FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

PHYSICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Two balls A and B are placed at the top of 180 m tall tower. Ball A is released from the top at t = 0 s. Ball B is thrown vertically down with an initial velocity 'u' at t = 2 s. After a certain time, both balls meet 100 m above the ground. Find the value of 'u' in ms^{-1}. [use g = 10 ms^{-2}] :

(A) 10 (B) 15

(C) 20 (D) 30

Official Ans. by NTA (D)

Sol. Let they meet at time t.

$$t = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2\times 80}{10}}$$

= 4 sec

Time taken by ball B to meet A = 2 sec

using $S = ut + \frac{1}{2}at^2$

$$-80 = -u \times 2 + \frac{1}{2}(-10)(2)^2$$

u = 30

2. A body of mass M at rest explodes into three pieces, in the ratio of masses 1 : 1 : 2. Two smaller pieces fly off perpendicular to each other with velocities of 30 ms^{-1} and 40 ms^{-1} respectively. The velocity of the third piece will be :

(A) 15 ms^{-1} (B) 25 ms^{-1}

(C) 35 ms^{-1} (D) 50 ms^{-1}

Official Ans. by NTA (B)

Sol. Mass of pieces by $\frac{M}{4}, \frac{M}{4}, \frac{M}{2}$

conserving momentum

$$\vec{P}_1 + \vec{P}_2 + \vec{P}_3 = 0$$

$$\vec{P}_3 = -\left(\vec{P}_1 + \vec{P}_2\right)$$

As $\vec{P}_1$ & $\vec{P}_2$ are perpendicular

so $P_3 = \sqrt{P_1^2 + P_2^2}$

$$P_3 = (50)\frac{M}{4}$$

$$\& \; P_3 = \frac{M}{2}v$$

so v = 25

3. The activity of a radioactive material is 2.56×10^{-3} Ci. If the half life of the material is 5 days, after how many days the activity will become 2×10^{-5} Ci?

(A) 30 days (B) 35 days

(C) 40 days (D) 25 days

Official Ans. by NTA (B)

Sol. $$\frac{A}{A_0} = \frac{N}{N_0}$$

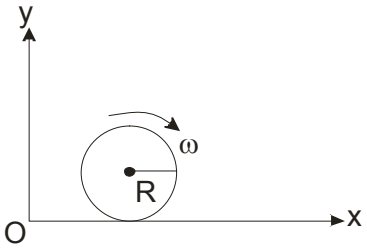
$$\frac{2\times 10^{-5}}{2.56\times 10^{-3}} = \frac{N}{N_0}$$

$$\frac{N}{N_0} = \frac{1}{128} \Rightarrow N = \frac{N_0}{128}$$

After 7 half life activity comes down to given value T = 7 × 5

= 35 days

4. A spherical shell of 1 kg mass and radius R is rolling with angular speed ω on horizontal plane (as shown in figure). The magnitude of angular momentum of the shell about the origin O is $\frac{a}{3}R^2\omega$. The value of a will be :



(A) 2 (B) 3

(C) 5 (D) 4

Official Ans. by NTA (C)

Sol. L_0 = angular momentum of shell about O.

As shell is rolling

so $V_{cm} = \omega R$

$L_0 = mV_{cm}R + I\omega$

$$= 1 \times \omega R \times R + \frac{2}{3}R^2\omega$$

$$= \frac{5}{3}R^2\omega$$

so a = 5

5. A cylinder of fixed capacity of 44.8 litres contains helium gas at standard temperature and pressure. The amount of heat needed to raise the temperature of gas in the cylinder by 20.0°C will be :

(Given gas constant R = 8.3 JK^{-1}-moI^{-1})

(A) 249 J (B) 415 J

(C) 498 J (D) 830 J

Official Ans. by NTA (C)

Sol. No of moles = $\frac{44.8}{22.4} = 2$

Gas is mono atomic so $C_V = \frac{3}{2}R$

$\Delta Q = nC_V\Delta T$

$= 2 \times \frac{3}{2}R(20)$

= 60 R

= 60 × 8.3

= 498 J

6. A wire of length L is hanging from a fixed support. The length changes to L_1 and L_2 when masses 1kg and 2 kg are suspended respectively from its free end. Then the value of L is equal to :

(A) $\sqrt{L_1L_2}$ (B) $\frac{L_1+L_2}{2}$

(C) $2L_1 - L_2$ (D) $3L_1 - 2L_2$

Official Ans. by NTA (C)

Sol. By Hooke's Law

so F α ΔL

$\frac{F_1}{F_2} = \frac{\Delta L_1}{\Delta L_2}$

$\frac{10}{20} = \frac{(L_1 - L)}{(L_2 - L)}$

$L = 2L_1 - L_2$

7. Given below are two statements : one is labelled as **Assertion A** and the other is labelled as **Reason R**.

Assertion A : The photoelectric effect does not take place, if the energy of the incident radiation is less than the work function of a metal.

Reason R : Kinetic energy of the photoelectrons is zero, if the energy of the incident radiation is equal to the work function of a metal.

In the light of the above statements, choose the **most appropriate** answer from the options given below.

(A) Both **A** and **R** are correct and **R** is the correct explanation of **A**

(B) Both **A** and **R** are correct but **R** is not the correct explanation of **A**

(C) **A** is correct but **R** is not correct

(D) **A** is not correct but **R** is correct

Official Ans. by NTA (B)

Sol. To free the electron from metal surface minimum energy required, is equal to the work function of that metal.

So Assertion A, is correct.

$hv = w_0 + K.E._{max}$

if $hv = w_0$

$\Rightarrow K.E._{max} = 0$

Hence reason R, is correct, But R is not the correct explanation of A.

8. A particle of mass 500 gm is moving in a straight line with velocity $\upsilon = b\,x^{5/2}$. The work done by the net force during its displacement from x = 0 to x = 4 m is : (Take b = 0.25 $m^{-3/2}$ s^{-1}).

(A) 2 J (B) 4 J

(C) 8 J (D) 16 J

Official Ans. by NTA (D)

Sol. By work energy theorem

work done by net force = ΔK.E.

$\Rightarrow w = \frac{1}{2}m\upsilon_f^2 - \frac{1}{2}m\upsilon_i^2$

$w = \frac{1}{2} \times 0.5 \times (0.25)^2 \times (4)^5$

$\boxed{w = 16J}$ (D)

9. A charged particle moves along circular path in a uniform magnetic field in a cyclotron. The kinetic energy of the charged particle increases to 4 times its initial value. What will be the ratio of new radius to the original radius of circular path of the charged particle :

(A) 1 : 1 (B) 1 : 2

(C) 2 : 1 (D) 1 : 4

Official Ans. by NTA (C)

Sol. radius of paerticle in cyclotron

$$r = \frac{\sqrt{2mK.E.}}{qB}$$

So ratio of new radius to original

$$\frac{r_n}{r_0} = \sqrt{\frac{(K.E.)_n}{(K.E)_0}} = \sqrt{4} \Rightarrow 2:1 \text{ (C)}$$

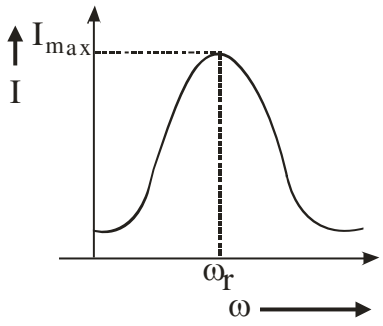
10. For a series LCR circuit, I vs ω curve is shown :

(a) To the left of ω_r, the circuit is mainly capacitive.

(b) To the left of ω_r, the circuit is mainly inductive.

(c) At ω_r, impedance of the circuit is equal to the resistance of the circuit.

(d) At ω_r, impedance of the circuit is 0.



Choose the **most appropriate** answer from the options given below :

(A) (a) and (d) only (B) (b) and (d) only

(C) (a) and (c) only (D) (b) and (c) only

Official Ans. by NTA (C)

Sol. at ω_r, $X_C = X_L$

$$\Rightarrow \frac{1}{\omega_r C} = \omega_r L$$

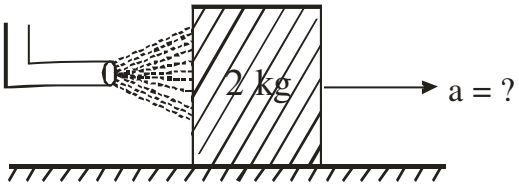
So if $\omega < \omega_r$ then x_C will increase and X_L will decrease.

Hence to left of ω_r circuit is capacitive

$$Z = \sqrt{R^2 + (X_C - X_L)^2}$$

at ω_r, $Z = \sqrt{R^2 + O^2} = R$ (C)

11. A block of metal weighing 2 kg is resting on a frictionless plane (as shown in figure). It is struck by a jet releasing water at a rate of 1 kgs^{-1} and at a speed of 10 ms^{-1}. Then, the initial acceleration of the block, in ms^{-2}, will be :



(A) 3 (B) 6

(C) 5 (D) 4

Official Ans. by NTA (C)

Sol. $F = \frac{dp}{dt} = \upsilon \frac{dm}{dt}$

$\Rightarrow Ma = 10 \times 1$

$\Rightarrow 2a = 10$

$a = 5 m/sec^2$

12. In Vander Waals equation $\left[P + \frac{a}{V^2}\right][V - b] = RT;$

P is pressure, V is volume, R is universal gas constant and T is temperature. The ratio of constants $\frac{a}{b}$ is dimensionally equal to :

(A) $\frac{P}{V}$ (B) $\frac{V}{P}$

(C) PV (D) PV^3

Official Ans. by NTA (C)

Sol. By principle of homogenity

$$[P] = \left[\frac{a}{v^2}\right] \text{ and } [b] = [v]$$

$$\Rightarrow \left[\frac{a}{b}\right] = [PV] \text{ (C)}$$

13. Two vectors $\vec{A}$ and $\vec{B}$ have equal magnitudes. If magnitude of $\vec{A}+\vec{B}$ is equal to two times the magnitude of $\vec{A}-\vec{B}$, then the angle between $\vec{A}$ and $\vec{B}$ will be :

(A) $\sin^{-1}\left(\frac{3}{5}\right)$ (B) $\sin^{-1}\left(\frac{1}{3}\right)$

(C) $\cos^{-1}\left(\frac{3}{5}\right)$ (D) $\cos^{-1}\left(\frac{1}{3}\right)$

Official Ans. by NTA (C)

Sol. $(a^2 + b^2 + 2ab\cos\theta) = 4\,(a^2 + b^2 - 2ab\cos\theta)$

put a = b we get

$2a^2 + 2a^2\cos\theta = 8a^2 - 8a^2\cos\theta$

$\cos\theta = \frac{3}{5}$

14. The escape velocity of a body on a planet 'A' is 12 kms^{-1}. The escape velocity of the body on another planet 'B', whose density is four times and radius is half of the planet 'A', is :

(A) 12 kms^{-1} (B) 24 kms^{-1}

(C) 36 kms^{-1} (D) 6 kms^{-1}

Official Ans. by NTA (A)

Sol. $V_{escape} = \sqrt{\frac{2Gm}{R}} \Rightarrow \sqrt{\frac{2G\rho \times \frac{4}{3}\pi R^3}{R}}$

$V_{escape} \propto \sqrt{\rho R^2}$

$\therefore$ if ρ is 4 times and Radius is halved.

$\Rightarrow V_{escape}$ will remain same $\therefore$ Ans (A)

15. At a certain place the angle of dip is 30° and the horizontal component of earth's magnetic field is 0.5 G. The earth's total magnetic field (in G), at that certain place, is :

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{1}{2}$

(C) $\sqrt{3}$ (D) 1

Official Ans. by NTA (A)

Sol. $B_H = B\cos\theta$

$\therefore B = \frac{B_H}{\cos\theta} = \frac{0.5G}{\cos 30°} \Rightarrow \frac{G}{\sqrt{3}}$

16. A longitudinal wave is represented by $x = 10\sin 2\pi\left(nt - \frac{x}{\lambda}\right)$ cm. The maximum particle velocity will be four times the wave velocity if the determined value of wavelength is equal to :

(A) 2π (B) 5π

(C) π (D) $\frac{5\pi}{2}$

Official Ans. by NTA (B)

Sol. $V_P \max = 4V_{wave}$

$\omega A = 4\left(\frac{\omega}{k}\right) \Rightarrow A = \frac{4\lambda}{2\pi}$

$\lambda = \frac{2\pi A}{4} \Rightarrow \frac{20\pi}{4} \Rightarrow 5\pi$

17. A parallel plate capacitor filled with a medium of dielectric constant 10, is connected across a battery and is charged. The dielectric slab is replaced by another slab of dielectric constant 15. Then the energy of capacitor will :

(A) increase by 50% (B) decrease by 15%

(C) increase by 25% (D) increase by 33%

Official Ans. by NTA (A)

Sol. $E \Rightarrow \frac{1}{2}(KC)v^2$

$\therefore$ % change

$\Rightarrow \frac{\frac{1}{2}K_2CV^2 - \frac{1}{2}K_1CV^2}{\frac{1}{2}K_1CV^2} = \frac{K_2 - K_1}{K_1} \times 100$

$\Rightarrow \frac{15-10}{10} \times 100 = 50\%$

18. A positive charge particle of 100 mg is thrown in opposite direction to a uniform electric field of strength 1×10^5 NC^{-1}. If the charge on the particle is 40 μC and the initial velocity is 200 ms^{-1}, how much distance it will travel before coming to the rest momentarily :

(A) 1 m (B) 5 m

(C) 10 m (D) 0.5 m

Official Ans. by NTA (D)

Sol. Distance travelled by particle before stopping

$\frac{V^2}{2a} = S \Rightarrow \frac{v^2m}{2qE} \Rightarrow \frac{(200)^2 \times 100 \times 10^{-6}}{2 \times 40 \times 10^{-6} \times 10^5} = 0.5\text{m}$

19. Using Young's double slit experiment, a monochromatic light of wavelength 5000 Å produces fringes of fringe width 0.5 mm. If another monochromatic light of wavelength 6000Å is used and the separation between the slits is doubled, then the new fringe width will be :

(A) 0.5 mm (B) 1.0 mm

(C) 0.6 mm (D) 0.3 mm

Official Ans. by NTA (D)

Sol. Fringe width $\beta = \frac{D\lambda}{d}$

$\lambda_1 = 5000$ Å

$\beta_1 = \frac{D}{d}(5000\times10^{-10}) = 5\times10^{-4}$ m ....(I)

$\beta_2 = \frac{D}{(2d)}(6000\times10^{-10}) = x$ (let) ....(II)

Divide (II) & (I)

$\frac{\beta_2}{\beta_1} = \frac{3000\times10^{-10}}{5000\times10^{-10}} = \frac{x}{5\times10^{-4}}$

$x = 3\times10^{-4}$ m or 0.3 mm

20. Only 2% of the optical source frequency is the available channel bandwidth for an optical communicating system operating at 1000 nm. If an audio signal requires a bandwidth of 8 kHz, how many channels can be accommodated for transmission :

(A) 375×10^7 (B) 75×10^7

(C) 375×10^8 (D) 75×10^9

Official Ans. by NTA (B)

Sol. Frequency at 1000 nm = $\frac{3\times10}{1000\times10^{-9}} \Rightarrow 3\times10^{14}$ Hz

available for channel band width

$= \frac{2}{100}\times3\times10^{14} \Rightarrow 6\times10^{12}$ Hz

Bandwidth for 1 channel = 8000 Hz

$\therefore$ No. of channel

$= \frac{6\times10^{12}}{8\times10^3} \Rightarrow \frac{600}{8}\times10^7 = 75\times10^7$

SECTION-B

1. Two coils require 20 minutes and 60 minutes respectively to produce same amount of heat energy when connected separately to the same source. If they are connected in parallel arrangement to the same source; the time required to produce same amount of heat by the combination of coils, will be_____min.

Official Ans. by NTA (15)

Sol. $\frac{dQ}{dt} = i^2R = \frac{V^2}{R}$ (we know)

$\Rightarrow$ In 't' time , $\Delta Q = \left(\frac{V^2}{R}\right)t$

Given that, (for same source, v = same)

$Q_0 = \frac{v^2}{R_1}\times20 = \frac{V^2}{R_2}\times60$ ....(1)

$\Rightarrow \boxed{R_2 = 3R_1}$ .....(ii)

If they are connected in parallel then

$Req = \frac{R_2R_1}{R_1+R_2} = \frac{3R_1.R_1}{3R_1+R_1} = \left(\frac{3R_1}{4}\right)$

To produce same heat, using equation ...(1)

$Q_0 = \frac{V^2}{R_1}\times20 = \frac{v^2}{\left(\frac{3R_1}{4}\right)}\times t$

$t = \frac{3\times20}{4} = 15\,\text{min}$

2. The intensity of the light from a bulb incident on a surface is 0.22 W/m^2. The amplitude of the magnetic field in this light-wave is_____$\times10^{-9}$ T.

(Given : Permittivity of vacuum $\epsilon_0 = 8.85\times10^{-12}C^2N^{-1}m^{-2}$, speed of light in vacuum c = 3×10^8 ms^{-1})

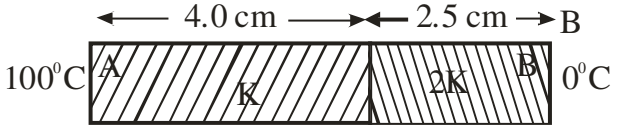
Official Ans. by NTA (43)

Sol. $I = \left(\frac{1}{2}\varepsilon_0E_0^2\right)C$

$\Rightarrow E_0 \Rightarrow \sqrt{\frac{2I}{\varepsilon_0C}} \Rightarrow \sqrt{\frac{2\times0.22}{8.85\times10^{-12}\times3\times10^8}} = 12.873$

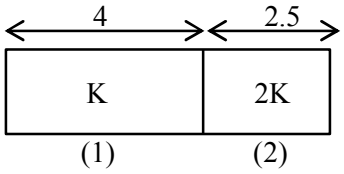
$B \Rightarrow \frac{E_0}{C} \Rightarrow \frac{12.873}{3\times10^8} = 4.291\times10^{-8} = 43\times10^{-9}$

3. As per the given figure, two plates A and B of thermal conductivity K and 2 K are joined together to form a compound plate. The thickness of plates are 4.0 cm and 2.5 cm respectively and the area of cross-section is 120 cm^2 for each plate. The equivalent thermal conductivity of the compound plate is $\left(1+\frac{5}{\alpha}\right)K$, then the value of α will be _____.



Official Ans. by NTA (21)

Sol.



$$\frac{\Delta Q}{\Delta t}=\left(\frac{1}{R}\right)\Delta T$$

R : Thermal resistivity

$$\therefore R_1=\frac{L_1}{K_1A}=\frac{L_1}{K(120)}$$

$L_1 = 4$ cm

$A = 120\ cm^2$

$$R_2=\frac{2.5}{(2K)(120)}$$

Now, R_{eq} of this series combination

$R_{eq} = R_1 + R_2$

where $L_{eq} = 4 + 2.5 = 6.5$

$$\frac{L_{eq}}{K_{eq}(A)}=\frac{4}{K(120)}+\frac{5}{\frac{2}{2K(120)}}$$

$$\frac{6.5}{K_{eq}(120)}=\frac{4}{K(120)}+\frac{5}{4K(120)}$$

$$\frac{6.5}{K_{eq}}=\frac{21}{4K}$$

$$K_{eq}=\frac{26}{21}K=\left(1+\frac{5}{21}\right)K$$

$\therefore a = 21$

4. A body is performing simple harmonic with an amplitude of 10 cm. The velocity of the body was tripled by air Jet when it is at 5 cm from its mean position. The new amplitude of vibration is $\sqrt{x}$ cm. The value of x is_____.

Official Ans. by NTA (700)

Sol. A = 10 cm

$$\therefore \text{ Total Energy} = \frac{1}{2}KA^2$$

By energy conservation we can final v at x = 5

$$\frac{1}{2}K(10)^2=\frac{1}{2}K(5)^2+\frac{1}{2}mv^2$$

$$V=\sqrt{\frac{75K}{m}}$$

Now, velocity is tripled through external mean so the amplitude of SHM will charge and so the total energy, (but potential) energy at this moment will remain same)

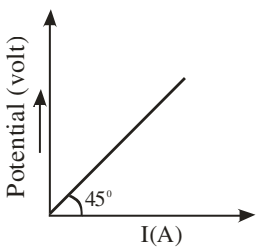
$$\therefore \frac{1}{2}K(5)^2+\frac{1}{2}m\left(3\sqrt{\frac{75K}{m}}\right)^2=\frac{1}{2}KA^2$$

$$\Rightarrow 25\ K + 675\ K = KA^2$$

$$\therefore A=\sqrt{700}$$

$\therefore x = 700$

5. The variation of applied potential and current flowing through a given wire is shown in figure. The length of wire is 31.4 cm. The diameter of wire is measured as 2.4 cm. The resistivity of the given wire is measured as $x \times 10^{-3}\ \Omega$ cm. The value of x is _____. [Take $\pi = 3.14$]



Official Ans. by NTA (144)

Sol. $1 = \rho\frac{\ell}{A}$

$1 = \frac{\rho \times 31.4}{\frac{\pi(2.4)^2}{4}}$

$\frac{\pi(2.4)^2}{4} = \rho \times 314$

$\frac{2.4 \times 2.4}{4} = \rho \times 10$

$\frac{0.6 \times 2.4}{10} = \rho$

$\frac{1.44}{10} = \rho$

$0.144 = \rho$

$144 \times 10^{-3} = \rho$

6. 300 cal. of heat is given to a heat engine and it rejects 225 cal. of heat. If source temperature is 227°C, then the temperature of sink will be __ ^{0}C.

Official Ans. by NTA (102)

Sol. $1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1}$

$\frac{Q_2}{Q_1} = \frac{T_2}{T_1}$

$\frac{225}{300} = \frac{T_2}{500}$

$\frac{500 \times 225}{300} = T_2$

$375 = T_2$

$102°C = T_2$

7. $\sqrt{d_1}$ and $\sqrt{d_2}$ are the impact parameters corresponding to scattering angles 60° and 90° respectively, when an α particle is approaching a gold nucleus. For $d_1 = x\ d_2$, the value of x will be_____.

Official Ans. by NTA (3)

Sol. $\sqrt{d} \propto \cot\frac{\theta}{2}$

$\cot^2 30° = x \cot^2 45°$

$3 = x$

8. A transistor is used in an amplifier circuit in common emitter mode. If the base current changes by 100 μA, it brings a change of 10 mA in collector current. If the load resistance is 2 kΩ and input resistance is 1 kΩ, the value of power gain is $x \times 10^4$. The value of x is _____.

Official Ans. by NTA (2)

Sol. $\Delta i_B = 100\ \mu A$ $\qquad \beta = \frac{\Delta i_C}{\Delta i_B}$

$\Delta i_C = 10\ mA$

$\text{power} = \beta^2 \times \frac{R_0}{R_{in}}$

$\text{Power} = \left(\frac{10}{0.1}\right)^2 \times \frac{2}{1}$

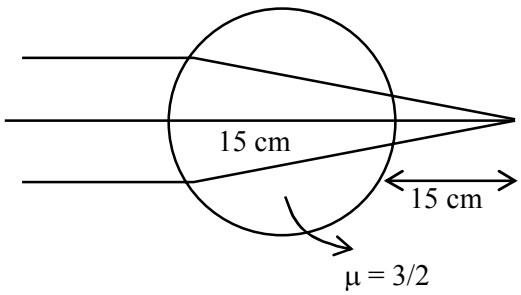
Power = 100 × 100 × 2

Gain = 2×10^4

9. A parallel beam of light is allowed to fall on a transparent spherical globe of diameter 30 cm and refractive index 1.5. The distance from the centre of the globe at which the beam of light can converge is _____ mm.

Official Ans. by NTA (225)

Sol.



$\frac{\frac{3}{2}}{V} - \frac{1}{\infty} = \frac{\frac{3}{2} - 1}{15}$

$\frac{3}{2V} = \frac{1}{30}$

V = 45 cm

$$\frac{1}{V}-\frac{3}{\frac{2}{15}}=\frac{1-\frac{3}{2}}{\frac{2}{-15}}$$

$$\frac{1}{V}-\frac{1}{10}=\frac{1}{30}$$

$$\frac{1}{V}=\frac{1}{10}+\frac{1}{30}=\frac{4}{30}$$

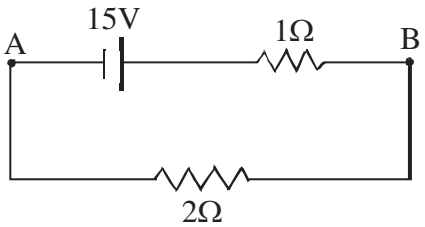
$$\boxed{V=7.5}$$

$$\boxed{V=22.5}$$

v = 225 mm

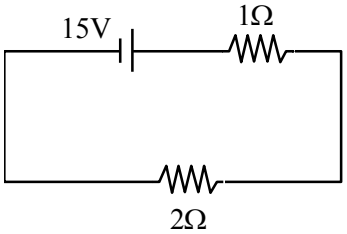
10. For the network shown below, the value $V_B - V_A$ is

_____V.



Official Ans. by NTA (10)

Sol.



$$i=\frac{15}{3}=5A$$

15 – 5 (1) = 10 Volt

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

CHEMISTRY

TEST PAPER WITH SOLUTION

SECTION-A

1. Production of iron in blast furnace follows the following equation

$Fe_3O_4(s) + 4CO(g) \rightarrow 3Fe(l) + 4CO_2(g)$

when 4.640 kg of Fe_3O_4 and 2.520 kg of CO are allowed to react then the amount of iron (in g) produced is :

[Given : Molar Atomic mass (g mol^{-1}): Fe = 56

Molar Atomic mass (g mol^{-1}) : 0 = 16

Molar Atomic mass (g mol^{-1}): = C = 12

(A) 1400 (B) 2200

(C) 3360 (D) 4200

Official Ans. by NTA (C)

Sol. Moles of $Fe_3O_4 = \frac{4.640 \times 10^3}{232} = 20$

Moles of $CO = \frac{2.52 \times 10^3}{28} = 90$

So limiting Reagent = Fe_3O_4

So moles of Fe formed = 60

Weight of Fe = 60 × 56 = 3360 gms

2. Which of the following statements are **correct** ?

(A) The electronic configuration of Cr is [Ar] $3d^5$ $4s^1$.

(B) The magnetic quantum number may have a negative value.

(C) In the ground state of an atom, the orbitals are filled in order of their increasing energies.

(D) The total number of nodes are given by n – 2 .

Choose the **most appropriate** answer from the options given below :

(A) (A), (C) and (D) only

(B) (A) and (B) only

(C) (A) and (C) only

(D) (A), (B) and (C) only

Official Ans. by NTA (D)

Sol. (A) Cr = [Ar]$3d^5$ $4s^1$

(B) m = – ℓ to +ℓ

(C) According to Aufbau principle, orbitals are filled in order of their increasing energies.

(D) Total nodes = n – 1

3. Arrange the following in the decreasing order of their covalent character :

(A) LiCl

(B) NaCl

(C) KCl

(D) CsCl

Question: Choose the **most appropriate** answer from the options given below :

(A) (A)> (C) > (B)> (D)

(B) (B) > (A) > (C) > (D)

(C) (A) > (B) > (C) > (D)

(D) (A) > (B) > (D) > (C)

Official Ans. by NTA (C)

Sol. LiCl > NaCl > KCl > CsCl (Covalent character)

4. The solubility of AgCl will be maximum in which of the following ?

(A) 0.01 M KCl

(B) 0.01 M HC1

(C) 0.01 M $AgNO_3$

(D) Deionised water

Official Ans. by NTA (D)

Sol. In deionized water no common ion effect will take place so maximum solubility

5. Which of the following is a **correct** statement ?

(A) Brownian motion destabilises sols.

(B) Any amount of dispersed phase can be added to emulsion without destabilising it.

(C) Mixing two oppositely charged sols in equal amount neutralises charges and stabilises colloids.

(D) Presence of equal and similar charges on colloidal particles provides stability to the colloidal solution.

Official Ans. by NTA (D)

Sol. As equal & similar charge particle will repel each other, hence will never precipitate.

6. The electronic configuration of Pt (atomic number 78) is:

(A) $[Xe]\ 4f^{14}\ 5d^{9}\ 6s^{1}$

(B) $[Kr]\ 4f^{14}\ 5d^{10}$

(C) $[Xe]\ 4f^{14}\ 5d^{10}$

(D) $[Xe]\ 4f^{14}\ 5d^{8}\ 6s^{2}$

Official Ans. by NTA (A)

Sol. $_{78}Pt = [Xe]\ 4f^{14}\ 5d^{9}\ 6s^{1}$ (Exceptional electronic configuration)

7. In isolation of which one of the following metals from their ores, the use of cyanide salt is not commonly involved ?

(A) Zinc

(B) Gold

(C) Silver

(D) Copper

Official Ans. by NTA (D)

Sol. For ZnS, KCN is used as depressant.

For Gold and silver $\Rightarrow$ leaching [Cyanide process]

8. Which one of the following reactions indicates the reducing ability of hydrogen peroxide in basic medium ?

(A) $HOCl + H_2O_2 \rightarrow H_3O^+ + Cl^- + O_2$

(B) $PbS + 4H_2O_2 \rightarrow PbSO_4 + 4H_2O$

(C) $2MnO_4^- + 3H_2O_2 \rightarrow 2MnO_2 + 3O_2 + 2H_2O + 2OH^-$

(D) $Mn^{2+} + H_2O_2 \rightarrow Mn^{4+} + 2OH^-$

Official Ans. by NTA (C)

Sol. In option (A) and (C) reducing action of hydrogen peroxide is shown.

In option (A) it is in acidic medium, in option (B) it is in basic medium.

or

For reducing ability H_2O_2 changes to O_2 , i.e. oxidize , so in option 'A' & 'C' O_2 is formed but 'A' is in acidic medium so option - C correct.

9. Match the **List-I** with **List- II.**

List-I (Metal)	**List-II (Emitted light wavelength (nm))**
(A) Li	(I) 670.8
(B) Na	(II) 589.2
(C) Rb	(III) 780.0
(D) Cs	(IV) 455.5

Choose the **most appropriate** answer from the options given below:

(A) (A)-(I), (B)-(II), (C)-(III), (D)-(IV)

(B) (A)-(III), (B)-(II), (C)-(I), (D)-(IV)

(C) (A)-(III), (B)-(I), (C)-(II), (D)-(IV)

(D) (A)-(IV), (B)-(II), (C)-(I), (D)-(III)

Official Ans. by NTA (A)

Sol. NCERT Table 10.1.5

Metal	**Li**	**Na**	**K**	**Rb**	**Cs**
Colour	Crimson red	Yellow	Violet	Red Violet	Blue
λ/nm	670.8	589.2	766.5	780.0	455.5

10. Match the **List-I** with **List- II.**

List-I (Metal)	List-II Application
(A) Cs	(I) High temperature thermometer
(B) Ga	(II) Water repellent sprays
(C) B	(III) Photoelectric cells
(D) Si	(IV) Bullet proof vest

Choose the most appropriate answer from the option given below:

(A) (A)-(III), (B)-(I), (C)-(IV), (D)-(II)

(B) (A)-(IV), (B)-(III), (C)-(II), (D)-(I)

(C) (A)-(II), (B)-(III), (C)-(IV), (D)-(I)

(D) (A)-(I), (B)-(IV), (C)-(II), (D)-(III)

Official Ans. by NTA (A)

Sol. Caesium is used in devising photoelectric cells.

Boron fibres are used in making bullet–proof vest.

Silicones being surrounded by non–polar alkyl groups are water repelling in nature.

Gallium is less toxic and has a very high boiling point, so it is used in high temperature thermometers.

11. The oxoacid of phosphorus that is easily obtained from a reaction of alkali and white phosphorus and has two P-H bonds, is :

(A) Phosphonic acid
(B) Phosphinic acid
(C) Pyrophosphorus acid
(D) Hypophosphoric acid

Official Ans. by NTA (B)

Sol. $P_4 + 3NaOH + 3H_2O \rightarrow PH_3 + 3NaH_2PO_2$

oxoacid = H_3PO_2 (hypo phosphorus acid) or (phosphinic acid)

12. The acid that is believed to be mainly responsible for the damage of Taj Mahal is

(A) Sulfuric acid (B) Hydrofluoric acid
(C) Phosphoric acid (D) Hydrochloric acid

Official Ans. by NTA (A)

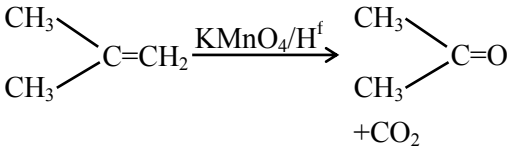
Sol. $CaCO_3 + H_2SO_4 \rightarrow CaSO_4 + H_2O + CO_2$

13. Two isomers 'A' and 'B' with molecular formula C_4H_8 give different products on oxidation with $KMnO_4$ in acidic medium. Isomer 'A' on reaction with $KMnO_4/H^+$ results in effervescence of a gas and gives ketone. The compound 'A' is

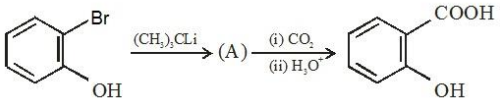
(A) But-1-ene (B) cis-But-2-ene
(C) trans-But-2ene (D) 2-methyl propene

Official Ans. by NTA (D)

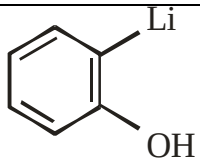
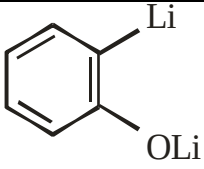
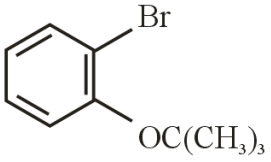
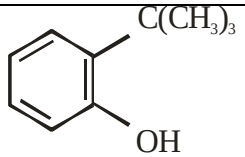
Sol.



14.

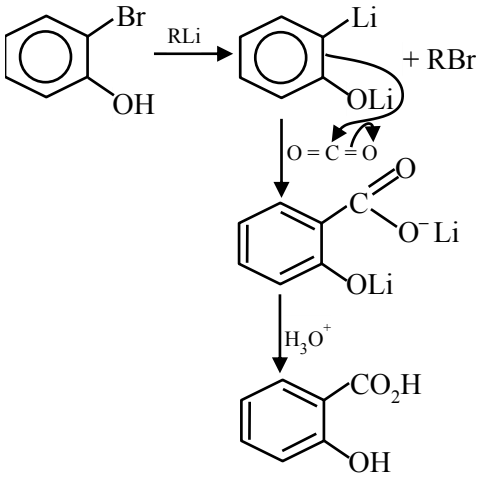


In the given conversion the compound A is:

(A)	
(B)	
(C)	
(D)	

Official Ans. by NTA (B)

Sol.



15. Given below are two statements :

Statement I : The esterification of carboxylic acid with an alcohol is a nucleophilic acyl substitution.

Statement II : Electron withdrawing groups in the carboxylic acid will increase the rate of esterification reaction.

Choose the **most appropriate** option :

(A) Both **Statement I** and **Statement II** are correct.

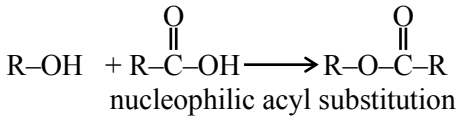
(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

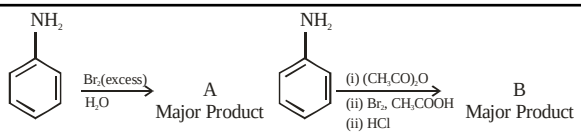
Official Ans. by NTA (A)

Sol.

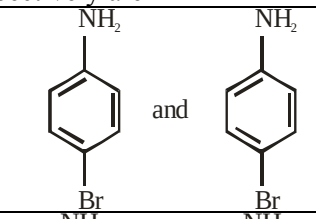
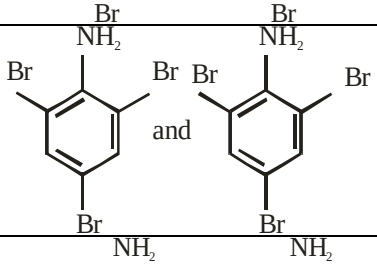
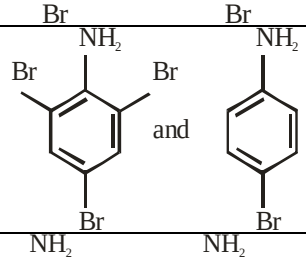
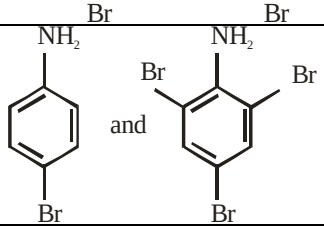


electron with drawing group on carboxylic acid will increase the rate of esterification

16.

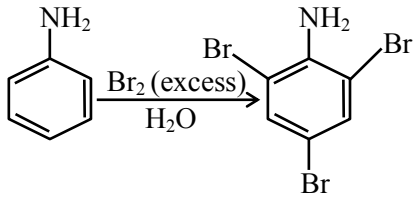


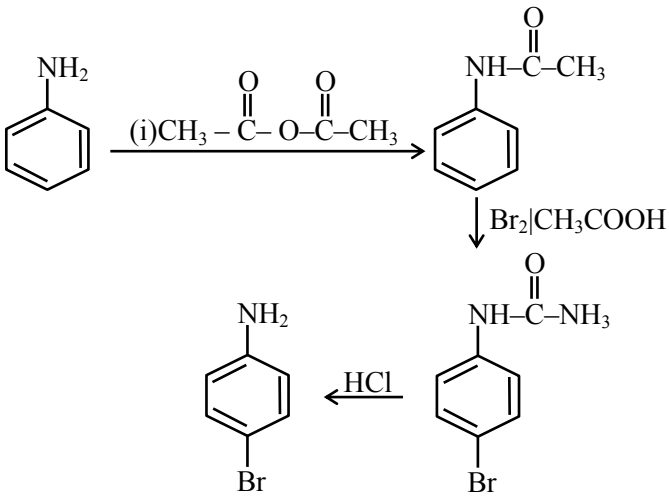
Consider the above reaction, the product A and product B respectively are

(A)	
(B)	
(C)	
(D)	

Official Ans. by NTA (C)

Sol.





17. The polymer, which can be stretched and retains its original status on releasing the force is

(A) Bakelite (B) Nylon 6,6

(C) Buna-N (D) Terylene

Official Ans. by NTA (C)

Buna – N is synthetic rubber which can be stretched and retains its original status on releasing the force.

18. Sugar moiety in DNA and RNA molecules respectively are

(A) β-D-2-deoxyribose, β -D-deoxyribose

(B) β-D-2-deoxyribose, β -D-ribose

(C) β-D-ribose, β -D-2-deoxyribose

(D) β-D-deoxyribose, β -D-2-deoxyribose

Official Ans. by NTA (B)

Sol. DNA contains $\Rightarrow$ β – D – 2 – deoxyribose

RNA contains $\Rightarrow$ β – D – ribose

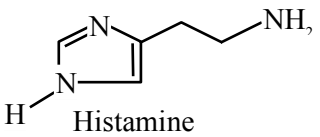
19. Which of the following compound **does not** contain sulphur atom ?

(A) Cimetidine (B) Ranitidine

(C) Histamine (D) Saccharin

Official Ans. by NTA (C)

Sol.



Histamine

Histamine is nitrogenous compound it does not contain sulpher.

20. Given below are two statements.

Statement I : Phenols are weakly acidic.

Statement II : Therefore they are freely soluble in NaOH solution and are weaker acids than alcohols and water.

Choose the **most appropriate** option:

(A) Both **Statement I** and **Statement II** are correct.

(B) Both **Statement I** and **Statement II** are incorrect.

(C) **Statement I** is correct but **Statement II** is incorrect.

(D) **Statement I** is incorrect but **Statement II** is correct.

Official Ans. by NTA (C)

Sol. Phenol are weakly acidic. Phenol is more acidic than alcohol & H_2O statement (I) is correct. (II) is incorrect.

SECTION-B

1. Geraniol, a volatile organic compound, is a component of rose oil. The density of the vapour is 0.46 gL^{-1} at 257°C and 100 mm Hg. The molar mass of geraniol is _____ (Nearest Integer)

[Given R = 0.082 L atm K^{-1} mol^{-1}]

Official Ans. by NTA (152)

Sol. Assuming ideal behaviour $P = \frac{dRT}{M}$

$P = \frac{100}{760}$ atm, $T = 257 + 273 = 530$ K

$d = 0.46$ gm/L

So $M = \frac{0.46 \times 0.082 \times 530}{100} \times 760$

$= 151.93 \approx 152$

2. 17.0 g of NH_3 completely vapourises at – 33.42°C and 1 bar pressure and the enthalpy change in the process is 23.4 kJ mol^{-1}. The enthalpy change for the vapourisation of 85 g of NH_3 under the same conditions is _____ kJ.

Official Ans. by NTA (117)

Sol. Given data is for 1 moles and asked for 5 moles so value is $23.4 \times 5 = 117$ kJ

3. 1.2 mL of acetic acid is dissolved in water to make 2.0 L of solution. The depression in freezing point observed for this strength of acid is 0.0198°C. The percentage of dissociation of the acid is _____ . (Nearest integer)

[Given : Density of acetic acid is 1.02 g mL^{-1}
Molar mass of acetic acid is 60 g mol^{-1}
$K_f (H_2O) = 1.85$ K kg mol^{-1}]

Official Ans. by NTA (5)

Sol. $M = d \times V = 1.02 \times 1.2 = 1.224$ gm

Moles of acetic acid = 0.0204 moles in 2L

So molality = 0.0102 mol/kg

Now $\Delta T_f = i \times K_f \times M$

$i = 1 + \alpha$ for acetic acid

$0.0198 = (1 + \alpha) \times 1.85 \times 0.0102$

$\alpha = 0.04928$

$\cong 5\%$

4. A dilute solution of sulphuric acid is electrolysed using a current of 0.10 A for 2 hours to produce hydrogen and oxygen gas. The total volume of gases produced at STP is _____ cm^3. (Nearest integer) [Given : Faraday constant F = 96500 C mol^{-1} at STP, molar volume of an ideal gas is 22.7 L mol^{-1}]

Official Ans. by NTA (127)

Sol. At anode

$2H_2O \rightarrow O_2(g) + 4H^+ + 4e^-$

At cathode

$2H^+ + 2e^- \rightarrow H_2(g)$

Now number of gm eq. $= \dfrac{i \times t}{96500}$

$= \dfrac{0.1 \times 2 \times 60 \times 60}{96500}$

$= 0.00746$

$V_{O_2} = \dfrac{0.00746}{4} \times 22.7 = 0.0423$

$V_{H_2} = \dfrac{0.00746}{2} \times 22.7 = 0.0846$

$V_{Total} \approx$ 127 ml or cc

5. The activation energy of one of the reactions in a biochemical process is 532611 J mol^{-1}. When the temperature falls from 310 K to 300 K, the change in rate constant observed is $k_{300} = x \times 10^{-3}\ k_{310}$. The value of x is _____.

[Given: ln10 = 2.3
R=8.3 J K^{-1} mol^{-1}]

Official Ans. by NTA (1)

Sol. $\ell n\left(\dfrac{K_2}{K_1}\right) = \dfrac{E_a}{R}\left(\dfrac{1}{T_1} - \dfrac{1}{T_2}\right)$

$\ell n\left(\dfrac{K_2}{K_1}\right) = \dfrac{532611}{8.3} \times \left(\dfrac{10}{310 \times 300}\right)$

where K_2 is at 310 K & K_1 is at 300 K

$\ell n\left(\dfrac{K_2}{K_1}\right) = 6.9$

$= 3 \times \ell n10$

$\ell n\dfrac{K_2}{K_1} = \ell n10^3$

$K_2 = K_1 \times 10^3$

$K_1 = K_2 \times 10^3$

So K = 1

6. The number of terminal oxygen atoms present in the product B obtained from the following reaction is _____.

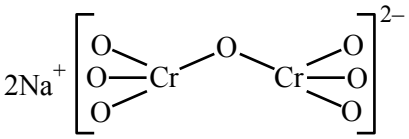
$FeCr_2O_4 + Na_2CO_3 + O_2 \rightarrow A + Fe_2O_3 + CO_2$

$A + H^+ \rightarrow B + H_2O + Na^+$

Official Ans. by NTA (6)

Sol. $4FeCr_2O_4 + 8Na_2CO_3 + 7O_2 \rightarrow 8Na_2CrO_4 + 2Fe_2O_3 + 8CO_2$

$2Na_2CrO_4 + 2H^+ \rightarrow \underbrace{Na_2Cr_2O_7}_{B} + 2Na^+ + H_2O$



7. An acidified manganate solution undergoes disproportionation reaction. The spin-only magnetic moment value of the product having manganese in higher oxidation state is _____ B.M. (Nearest integer)

Official Ans. by NTA (0)

Sol. $3MnO_4^{2-} + 4H^+ \longrightarrow 2\overset{+7}{Mn}O_4^- + \overset{+4}{Mn}O_2 + 2H_2O$

$\overset{+7}{Mn}$ = no. of unpaired electrons is '0'

$\mu = 0$ B.M.

8. Kjeldahl's method was used for the estimation of nitrogen in an organic compound. The ammonia evolved from 0.55 g of the compound neutralised 12.5 mL of 1 M H_2SO_4 solution. The percentage of nitrogen in the compound is _____. (Nearest integer)

Official Ans. by NTA (64)

Sol. Meq of H_2SO_4 used by NH_3 = $12.5 \times 1 \times 2 = 25$

$$\% \text{ of N in the compound} = \frac{25\times10^{-3}\times14\times100}{0.55} = 63.6$$

or

Meq. of H_2SO_4 = Meq. of NH_3

$12.5 \times 1 \times 2 = 25$ meq. of NH_3

= 25 millimoles of NH_3

So Millimoles of 'N' = 25

Moles of 'N' = 25×10^{-3}

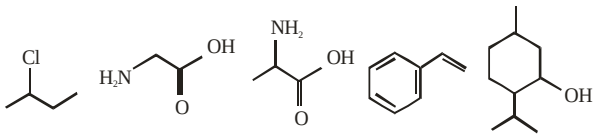
wt. of N = $14 \times 25\times10^{-3}$

$$\%N = \frac{14\times25\times10^{-3}}{0.55}\times100$$

= 63.66

≈ 64%

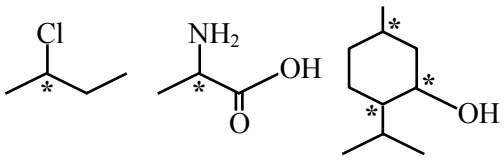
9. Observe structures of the following compounds



The total number of structures/compounds which possess asymmetric carbon atoms is _____.

Official Ans. by NTA (3)

Sol.



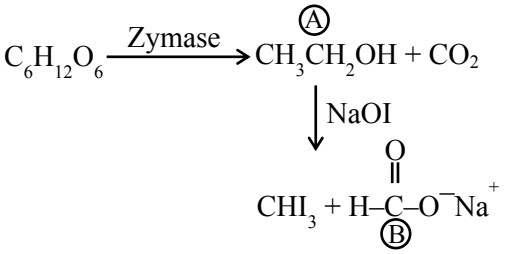
Number of compounds containing asymmetric carbons are three.

10. $C_6H_{12}O_6 \xrightarrow{\text{Zymase}} A \xrightarrow[\Delta]{\text{NaOI}} B + CHI_3$

The number of carbon atoms present in the product B is _____.

Official Ans. by NTA (1)

Sol.



no. of carbon atoms present in B is 1

FINAL JEE–MAIN EXAMINATION – JUNE, 2022

(Held On Wednesday 29th June, 2022) **TIME : 9 : 00 AM to 12 : 00 PM**

MATHEMATICS

TEST PAPER WITH SOLUTION

SECTION-A

1. Question ID: 101761

The probability that a randomly chosen 2 × 2 matrix with all the entries from the set of first 10 primes, is singular, is equal to :

(A) $\frac{133}{10^4}$ (B) $\frac{18}{10^3}$

(C) $\frac{19}{10^3}$ (D) $\frac{271}{10^4}$

Official Ans. by NTA (C)

Sol. Let matrix A is singular then |A| = 0

Number of singular matrix = All entries are same + only two prime number are used in matrix

$= 10 + 10 \times 9 \times 2$

$= 190$

Required probability $= \frac{190}{10^4} = \frac{19}{10^3}$

2. Question ID: 101762

Let the solution curve of the differential equation

$x\frac{dy}{dx} - y = \sqrt{y^2 + 16x^2}$, $y(1) = 3$ be $y = y(x)$.

Then y(2) is equal to :

(A) 15 (B) 11

(C) 13 (D) 17

Official Ans. by NTA (A)

Sol. $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow x\frac{dv}{dx} = \sqrt{v^2 + 16}$

$\Rightarrow \int \frac{dv}{\sqrt{v^2 + 16}} = \int \frac{dx}{x}$

$\Rightarrow \ell n\left|v + \sqrt{v^2 + 16}\right| = \ln x + \ln C$

$\Rightarrow y + \sqrt{y^2 + 16x^2} = Cx^2$

As $y(1) = 3 \Rightarrow C = 8$

$\Rightarrow y(2) = 15$

3. Question ID: 101763

If the mirror image of the point (2, 4, 7) in the plane $3x - y + 4z = 2$ is (a, b, c), the $2a + b + 2c$ is equal to :

(A) 54 (B) 50

(C) -6 (D) -42

Official Ans. by NTA (C)

Sol. $\frac{a-2}{3} = \frac{b-4}{-1} = \frac{c-7}{4} = \frac{-2(6-4+28-2)}{3^2+1^2+4^2}$

$\Rightarrow a = \frac{-84}{13} + 2, b = \frac{28}{13} + 4, C = \frac{-112}{13} + 7$

$\Rightarrow 2a + b + 2c = -6$

4. Question ID: 101764

Let $f : R \to R$ be a function defined by :

$$f(x) = \begin{cases} \max\limits_{t \le x}\{t^3 - 3t\}; & x \le 2 \\ x^2 + 2x - 6; & 2 < x < 3 \\ [x-3] + 9; & 3 \le x \le 5 \\ 2x + 1; & x > 5 \end{cases}$$

Where [t] is the greatest integer less than or equal to t. Let m be the number of points where f is not differentiable and $I = \int_{-2}^{2} f(x)dx$. Then the ordered pair (m, I) is equal to :

(A) $\left(3, \frac{27}{4}\right)$ (B) $\left(3, \frac{23}{4}\right)$

(C) $\left(4, \frac{27}{4}\right)$ (D) $\left(4, \frac{23}{4}\right)$

Official Ans. by NTA (C)

Sol.

$$\begin{cases} f(x)=x^3-3x, x\le -1 \\ 2, -1<x<2 \\ x^2+2x-6, 2<x<3 \\ 9, 3\le x<4 \\ 10, 4\le x<5 \\ 11, x=5 \\ 2x+1, x>5 \end{cases}$$

Clearly f (x) is not differentiable at

$x = 2, 3, 4, 5 \Rightarrow m = 4$

$$I=\int_{-2}^{-1}(x^3-3x)dx+\int_{-1}^{2}2\cdot dx=\frac{27}{4}$$

5. Question ID: 101765

Let $\vec{a}=\alpha\hat{i}+3\hat{j}-\hat{k}, \vec{b}=3\hat{i}-\beta\hat{j}+4\hat{k}$ and $\vec{c}=\hat{i}+2\hat{j}-2\hat{k}$ where α, β ∈ R, be three vectors. If the projection of $\vec{a}$ on $\vec{c}$ is $\frac{10}{3}$ and $\vec{b}\times\vec{c}=-6\hat{i}+10\hat{j}+7\hat{k}$, then the value of α + β equal to :

(A) 3 (B) 4

(C) 5 (D) 6

Official Ans. by NTA (A)

Sol.

$$\frac{a\cdot c}{|\vec{c}|}=\frac{10}{3}$$

$$\Rightarrow \frac{\alpha+6+2}{\sqrt{1+4+4}}=\frac{10}{3}\Rightarrow \alpha=2$$

and $$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -\beta & 4 \\ 1 & 2 & -2 \end{vmatrix}=-6\hat{i}+\hat{j}+\hat{k}$$

$$\Rightarrow 2\beta-8=-6\Rightarrow \beta=1$$

$$\Rightarrow \alpha+\beta=3$$

6. Question ID : 101766

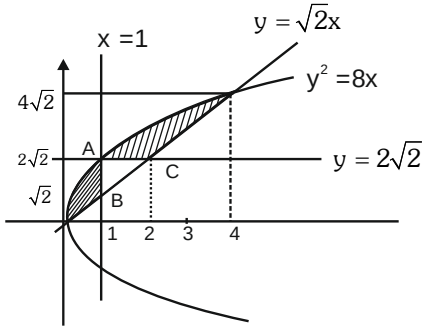
The area enclosed by $y^2 = 8x$ and $y=\sqrt{2}x$ that lies outside the triangle formed by $y=\sqrt{2}x, x=1, y=2\sqrt{2}$, is equal to :

(A) $\frac{16\sqrt{2}}{6}$ (B) $\frac{11\sqrt{2}}{6}$

(C) $\frac{13\sqrt{2}}{6}$ (D) $\frac{5\sqrt{2}}{6}$

Official Ans. by NTA (C)

Sol.



Area of $\Delta ABC=\frac{1}{2}(\sqrt{2})\cdot 1=\frac{\sqrt{2}}{2}$

So required Area $=\int_0^4\left(\sqrt{8x}-\sqrt{2}x\right)dx-\frac{\sqrt{2}}{2}$

$$=\frac{32\sqrt{2}}{3}-8\sqrt{2}-\frac{\sqrt{2}}{2}=\frac{13\sqrt{2}}{6}$$

7. Question ID: 101767

If the system of linear equations

2x + y – z = 7

x – 3y + 2z = 1

x + 4y + δz = k, where δ, k ∈ R

has infinitely many solutions, then δ + k is equal to:

(A)−3 (B) 3 (C) 6 (D) 9

Official Ans. by NTA (B)

Sol.

$$\begin{vmatrix} 2 & 1 & -1 \\ 1 & -3 & 2 \\ 1 & 4 & \delta \end{vmatrix}=0$$

$$\Rightarrow \delta=-3$$

And $$\begin{vmatrix} 7 & 1 & -1 \\ 1 & -3 & 2 \\ K & 4 & -3 \end{vmatrix}=0\Rightarrow K=6$$

$$\Rightarrow \delta+K=3$$

Alternate

2x + y – z = 7 .....(1)

x – 3y + 2z = 1 .....(2)

x + 4y + δz = k .....(3)

Equation (2) + (3)

We get 2x + y + (2 + δ) z = 1 + K .....(4)

For infinitely solution

Form equation (1) and (4)

$2 + \delta = -1 \Rightarrow \boxed{\delta = -3}$

$1 + k = 7 \Rightarrow \boxed{k = 6}$

δ + k = 3

8. Question ID: 101768

Let α and β be the roots of the equation $x^2 + (2i - 1) = 0$. Then, the value of $|\alpha^8 + \beta^8|$ is equal to :

(A) 50 (B) 250

(C) 1250 (D) 1500

Official Ans. by NTA (A)

Sol. $X^2 = 1 - 2i \Rightarrow \alpha^2 = 1 - 2i, \ \beta^2 = 1 - 2i$

Hence $\alpha^8 = \beta^8$

$|\alpha^8 + \beta^8| = |2\alpha^8| = 2|\alpha^2|^4$

$= 2\sqrt{5}^4 = 50$

9. Question ID: 101769

Let $\Delta \in \{\wedge, \vee, \Rightarrow, \Leftrightarrow\}$ be such that $(p \wedge q)\Delta((p \vee q) \Rightarrow q)$ is a tautology. Then Δ is equal to :

(A) $\wedge$ (B) $\vee$

(C) $\Rightarrow$ (D) $\Leftrightarrow$

Official Ans. by NTA (C)

Sol. $p \vee q \Rightarrow q$

$\Rightarrow \sim (p \vee q) \vee q$

$\Rightarrow (\sim p \wedge \sim q) \vee q$

$\Rightarrow (\sim p \vee q) \wedge (\sim q \vee q)$

$\Rightarrow (\sim p \vee q) \wedge t = \sim p \vee q$

Now by taking option C

$(p \wedge q) \Rightarrow \sim p \vee q$

$\Rightarrow \sim p \vee \sim q \vee \sim p \vee q$

$\Rightarrow t$

Hence C

10. Question ID: 101770

Let $A = [a_{ij}]$ be a square matrix of order 3 such that $a_{ij} = 2^{j-i}$, for all i, j = 1, 2, 3. Then, the matrix $A^2 + A^3 + \ldots + A^{10}$ is equal to :

(A) $\left(\frac{3^{10}-3}{2}\right)A$ (B) $\left(\frac{3^{10}-1}{2}\right)A$

(C) $\left(\frac{3^{10}+1}{2}\right)A$ (D) $\left(\frac{3^{10}+3}{2}\right)A$

Official Ans. by NTA (A)

Sol. $A = \begin{pmatrix} 1 & 2 & 2^2 \\ 1/2 & 1 & 2 \\ 1/2^2 & 1/2 & 1 \end{pmatrix}$

$A^2 = 3A$

$A^3 = 3^2A$

$A^2 + A^3 + \ldots A^{10}$

$= 3A + 3^2A + \ldots + 3^9A = \frac{3(3^9 - 1)}{3 - 1}A$

$= \frac{3^{10} - 3}{2}A$

11. Question ID: 101771

Let a set $A = A_1 \cup A_2 \cup \ldots \cup A_k$, where $A_i \cap A_j = \phi$ for $i \neq j \ 1 \leq i, j \leq k$. Define the relation R from A to A by $R = \{(x, y) : y \in A_i$ if and only if $x \in A_i, 1 \leq i \leq k\}$. Then, R is :

(A) reflexive, symmetric but not transitive

(B) reflexive, transitive but not symmetric

(C) reflexive but not symmetric and transitive

(D) an equivalence relation

Official Ans. by NTA (D)

Sol. $A = \{1, 2, 3\}$

$R = \{(1, 1), (1, 2), (1, 3) (2, 1), (2, 2), (2, 3) (3,1), (3, 2) (3, 3)\}$

12. Question ID: 101772

Let $\{a_n\}_{n=0}^{\infty}$ be a sequence such that $a_0 = a_1 = 0$ and $a_{n+2} = 2a_{n+1} - a_n + 1$ for all $n \geq 0$. Then, $\sum_{n=2}^{\infty} \frac{a_n}{7^n}$ is equal to

(A) $\frac{6}{343}$ (B) $\frac{7}{216}$

(C) $\frac{8}{343}$ (D) $\frac{49}{216}$

Official Ans. by NTA (B)

Sol. $a_2 = 1,\ a_3 = 3\ a_4 = 6$

$$a_n = \frac{n(n-1)}{2}$$

$$S = \sum_{n=2}^{\infty} \frac{n(n-1)}{2(7^n)}$$

$$S = \frac{1}{7^2} + \frac{3}{7^3} + \frac{6}{7^4} + \frac{10}{7^5} + \frac{15}{7^5} + \ldots\ldots$$

$$\frac{S}{7} = \frac{1}{7^3} + \frac{3}{7^4} + \frac{6}{7^5} + \frac{10}{7^6} + \ldots$$

$$6\frac{S}{7} = \frac{1}{7^2} + \frac{2}{7^3} + \frac{3}{7^4} + \frac{4}{7^5} + \ldots$$

$$6\frac{S}{7^2} = \frac{1}{7^3} + \frac{2}{7^4} + \frac{3}{7^5} + \ldots$$

$$6\frac{S}{7}\cdot\frac{6}{7} = \frac{1}{7^2} + \frac{1}{7^3} + \ldots = \frac{1/7^2}{1-1/7}$$

$$6\times 6\frac{S}{7^2} = \cdot\frac{1}{7\times 6}$$

$$S = \frac{7}{6^3} = \frac{7}{216}$$

Alternate

$a_{n+2} = 2a_{n+1} - a_n + 1$

$$\Rightarrow \frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\frac{a_n}{7^n} + \frac{1}{7^{n+2}}$$

$$\Rightarrow \sum_{n=2}^{\infty}\frac{a_{n+2}}{7^{n+2}} = \frac{2}{7}\sum_{n=2}^{\infty}\frac{a_{n+1}}{7^{n+1}} - \frac{1}{49}\sum_{n=2}^{\infty}\frac{a_n}{7^n} + \sum_{n=2}^{\infty}\frac{1}{7^{n+2}}$$

Let $\sum_{n=2}^{\infty}\frac{a_n}{7^n} = p$

$$\Rightarrow \left(p - \frac{a_2}{7^2} - \frac{a_3}{7^3}\right) = \frac{2}{7}\left(p - \frac{a_2}{7^2}\right) - \frac{1}{49}p + \frac{1/7^4}{1-\frac{1}{7}}$$

$\because a_2 = 1,\ a_3 = 3$

$$\Rightarrow p - \frac{1}{49} - \frac{3}{343} = \frac{2}{7}p - \frac{2}{7^3} - \frac{p}{49} + \frac{1}{6.7^3}$$

$$\Rightarrow p = \frac{7}{216}$$

13. Question ID: 101773

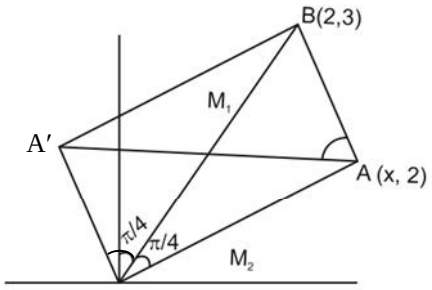
The distance between the two points A and A' which lie on y = 2 such that both the line segments AB and A' B (where B is the point (2, 3)) subtend angle $\frac{\pi}{4}$ at the origin, is equal to :

(A) 10 (B) $\frac{48}{5}$

(C) $\frac{52}{5}$ (D) 3

Official Ans. by NTA (C)

Sol.



$M_1 = 3/2$ $M_2 = 2/x$

$$\tan\pi/4 = \left|\frac{3/2 - 2/x}{1 + 6/2x}\right| = 1$$

$\Rightarrow x_1 = 10,\quad x_2 = -2/5$

$\Rightarrow AA^1 = 52/5$

14. Question ID: 101774

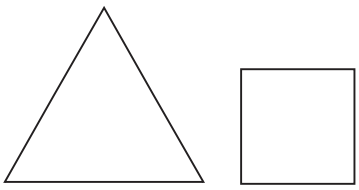
A wire of length 22 m is to be cut into two pieces. One of the pieces is to be made into a square and the other into an equilateral triangle. Then, the length of the side of the equilateral triangle, so that the combined area of the square and the equilateral triangle is minimum, is :

(A) $\frac{22}{9+4\sqrt{3}}$ (B) $\frac{66}{9+4\sqrt{3}}$

(C) $\frac{22}{4+9\sqrt{3}}$ (D) $\frac{66}{4+9\sqrt{3}}$

Official Ans. by NTA (B)

Sol.



3a = x 4b = 22 – x

a = 2/13

$$A_T = \frac{\sqrt{3}}{4}a^2 + b^2$$

$$= \frac{\sqrt{3}}{4}x^2/9 + \frac{(22-x)^2}{16}$$

$$\frac{dA}{dx} = 0 \Rightarrow x\left(\frac{\sqrt{3}}{2\times 9} + \frac{1}{8}\right) - \frac{22}{8} = 0$$

$$\Rightarrow x\left(\frac{4\sqrt{3}+9}{36}\right) = \frac{11}{2}$$

a = x/3

$$a = \left(\frac{11/2}{\frac{4\sqrt{3}+9}{36}}\right)\left(\frac{1}{3}\right) = \frac{66}{4\sqrt{3}+9}$$

15. Question ID: 101775

The domain of the function $\cos^{-1}\left(\frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi}\right)$ is :

(A) $R - \left\{-\frac{1}{2}, \frac{1}{2}\right\}$

(B) $(-\infty, -1] \cup [1, \infty) \cup \{0\}$

(C) $\left(-\infty, \frac{-1}{2}\right) \cup \left(\frac{1}{2}, \infty\right) \cup \{0\}$

(D) $\left(-\infty, \frac{-1}{\sqrt{2}}\right] \cup \left[\frac{1}{\sqrt{2}}, \infty\right) \cup \{0\}$

Official Ans. by NTA (D)

Sol.

$$-1 \le \frac{2\sin^{-1}\left(\frac{1}{4x^2-1}\right)}{\pi} \le 1$$

$$-\pi/2 \le \sin^{-1}\frac{1}{4x^2-1} \le \pi/2$$

Always $-1 \le \frac{1}{4x^2-1} \le 1$

$$x \in \left(\infty, \frac{1}{\sqrt{2}}\right) \cup \left[\frac{1}{\sqrt{2}}, \infty\right)$$

16. Question ID: 101776

If the constant term in the expansion of $\left(3x^3 - 2x^2 + \frac{5}{x^5}\right)^{10}$ is $2^k \cdot l$, where l is an odd integer, then the value of k is equal to :

(A) 6 (B) 7

(C) 8 (D) 9

Official Ans. by NTA (D)

Sol. General term

$$T_{r+1} = \frac{\lfloor 10}{\lfloor r_1 \lfloor r_2 \lfloor r_3} (3)^{r_1}(-2)^{r_2}(5)^{r_3}(x)^{3r_1+2r_2-5r_3}$$

$3r_1 + 2r_2 - 5r_3 = 0$ ...(1)

$r_1 + r_2 + r_3 = 10$ ...(2)

from equation (1) and (2)

$r_1 + 2(10 - r_3) - 5r_3 = 0$

$r_1 + 20 = 7r_3$

$(r_1, r_2, r_3) = (1, 6, 3)$

constant term = $\frac{\lfloor 10}{\lfloor 1 \lfloor 6 \lfloor 3}(3)^1(-2)^6(5)^3$

$= 2^9.3^2.5^4.7^1$

$l = 9$

17. Question ID: 101777

$$\int_0^5 \cos\left(\pi\left(x - \left[\frac{x}{2}\right]\right)\right) dx,$$

Where [t] denotes greatest integer less than or equal to t, is equal to :

(A) -3 (B) -2

(C) 2 (D) 0

Official Ans. by NTA (D)

Sol. $I = \int_0^5 \cos\left(\pi x - \pi\left[\frac{x}{2}\right]\right) dx$

$$\Rightarrow I = \int_0^2 \cos(\pi x)\,dx + \int_2^4 \cos(\pi x - \pi)\,dx + \int_4^5 \cos(\pi x - 2\pi)\,dx$$

$$\Rightarrow I = \left[\frac{\sin \pi x}{\pi}\right]_0^2 + \left[\frac{\sin(\pi x - \pi)}{\pi}\right]_2^4 + \left[\frac{\sin(\pi x - 2\pi)}{\pi}\right]_4^5$$

$$\Rightarrow I = 0$$

18. Question ID: 101778

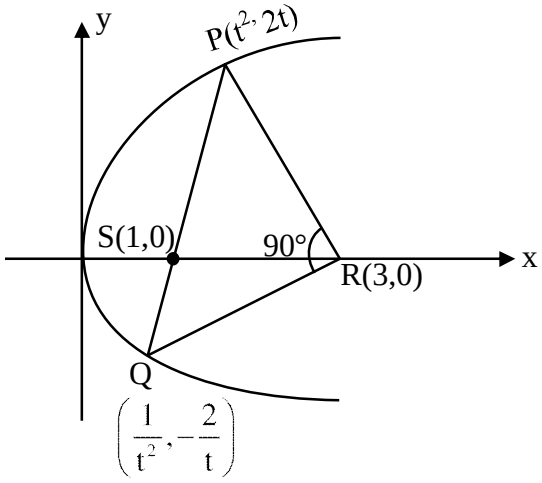
Let PQ be a focal chord of the parabola $y^2 = 4x$ such that it subtends an angle of $\frac{\pi}{2}$ at the point (3, 0). Let the line segment PQ be also a focal chord of the ellipse E: $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1, a^2>b^2$. If e is the eccentricity of the ellipse E, then the value of $\frac{1}{e^2}$ is equal to :

(A) $1+\sqrt{2}$ (B) $3+2\sqrt{2}$

(C) $1+2\sqrt{3}$ (D) $4+5\sqrt{3}$

Official Ans. by NTA (B)

Sol. PQ is focal chord



$m_{PR} \cdot m_{PQ} = -1$

$$\frac{2t}{t^2-3}\times\frac{-2/t}{\frac{1}{t^2}-3}=-1$$

$(t^2-1)^2=0$

$\Rightarrow t=1$

$\Rightarrow$ P & Q must be end point of latus rectum:

P(1, 2) & Q(1, –2)

$\therefore \frac{2b^2}{a}=4$ & ae =1

$\because$ We know that $b^2=a^2(1-e^2)$

$\therefore a=1+\sqrt{2}$

$\because e^2=1-\frac{b^2}{a^2}$

$\therefore e^2=3-2\sqrt{2}$

$\frac{1}{e^2}=3+2\sqrt{2}$

19. Question ID: 101779

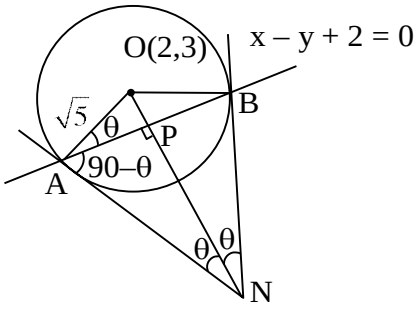
Let the tangent to the circle $C_1: x^2+y^2=2$ at the point $M(-1,1)$ intersect the circle $C_2: (x-3)^2+(y-2)^2=5$, at two distinct points A and B. If the tangents to C_2 at the points A and B intersect at N, then the area of the triangle ANB is equal to :

(A) $\frac{1}{2}$ (B) $\frac{2}{3}$

(C) $\frac{1}{6}$ (D) $\frac{5}{3}$

Official Ans. by NTA (C)

Sol. $OP=\left|\frac{2-3+2}{\sqrt{2}}\right|$



$OP=\frac{3}{\sqrt{2}}$

$AP=\sqrt{OA^2-OP^2}$

$=\frac{1}{\sqrt{2}}$

$\tan\theta = 3$

$\therefore \sin\theta=\frac{3}{\sqrt{10}}=\frac{AP}{AN}$

$\Rightarrow AN=\frac{\sqrt{5}}{3}=BN$

Area of $\Delta ANB=\frac{1}{2}.\left(AN^2\right)\sin 2\theta=\frac{1}{6}$

20. Question ID: 101780

Let the mean and the variance of 5 observations x_1, x_2, x_3, x_4, x_5 be $\frac{24}{5}$ and $\frac{194}{25}$ respectively.

If the mean and variance of the first 4 observation are $\frac{7}{2}$ and a respectively, then $(4a+x_5)$ is equal to:

(A) 13 (B) 15

(C) 17 (D) 18

Official Ans. by NTA (B)

Sol. $\bar{x} = \frac{\sum x_i}{5} = \frac{24}{5} \Rightarrow \sum x_i = 24$

$\sigma^2 = \frac{\sum x_i^2}{5} - \left(\frac{24}{5}\right)^2 = \frac{194}{25}$

$\Rightarrow \sum x_i^2 = 154$

$x_1 + x_2 + x_3 + x_4 = 14$

$\Rightarrow x_5 = 10$

$\sigma^2 = \frac{x_1^2 + x_2^2 + x_3^2 + x_4^2}{4} - \frac{49}{4} = a$

$x_1^2 + x_2^2 + x_3^2 + x_4^2 = 4a + 49$

$x_5^2 = 154 - 4a - 49$

$\Rightarrow 100 = 105 - 4a \Rightarrow 4a = 5$

$4a + x_5 = 15$

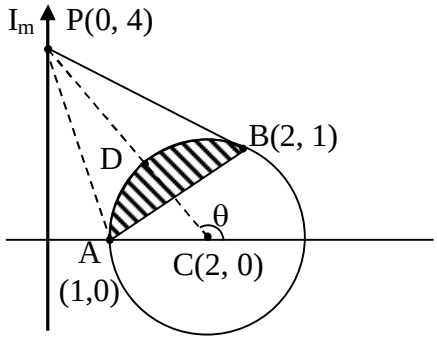
SECTION-B

1. Question ID: 101781

Let $S = \{z \in C : |z-2| \le 1, z(1+i) + \bar{z}(1-i) \le 2\}$. Let $|z - 4i|$ attains minimum and maximum values, respectively, at $z_1 \in S$ and $z_2 \in S$. If $5(|z_1|^2 + |z_2|^2) = \alpha + \beta\sqrt{5}$, where α and β are integers, then the value of $\alpha + \beta$ is equal to _____.

Official Ans. by NTA (26)

Sol. $|z-2| \le 1$



$(x-2)^2 + y^2 \le 1$ ….. (1)

&

$z(1+i) + \bar{z}(1-i) \le 2$

Put $z = x + iy$

$\therefore x - y \le 1$ ….. (2)

PA = $\sqrt{17}$, PB = $\sqrt{13}$

Maximum is PA & Minimum is PD

Let D $(2 + \cos\theta, 0 + \sin\theta)$

$\therefore m_{cp} = \tan\theta = -2$

$\cos\theta = -\frac{1}{\sqrt{5}}, \sin\theta = \frac{2}{\sqrt{5}}$

$\therefore D\left(2 - \frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\right)$

$\Rightarrow z_1 = \left(2 - \frac{1}{\sqrt{5}}\right) + \frac{2i}{\sqrt{5}}$

$|z_1| = \frac{25 - 4\sqrt{5}}{5}$ & $z_2 = 1$

$\therefore |z_2|^2 = 1$

$\therefore 5\left(|z_1|^2 + |z_2|^2\right) = 30 - 4\sqrt{5}$

$\therefore \alpha = 30$

$\beta = -4$

$\therefore \alpha + \beta = 26$

2. Question ID: 101782

Let $y = y(x)$ be the solution of the differential equation

$\frac{dy}{dx} + \frac{\sqrt{2}y}{2\cos^4 x - \cos 2x} = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}, 0 < x < \pi/2$ with $y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32}$.

If $y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{-\tan^{-1}(\alpha)}$, then the value of $3\alpha^2$ is equal to _____.

Official Ans. by NTA (2)

Sol. $\frac{dy}{dx} + \frac{\sqrt{2}}{2\cos^4 x - \cos 2x} y = xe^{\tan^{-1}(\sqrt{2}\cot 2x)}$

$\int \frac{dx}{2\cos^4 x - \cos 2x}$

$= \int \frac{dx}{\cos^4 x + \sin^4 x} = \int \frac{\operatorname{cosec}^4 x\, dx}{1 + \cot^4 x}$

$= -\int \frac{t^2 + 1}{t^4 + 1} dt = -\int \frac{\left(1 + \frac{1}{t^2}\right)}{\left(t - \frac{1}{t}\right)^2 + 2} dt = \frac{-1}{\sqrt{2}} \tan^{-1}\left(\frac{t - \frac{1}{t}}{\sqrt{2}}\right)$

Cotx = t

$= -\frac{1}{\sqrt{2}} \tan^{-1}\left(\sqrt{2}\cot 2x\right)$

$\therefore$ IF $= e^{-\tan^{-1}(\sqrt{2}\cot 2x)}$

$$ye^{-\tan^{-1}(\sqrt{2}\cot 2x)} = \int x\,dx$$

$$ye^{-\tan^{-1}\left(\sqrt{2}\cot 2x\right)} = \frac{x^2}{2} + c$$

$$y\left(\frac{\pi}{4}\right) = \frac{\pi^2}{32} + c \Rightarrow c = 0$$

$$y = \frac{x^2}{2} e^{\tan^{-1}(\sqrt{2}\cot 2x)}$$

$$y\left(\frac{\pi}{3}\right) = \frac{\pi^2}{18} e^{\tan^{-1}\left(\sqrt{2}\cot\frac{2\pi}{3}\right)}$$

$$= \frac{\pi^2}{18} e^{-\tan^{-1}\left(\sqrt{\frac{2}{3}}\right)}$$

$$\alpha = \sqrt{\frac{2}{3}} \Rightarrow 3\alpha^2 = 2$$

3. Question ID: 101783

Let d be the distance between the foot of perpendiculars of the points P(1, 2 – 1) and Q(2, – 1, 3) on the plane – x + y + z = 1. Then d^2 is equal to _____.

Official Ans. by NTA (26)

Sol. Points P(1, 2, –1) and Q (2, –1, 3) lie on same side of the plane.

Perpendicular distance of point P from plane is

$$\left|\frac{-1+2-1-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

Perpendicular distance of point Q from plane is

$$= \left|\frac{-2-1+3-1}{\sqrt{1^2+1^2+1^2}}\right| = \frac{1}{\sqrt{3}}$$

$\Rightarrow \overrightarrow{PQ}$ is parallel to given plane. So, distance between P and Q = distance between their foot of perpendiculars.

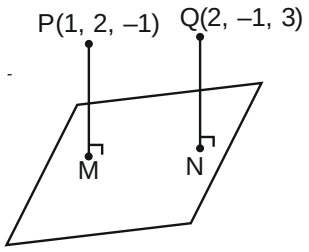
$$\Rightarrow \left|\overrightarrow{PQ}\right| = \sqrt{(1-2)^2 + (2+1)^2 + (-1-3)^2}$$

$$= \sqrt{26}$$

$$\left|\overrightarrow{PQ}\right|^2 = 26 = d^2$$

Alternate

$$-x + y + z - 1 = 0$$



$$M(x_1, y_1, z_1)$$

$$\frac{x_1 - 1}{-1} = \frac{y_1 - 2}{1} = \frac{z_1 + 1}{1} = \frac{1}{3}$$

$$x_1 = \frac{2}{3}, y_1 = \frac{7}{3}, z_1 = \frac{-2}{3}$$

$$M\left(\frac{2}{3}, \frac{7}{3}, \frac{-2}{3}\right)$$

$$N(x_2, y_2, z_2)$$

$$\frac{x_2 - 2}{-1} = \frac{y_2 + 1}{1} = \frac{z_2 - 3}{1} = \frac{1}{3}$$

$$x_2 = \frac{5}{3}, y_2 = \frac{-2}{3}, \; z_2 = \frac{10}{3}$$

$$N = \left(\frac{5}{3}, \frac{-2}{3}, \frac{10}{3}\right)$$

$d^2 = 1^2 + 3^2 + 4^2 = 26$

4. Question ID: 101784

The number of elements in the set $S = \{\theta \epsilon\, [-4\pi, 4\pi] : 3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0\}$ is _____.

Official Ans. by NTA (32)

Sol. $3\cos^2 2\theta + 6\cos 2\theta - 10\cos^2\theta + 5 = 0$

$3\cos^2 2\theta + 6\cos 2\theta - 5(1 + \cos 2\theta) + 5 = 0$

$3\cos^2 2\theta + \cos 2\theta = 0$

$\cos 2\theta = 0$ OR $\cos 2\theta = -1/3$

$\theta \in [-4\pi, 4\pi]$

$2\theta = (2n + 1).\frac{\pi}{2}$

$\therefore \quad \theta = \pm\pi/4, \pm 3\pi/4 ..... \pm 15\pi/4$

Similarly $\cos 2\theta = -1/3$ gives 16 solution

5. **Question ID: 101785**

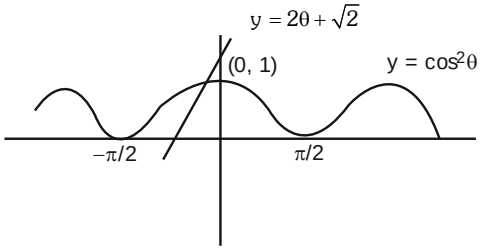
The number of solutions of the equation $2\theta - cos^2\theta + \sqrt{2} = 0$ is R is equal to _____.

Official Ans. by NTA (1)

Sol. $2\theta - \cos^2\theta + \sqrt{2} = 0$

$\Rightarrow \cos^2\theta = 2\theta + \sqrt{2}$

$y = 2\theta + \sqrt{2}$



Both graphs intersect at one point.

6. **Question ID: 101786**

$50\ tan\left(3tan^{-1}\left(\frac{1}{2}\right) + 2cos^{-1}\left(\frac{1}{\sqrt{5}}\right)\right) +$

$4\sqrt{2}\tan\left(\frac{1}{2}tan^{-1}(2\sqrt{2})\right)$ is equal to _____.

Official Ans. by NTA (29)

Sol. $50\tan\left(3\tan^{-1}\frac{1}{2} + 2\cos^{-1}\frac{1}{\sqrt{5}}\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2(\tan^{-1}\frac{1}{2} + \tan^{-1}2)\right)$

$+4\sqrt{2}\tan\left(\frac{1}{2}\tan^{-1}2\sqrt{2}\right)$

$= 50\tan\left(\tan^{-1}\frac{1}{2} + 2.\frac{\pi}{2}\right) + 4\sqrt{2}\times\frac{1}{\sqrt{2}}$

$= 50\left(\tan\tan^{-1}\frac{1}{2}\right) + 4$

$= 25 + 4 = 29$

7. **Question ID: 101787**

Let c, k ∈ R. If $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ and $f(x + y) = f(x) + f(y) - xy$, for all x, y ∈ R, then the value of $|2(f(1) + f(2) + f(3) + \ldots\ldots + f(20))|$ is equal to _____.

Official Ans. by NTA (3395)

Sol. $f(x) = (c + 1)x^2 + (1 - c^2)x + 2k$ ....(1)

& $f(x + y) = f(x) + f(y) - xy \quad \forall\ xy \in R$

$$\lim_{y\to0}\frac{f(x+y)-f(x)}{y} = \lim_{y\to0}\frac{f(y)-xy}{y} \Rightarrow f'(x) = f'(0) - x$$

$f(x) = -\frac{1}{2}x^2 + f'(0).x + \lambda$ but $f(0) = 0 \Rightarrow \lambda = 0$

$f(x) = -\frac{1}{2}x^2 + (1 - c^2).x$ ....(2)

$\therefore$ as $f'(0) = 1 - c^2$

Comparing equation (1) and (2)

We obtain, $c = -\frac{3}{2}$

$\therefore$ $f(x) = -\frac{1}{2}x^2 - \frac{5}{4}x$

Now $|2\sum_{x=1}^{20} f(x)| = \sum_{x=1}^{20} x^2 + \frac{5}{2}.\sum_{x=1}^{20} x$

$= 2870 + 525$

$= 3395$

8. Question ID: 101788

Let $H: \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, a > 0, b > 0, be a hyperbola such that the sum of lengths of the transverse and the conjugate axes is $4\left(2\sqrt{2} + \sqrt{14}\right)$. If the eccentricity H is $\frac{\sqrt{11}}{2}$, then value of $a^2 + b^2$ is equal to _____.

Official Ans. by NTA (88)

Sol. $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

Given $e^2=1+\frac{b^2}{a^2}\Rightarrow\frac{11}{4}=1+\frac{b^2}{a^2}\Rightarrow b^2=\frac{7}{4}a^2$

$\therefore\ \frac{x^2}{(a)^2}-\frac{y^2}{\left(\frac{\sqrt{7}}{2}a\right)^2}=1$ Now given

$2a+2.\frac{\sqrt{7}a}{2}=4\left(2\sqrt{2}+\sqrt{14}\right)$

$a\left(2+\sqrt{7}\right)=4\sqrt{2}(2+\sqrt{7})$

$a=4\sqrt{2}\Rightarrow a^2=32$

$b^2=\frac{7}{4}\times16\times2=56$

9. Question ID: 101789

Let $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$ be a plane. Let P_2 be another plane which passes through the points (2, -3, 2) (2, − 2, − 3) and (1, −4, 2). If the direction ratios of the line of intersection of P_1 and P_2 be 16, α, β, then the value of α + β is equal to _____.

Official Ans. by NTA (28)

Sol. $P_1:\vec{r}.(2\hat{i}+\hat{j}-3\hat{k})=4$

P_1: $2x+y-3z=4$

$P_2\begin{vmatrix}x-2 & y+3 & z-2\\ 0 & 1 & -5\\ -1 & -1 & 0\end{vmatrix}=0$

$\Rightarrow -5x+5y+z+23=0$

Let a, b, c be the d'rs of line of intersection

Then $a=\frac{16\lambda}{15};\ b=\frac{13\lambda}{15};\ c=\frac{15\lambda}{15}$

$\therefore\ \alpha=13:\ \beta=15$

10. Question ID: 101790

Let $b_1b_2b_3b_4$ be a 4-element permutation with $b_i\in\{1, 2, 3, \ldots\ldots, 100\}$ for $1\le i\le4$ and $b_i\ne b_j$ for $i\ne j$, such that either b_1, b_2, b_3 are consecutive integers or b_2, b_3, b_4 are consecutive integers.

Then the number of such permutations $b_1b_2b_3b_4$ is equal to _____.

Official Ans. by NTA (18915)

Sol. $b_i\in\{1,2,3.....100\}$

Let A = set when b_1 b_2 b_3 are consecutive

$n(A)=\frac{97+97+.....+97}{98\text{times}}=97\times98$

Similarly when b_2b_3 b_4 are consecutive

N(A) = 97 × 98

$n(A\cap B)=\frac{97+97+-------97}{98\text{times}}=97\times98$

Similarly when b_2 b_3 b_4 are consecutive

n(B) = 97 × 98

n(A∩B) = 97

n(AUB) = n(A)+n(B) − n(A∩B)

Number of permutation = 18915