Ph. D. MATH

1) If $K = \{(x, y): x \geq 0, y \geq 0\} \subset \mathbb{R}^2$, then K is
   (a) bounded
   (b) closed
   (c) open
   (d) None of the above.

2) The sphere $S^n = \{x \in \mathbb{R}^{n+1}: \|x\| = 1\}$ is a
   (a) Vector space
   (b) Connected and convex
   (c) Closed and convex
   (d) Closed *n*-manifolds.

3) If $f, g \in \mathbb{R}[a, b]$, then the value of the integral $\int_a^b f(x)g(x)dx$ is less than or equal to
   (a) $\sqrt{\int_a^b f(x)dx}\sqrt{\int_a^b g(x)dx}$
   (b) $\sqrt{\int_a^b f^2(x)dx}\sqrt{\int_a^b g^2(x)dx}$
   (c) $\int_a^b f(x)dx \int_a^b g(x)dx$
   (d) None of the above.

4) If there exist a finite number of open sets $A_i \subset A, i = 1,2, \dots, n$ such that $\bigcup_{i=1}^n A_i = A$ then A is
   (a) closed
   (b) bounded
   (c) countable
   (d) compact.

5) The dual space of X is X if X is a
   (a) Banach space
   (b) Hilbert space
   (c) Housdorf space
   (d) none of the above.

6) A Banach space X is reflexive if the map $J: X \to X^{**}$ (X^* is dual of X) is
   (a) Surjective
   (b) Bijective
   (c) Injective
   (d) none of the above.