m$$

$= 1 \times 10^5$ N/C

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Which of the following pair of molecules contain odd electron molecule and an expanded octet molecule?

(A) BCl_3 and SF_6 (B) NO and H_2SO_4

(C) SF_6 and H_2SO_4 (D) BCl_3 and NO

Official Ans. by NTA (B)

Sol. **(A)** $BCl_3 \rightarrow$ Even Electron molecule

$SF_6 \rightarrow$ Expanded octet molecule

(B) NO $\rightarrow$ Odd Electron molecule

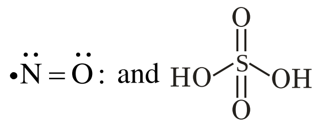
$H_2SO_4 \rightarrow$ Expanded octet.

(C) $SF_6 \rightarrow$ Even Electron molecule

$H_2SO_4 \rightarrow$ Expanded octet.

(D) $BCl_3 \rightarrow$ Even Electron molecule

NO $\rightarrow$ Odd Electron molecule



S $\rightarrow$ 12e$^-$ in outer orbit.

2. $N_{2(g)} + 3H_{2(g)} \rightleftharpoons 2NH_{3(g)}$

20 g 5 g

Consider the above reaction, the limiting reagent of the reaction and number of moles of NH_3 formed respectively are:

(A) H_2, 1.42 moles (B) H_2, 0.71 moles

(C) N_2, 1.42 moles (D) N_2, 0.71 moles

Official Ans. by NTA (C)

Sol.

$$N_2(g) + 3H_2(g) \rightleftharpoons 2NH_3(g)$$

$W_2 =$ 20g 5g.

$n = \frac{20}{28} \quad \frac{5}{2}$

Stoichiometric Amount:

$$N_2 \rightarrow \frac{20/28}{1} = \frac{20}{28} \qquad H_2 \rightarrow \frac{5/2}{3} = \frac{5}{6}$$

$\therefore$ N_2 is the Limiting Reagent.

$$\therefore \quad n(NH_3) = 2 \times n(N_2) = 2 \times \frac{20}{28}$$

$= 1.42$

3. 100 mL of 5% (w/v) solution of NaCl in water was prepared in 250 mL beaker. Albumin from the egg was poured into NaCl solution and stirred well. This resulted in a/ an :

(A) Lyophilic sol (B) Lyophobic sol

(C) Emulsion (D) Precipitate

Official Ans. by NTA (A)

Sol. Standard method for the preparation of lyophilic sol. (Discussed in lab Manual)

4. The first ionization enthalpy of Na, Mg and Si, respectively, are: 496, 737 and 786 kJ mo1^{-1}. The first ionization enthalpy (kJ mol^{-1}) of Al is:

(A) 487 (B) 768

(C) 577 (D) 856

Official Ans. by NTA (C)

Sol. I. E : Na < Al < Mg < Si

$\therefore$ 496 < IE (Al) < 737

Option (C), matches the condition.

i.e $IE(Al) = 577 \text{ kJmol}^{-1}$

5. In metallurgy the term "gangue" is used for:

(A) Contamination of undesired earthy materials.

(B) Contamination of metals, other than desired metal

(C) Minerals which are naturally occuring in pure form

(D) Magnetic impurities in an ore.

Official Ans. by NTA (A)

Sol. Earthy and undesired materials present in the ore, other then the desired metal, is known as gangue.

6. The reaction of zinc with excess of aqueous alkali, evolves hydrogen gas and gives :

(A) $Zn(OH)_2$ (B) ZnO

(C) $[Zn(OH)_4]^{2-}$ (D) $[ZnO_2]^{2-}$

Official Ans. by NTA (D)

Sol. Zinc dissolves in excess of aqueous alkali

$$Zn + 2OH^- + 2H_2O \rightarrow \underset{\text{Tetrahydroxozincate(II) ion}}{[Zn(OH)_4]^{2-}} + H_2 \uparrow$$

However, this reaction in NCERT is given as

$$Zn + 2NaOH \rightarrow Na_2ZnO_2 + H_2 \uparrow$$

ZnO_2^{2-} is anhydrous form of $[Zn(OH)_4]^{2-}$

So in aqueous medium best answer of this question is $[Zn(OH)_4]^{2-}$

7. Lithium nitrate and sodium nitrate, when heated separately, respectively, give :

(A) $LiNO_2$ and $NaNO_2$

(B) Li_2O and Na_2O

(C) Li_2O and $NaNO_2$

(D) $LiNO_2$ and Na_2O

Official Ans. by NTA (C)

Sol. Li_2O, $NaNO_2$

As per NCERT Lithium nitrate when heated gives lithium oxide, Li_2O, whereas other alkali metal nitrates decompose to give the corresponding nitrite.

$$4LiNO_3 \longrightarrow 2Li_2O + 4NO_2 + O_2$$

$$2NaNO_3 \longrightarrow 2NaNO_2 + O_2$$

However, the decomposition product of $NaNO_3$ are temperature dependent process as shown in the below reaction.

$$NaNO_3 \xrightarrow[500^\circ C]{\Delta} NaNO_2\,(s) + \frac{1}{2}O_2\,(g)$$

$$\downarrow \Delta\ 800^\circ C$$

$$Na_2O\,(s) + N_2(g) + O_2\,(g)$$

As temperature is not mentioned, we can go by **Ans. (C)**

8. Number of lone pairs of electrons in the central atom of SCl_2, O_3, ClF_3 and SF_6, respectively, are :

(A) 0, 1, 2 and 2

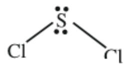
(B) 2, 1, 2 and 0

(C) 1, 2, 2 and 0

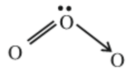
(D) 2, 1, 2 and 0

Official Ans. by NTA (B)

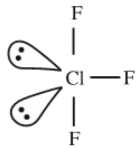
Sol.



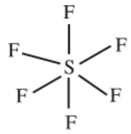
(2 ℓ .p.)



(1 ℓ .p.)



(2 ℓ .p.)



(0 ℓ .p.)

9. In following pairs, the one in which both transition metal ions are colourless is :

(A) Sc^{3+}, Zn^{2+}

(B) Ti^{4+}, Cu^{2+}

(C) V^{2+}, Ti^{3+}

(D) Zn^{2+}, Mn^{2+}

Official Ans. by NTA (A)

Sol. (A) Sc^{3+}, Zn^{2+} (B) Ti^{4+}, Cu^{2+}

$3d^0$ $3d^{10}$ $3d^0$ $3d^9$

(C) V^{2+}, Ti^{3+} (D) Zn^{2+}, Mn^{2+}

$3d^3$ $3d^1$ $3d^{10}$ $3d^5$

No d-d transitions in ions with d^o & d^{10} configuration. Therefore they are colourless.

10. In neutral or faintly alkaline medium, $KMnO_4$ being a powerful oxidant can oxidize, thiosulphate almost quantitatively, to sulphate. In this reaction overall change in oxidation state of manganese will be :

(A) 5 (B) 1 (C) 0 (D) 3

Official Ans. by NTA (D)

Sol. $$8\overset{+7}{Mn}O_4^- + 3S_2O_3^{2-} + H_2O \rightarrow 8\overset{+4}{Mn}O_2 + 6SO_4^{2-} + 2OH^-$$

Change in oxidation state of Mn is from +7 to +4 which is 3.

11. Which among the following pairs has only herbicides ?

(A) Aldrin and Dieldrin

(B) Sodium chlorate and Aldrin

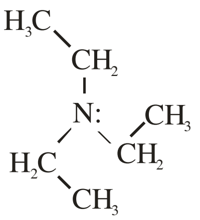
(C) Sodium arsinate and Dieldrin

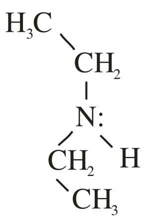
(D) Sodium chlorate and sodium arsinite.

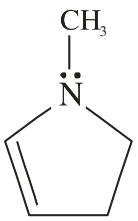
Official Ans. by NTA (D)

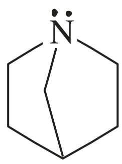
Sol. Both sodium chlorate and sodium arsenite behave as herbicide.

12. Which among the following is the strongest Bronsted base ?

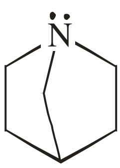
(A) 

(B) 

(C) 

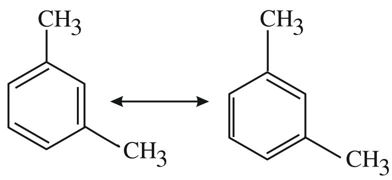
(D) 

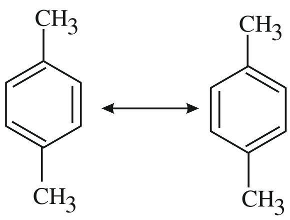
Official Ans. by NTA (D)

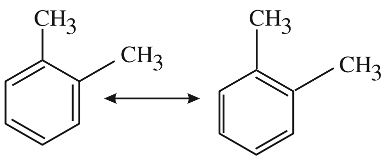
Sol. 

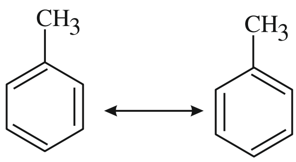
It is most basic because there is no amine inversion.

13. Which among the following pairs of the structures will give different products on ozonolysis? (Consider the double bonds in the structures are rigid and not delocalized.)

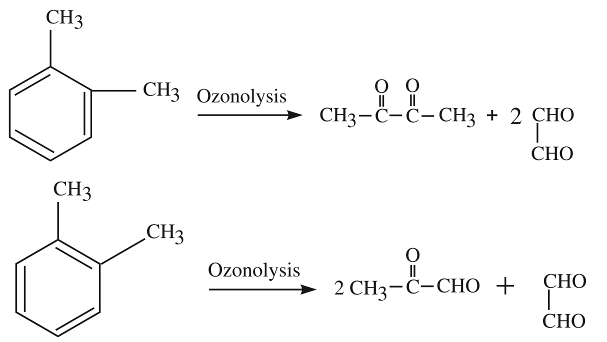
(A) 

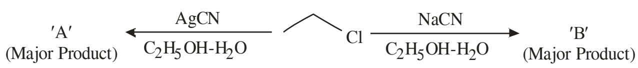
(B) 

(C) 

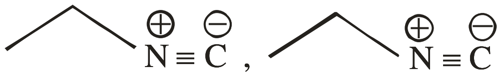
(D) 

Official Ans. by NTA (C)

Sol. 

14. 

Considering the above reactions, the compound 'A' and compound 'B' respectively are :

(A) 

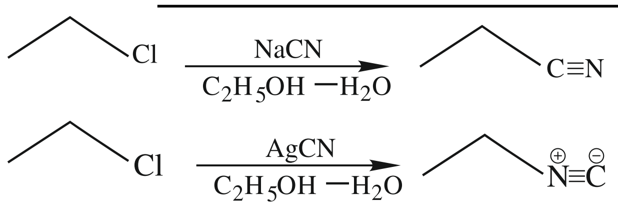
(B) 

(C) 

(D) 

Official Ans. by NTA (C)

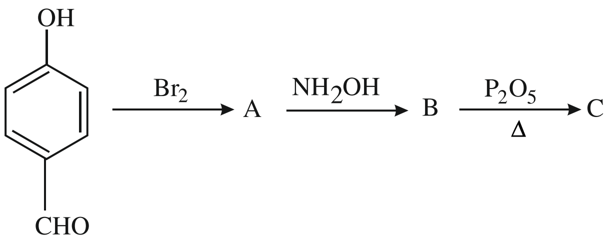
Sol.



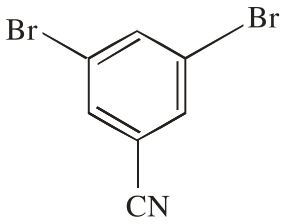
In NaCN; carbon is more nucleophilic atom.

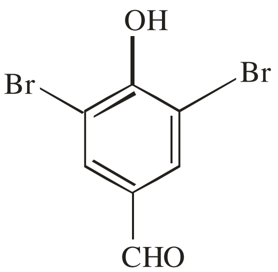
Whereas in AgCN; Ag – C has covalent bond.

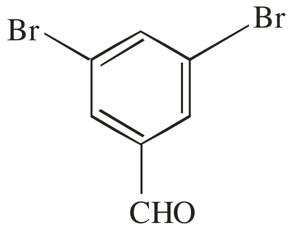
15.

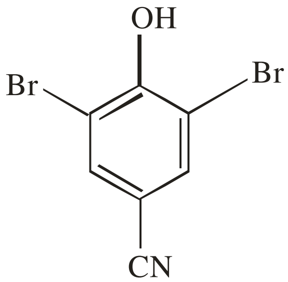


Consider the above reaction sequence, the Product 'C' is :

(A) 

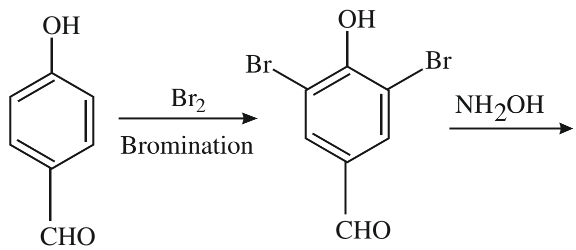
(B) 

(C) 

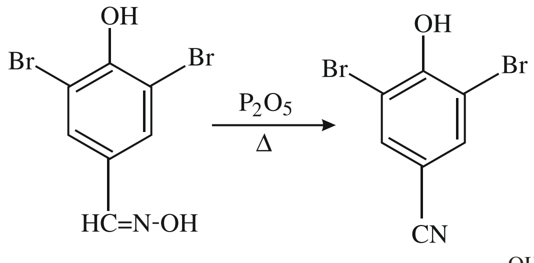
(D) 

Official Ans. by NTA (D)

Sol.

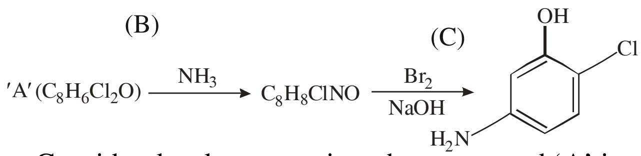


(A)

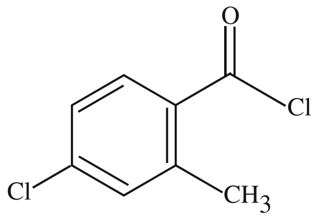


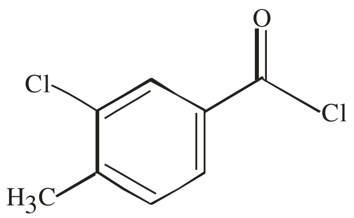
(B) (C)

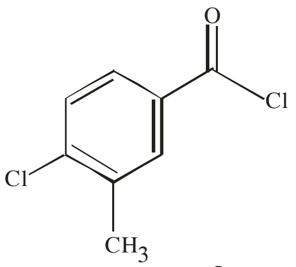
16.

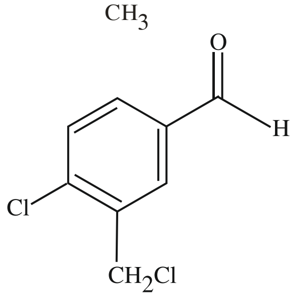


Consider the above reaction, the compound 'A' is :

(A) 

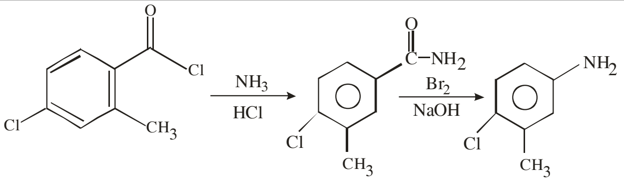
(B) 

(C) 

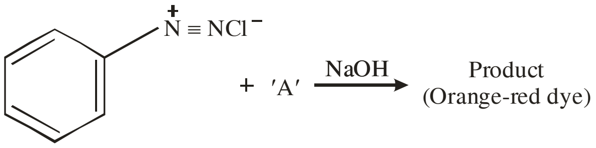
(D) 

Official Ans. by NTA (C)

Sol.

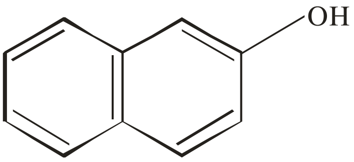


17.

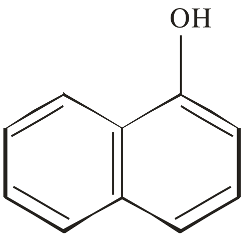


Which among the following represent reagent 'A'?

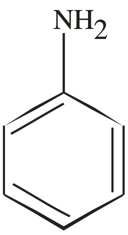
(A)



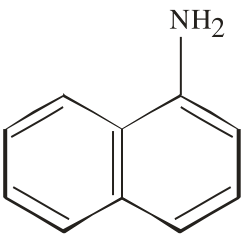
(B)



(C)

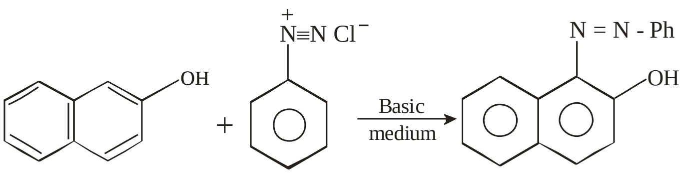


(D)

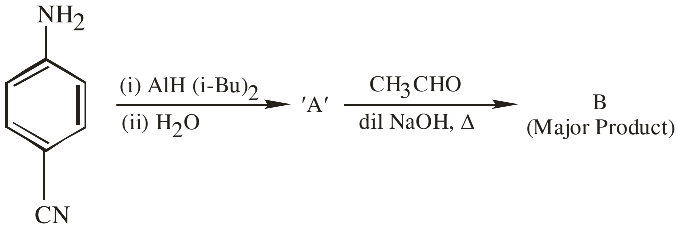


Official Ans. by NTA (A)

Sol.

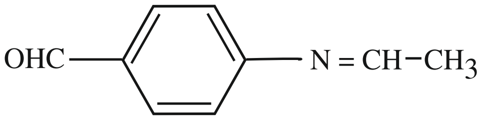


18. Consider the following reaction sequence :

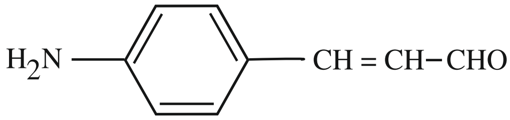


The product 'B' is :

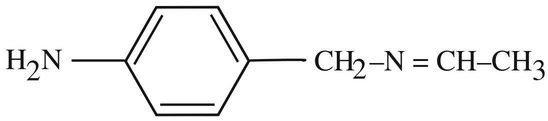
(A)



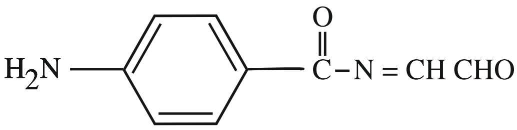
(B)



(C)

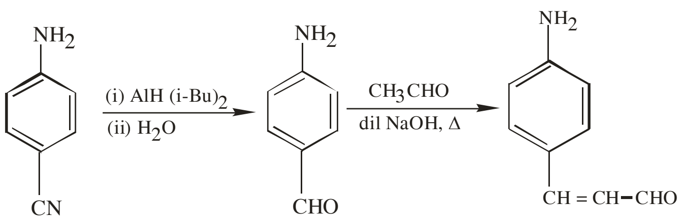


(D)



Official Ans. by NTA (B)

Sol.



Cross aldol condensation

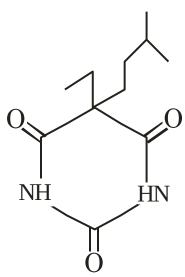
19. Which of the following compounds is an example of hypnotic drug ?

(A) Seldane (B) Amytal

(C) Aspartame (D) Prontosil

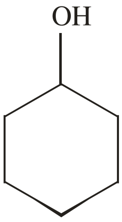
Official Ans. by NTA (B)

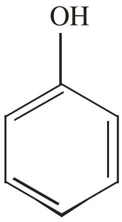
Sol. Amytal is hypnotic drug used to treat sleeping disorder.

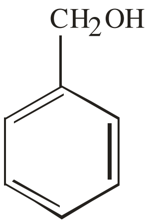


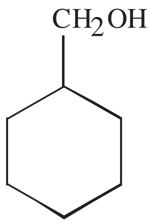
Amytal

20. A compound 'X' is acidic and it is soluble in NaOH solution, but insoluble in $NaHCO_3$ solution. Compound 'X' also gives violet colour with neutral $FeCl_3$ solution. The compound 'X' is :

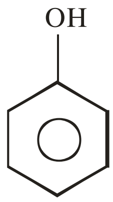
(A) 

(B) 

(C) 

(D) 

Official Ans. by NTA (B)

Sol. 6 

$+ FeCl_3 \longrightarrow [Fe(C_6H_5O)_6]^{3-}$

violet colour

SECTION-B

1. Resistance of a conductivity cell (cell constant 129 m^{-1}) filled with 74.5 ppm solution of KCl is 100 Ω (labelled as solution 1). When the same cell is filled with KCl solution of 149 ppm, the resistance is 50 Ω (labelled as solution 2). The ratio of molar conductivity of solution 1 and solution 2 is i.e. $\frac{\wedge_1}{\wedge_2} = x \times 10^{-3}$. The value of x is _____.

(Nearest integer)

Given, molar mass of KCl is 74.5 g mol^{-1}

Official Ans. by NTA (1000)

Sol. $\frac{\ell}{A} = 129 m^{-1}$

KCl solution 1 :

74.5 ppm, $R_1 = 100\,\Omega$

KCl solution 2 :

149 ppm, $R_2 = 50\,\Omega$

149 ppm, $R_2 = 50\,\Omega$

Here, $\frac{ppm_1}{ppm_2} = \frac{M_1}{M_2}\left(= \frac{W_1/M_0}{V} \times \frac{V}{W_2/M_0}\right)$

$$\frac{\wedge_1}{\wedge_2} = \frac{\kappa_1 \times \frac{1000}{M_1}}{\kappa_2 \times \frac{1000}{M_2}}$$

$$= \frac{\kappa_1}{\kappa_2} \times \frac{M_2}{M_1}$$

$$= \frac{50}{100} \times 2$$

$$= \frac{\wedge_1}{\wedge_2} = 1{,}000 \times 10^{-3}$$

Ans. 1,000

2. Ionic radii of cation A^+ and anion B^- are 102 and 181 pm respectively. These ions are allowed to crystallize into an ionic solid. This crystal has cubic close packing for B^-. A^+ is present in all octahedral voids. The edge length of the unit cell of the crystal AB is _____ pm. (Nearest Integer)

Official Ans. by NTA (512)

Sol. $a = 2(r_+ + r_-)$

$a = 2\,(102 + 181)$

$a = 2(283)$

$a = 566$ pm

3. The minimum uncertainty in the speed of an electron in an one dimensional region of length $2a_O$ (Where a_0 = Bohr radius 52.9 pm) is _____km s^{-1}.

(Given : Mass of electron = 9.1×10^{-31} kg, Planck's constant h = 6.63×10^{-34} Js)

Official Ans. by NTA (548)

Sol. **Heisenberg's uncertainty principle**

$$\Delta x \times \Delta p_x \geq \frac{h}{4\pi}$$

$$\Rightarrow 2a_0 \times m\Delta v_x = \frac{h}{4\pi}(\text{minimum})$$

$$\Rightarrow \Delta v_x = \frac{h}{4\pi} \times \frac{1}{2a_0} \times \frac{1}{m}$$

$$= \frac{6.63 \times 10^{-34}}{4 \times 3.14 \times 2 \times 52.9 \times 10^{-12} \times 9.1 \times 10^{-31}}$$

= 548273 ms^{-1}

= 548.273 km s^{-1}

= $\boxed{548}$ km s^{-1}

4. When 600 mL of 0.2 M HNO_3 is mixed with 400 mL of 0.1M NaOH solution in a flask, the rise in temperature of the flask is _______ $\times 10^{-2}$ ^{0}C.

(Enthalpy of neutralisation = 57 kJ mol^{-1} and Specific heat of water = 4.2 JK^{-1} g^{-1})

(Neglect heat capacity of flask)

Official Ans. by NTA (54)

Sol.

	HNO_3		NaOH
	600 mL × 0.2 M		400 mL × 0.1 M
	= 120 m mol		= 40 m mol

$HNO_3 + NaOH \rightarrow NaNO_3 + H_2O$

Bef.	120	40		
Aft.	80	0		40 m mol

$$\Delta_r H = 40\,\text{m mol} \times (57 \times 10^3)\frac{J}{mol}$$

$$= 40 \times 10^{-3}\,mol \times 57 \times 10^3\,\frac{J}{mol}$$

= 2280 J

$m\,S\Delta T = 2280$

$$\Rightarrow 1000\text{ mL} \times \frac{1gm}{mL} \times 4{,}2 \times \Delta T = 2280$$

$$\Delta T = \frac{2280}{4.2} \times 10^{-3}$$

$$= \frac{22800}{42} \times 10^{-3}$$

= 542.86×10^{-3}

$\Delta T = 54.286 \times 10^{-2}\,K$

$\Delta T = 54.286 \times 10^{-2}\,{}^0C$

Ans. $\boxed{54.286}$

Answer mentioned as 54 (Closest integer)

5. If O_2 gas is bubbled through water at 303 K, the number of millimoles of O_2 gas that dissolve in 1 litre of water is_______. (Nearest Integer)

(Given : Henry's Law constant for O_2 at 303 K is 46.82 k bar and partial pressure of O_2 = 0.920 bar)

(Assume solubility of O_2 in water is too small, nearly negligible)

Official Ans. by NTA (1)

Sol. $p = K_H \times x$

$$\underset{bar}{0.920} = 46.82 \times 10^3\text{ bar} \times \frac{\text{mol of }O_2}{\text{mol of }H_2O}$$

$$0.920 = 46.82 \times \cancel{10^3} \times \frac{\text{mol of }O_2}{\cancel{1000}/18}$$

$0.920 = 46.82 \times n_{o_2}$

$$p = \frac{0.920}{46.82 \times 18} = n_{O_2}$$

$\Rightarrow 1.09 \times 10^{-3} = n_{O_2}$

$\Rightarrow$ m mol of $O_2 = 1$

6. If the solubility product of PbS is 8×10^{-28}, then the solubility of PbS in pure water at 298 K is $x \times 10^{-16}$ mol L^{-1}. The value of x is ________. (Nearest Integer)

[Given $\sqrt{2} = 1.41$]

Official Ans. by NTA (282)

Sol. $K_{sp} = S^2$

$$S = \sqrt{K_{sp}} = \sqrt{8\times10^{-28}} = 2\sqrt{2}\times10^{-14}$$

$$= 2.82 \times 10^{-14}$$

$$= 282 \times 10^{-16}$$

Ans. = 282

7. The reaction between X and Y is first order with respect to X and zero order with respect to Y.

Experiment	$\frac{[X]}{\text{mol L}^{-1}}$	$\frac{[Y]}{\text{mol L}^{-1}}$	$\frac{\text{Initial rate}}{\text{mol L}^{-1}\text{ min}^{-1}}$
I.	0.1	0.1	2×10^{-3}
II.	L	0.2	4×10^{-3}
III.	0.4	0.4	$M\times10^{-3}$
IV.	0.1	0.2	2×10^{-3}

Examine the data of table and calculate ratio of numerical values of M and L. (Nearest Inetger)

Official Ans. by NTA (40)

Sol. r = k [x] $[y]^0$ = k [x]

Using I & II

$$\frac{4\times10^{-3}}{2\times10^{-3}} = \left(\frac{L}{0.1}\right) \Rightarrow L = 0.2$$

Using I & III

$$\frac{M\times10^{-3}}{2\times10^{-3}} = \frac{0.4}{0.1} \Rightarrow M = 8$$

$$\frac{M}{L} = \frac{8}{0.2} = 40$$

Ans. 40

8. In a linear tetrapeptide (Constituted with different amino acids), (number of amino acids) - (number of peptide bonds) is______.

Official Ans. by NTA (1)

Sol. In Tetrapeptide,

No. of Amino Acids = 4

No. of Peptide bonds = 3

Hence

Ans. = 1

9. In bromination of Propyne, with Bromine 1, 1, 2, 2-tetrabromopropane is obtained in 27% yield. The amount of 1, 1, 2, 2 tetrabromopropane obtained from 1 g of Bromine in this reaction is ______ $\times$ 10^{-1} g. (Nearest integer)

(Molar Mass : Bromine = 80 g/mol)

Official Ans. by NTA (3)

Sol. $CH_3-C\equiv CH + 2Br_2 \rightarrow CH_3-CBr_2-CHBr_2$

$$= \frac{1}{160}\times\frac{1}{2}\times360\times0.27$$

$$= 0.30375$$

$$= 3.0375 \times 10^{-1}$$

Ans. = 3

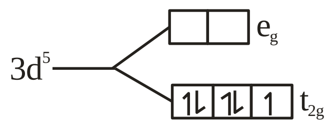
10. $[Fe(CN)_6]^{3-}$ should be an inner orbital complex. Ignoring the pairing energy, the value of crystal field stabilization energy for this complex is (–) _________ Δ_o. (Nearest integer)

Official Ans. by NTA (2)

Sol. $[Fe(CN)_6]^{3-}$

CN^- is strong field ligand

Fe^{+3} $3d^5$ (t_{2g}^5 e_g^0)



CFSE = 5 (–0.4 Δ_0) = –2.0 Δ_0

Ans. (2)

FINAL JEE–MAIN EXAMINATION – JULY, 2022

(Held On Friday 29th July, 2022) **TIME : 9 : 00 AM to 12 : 00 NOON**

MATHEMATICS | TEST PAPER WITH SOLUTION

SECTION-A

1. Let R be a relation from the set {1, 2, 3………,60} to itself such that R = {(a, b) : b = pq, where p, q ≥ 3 are prime numbers}. Then, the number of elements in R is :

(A) 600 (B) 660

(C) 540 (D) 720

Official Ans. by NTA (B)

Sol. Number of possible values of a = 60, for b = pq,

If p = 3, q = 3, 5, 7, 11, 13, 17, 19

If p = 5 q = 5, 7, 11

If p = 7 q = 7

Total cases = 60 × 11 = 660

2. If z = 2 + 3i, then $z^5 + (\overline{z})^5$ is equal to :

(A) 244 (B) 224

(C) 245 (D) 265

Official Ans. by NTA (A)

Sol. $z^5 + (\overline{z})^5 = (2+3i)^5 + (2-3i)^5$

$= 2({}^5C_0 2^5 + {}^5C_2 2^3 (3i)^2 + {}^5C_4 2^1 (3i)^4)$

= 2 (32 + 10 × 8 (– 9) + 5 ×2 × 81) = 244

3. Let A and B be two 3 × 3 non-zero real matrices such that AB is a zero matrix. Then

(A) The system of linear equations AX = 0 has a unique solution

(B) The system of linear equations AX = 0 has infinitely many solutions

(C) B is an invertible matrix

(D) adj (A) is an invertible matrix

Official Ans. by NTA (B)

Sol. AB = 0 ⇒ |AB| = 0

|A| |B| = 0

|A| = 0 |B| = 0

If |A| ≠ 0, B = 0 (not possible)

If |B| ≠ 0, A = 0 (not possible)

Hence |A| = |B| = 0

⇒ AX = 0 has infinitely many solutions

4. If $\frac{1}{(20-a)(40-a)} + \frac{1}{(40-a)(60-a)} + \ldots\ldots + \frac{1}{(180-a)(200-a)} = \frac{1}{256}$, then the maximum value of a is :

(A) 198 (B) 202

(C) 212 (D) 218

Official Ans. by NTA (C)

Sol. By splitting

$$\frac{1}{20}\left[\left(\frac{1}{20-a}-\frac{1}{40-a}\right)+\left(\frac{1}{40-a}-\frac{1}{60-a}\right) +\ldots+\left(\frac{1}{180-a}-\frac{1}{200-a}\right)\right]$$

$$\Rightarrow \frac{1}{20}\left(\frac{1}{20-a}-\frac{1}{200-a}\right)=\frac{1}{256}$$

(20 – a) (200 – a) = 256 × 9

$a^2 - 220a + 1696 = 0$

a = 8, 212

Hence maximum value of a is 212.

5. If $\lim\limits_{x\to 0}\frac{\alpha e^x + \beta e^{-x} + \gamma \sin x}{x\sin^2 x} = \frac{2}{3}$,

where α, β,γ ∈ R, then which of the following is NOT correct ?

(A) $\alpha^2 + \beta^2 + \gamma^2 = 6$

(B) $\alpha\beta + \beta\gamma + \gamma\alpha + 1 = 0$

(C) $\alpha\beta^2 + \beta\gamma^2 + \gamma\alpha^2 + 3 = 0$

(D) $\alpha^2 - \beta^2 + \gamma^2 = 4$

Official Ans. by NTA (C)

Sol.

$$\lim_{x\to 0}\frac{\alpha\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+...\right)+\beta\left(1-x+\frac{x^2}{2!}-\frac{x^3}{3!}+...\right)+\gamma\left(x-\frac{x^3}{3!}+...\right)}{x^3}$$

constant terms should be zero

$\Rightarrow a + \beta = 0$

coeff of x should be zero

$\Rightarrow \alpha - \beta + \gamma = 0$

coeff of x^2 should be zero

$$\lim_{x\to 0}\frac{x^3\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)+x^4\left(\frac{\alpha}{3!}-\frac{\beta}{3!}-\frac{\gamma}{3!}\right)}{x^3}=\frac{}{3}$$

$\Rightarrow \frac{\alpha}{2}+\frac{\beta}{2}=0$

$\frac{\alpha}{6}-\frac{\beta}{6}-\frac{\gamma}{6}=2/3$

$\Rightarrow \alpha = 1, \beta = -1, \gamma = -2$

6. The integral $\int_0^{\frac{\pi}{2}}\frac{1}{3+2\sin x+\cos x}dx$ is equal to:

(A) $\tan^{-1}(2)$ (B) $\tan^{-1}(2)-\frac{\pi}{4}$

(C) $\frac{1}{2}\tan^{-1}(2)-\frac{\pi}{8}$ (D) $\frac{1}{2}$

Official Ans. by NTA (B)

Sol.

$$I=\int_0^{\frac{\pi}{2}}\frac{dx}{3+2\sin x+\cos x}=\int_0^{\frac{\pi}{2}}\frac{\sec^2\frac{x}{2}.dx}{2\tan^2\frac{x}{2}+4\tan\frac{x}{2}+4}$$

Put $\tan\frac{x}{2}=t$, so

$$I=\int_0^1\frac{dt}{(t+1)^2+1}=\tan^{-1}(x+1)\Big|_0^1=\tan^{-1}2-\frac{\pi}{4}$$

7. Let the solution curve y = y (x) of the differential equation $(1 + e^{2x})\left(\frac{dy}{dx}+y\right) = 1$ pass through the point $\left(0,\frac{\pi}{2}\right)$. Then, $\lim_{x\to\infty} e^x y(x)$ is equal to :

(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$

(C) $\frac{\pi}{2}$ (D) $\frac{3\pi}{2}$

Official Ans. by NTA (B)

Sol. $\frac{dy}{dx}+y=\frac{1}{1+e^{2x}}$

So integrating factor is $e^{\int 1.dx}=e^x$

So solution is $y\cdot e^x=\tan^{-1}\left(e^x\right)+c$

Now as curve is passing through $\left(0,\frac{\pi}{2}\right)$ so

$\Rightarrow c=\frac{\pi}{4}$

$$\Rightarrow \lim_{x\to\infty}\left(y\cdot e^x\right)=\lim_{x\to\infty}\left(\tan^{-1}\left(e^x\right)+\frac{\pi}{4}\right)=\frac{3\pi}{4}$$

8. Let a line L pass through the point of intersection of the lines bx + 10y – 8 = 0 and 2x – 3y = 0, $b \in R-\left\{\frac{4}{3}\right\}$. If the line L also passes through the point (1, 1) and touches the circle $17(x^2 + y^2) = 16$, then the eccentricity of the ellipse $\frac{x^2}{5}+\frac{y^2}{b^2}=1$ is :

(A) $\frac{2}{\sqrt{5}}$ (B) $\sqrt{\frac{3}{5}}$

(C) $\frac{1}{\sqrt{5}}$ (D) $\sqrt{\frac{2}{5}}$

Official Ans. by NTA (B)

Sol. Line is passing through intersection of $bx+10y-8=0$ and $2x-3y=0$ is $(bx+10y-8)+\lambda(2x-3y)=0$. As line is passing through $(1,1)$ so $\lambda=b+2$

Now line $(3b+4)x-(3b-4)y-8=0$ is tangent to circle $17(x^2+y^2)=16$

So $\dfrac{8}{\sqrt{(3b+4)^2+(3b-4)^2}}=\dfrac{4}{\sqrt{17}}$

$\Rightarrow b^2=2 \Rightarrow e=\sqrt{\dfrac{3}{5}}$

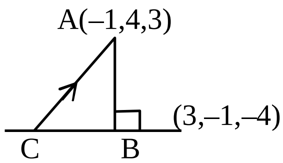
9. If the foot of the perpendicular from the point A(– 1, 4, 3) on the plane P : 2x + my + nz = 4, is $\left(-2,\frac{7}{2},\frac{3}{2}\right)$, then the distance of the point A from the plane P, measured parallel to a line with direction ratios 3, – 1, – 4, is equal to :

(A) 1 (B) $\sqrt{26}$

(C) $2\sqrt{2}$ (D) $\sqrt{14}$

Official Ans. by NTA (B)

Sol.



Let B be foot of $\perp$ coordinates of $B=\left(-2,\frac{7}{2},\frac{3}{2}\right)$

Direction ratio of line AB is $<2,1,3>$ so $m=1,\ n=3$

So equation of AC is $\dfrac{x+1}{3}=\dfrac{y-4}{-1}=\dfrac{z-3}{-4}=\lambda$

So point C is $(3\lambda-1,-\lambda+4,-4\lambda+3)$. But C lies on the plane, so

$6\lambda-2-\lambda+4-12\lambda+9=4$

$\Rightarrow \lambda=1 \Rightarrow C(2,3,-1)$

$\Rightarrow AC=\sqrt{26}$

10. Let $\vec{a}=3\hat{i}+\hat{j}$ and $\vec{b}=\hat{i}+2\hat{j}+\hat{k}$. Let $\vec{c}$ be a vector satisfying $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$. If $\vec{b}$ *and* $\vec{c}$ are non-parallel, then the value of λ is :

(A) – 5 (B) 5

(C) 1 (D) – 1

Official Ans. by NTA (A)

Sol. $a=3\hat{i}+\hat{j},\ \vec{b}=\hat{i}+2\hat{j}+\hat{k}$

As $\vec{a}\times(\vec{b}\times\vec{c})=\vec{b}+\lambda\vec{c}$

$\Rightarrow \vec{a}.\vec{c}(\vec{b})-(\vec{a}.\vec{b})\vec{c}=\vec{b}+\lambda\vec{c}$

$\Rightarrow \vec{a}\cdot\vec{c}=1, \vec{a}\cdot\vec{b}=-\lambda$

$\Rightarrow (3\hat{i}+\hat{j}).(\hat{i}+2\hat{j}+\hat{k})=-\lambda$

$\Rightarrow \lambda=-5$

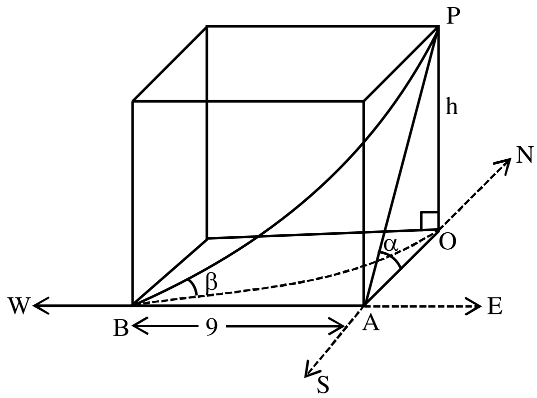
11. The angle of elevation of the top of a tower from a point A due north of it is α and from a point B at a distance of 9 units due west of A is $\cos^{-1}\left(\frac{3}{\sqrt{13}}\right)$. If the distance of the point B from the tower is 15 units, then cot α is equal to :

(A) $\frac{6}{5}$ (B) $\frac{9}{5}$

(C) $\frac{4}{3}$ (D) $\frac{7}{3}$

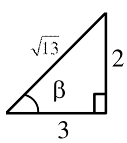
Official Ans. by NTA (A)

Sol.

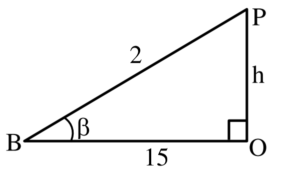


given OB = 15

$\cos\beta=\dfrac{3}{\sqrt{13}}$



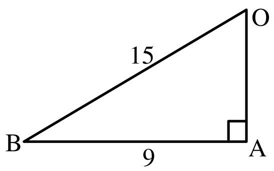
$\tan\beta=\dfrac{2}{3}$



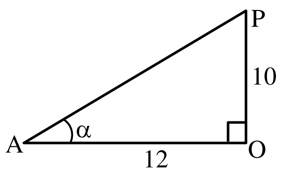
$$\tan\beta = \frac{h}{15}$$

$$\frac{2}{3} = \frac{h}{15}$$

$$\boxed{10 = h}$$



$OA^2 + AB^2 = 225$
$OA^2 + 81 = 225$
$\boxed{OA = 12}$



$$\tan\alpha = \frac{10}{12}$$

$$\cot\alpha = \frac{12}{10} = \frac{6}{5}$$

12. The statement $(p \wedge q) \Rightarrow (p \wedge r)$ is equivalent to :

(A) $q \Rightarrow (p \wedge r)$ (B) $p \Rightarrow (p \wedge r)$

(C) $(p \wedge r) \Rightarrow (p \wedge q)$ (D) $(p \wedge q) \Rightarrow r$

Official Ans. by NTA (D)

Sol. $(p \wedge q) \Rightarrow (p \wedge r)$

$\sim(p \wedge q) \vee (p \wedge r)$

$(\sim p \vee \sim q) \vee (p \wedge r)$

$(\sim p \vee (p \wedge r)) \vee \sim q$

$(\sim p \vee p) \wedge (\sim p \vee r) \vee \sim q$

$(\sim p \vee r) \vee \sim q$

$(\sim p \vee \sim q) \vee r$

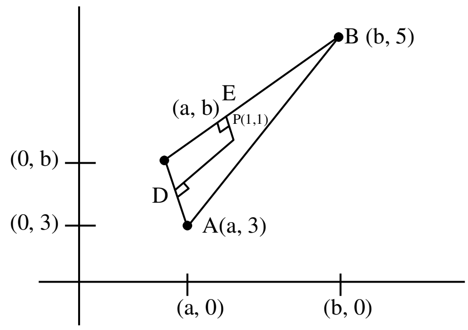
$\sim(p \wedge q) \vee r$

$(p \wedge q) \Rightarrow r$

13. Let the circumcentre of a triangle with vertices A(a, 3), B(b, 5) and C(a, b), ab > 0 be P(1, 1). If the line AP intersects the line BC at the point Q(k_1, k_2), then $k_1 + k_2$ is equal to :

(A) 2 (B) $\frac{4}{7}$ (C) $\frac{2}{7}$ (D) 4

Official Ans. by NTA (B)



$m_{AC} \longrightarrow \infty$

$m_{PD} = 0$

$$D\left(\frac{a+a}{2}, \frac{b+3}{2}\right)$$

$$D\left(a, \frac{b+3}{2}\right)$$

$m_{PD} = 0$

$$\frac{b+3}{2} - 1 = 0$$

$b + 3 - 2 = 0$

$\boxed{b = -1}$

$$E\left(\frac{b+a}{2}, \frac{5+b}{2}\right) = \left(\frac{af}{2}, 2\right)$$

$m_{CB} \cdot m_{EP} = -1$

$$\left(\frac{5-b}{b-a}\right) = \left(\frac{2-1}{\frac{a-1}{2} - 1}\right) = -1$$

$$\left(\frac{6}{-1-a}\right) = \left(\frac{2}{a-3}\right) = -1$$

$12 = (1 + a)(a - 3)$

$12 = a^2 - 3a + a - 3$

$\Rightarrow a^2 - 2a - 15 = 0$

$(a - 5)(a + 3) = 0$

a = 5 or a = –3

Given ab > 0

a(–1) > 0

–a > 0

a < 0

$\boxed{a=-3}$ Accept

AP line A (–3, 3) P(1, 1)

$$y-1=\left(\frac{3-1}{-3-1}\right)(x-1)$$

–2y + 2 = x – 1

$\Rightarrow$ $\boxed{x+2y=3}$ Appling .....(1)

Line BC B(–1, 5)

C(–3, –1)

$$(y-5)=\frac{6}{2}(x+1)$$

y –5 = 3x + 3

$\boxed{y=3x+8}$ .....(2)

Solving (1) & (2)

x + 2 (3x + 8) = 3

$\Rightarrow$ 7x + 16 = 3

7x = –13

$$x=-\frac{13}{7}$$

$$y=3\left(-\frac{13}{7}\right)+8$$

$$=\frac{-39+56}{7}$$

$$y=\frac{17}{7}$$

$$x+y=\frac{-13+17}{7}=\frac{4}{7}$$

14. Let $\hat{a}$ and $\hat{b}$ be two unit vectors such that the angle between them is $\frac{\pi}{4}$. If θ is the angle between the vectors $\left(\hat{a}+\hat{b}\right)$ and $\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)$, then the value of 164 $\cos^2\theta$ is equal to :

(A) $90+27\sqrt{2}$ (B) $45+18\sqrt{2}$

(C) $90+3\sqrt{2}$ (D) $54+90\sqrt{2}$

Official Ans. by NTA (A)

Sol. $\hat{a}\wedge\hat{b}=\frac{\pi}{4}=\phi$

$$\hat{a}.\hat{b}=|\hat{a}||\hat{b}|\cos\phi$$

$$\hat{a}.\hat{b}=\cos\phi=\frac{1}{\sqrt{2}}$$

$$\cos\theta=\frac{\left(\hat{a}+\hat{b}\right).\left(\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right)}{\left|\hat{a}+\hat{b}\right|\left|\hat{a}+2\hat{b}+2\left(\hat{a}\times\hat{b}\right)\right|}$$

$$\left|\hat{a}+\hat{b}\right|^2=\left(\hat{a}+\hat{b}\right).\left(\hat{a}+\hat{b}\right)$$

$$\left|\hat{a}+\hat{b}\right|^2=2+2\hat{a}.\hat{b}$$

$$=2+\sqrt{2}$$

$$\hat{a}\times\hat{b}=|\hat{a}||\hat{b}|\sin\phi\,\hat{n}$$

$\hat{a}\times\hat{b}=\frac{\hat{n}}{\sqrt{2}}$ when $\hat{n}$ is vector $\perp$ $\hat{a}$ and $\hat{b}$

let $\vec{c}=\hat{a}\times\hat{b}$

We know.

$\vec{c}.\vec{a}=0$

$\vec{c}.\vec{b}=0$

$$\left|\hat{a}+2\hat{b}+2\vec{c}\right|^2$$

$$=1+4+\frac{(4)}{2}+4\,\hat{a}.\hat{b}+8\hat{b}.\vec{c}+4\vec{c}.\hat{a}$$

$$=7+\frac{4}{\sqrt{2}}=7+2\sqrt{2}$$

Now

$$\left(\hat{a}+\hat{b}\right).\left(\hat{a}+2\hat{b}+2\vec{c}\right)$$

$$=|\hat{a}|^2+2\hat{a}.\hat{b}+0+\hat{b}.\hat{a}+2|\hat{b}|^2+0$$

$$=1+\frac{2}{\sqrt{2}}+\frac{1}{\sqrt{2}}+2$$

$$=3+\frac{3}{\sqrt{2}}$$

$$\cos\theta=\frac{3+\frac{3}{\sqrt{2}}}{\sqrt{2+\sqrt{2}}\sqrt{7+2\sqrt{2}}}$$

$$\cos^2\theta=\frac{9\left(\sqrt{2}+1\right)^2}{2\left(2+\sqrt{2}\right)\left(7+2\sqrt{2}\right)}$$

$$\cos^2\theta = \left(\frac{9}{2\sqrt{2}}\right)\frac{\left(\sqrt{2}+1\right)}{\left(7+2\sqrt{2}\right)}$$

$$164\cos^2\theta = \frac{(82)(9)}{\sqrt{2}}\frac{\left(\sqrt{2}+1\right)\left(7-2\sqrt{2}\right)}{\left(7+2\sqrt{2}\right)\left(7-2\sqrt{2}\right)}$$

$$= \frac{(82)}{\sqrt{2}}\ \frac{(9)\left[7\sqrt{2}-4+7-2\sqrt{2}\right]}{(41)}$$

$$= \left(9\sqrt{2}\right)\left[5\sqrt{2}+3\right]$$

$$= 90 + 27\sqrt{2}$$

15. If $f(\alpha) = \int_1^{\alpha}\frac{\log_{10}t}{1+t}dt, \alpha > 0$, then $f(e^3) + f(e^{-3})$ is equal to :

(A) 9 (B) $\frac{9}{2}$

(C) $\frac{9}{\log_e(10)}$ (D) $\frac{9}{2\log_e(10)}$

Official Ans. by NTA (D)

Sol. $f(e^3) = \int_1^{e^3}\frac{\ell n\, t}{\ell n 10(1+t)}dt$......(1)

$$f(\alpha) = \int_1^{\alpha}\frac{\ell n\, t}{(\ell n 10)(1+t)}dt$$

$$t = \frac{1}{x} \Rightarrow x = \frac{1}{t}$$

$$dt = \frac{-1}{x^2}dx$$

$$= \int_1^{\frac{1}{\alpha}}\frac{-\ell n\, x}{(\ell n 10)\left(1+\frac{1}{x}\right)}\left(-\frac{1}{x^2}\right)dx$$

$$f(\alpha) = \frac{1}{\ell n 10}\int_1^{\frac{1}{\alpha}}\frac{\ell n x}{x(x+1)}dx$$

$f(e^{-3}) = \frac{1}{\ell n 10}\int_1^{e^3}\frac{\ell n t}{t(t+1)}dt$........(2)

Add (1) & (2)

$f(e^3) + f(e^{-3})$

$$= \left(\frac{1}{\ell n 10}\right)\int_1^{e^3}\frac{\ell n t}{(1+t)}\left[1+\frac{1}{t}\right]dt$$

$$= \left(\frac{1}{\ell n 10}\right)\int_1^{3}\frac{\ell n t}{t}dt$$

$\ell n t = r$

$$\frac{dt}{t} = dr$$

$$= \frac{1}{\ell n 10}\int_0^3 r\,dr$$

$$= \left(\frac{1}{\ell n 10}\right)\left(\frac{r^2}{2}\right)\Bigg|_0^3$$

$$= \left(\frac{1}{\log 10}\right)\left(\frac{9}{2}\right)$$

$$= \frac{9}{2\log_e 10}$$

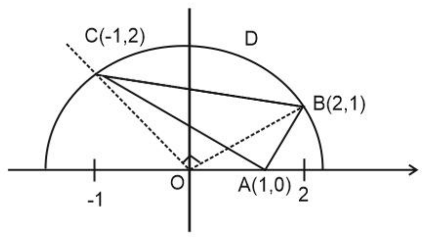
16. The area of the region $\left\{(x,y): |x-1| \le y \le \sqrt{5-x^2}\right\}$ is equal to :

(A) $\frac{5}{2}\sin^{-1}\left(\frac{3}{5}\right)-\frac{1}{2}$ (B) $\frac{5\pi}{4}-\frac{3}{2}$

(C) $\frac{3\pi}{4}+\frac{3}{2}$ (D) $\frac{5\pi}{4}-\frac{1}{2}$

Official Ans. by NTA (D)

Sol.



$$|x-1| < y < \sqrt{5-x^2}$$

When $|x-1| = \sqrt{5-x^2}$

$$\Rightarrow (x-1)^2 = 5-x^2$$

$$\Rightarrow x^2 - x - 2 = 0$$

$$\Rightarrow x = 2, -1$$

Required Area = Area of ΔABC + Area of region BCD

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 2 & 1 & 1 \\ -1 & 2 & 1 \end{vmatrix} + \frac{\pi}{4}\left(\sqrt{5}\right)^2 - \frac{1}{2}\left(\sqrt{5}\right)^2$$

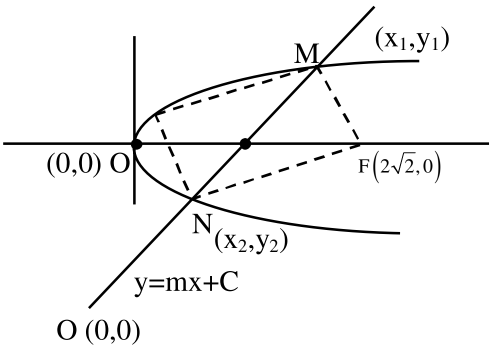
$$= \frac{5\pi}{4} - \frac{1}{2}$$

17. Let the focal chord of the parabola P : $y^2 = 4x$ along the line L : $y = mx + c$, $m > 0$ meet the parabola at the points M and N. Let the line L be a tangent to the hyperbola H : $x^2 - y^2 = 4$. If O is the vertex of P and F is the focus of H on the positive x-axis, then the area of the quadrilateral OMFN is :

(A) $2\sqrt{6}$ (B) $2\sqrt{14}$

(C) $4\sqrt{6}$ (D) $4\sqrt{14}$

Official Ans. by NTA (B)



Sol.

$$H : \frac{x^2}{4} - \frac{y^2}{4} = 1$$

Focus (ae, 0)

$$F\left(2\sqrt{2},0\right)$$

Line L : y = mx + c pass (1,0)

$$\boxed{o = m + C} \quad ....(1)$$

Line L is tangent to Hyperbola. $\frac{x^2}{4} - \frac{y^2}{4} = 1$

$$C = \pm\sqrt{a^2m^2 - \ell^2}$$

$$C = \pm\sqrt{4m^2 - 4}$$

From (1)

$$-m = \pm\sqrt{4m^2 - 4}$$

Squaring

$m^2 = 4m^2 - 4$

$4 = 3m^2$

$$\boxed{\frac{2}{\sqrt{3}} = m} \quad (\text{as } m > 0)$$

$C = -m$

$$C = \frac{-2}{\sqrt{3}}$$

$$y = \frac{2x}{\sqrt{3}} - \frac{2}{\sqrt{3}}$$

$y^2 = 4x$

$$\Rightarrow \left(\frac{2x-2}{\sqrt{3}}\right)^2 = 4x$$

$\Rightarrow x^2 + 1 - 2x = 3x$

$$\Rightarrow \boxed{x^2 - 5x + 1 = 0}$$

$$y^2 = 4\left(\frac{\sqrt{3}y + 2}{2}\right)$$

$$y^2 = 2\sqrt{3}\,y + 4$$

$$\Rightarrow \boxed{y^2 - 2\sqrt{3}y - 4 = 0}$$

Area

$$\left|\frac{1}{2}\begin{vmatrix} 0 & x_1 & 2\sqrt{2} & x_2 & 0 \\ 0 & y_1 & 0 & y_2 & 0 \end{vmatrix}\right|$$

$$= \left|\frac{1}{2}\left[-2\sqrt{2}y_1 + 2\sqrt{2}y_2\right]\right|$$

$$= \sqrt{2}\left|y_2 - y_1\right| = \frac{\left(\sqrt{2}\right)\sqrt{12+16}}{111}$$

$$= \sqrt{56}$$

$$= 2\sqrt{14}$$

18. The number of points, where the function f : **R** → **R**, f (x) = |x – 1| cos |x – 2| sin |x – 1| + (x – 3) $|x^2 - 5x + 4|$, is **NOT** differentiable, is :

(A) 1 (B) 2 (C) 3 (D) 4

Official Ans. by NTA (B)

Sol. f(x) = |x – 1| cos |x – 2| sin |x – 1| + (x – 3)| $x^2 - 5x$ + 4|

= |x – 1| cos |x – 2| sin |x – 1| + (x – 3)| x – 1||x – 4|

= |x – 1| [cos |x – 2| sin |x – 1| + (x – 3) |x – 4|]

Non differentiable at x = 1 and x = 4.

19. Let S = {1, 2, 3, …, 2022}. Then the probability, that a randomly chosen number n from the set S such that HCF (n, 2022) = 1, is :

(A) $\frac{128}{1011}$ (B) $\frac{166}{1011}$

(C) $\frac{127}{337}$ (D) $\frac{112}{337}$

Official Ans. by NTA (D)

Sol. Total number of elements = 2022

2022 = 2×3×337

HCF (n, 2022) = 1

is feasible when the value of 'n' and 2022 has no common factor.

A = Number which are divisible by 2 from {1,2,3.....2022}

n(A) = 1011

B = Number which are divisible by 3 by 3 from {1,2,3......2022}

n(B) = 674

$A \cap B =$ Number which are divisible by 6 from {1,2,3........2022}

6,12,18........., 2022

$\boxed{337 = n(A \cap B)}$

$n(A \cup B) = n(A) + n(B) - n(A \cap B)$

= 1011 + 674 – 337

= 1348

C= Number which divisible by 337 from {1,........1022}

C= {337,674,1011,1348,1685,20222}

Already counted in Set $(A \cup B)$ Already counted in Set $(A \cup B)$ Already counted in Set $(A \cup B)$

Total elements which are divisible by 2 or 3 or 337

= 1348 + 2 = 1350

Favourable cases = Element which are neither divisible by 2, 3 or 337

= 2022 – 1350

= 672

Required probability $= \frac{672}{2022} = \frac{112}{337}$

20. Let $f(x) = 3^{(x^2-2)^3+4}$, x ∈ **R**. Then which of the following statements are true ?

P : x = 0 is a point of local minima of f

Q : $x = \sqrt{2}$ is a point of inflection of f

R : f' is increasing for $x > \sqrt{2}$

(A) Only P and Q   (B) Only P and R

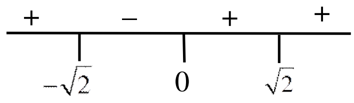
(C) Only Q and R   (D) All, P, Q and R

Official Ans. by NTA (D)

Sol. $f(x) = 81.3^{(x^2-2)^3}$

$f'(x) = 81.3^{(x^2-2)^3}.\ell n3.3(x^2-2)^2.2x$

$= (81 \times 6)3^{(x^2-2)^3} x(x^2-2)^2 \ell n3$



x = 6 is point of local min

$f'(x) = \underbrace{(486.\ell n3)}_{k}\underbrace{3^{(x^2-2)^3} x(x^2-2)^2}_{g(x)}$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)^2 + x.3^{(x^2-2)^3}.4x.(x^2-2)$

$+x.(x^2-2)^2.3^{(x^2-2)^3}\ell n3.3(x^2-2)^2.2x$

$= 3^{(x^2-2)^3}(x^2-2)\left[x^2-2+4x^2+6x^2\ell n3(x^2-2)^3\right]$

$g'(x) = 3^{(x^2-2)^3}(x^2-2)\left[5x^2-2+6x^2\ell n3(x^2-2)^3\right]$

$f''(x) = k.g'(x)$

$f''(\sqrt{2}) = 0, f''(\sqrt{2}^+) > 0, f''(\sqrt{2}^-) < 0$

$x = \sqrt{2}$ is point of inflection

$f''(x) > 0$ for $x > \sqrt{2}$ so $f'(x)$ is increasing

SECTION-B

1. Let S = {θ ∈ (0, 2π) : 7 $\cos^2\theta$ – 3 $\sin^2\theta$ – 2 $\cos^2 2\theta$ = 2}. Then, the sum of roots of all the equations $x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$ θ ∈ S, is________.

Official Ans. by NTA (16)

Sol. $7\cos^2\theta - 3\sin^2\theta - 2\cos^2 2\theta = 2$

$4\cos^2\theta + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2(1 + \cos 2\theta) + 3\cos 2\theta - 2\cos^2 2\theta = 2$

$2\cos^2 2\theta - 5\cos 2\theta = 0$

$\cos 2\theta(2\cos 2\theta - 5) = 0$

$\cos 2\theta = 0$

$2\theta = (2n+1)\dfrac{\pi}{2}$

$\theta = (2n+1)\dfrac{\pi}{4}$

$S = \left\{\dfrac{\pi}{4}, \dfrac{3\pi}{4}, \dfrac{5\pi}{4}, \dfrac{7\pi}{4}\right\}$

For all four values of θ

$x^2 - 2(\tan^2\theta + \cot^2\theta)x + 6\sin^2\theta = 0$

$\Rightarrow x^2 - 4x + 3 = 0$

Sum of roots of all four equations $= 4 \times 4 = 16$.

2. Let the mean and the variance of 20 observations $x_1, x_2, \ldots x_{20}$ be 15 and 9, respectively. For $\alpha \in R$, if the mean of $(x_1+\alpha)^2, (x_2+\alpha)^2, \ldots, (x_{20}+\alpha)^2$ is 178, then the square of the maximum value of α is equal to ______.

Official Ans. by NTA (4)

Sol. $\sum x_1 = 15 \times 20 = 300 \quad \ldots(\ )$

$\dfrac{\sum x_1^2}{20} - (15)^2 = 9 \quad \ldots(ii)$

$\sum x_1^2 = 234 \times 20 = 4680$

$\dfrac{\sum (x_1+\alpha)^2}{20} = 178 \Rightarrow \sum (x_1+\alpha)^2 = 3560$

$\Rightarrow \sum x_1^2 + 2\alpha \sum x_1 + \sum \alpha^2 = 3560$

$4680 + 600\alpha + 20\alpha^2 = 3560$

$\Rightarrow \alpha^2 + 30\alpha + 56 = 0$

$\Rightarrow (\alpha+28)(\alpha+2) = 0$

$\alpha = -2, -28$

Square of maximum value of α is 4

3. Let a line with direction ratios a, –4a, –7 be perpendicular to the lines with direction ratios 3, –1, 2b and b, a, –2. If the point of intersection of the line $\dfrac{x+1}{a^2+b^2} = \dfrac{y-2}{a^2-b^2} = \dfrac{z}{1}$ and the plane $x - y + z = 0$ is (α, β, γ), then $\alpha + \beta + \gamma$ is equal to __________.

Official Ans. by NTA (10)

Sol. $(a, -4a, -7) \perp$ to $(3, -1, 2b)$

$a = 2b \quad \ldots(i)$

$(a, -4a, -7) \perp$ to $(b, a, -2)$

$3a + 4a - 14b = 0$

$ab - 4a^2 + 14 = 0 \quad \ldots(ii)$

From Equations (i) and (ii)

$2b^2 - 16b^2 + 14 = 0$

$b^2 = 1$

$a^2 = 4b^2 = 4$

$\dfrac{x+1}{5} = \dfrac{y-2}{3} = \dfrac{z}{1} = k$

$\alpha = 5k - 1, \beta = 3k + 2, \gamma = k$

As (α, β, γ) satisfies $x - y + z = 0$

$5k - 1 - (3k+2) + k = 0$

$k = 1$

$\therefore\ \alpha + \beta + \gamma = 9k + 1 = 10$

4. Let $a_1, a_2, a_3, \ldots$ be an A.P. If $\sum_{r=1}^{\infty} \dfrac{a_r}{2^r} = 4$, then $4a_2$ is equal to ________.

Official Ans. by NTA (16)

Sol. $S = \dfrac{a_1}{2} + \dfrac{a_2}{2^2} + \dfrac{a_3}{2^3} + \ldots$

$\dfrac{S}{2} = \dfrac{a_1}{2^2} + \dfrac{a_2}{2^3} + \ldots$

$\dfrac{S}{2} = \dfrac{a_1}{2} + d\left(\dfrac{1}{2^2} + \dfrac{1}{2^3} + \ldots\right)$

$\dfrac{S}{2} = \dfrac{a_1}{2} + d\left(\dfrac{\frac{1}{4}}{1-\frac{1}{2}}\right)$

$\therefore S = a_1 + d = a_2 = 4$

Or $4a_2 = 16$

5. Let the ratio of the fifth term from the beginning to the fifth term from the end in the binomial expansion of $\left(\sqrt[4]{2} + \dfrac{1}{\sqrt[4]{3}}\right)^n$, in the increasing powers of $\dfrac{1}{\sqrt[4]{3}}$ be $\sqrt[4]{6} : 1$. If the sixth term from the beginning is $\dfrac{\alpha}{\sqrt[4]{3}}$, then α is equal to _______.

Official Ans. by NTA (84)

Sol. $\dfrac{T_5}{T_{n-3}}=\dfrac{{}^nC_4(2^{1/4})^{n-4}(3^{-1/4})^4}{{}^nC_{n-4}(2^{1/4})^4(3^{-1/4})^{n-4}}=\dfrac{\sqrt[4]{6}}{1}$

$\Rightarrow 2^{\frac{n-8}{4}}3^{\frac{n-8}{4}}=6^{1/4}$

$\Rightarrow 6^{n-8}=6$

$\Rightarrow n-8=1\Rightarrow n=9$

$T_6={}^9C_5\,(2^{1/4})^4(3^{-1/4})^5=\dfrac{84}{\sqrt[4]{3}}$

$\therefore \alpha=84$

6. The number of matrices of order 3 × 3, whose entries are either 0 or 1 and the sum of all the entries is a prime number, is______.

Official Ans. by NTA (282)

Sol. $A=\begin{bmatrix} a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix} a_{ij}\in\{0,1\}$

$\sum a_{ij}=2,3,5,7$

Total matrix = ${}^9C_2+{}^9C_3+{}^9C_5+{}^9C_7$

= 282

7. Let p and p + 2 be prime numbers and let

$$\Delta=\begin{vmatrix} p! & (p+1)! & (p+2)!\\ (p+1)! & (p+2)! & (p+3)!\\ (p+2)! & (p+3)! & (p+4)!\end{vmatrix}$$

Then the sum of the maximum values of α and β, such that p^{α} and $(p + 2)^{\beta}$ divide Δ, is _________.

Official Ans. by NTA (4)

Sol. $\Delta=\begin{vmatrix} P! & (P+1)! & (P+2)!\\ (P+1)! & (P+2)! & (P+3)!\\ (P+2)! & (P+3)! & (P+4)!\end{vmatrix}$

$\Delta=P!(P+1)!(P+2)!\begin{vmatrix} 1 & 1 & 1\\ P+1 & P+2 & P+3\\ (P+2)(P+1) & (P+3)(P+2) & (P+4)(P+3)\end{vmatrix}$

$\Delta=2P!(P+1)!(P+2)!$

Which is divisible by $P^{\alpha}\,\&(P+2)^{\beta}$

$\therefore \alpha=3,\beta=1$

Ans. 4

8. If $\dfrac{1}{2\times3\times4}+\dfrac{1}{3\times4\times5}+\dfrac{1}{4\times5\times6}+\ldots+\dfrac{1}{100\times101\times102}=\dfrac{k}{101}$, then 34 k is equal to _____.

Official Ans. by NTA (286)

Sol. $\dfrac{1}{2.3.4}+\dfrac{1}{3.4.5}+.....+\dfrac{1}{100.101.102}=\dfrac{k}{101}$

$\dfrac{4-2}{2.3.4}+\dfrac{5-3}{3.4.5}+.....+\dfrac{102-100}{100.101.102}=\dfrac{2k}{101}$

$\dfrac{1}{2.3}-\dfrac{1}{3.4}+\dfrac{1}{3.4}-\dfrac{1}{4.5}+.....+\dfrac{1}{100.101}-\dfrac{1}{101.102}=\dfrac{2k}{101}$

$\dfrac{1}{2.3}-\dfrac{1}{101.102}=\dfrac{2k}{101}$

$\therefore 2k=\dfrac{101}{6}-\dfrac{1}{102}$

$\therefore 34k=286$

9. Let S = {4, 6, 9} and T = {9, 10, 11, …, 1000}. If $A=\{a_1+a_2+\ldots+a_k : k\in N, a_1, a_2, a_3, \ldots, a_k\in S\}$, then the sum of all the elements in the set T – A is equal to ______.

Official Ans. by NTA (11)

Sol. $S=\{4,6,9\}$ $T=\{9,10,11....1000\}$

$A\{a_1+a_2+.....+a_k : K\in N\}$ & $a_i \in S$

Here by the definition of set 'A'

$A=\{a : a=4x+6y+9z\}$

Except the element 11, every element of set T is of of the form 4x + 6y + 9z for some x, y, z $\in W$

$\therefore$ T – A = {11}

Ans. 11

10. Let the mirror image of a circle $c_1 : x^2 + y^2 - 2x - 6y + \alpha = 0$ in line y = x + 1 be $c_2 : 5x^2 + 5y^2 + 10gx + 10fy + 38 = 0$. If r is the radius of circle c_2, then $\alpha + 6r^2$ is equal to ________

Official Ans. by NTA (12)

Sol. Image of centre $c_1 \equiv (1,3)$ in x – y + 1 = 0 is given by

$$\frac{x_1-1}{1}=\frac{y_1-3}{-1}=\frac{-2(1-3+1)}{1^2+1^2}$$

$\Rightarrow x_1=2, y_1=2$

$\therefore$ Centre of circle $c_2 \equiv (2,2)$

$\therefore$ Equation of c_2 be $x^2+y^2-4x-4y+\frac{38}{5}=0$

Now radius of c_2 is $\sqrt{4+4-\frac{38}{5}}=\sqrt{\frac{2}{5}}=r$

(radius of $c_1)^2$ = (radius of $c_2)^2$

$\Rightarrow 10-\alpha=\frac{2}{5} \Rightarrow \alpha=\frac{48}{5}$

$\therefore \alpha+6r^2=\frac{48}{5}+\frac{12}{5}=12$